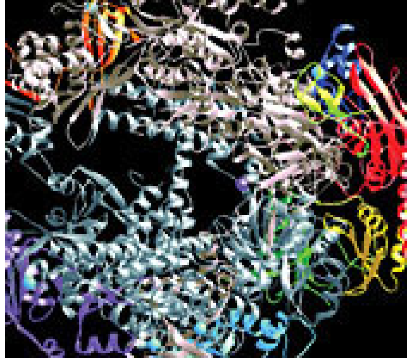
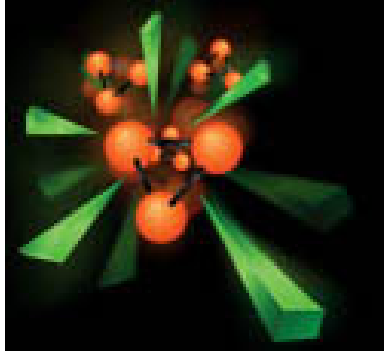




Accelerator Power System Engineering

US Particle Accelerator School

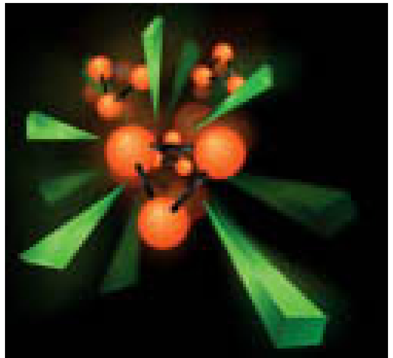
Texas A & M University

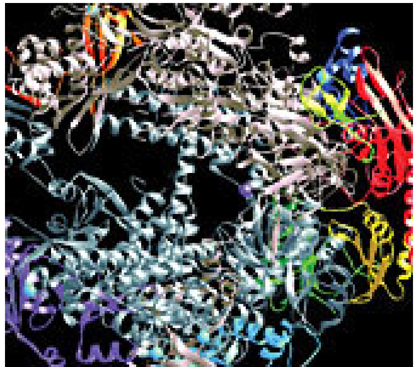
January 24 – February 4, 2022

James Sebek, PhD, SLAC National Accelerator Laboratory

Paul Bellomo, P.E. (SLAC Retired)

Yugang Tan, Oak Ridge National Laboratory



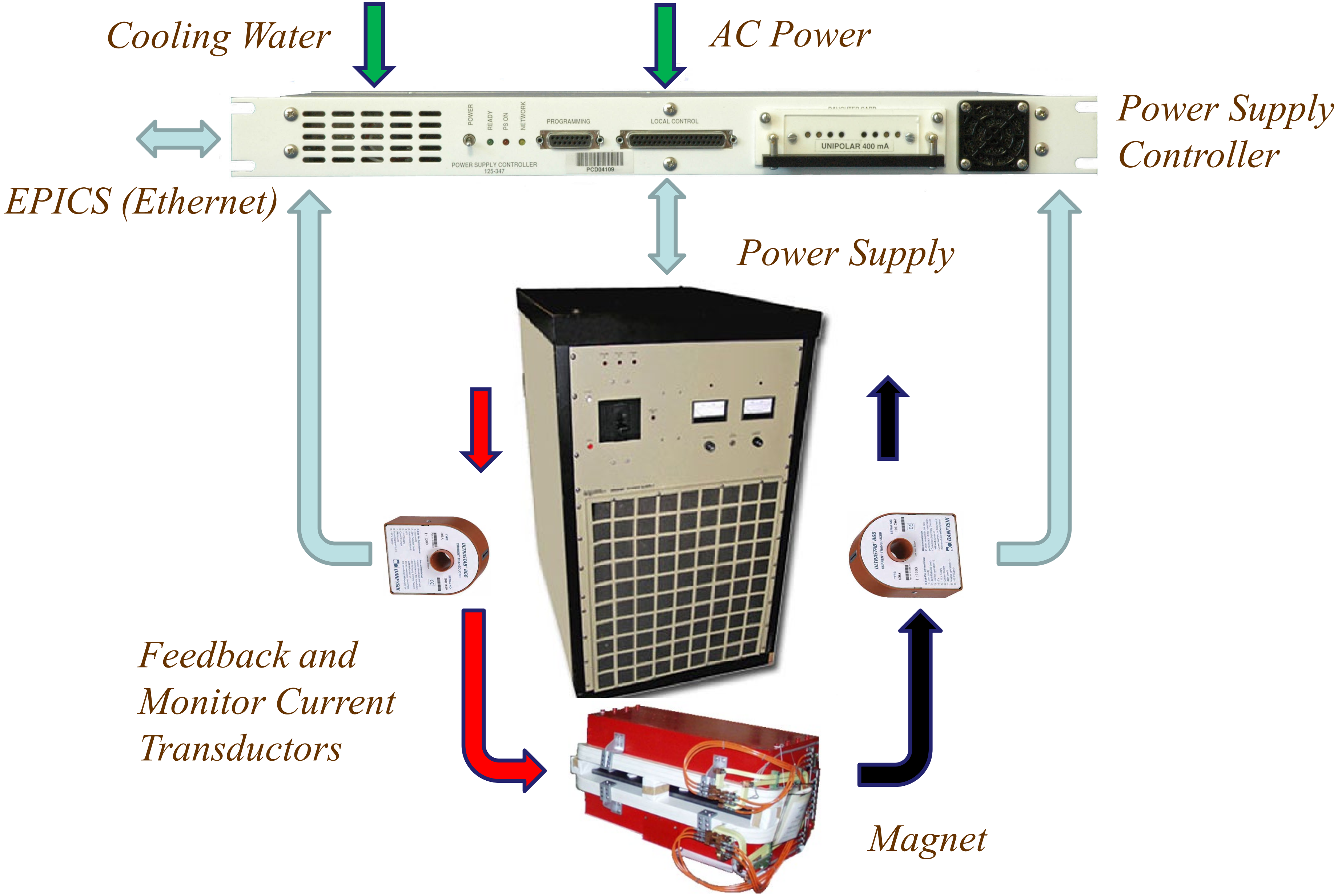


Sections

Section 1

- *Introduction*

A Typical DC Magnet Power System



Section 2

- *Purpose*
- *Goals*
- *Intended Audience*
  - *Civil, Mechanical Designers*
  - *Control Engineers*
  - *Electrical Distribution System Designers*
  - *Maintenance Personnel*
  - *Magnet Designers*
  - *Operators*
  - *Physicists*
  - *Power Conversion / Power Supply Designers*
  - *Project Engineers / Managers*
  - *Safety Engineers / Designers*

Purpose

- *Provide an overview of Accelerator Power Electronics Engineering with an emphasis on DC and an overview of pulsed power supplies*

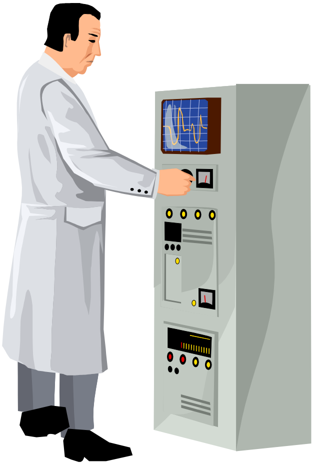
Goals

- *Provide a historical overview of Accelerator Power Supplies from early designs, to presently employed technology, to some promising future developments now in incubation*
- *Survey the most pertinent power supply topologies from the perspectives of accelerators, load type and rating*
- *Give other, non-power conversion disciplines a glimpse of, and a better understanding of, Power Electronics Engineering*
- *Define the information needed for the power supply designer, or user, to make appropriate choices for power supply type, design, and rating*

Intended Audience

- ***Civil, Mechanical Designers*** *– interest in facility space, weight, mounting, cooling*

- ***Control Engineers*** *– an insight into some interface requirements*



Intended Audience

- ***Electrical Distribution System Designers*** *– AC distribution requirements, address and reduce harmonics and EMI*

- ***Maintenance Personnel*** *– power system reliability and maintainability*

- ***Magnet Designers*** *– tradeoffs between power supply output voltage, current and stability limitations and the magnet design. The power supply role in magnet protection via cooling interlocks and ground fault detection and protection*

Intended Audience

• ***Accelerator Operators*** *– Power supply control and operating characteristics*

• ***Physicists*** *– Power system rating limitations, magnet configuration options vs. physics tradeoffs, long and short-term current stability limitations*

Intended Audience

- ***Power Conversion / Power Supply Designers*** – *power systems from another point of view*

- ***Project Engineers and Managers*** – *Power conversion system costs*

- ***Safety Engineers / Designers*** – *Personnel and equipment safety in an electrical power environment. General power safety provisions*

Section 3

- *Mathematical Preliminaries*
  - *Why Mathematical Preliminaries*
  - *Average and RMS Values*
  - *Complex Exponentials*
  - *Differential Equations*
  - *Linear Systems*
  - *Impulse and Step Functions*
  - *System Transfer Function*
  - *Fourier Series and Transforms*
  - *Laplace Transforms*
  - *Exponential Approximations*
  - *Simple Circuit Equations*

Why Mathematical Preliminaries?

- *We need to use circuits and understand their behavior*
  - *Power supply loads*
  - *Filter circuits*
  - *Pulse shaping circuits*
  - *Feedback and control circuits*

- *Many important circuits are passive, consisting of*
  - *Resistors*
  - *Capacitors*
  - *Inductors*

- *For these circuits we know the voltage-current relations for each element*
  - $v_R = Ri_R$
  - $v_L = L\frac{di_L}{dt}$
  - $i_C = C\frac{dv_C}{dt}$

- *And*

  Kirchoff's Voltage Law for each loop:

  $$\sum_{n=1}^{N} v_n = 0$$

  Kirchoff's Current Law for each node

  $$\sum_{n=1}^{N} i_n = 0$$

Why Mathematical Preliminaries?

- *Solving circuit equations involves calculus, which includes solving differential equations, integration, and convolution*

- *Fortunately circuits containing only passive elements can be well-approximated by linear systems*

- *If we learn the mathematics behind linear systems*
  - *Fourier and Laplace transforms and their inverses*
  - *Impulse and step functions*

- *We can trade*
  - *Calculus for algebra*
  - *Convolution for multiplication*

$$< F > = \frac{1}{T}\int_0^T f(t)dt$$ *Average value*

$$f(t) = A\sin\frac{2\pi}{T}t \Rightarrow < F > = \frac{1}{T}A\int_0^T \sin\frac{2\pi}{T}t\,dt = 0$$

$$f(t) = A\left|\sin\frac{2\pi}{T}t\right| \Rightarrow < F > = \frac{2}{T}A\int_0^{\frac{T}{2}} \sin\frac{2\pi}{T}t\,dt = \frac{2}{\pi}A \approx 0.6366\cdot A$$

DC value of rectified sine wave

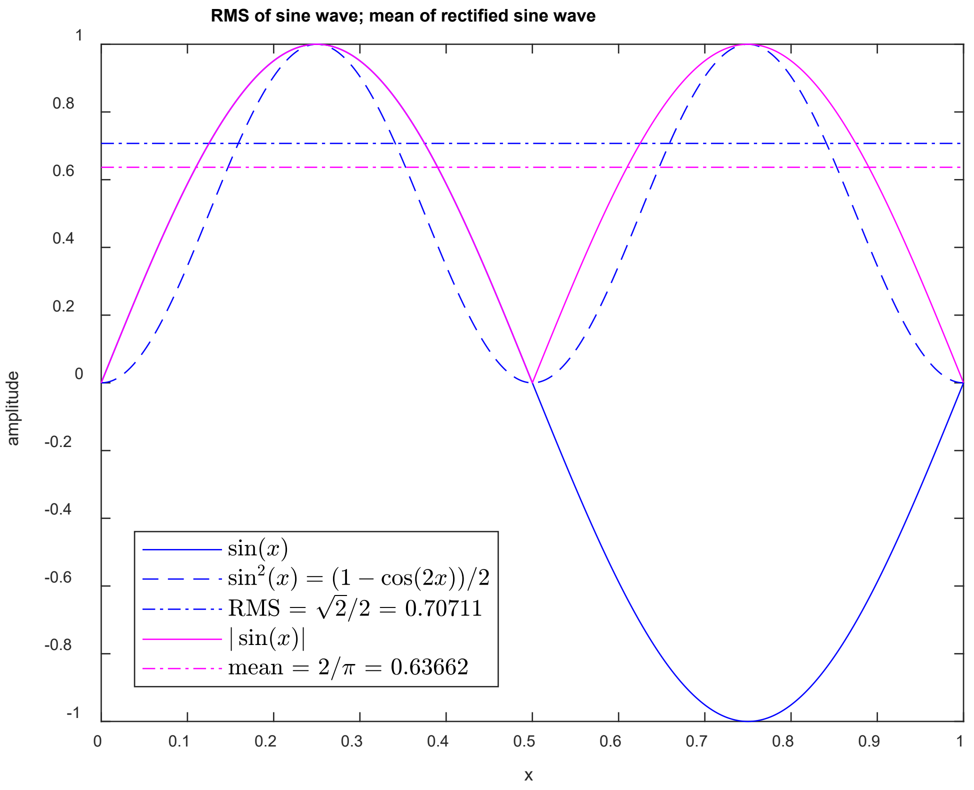
$$F_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt}$$

$$f(t) = A\sin\frac{2\pi}{T}t$$

$$F_{rms} = \sqrt{\frac{A^2}{T}\int_0^T \frac{1}{2}(1-\cos\frac{4\pi}{T})\,dt}$$

$$F_{rms} = \frac{A}{\sqrt{2}} \approx 0.707\cdot A$$

rms used for average power



Why Mathematical Preliminaries?

- *Often we will encounter symmetric signals that have an average value of zero, but whose RMS value is non-zero.*
  - *In such cases the average rectified value is also non-zero and of interest.*
- *Often in power systems terminology, the term "average value" is a short-hand notation used to mean "average rectified value".*
- *When working with power systems, including in this course, when asked for an "average value" of a voltage or current, it is therefore good practice to calculate both the "average" (arithmetic average) and the "average rectified value" of these quantities and label the quantities accordingly.*

Average value:

$$< F > = \frac{1}{T}\int_0^T f(t)dt$$

Average Rectified Value:

$$< |F| > = \frac{1}{T}\int_{t=0}^{t=T} |f(t)|\, dt$$

$$Duty\ Factor = DF = \frac{T_{on}}{T_{on} + T_{off}} = \frac{T_{on}}{T}$$

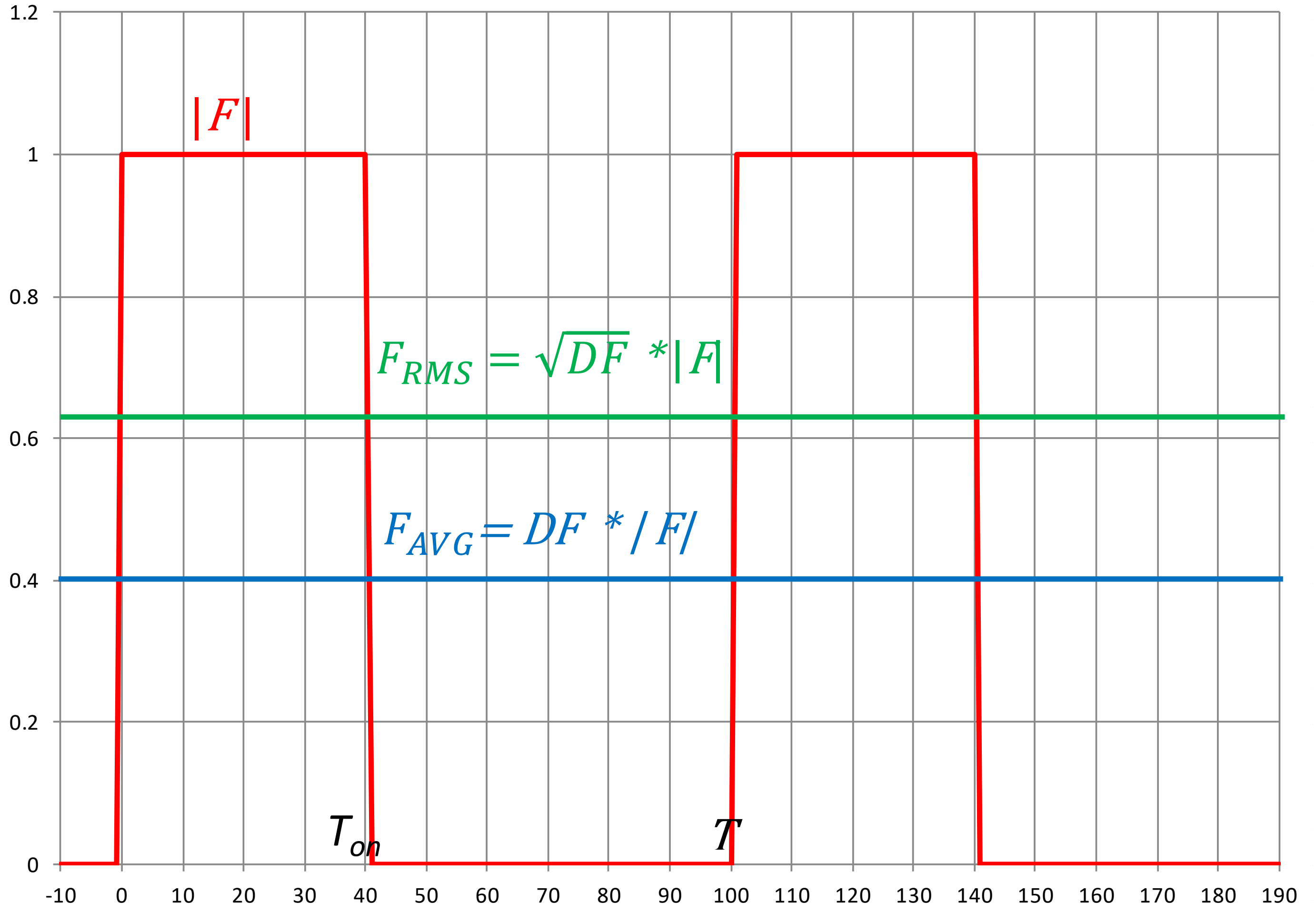
$$F_{ave} = \frac{1}{T}\int_{t=0}^{t=T} f(t)\,dt, \quad if\ \ f(t) = F_m\ during\ T_{on}$$

$$F_{ave} = \frac{1}{T}\int_{t=0}^{t=T_{on}} F_m\ dt = \frac{T_{on}}{T} * F_m = DF * F_m$$

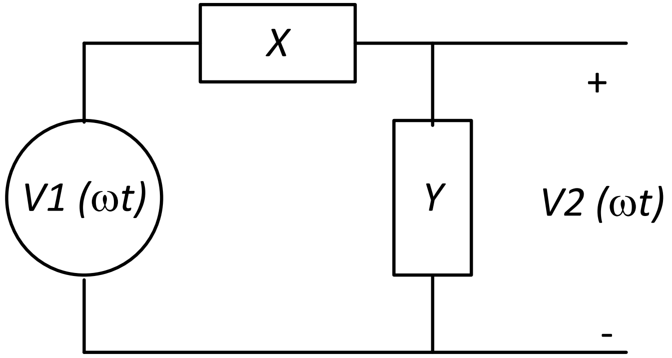
$$F_{rms} = \sqrt{\frac{1}{T}\int_{t=0}^{t=T} f(t)^2\,dt}\ , \quad if\ \ f(t) = F_m\ during\ T_{on}$$

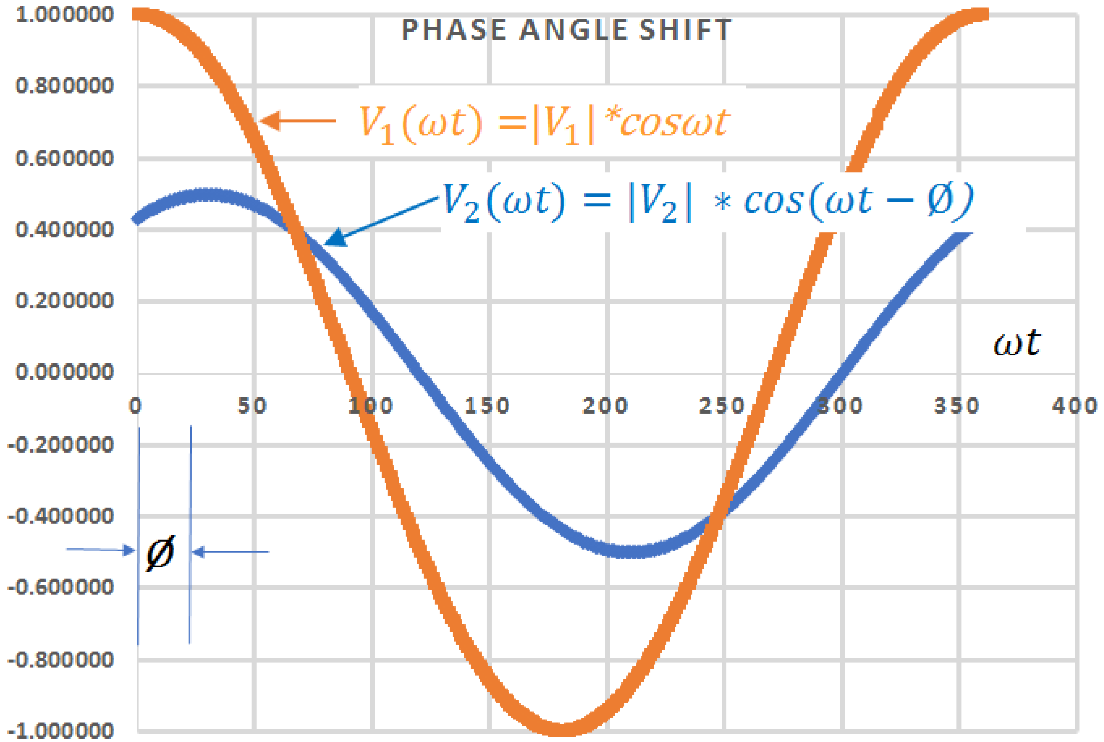
$$F_{rms} = \sqrt{\frac{1}{T}\int_{t=0}^{t=T_{on}} {F_m}^2\,dt} = \sqrt{\frac{T_{on}}{T}} * F_m = \sqrt{DF} * F_m$$

$$Duty\ Factor = DF = \frac{T_{on}}{T}$$



Given $\omega = 2\pi f \ (\frac{\text{rad}}{\text{sec}})$, *t= time (sec),* $V = |V| \cdot \angle(\omega t \pm \emptyset)$





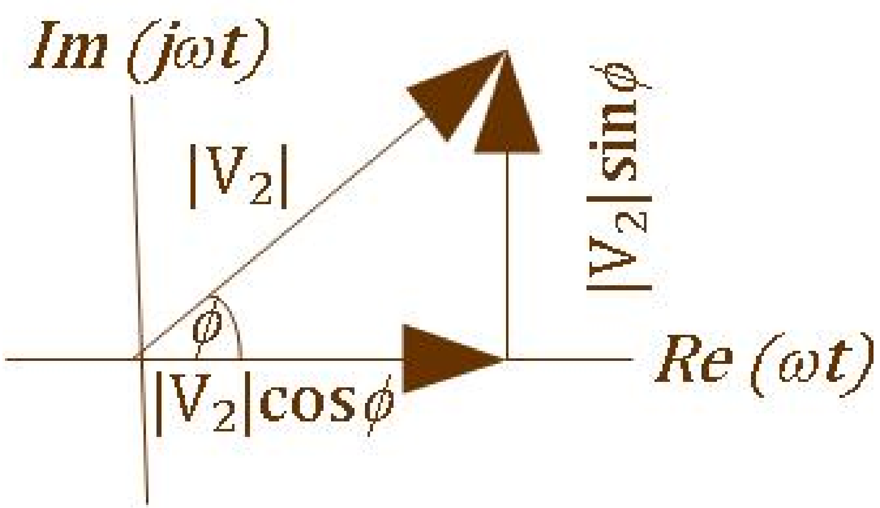
$V_1(\omega t) = |V_1| \cdot \cos \omega t$, *Real, in-phase component only*

$V_1(\omega t) = |V_1| \cdot \angle(\omega t + 0) = |V_1| \cdot \angle 0$ *phasor form*

$V_2(\omega t) = |V_2| \cdot \cos(\omega t - \emptyset)$, *in-phase and out-of-phase components*

$V_2(\omega t) = |V_2| \cdot \angle(\omega t - \emptyset)$, *or* $V_2 = |V_2| \cdot \angle - \emptyset$ *phasor form*

$$V_2(\omega t) = |V_2| \cdot cos(\omega t + \emptyset)$$



$$Re\ V_2 = |V_2| \cos \emptyset$$

$$Im\ V_2 = |V_2|\ sin\ \emptyset$$

$$V_2 = |V_2| \cos \emptyset + j\ |V_2| \sin \emptyset$$

Euler's Identity: $Ae^{jX} = \mathrm{A}(\cos X + j \sin X)$

$$V_2 = |V_2| e^{j\emptyset}$$

$V_2(\omega t) = |V_2|e^{j(\omega t \pm \phi)} = |V_2|e^{\pm j\phi}$ *(exponential form: $e^{j\omega t}$ understood)*

$|V_2| = |V_2| \cdot \sqrt{cos\,\emptyset^2 + sin\,\emptyset^2} = |V_2| \cdot 1$

Since the magnitude of the complex exponential is always 1, this function gives us a steady state eigenfunction of the constant, differential and integral operators we will later need to analyze circuits

*An **eigenfunction** is a function that, when operated on by the differential equation, returns itself multiplied by a constant, possibly complex*

$$D \cdot f(t) = \left(a_n \frac{d^n}{dt^n} + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} + \cdots + a_1 \frac{d}{dt} + a_0\right) f(t) = \alpha f(t)$$

*α is the **eigenvalue** of this function with respect to the differential equation*

- *Ex:* $D = \frac{d}{dt}; f(t) = e^{j\omega t};\ D \cdot f(t) = \frac{d}{dt} e^{j\omega t} = j\omega e^{j\omega t} = \alpha f(t);\ \alpha = j\omega$
- *For example if the behavior of a system is determined by the equation*

$$\left(a \frac{d^2}{dt^2} + b \frac{d}{dt} + c\right) e^{st} = 0$$

one finds $(as^2 + bs + c)e^{st} = 0 \ \Rightarrow (as^2 + bs + c) = 0$

The roots given by the quadratic formula $s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *are the eigenvalues, or the roots, of the system.*

Differential equations (diffeq) describe dynamic systems that change with time

For a system with time-varying quantities, $u(t), y(t)$ *that satisfy the diffeq*

$$\frac{dy(t)}{dt} = au(t)$$

$y(t)$ *depends on its past values as well as those of* $u(t)$

$$\frac{dy(t)}{dt} = \lim_{\Delta t \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t} = au(t)$$

$$y(t+\Delta t) \approx y(t) + \Delta t \cdot au(t)$$

$$y(t+2\Delta t) \approx y(t+\Delta t) + \Delta t \cdot au(t+\Delta t) \approx y(t) + \Delta t \cdot a(u(t) + u(t+\Delta t))$$

Continuing this for arbitrary times in the past

$$y(t+N\Delta t) \approx y(t) + a\sum_{n=0}^{N-1} \Delta t \cdot u(t+n\Delta t)$$

As $\Delta t \to 0$, *the sum becomes infinite and turns into the integral equation*

$$y(t) = y(t_0) + a\int_{t_0}^{t} u(\tau)\, d\tau$$

Differential equations describe systems that evolve with time

In general, given a driving term $u(t)$ and a driven term $y(t)$, one can define a differential equation for the evolution of $y(t)$

$$\frac{dy(t)}{dt} + ay(t) = bu(t)$$

$y(t)$ depends on the past and current values of itself and $u(t)$

The derivative is defined as

$$\frac{dy(t)}{dt} = \lim_{\Delta t \to 0} \frac{y(t) - y(t - \Delta t)}{\Delta t} = -ay(t) + bu(t)$$

so that

$$y(t) \approx y(t - \Delta t) + \Delta t[-ay(t) + bu(t)]$$

$$\approx \frac{1}{1 + a\Delta t} y(t - \Delta t) + \frac{b\Delta t}{1 + a\Delta t} u(t)$$

If we continue this construction

$$y(t+\Delta t) \approx y(t) + \Delta t[-ay(t+\Delta t) + bu(t+\Delta t)]$$

$$(1+a\Delta t)y(t+\Delta t) \approx \frac{1}{1+a\Delta t}y(t-\Delta t) + \frac{b\Delta t u(t)}{1+a\Delta t} + b\Delta t u(t+\Delta t)$$

$$y(t+\Delta t) \approx \frac{1}{(1+a\Delta t)^2}y(t-\Delta t) + b\Delta t\left[\frac{u(t)}{(1+a\Delta t)^2} + \frac{u(t+\Delta t)}{1+a\Delta t}\right]$$

$$y(t+(N+1)\Delta t) \approx \frac{1}{(1+a\Delta t)^N}y(t-\Delta t) + b\sum_{n=0}^{N+1}\frac{u(t+n\Delta t)}{(1+a\Delta t)^{N-n}}$$

From this we can continue on to obtain the exact solution

$$y(t) = e^{-at}[y(t_0) + \int_{t_0}^{t} e^{a\tau}u(\tau)d\tau]$$

as obtained from the method of variation of parameters.

Note that the ay *term in the differential equation gives rise to a term*

$$e^{-at}$$

that acts to damp out initial conditions and past inputs.

A linear system, $h[x]$ is defined such that for inputs x_1 and x_2, if
$y_1 = h[x_1]$ *and* $y_2 = h[x_2]$
then

$$ay_1 + by_2 = h[ax_1 + bx_2]$$

This is the principle of linear superposition.

Examples of linear systems:

Constant gain system	$h_1[x] = A_1 x$	$V = R_1 I$
Sum of two constant gains	$h_2[x] = A_2 x + A_3 x$	$V = R_2 I + R_3 I$
Derivatives	$h_3[x] = A_4 \frac{dx}{dt}$	$V = L_4 \frac{dI}{dt}$
Integrals	$h_4[x] = A_5 \int x\, dt$	$V = \frac{1}{C_5} \int I\, dt$

We are interested in linear systems because there are many mathematical tools available for use on linear systems and because many common physical systems and components are linear: Resistors, Inductors, Capacitors

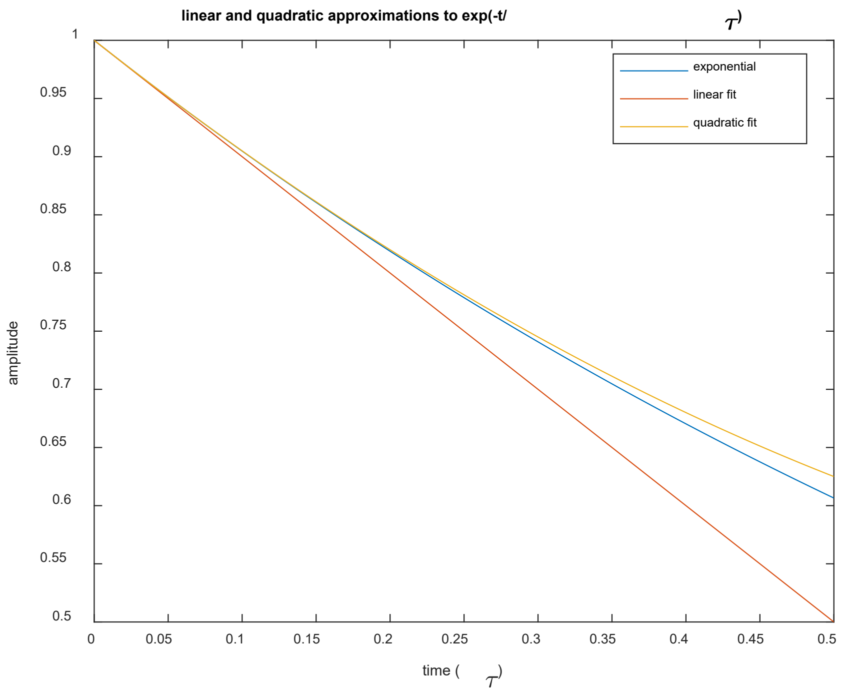
$h(x) = e^x$ *is a nonlinear system.*

Proof:

$$e^{ax+by} = e^{ax}e^{by} \neq ae^x + be^y$$

We note that non-linear systems can often be approximated by linear systems.

As we will show later, slow exponentials are well approximated by linear systems



- *The problems we investigate involve a control signal acting on a system*
- *We simplify the solution by representing the control signal as a sequence of elementary functions*
- *Then we need to characterize the response of our system to these elementary functions*
- *Finally, we use the properties of linear systems to obtain the response of the system with the control signal acting on it*
- *Two such commonly used elementary functions are the impulse function and the step function*

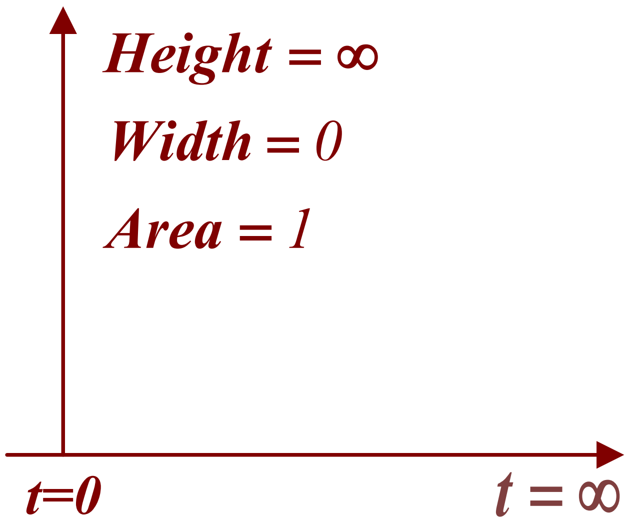
Mathematical Preliminaries - Impulse Functions - Discrete and Continuous

Continuous impulse (Dirac delta) function, $\delta(t)$

Properties: This is a generalized function

$$\delta(t) = 0, \qquad t \neq 0$$
$$\delta(t) = \infty, \qquad t = 0$$
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

Shifting property: $f(t_0) = \int_{-\infty}^{\infty} f(t)\delta(t_0 - t)dt$

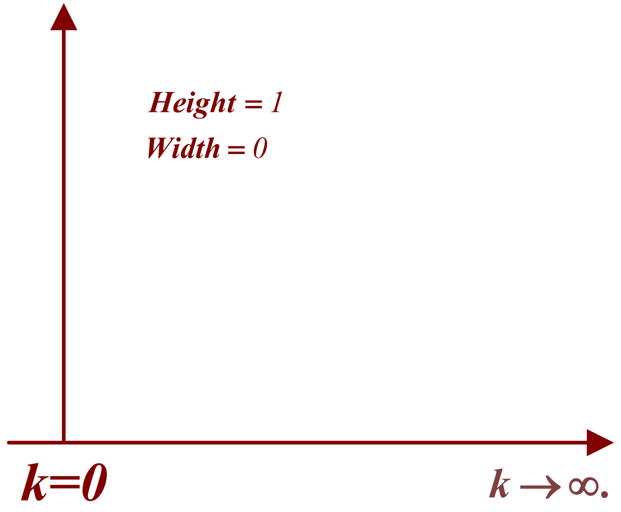


Discrete impulse function, $\delta[n]$

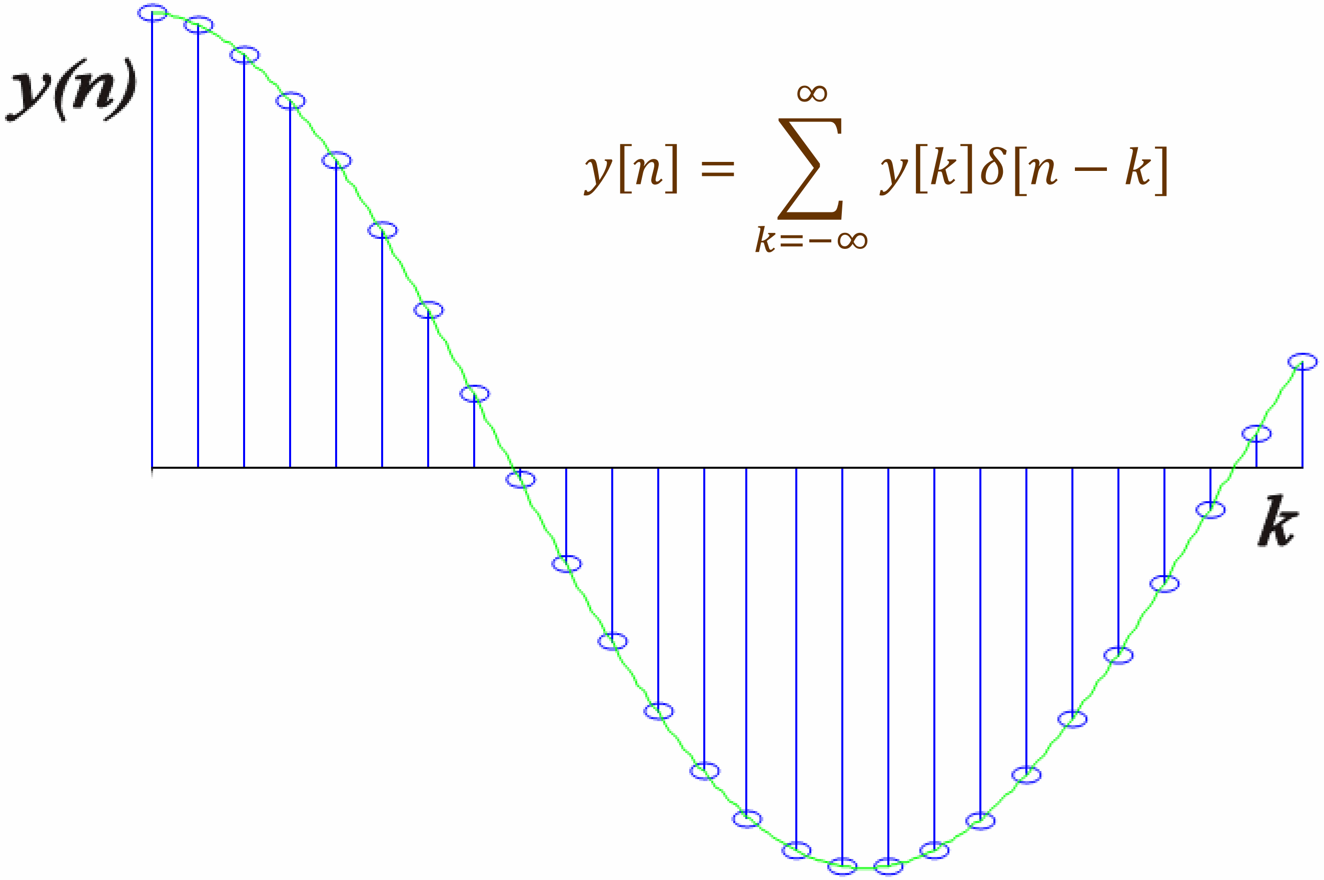
Properties:

$$\delta[n] = 0, \qquad n \neq 0$$
$$\delta[n] = 1, \qquad n = 0$$
$$\sum_{n=-\infty}^{\infty} \delta[n] = 1$$

Shifting property: $f[k] = \sum_{n=-\infty}^{\infty} f[n]\delta[k - n]$

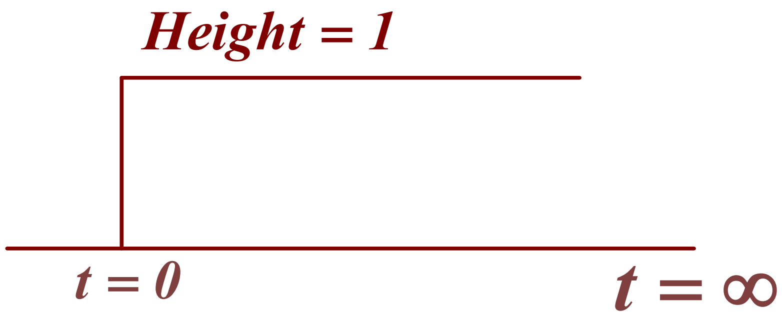


$$y[n] = \sum_{k=-\infty}^{\infty} y[k]\delta[n-k]$$



Properties

$$U(t) = 0, \qquad t < 0$$
$$U(t) = 1, \qquad t \geq 0$$



Relation to Impulse

$$\delta(t) = \frac{d}{dt} U(t)$$

Functional Representation

$$f(t_0) = \int_{-\infty}^{\infty} f(t)\delta(t - t_0)\, dt$$

$$= \int_{-\infty}^{\infty} f(t) \left(\frac{d}{dt} U(t - t_0) \right) dt$$

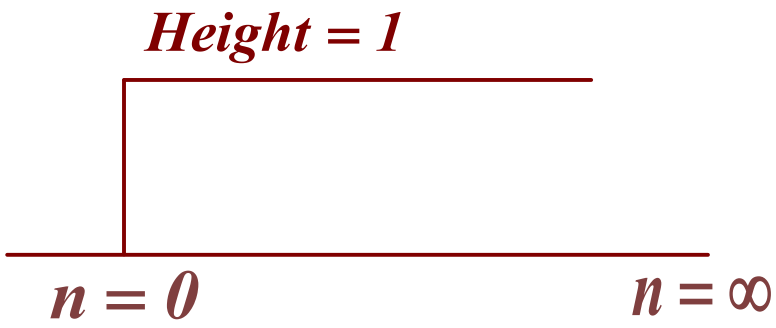
$$= f(t)U(t - t_0) \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} U(t - t_0) \frac{df(t)}{dt}\, dt$$

$$= f(\infty) - \int_{t_0}^{\infty} \frac{df(t)}{dt}\, dt = f(\infty) - \big(f(\infty) - f(t_0)\big) = f(t_0)$$

Heaviside step function

$U[n] = 0, \qquad n < 0$

$U[n] = 1, \qquad n \geq 0$



Relation to impulse

$\delta[n] = U[n] - U[n-1]$

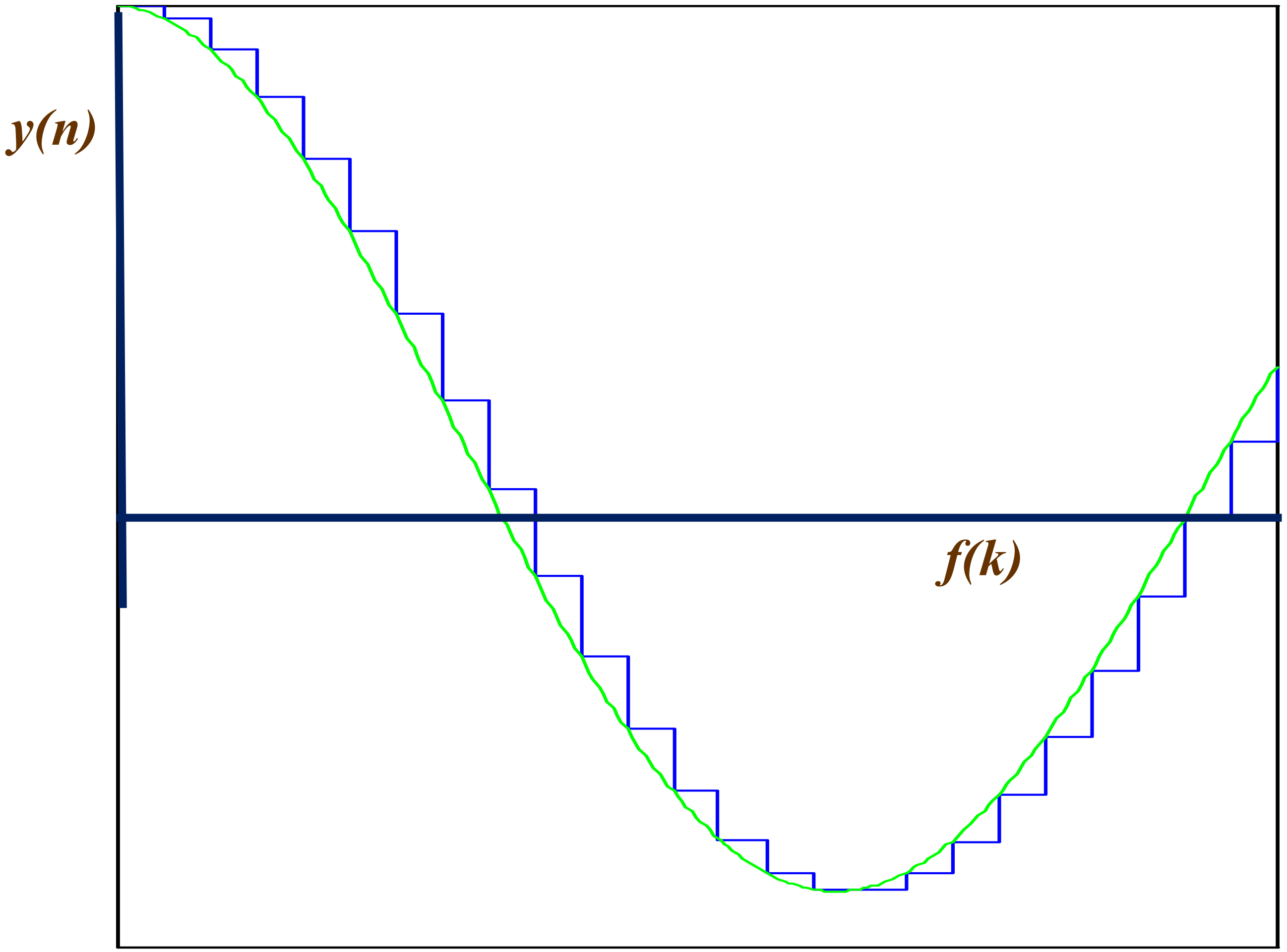
Functional representation

$$f[n] = \sum_{k=-\infty}^{\infty} f[k]\delta[n-k] = \sum_{k=-\infty}^{\infty} f[k]\delta[k-n]$$

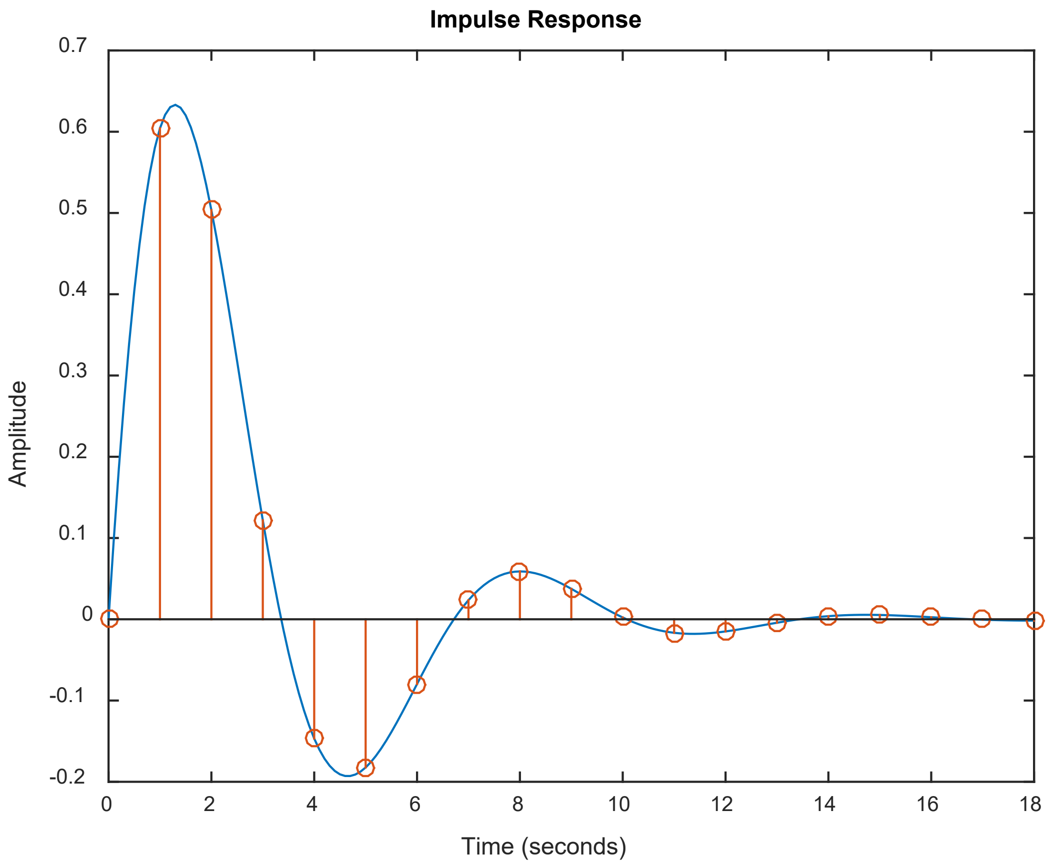
$$= \sum_{k=-\infty}^{\infty} f[k]\,(U[k-n] - U[k-n-1])$$

$$= \sum_{k=-\infty}^{\infty} (f[k] - f[k+1])\,U[k-n]$$

$$= \sum_{k=n}^{\infty} (f[k] - f[k+1]) = f[n]$$



$$y[n] = \sum_{k=-\infty}^{\infty} (y[k] - y[k+1]) \cdot U[k-n]$$



The impulse response of a general system is causal

There is no response before the impulse occurs

The impulse response, in general, also lasts after the impulse ends

The input to the system can be represented as a series of impulses, $x[k]$.
For each impulse, the output at any later time is the system response to that impulse

$$y[n] = h[n-k]x[k]$$

The total system output for the total system input is

$$y[n] = \sum_{k=-\infty}^{\infty} h[n-k]x[k]$$

where $h[n]$ *is causal, so vanishes for* $n < 0$.

For continuous systems, this is

$$y(t) = \int_{-\infty}^{\infty} h(t-u)x(u)du$$

These are convolution integrals and sums.
If one Fourier-transforms this integral relationship, the convolution integral in the time domain becomes a product in the frequency domain

$$Y(\omega) = H(\omega)X(\omega)$$

We want to convert the convolution integral of the input with the impulse response into the frequency domain.

We first need to define, and then use, a representation of the delta function

$$\delta(\omega - \omega_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j(\omega-\omega_0)t}dt$$

Intuition:

For $\omega \neq \omega_0$*, the integrand oscillates, so the average value vanishes*

For $\omega = \omega_0$*, the integrand is unity, and the integral is infinite*

Definition of the Fourier transform pair $f(t), F(\omega)$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t} \frac{d\omega}{2\pi}$$

Representation of the Dirac delta function

$$\delta(t-\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega(t-\tau)}\, d\omega$$

Two expressions for $f(t)$

$f(t) = \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)\, d\tau$ *definition of delta function*

$f(t) = \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t} \frac{d\omega}{2\pi}$ *definition of Fourier transform for* $f(t)$

$f(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tau)\, e^{-j\omega\tau}\, d\tau\, e^{j\omega t} \frac{d\omega}{2\pi}$ *definition of Fourier transform for* $F(\omega)$

$$= \int_{-\infty}^{\infty} f(\tau) \int_{-\infty}^{\infty} e^{j\omega(t-\tau)} \frac{d\omega}{2\pi}\, d\tau \;\Rightarrow\; \delta(t-\tau) = \int_{-\infty}^{\infty} e^{j\omega(t-\tau)} \frac{d\omega}{2\pi}$$

Transform the convolution of the input with the impulse response

Starting with the convolution integral

$$y(t) = \int_{-\infty}^{\infty} h(t-u)x(u)du$$

insert the Fourier transforms for $h(t-u)$ *and* $x(u)$

$$y(t) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(\omega)e^{j\omega(t-u)}X(\omega_1)e^{j\omega_1 u}du\, d\omega_1 d\omega$$

$$= \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(\omega)X(\omega_1)e^{j\omega t}e^{j(\omega_1-\omega)u}du\, d\omega_1 d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(\omega)X(\omega_1)e^{j\omega t}\delta(\omega_1-\omega)d\omega_1 d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)X(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)e^{j\omega t}\, d\omega$$

Therefore $\quad Y(\omega) = H(\omega)X(\omega)$

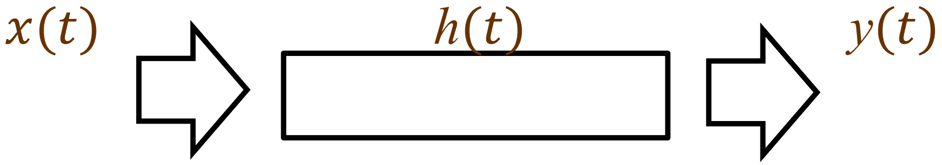
The transform of the output equals the product of the transform of the input multiplied by the transform of the impulse response.

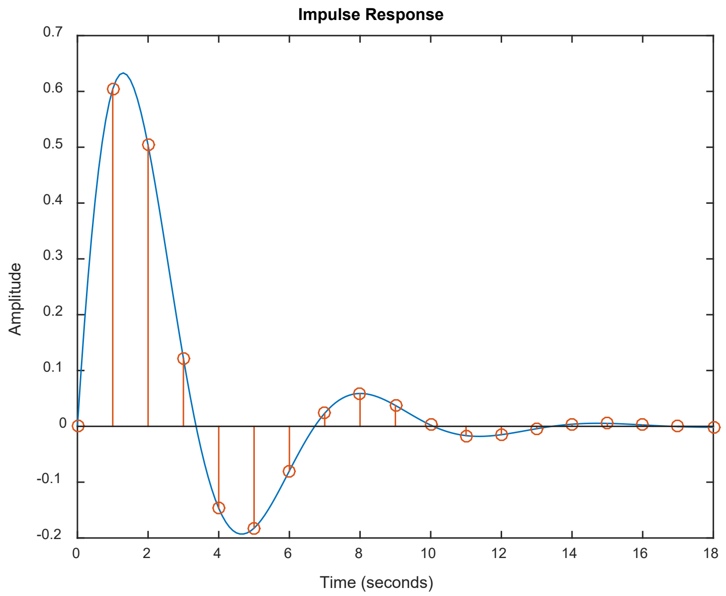
Given an input $x(t)$, a system $h(t)$, and an output $y(t)$, the Transfer Function of the system is the Fourier Transform of $h(t)$

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

and $H(\omega) = \dfrac{Y(\omega)}{X(\omega)}$

where $Y(\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$ *and* $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$





- *Fourier transforms represent functions as combinations (sums/integrals) of complex exponentials.*
- *When working with aperiodic continuous functions, we need the standard Fourier transform pair* $(f(t), F(\omega))$

$$f(t) = \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t} \frac{d\omega}{2\pi}; \qquad F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

- *For periodic systems, with a period,* T, *the only complex eigenvectors that can be used to represent the signals are those whose frequencies are multiples of the "fundamental harmonic",* $\omega = 2\pi/T$ *(including* $\omega = 0$*).*
- *Periodic functions are represented by the infinite sums of the appropriately weighted discrete harmonics. In this case the Fourier transforms between the pair* $(f(t), F_n)$ *are*

$$f(t) = \sum_{n=-\infty}^{\infty} F_n\, e^{j\left(\frac{2\pi n}{T}\right)t}; \qquad F_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-j\left(\frac{2\pi n}{T}\right)t}$$

- *We will work with many periodic systems and often will concentrate on just one harmonic, either DC or the fundamental*

Using Euler's formula, $e^{jx} = \cos x + j \sin x$, *we can also represent these relations as*

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi n}{T} t + \sum_{n=1}^{\infty} b_n \sin \frac{2\pi n}{T} t$$

where the coefficients a_n, b_n *are defined as*

$$a_0 = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, dt$$

$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos \frac{2\pi n}{T} t\, dt\,, \qquad n \neq 0$$

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin \frac{2\pi n}{T} t\, dt$$

Both the exponential and trigonometric series are ***complete****. That is, they can faithfully represent any function.*

Also the individual terms in the series are orthogonal to each other.

The representation of any function in terms of the Fourier series is unique.

- *The DC term is orthogonal to all others*

$$\frac{1}{T}\int_{-T/2}^{T/2}\cos\frac{2\pi nt}{T}dt \; = \frac{1}{T}\int_{-T/2}^{T/2}\sin\frac{2\pi nt}{T}dt = 0$$

- *The sinusoidal terms are periodic, so the integral over one period vanishes.*

- *All cosine terms are orthogonal to all sine terms. Using*

$$\sin A\cos B = 1/2[\sin(A+B)+\sin(A-B)]$$

$$\frac{2}{T}\int_{-T/2}^{T/2}\cos\frac{2\pi nt}{T}\sin\frac{2\pi mt}{T}dt$$

$$= \frac{2}{T}\frac{1}{2}\,[\int_{-T/2}^{T/2}\sin\frac{2\pi(m+n)t}{T}dt + \int_{-T/2}^{T/2}\sin\frac{2\pi(m-n)t}{T}dt] = 0$$

- *Cosine terms are orthogonal to other cosine terms. Using* $\cos A \cos B = 1/2[\cos(A+B) + \cos(A-B)]$ *we get*

$$\frac{2}{T}\int_{-T/2}^{T/2} \cos\frac{2\pi nt}{T}\cos\frac{2\pi mt}{T}dt$$

$$= \frac{2}{T}\frac{1}{2}[\int_{-T/2}^{T/2} \cos\frac{2\pi(n+m)t}{T}dt + \int_{-T/2}^{T/2} \cos\frac{2\pi(n-m)t}{T}dt]$$

$$= \{\begin{matrix} 1/2 & n = m \\ 0 & n \neq m \end{matrix}$$

- *Using* $\sin A \sin B = 1/2[\cos(A-B) - \cos(A+B)]$, *we find the same relationship for products of sine terms.*

- *Using the calculations above, we represent the Fourier series of the periodic function* $f(t)$

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2\pi nt}{T} + b_n \sin\frac{2\pi nt}{T}\right)$$

where

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt$$

$$a_n = \frac{2}{T}\int_{T/2}^{T/2} f(t)\cos\frac{2\pi nt}{T}\,dT$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin\frac{2\pi nt}{T}\,dT$$

$$f(t) = \begin{cases} 0 & -T/2 \le t < 0 \\ 1 & 0 \le t < T/2 \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^{T/2} dt = \frac{1}{2}$$

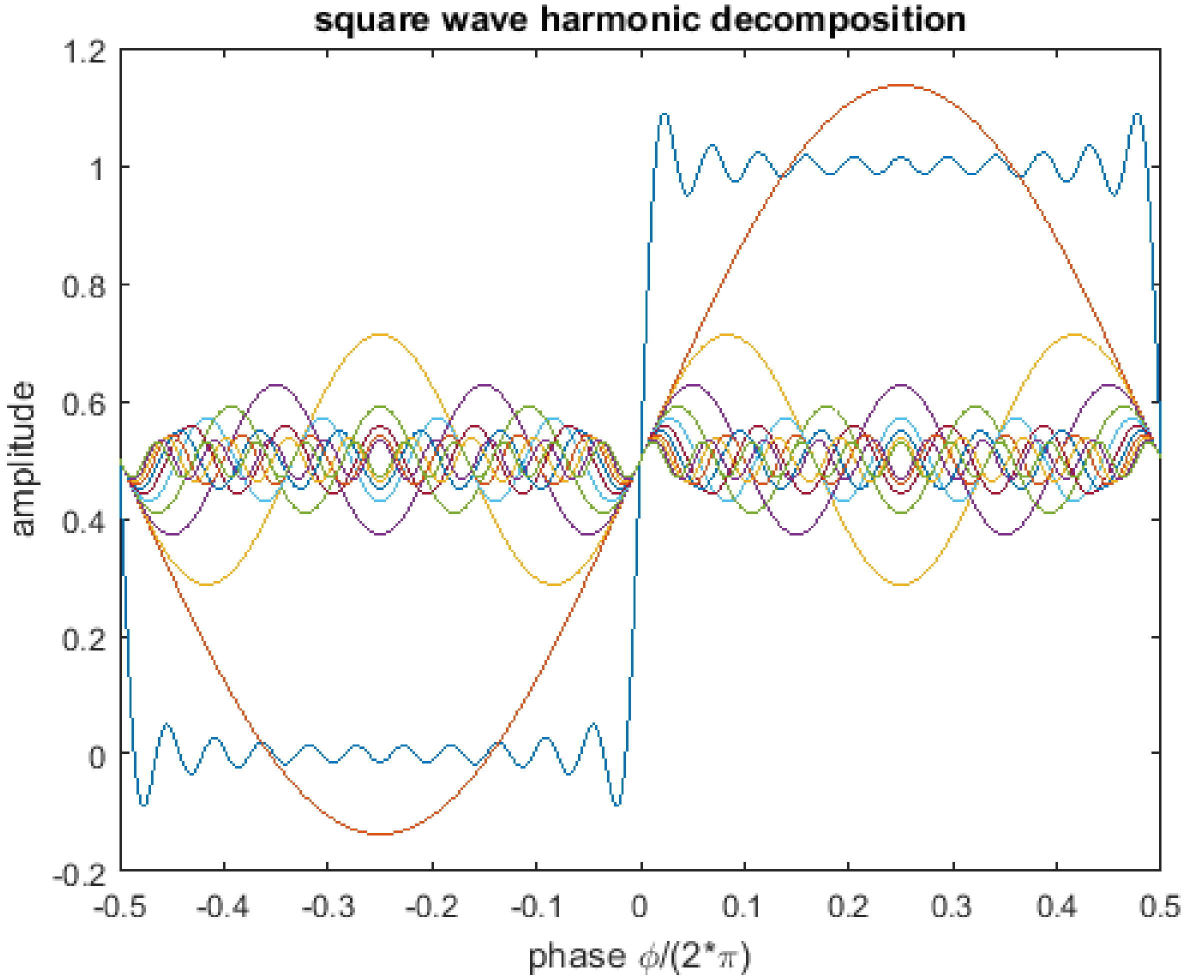
$$a_n = \frac{2}{T}\int_0^{T/2} \cos\frac{2\pi nt}{T}\,dt = \frac{2}{T}\frac{T}{2\pi n}\sin\frac{2\pi nt}{T}\Big|_0^{T/2} = \frac{1}{\pi n}\sin n\pi = 0$$

$$b_n = \frac{2}{T}\int_0^{T/2} \sin\frac{2\pi nt}{T}\,dt = -\frac{1}{\pi n}\cos\frac{2\pi nt}{T}\Big|_0^{T/2} = \frac{1}{\pi n}(1 - \cos n\pi)$$

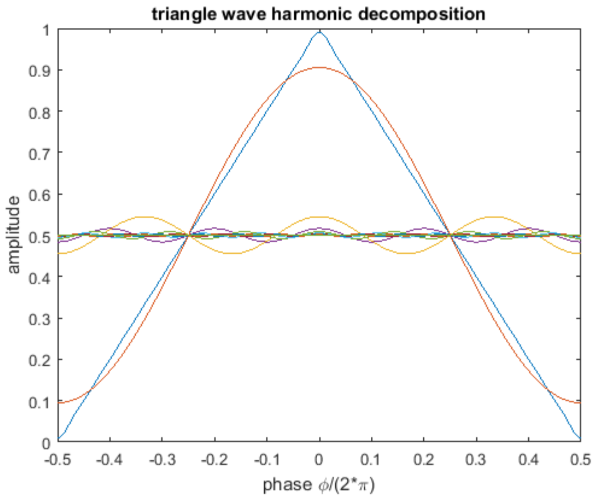
$$= \frac{1}{\pi n}(1 - (-1)^n)$$

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{1}{\pi n}(1 - (-1)^n)\sin\frac{2\pi nt}{T}$$

$$f(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1}\sin\frac{2\pi(2n+1)t}{T}$$



- *Fundamental term dominates*
- *Harmonic terms mainly contribute to sharp transition at edge*
- *Since the square wave function is discontinuous, the edges have "Gibbs ears"*



$$f(t) = \begin{cases} 1 + 2t/T & -T/2 \le t < 0 \\ 1 - 2t/T & 0 \le t < \mathrm{T}/2 \end{cases}$$

$$a_0 = \frac{1}{T}\left[\int_{-T/2}^{0} (1 + 2t/T)\, dt + \int_{0}^{T/2} (1 - 2t/T)\, dt\right]$$

$$= \frac{1}{T}\left[\left(t + \frac{2}{T}\frac{t^2}{2}\right)\Big|_{-T/2}^{0} + \left(t - \frac{2}{T}\frac{t^2}{2}\right)\Big|_{0}^{T/2}\right]$$

$$= \frac{1}{T}\left[\left(\frac{T}{2} - \frac{1}{T}\left(\frac{T}{2}\right)^2\right) + \left(\frac{T}{2} - \frac{1}{T}\left(\frac{T}{2}\right)^2\right)\right] = \frac{1}{2}$$

$$a_n = \frac{2}{T}[\int_{-T/2}^{0}\left(1+\frac{2t}{T}\right)\cos\frac{2\pi nt}{T}dt + \int_{0}^{T/2}\left(1-\frac{2t}{T}\right)\cos\frac{2\pi nt}{T}dt]$$

$$= \frac{2}{T}\left[\int_{-T/2}^{T/2}\cos\frac{2\pi nt}{T}dt + \frac{2}{T}\int_{-T/2}^{0} t\cos\frac{2\pi nt}{T}dt - \frac{2}{T}\int_{0}^{T/2} t\cos\frac{2\pi nt}{T}dt\right]$$

$$= \left(\frac{2}{T}\right)^2\left[\int_{-T/2}^{0} t\cos\frac{2\pi nt}{T}dt - \int_{0}^{T/2} t\cos\frac{2\pi nt}{T}dt\right]$$

$$= -2\left(\frac{2}{T}\right)^2\int_{0}^{T/2} t\cos\frac{2\pi nt}{T}dt$$

$$= -2\left(\frac{2}{\mathrm{T}}\right)^2[\frac{\mathrm{T}}{2\pi n}\mathrm{t}\sin\frac{2\pi nt}{T}\Big|_0^{T/2} - \frac{T}{2\pi n}\int_{0}^{T/2}\sin\frac{2\pi nt}{T}dt]$$

$$= 2\left(\frac{2}{T}\right)^2\left(\frac{T}{2\pi n}\right)^2(1-\cos n\pi) = \frac{2}{(\pi n)^2}(1-(-1)^n)$$

$$b_n = \frac{2}{T}\left[\int_{-T/2}^{T/2} \sin\frac{2\pi nt}{T}dt + \frac{2}{T}\int_{-T/2}^{0} t\sin\frac{2\pi nt}{T}dt - \frac{2}{T}\int_{0}^{T/2} t\sin\frac{2\pi nt}{T}dt\right]$$

$$= \frac{2}{T}\left[\int_{-T/2}^{T/2} \sin\frac{2\pi nt}{T}dt - \frac{4}{T}\int_{0}^{T/2} t\sin\frac{2\pi nt}{T}dt\right]$$

$$= \left(\frac{2}{T}\right)^2 [\frac{T}{\pi n} t\cos\frac{2\pi nt}{T}\Big|_0^{T/2} - \frac{T}{\pi n}\int_0^{T/2} \cos\frac{2\pi nt}{T}dt]$$

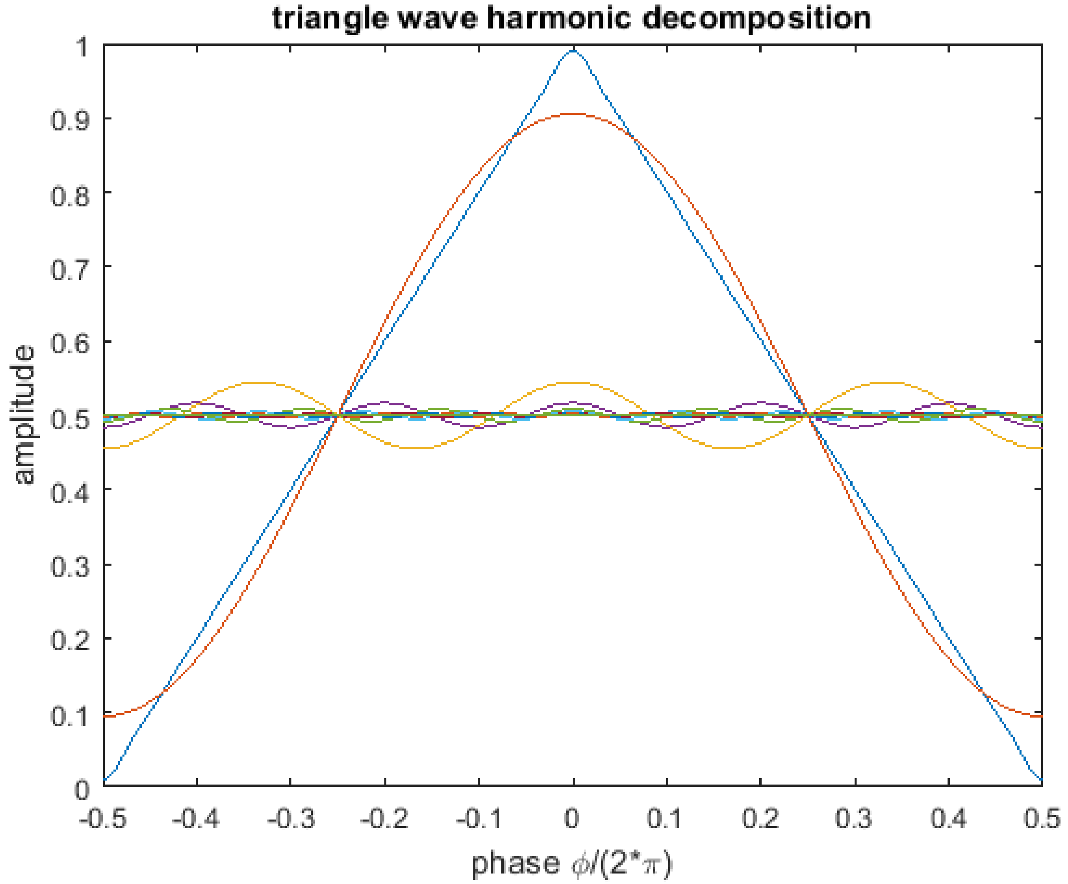
$$= \left(\frac{2}{T}\right)^2 \left[\frac{\mathrm{T}}{\pi n}\frac{\mathrm{T}}{2}\cos(n\pi) - \frac{1}{2}\left(\frac{T}{\pi n}\right)^2 \sin n\pi\right] = 0$$

where we have used the symmetry of the terms in the first step and integration by parts to go from the second to third step.

We are left with the Fourier expansion of the triangle wave

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{(n\pi)^2}(1-(-1)^n)\cos\frac{2\pi nt}{T}$$

$$f(t) = \frac{1}{2} + \left(\frac{2}{\pi}\right)^2 \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}\cos\frac{2\pi(2n+1)t}{T}$$



- *Fundamental term again dominates*
- *Now harmonics mainly contribute to peaks*
- *Sharp corners require high harmonics*
- *Triangle wave is continuous, so expansion approaches waveform everywhere*

- *When working with linear, time-invariant systems, there are several advantages to moving from the time domain to the frequency domain.*
- *If* $x_1 \rightarrow y_1$ *and* $x_2 \rightarrow y_2$ *and if* $ax_1 + bx_2 \rightarrow ay_1 + by_2$
  then the system is ***linear***.
- *If* $x(t) \rightarrow y(t)$ *and if* $x(t - t_0) \rightarrow y(t - t_0)$
  then the system is ***time-invariant***.
- *Each frequency corresponds to a unique eigenfunction of the system and the system response for each frequency can be calculated independently.*

- *There is another transform often used in system analysis, the Laplace transform.*
- *It is closely related to the Fourier transform in that it is also based on system eigenfunctions.*
- *In addition to "real" frequencies, it also uses complex frequencies that allow it to also study decaying solutions.*
- *As with Fourier transform, integral must converge in order for the Laplace transform to exist.*
- *It is convenient to use Laplace transforms for the study of solutions to problems with initial conditions.*
- *The variable used in Laplace transforms is often* $s = j\omega$

- *The Laplace transform is used for analysis of systems with given initial conditions*

- *For a given function of time,* $f(t)$*, its Laplace transform,* $F(s)$*, is defined as*

$$\mathcal{L}(f(t)) = F(s) = \int_0^\infty f(t)e^{-st}dt$$

- $f(t)$ *has to grow less quickly than* e^{-st} *decreases as* $t \to \infty$

- *When working in the frequency* (s) *domain, we express transfer functions in terms of known Laplace transforms and take the inverse transform,* $\mathcal{L}^{-1}(F(s))$*, to obtain the time domain solution.*

- ***Delta function:*** $f(t) = \delta(t)$

$$F(s) = \int_{-\epsilon}^{\infty} \delta(t) e^{-st} dt = 1$$

- ***Delta function with delay:*** $f(t) = \delta(t - t_0), t_0 \geq 0$

$$F(s) = \int_0^{\infty} \delta(t - t_0) e^{-st} dt = e^{-st_0}$$

- ***Step function:*** $f(t) = U(t)$

$$F(s) = \int_0^{\infty} U(t) e^{-st} dt = \int_0^{\infty} e^{-st} dt = -\frac{1}{s} e^{-st} \Big|_0^{\infty} = \frac{1}{s}$$

- ***Step function with delay:*** $f(t) = U(t - t_0), t \geq 0$

$$F(s) = \int_0^{\infty} U(t - t_0) e^{-st} dt = \int_{t_0}^{\infty} e^{-st} dt = -\frac{1}{s} e^{-st} \Big|_{t_0}^{\infty} = \frac{e^{-st_0}}{s}$$

- ***Ramp function:*** $f(t) = t$
  Use integration by parts (IBP)
  $$\int_a^b u dv = uv\Big|_a^b - \int_a^b v du$$
  $$u = t; du = dt; dv = e^{-st}dt; v = -e^{-st}/s$$
  $$F(s) = \int_0^\infty te^{-st}dt = -\frac{t}{s}e^{-st}\Big|_0^\infty + \frac{1}{s}\int_0^\infty e^{-st}dt = \frac{1}{s^2}$$

- ***Exponential function:*** $f(t) = e^{at}$ *with a an arbitrary complex number*
  $$F(s) = \int_0^\infty e^{at}e^{-st}dt = \int_0^\infty e^{-(s-a)t}dt$$
  $$F(s) = -\frac{1}{s-a}e^{-(s-a)t}\Big|_0^\infty = \frac{1}{s-a}$$
  Only if $\lim_{t\to\infty} e^{-(s-a)t}$ *exists does F(s) exist. Therefore* $Re(a) \le 0$. *Note that a can be imaginary.*

- ***Sinusoidal functions:*** $f(t) = \cos\omega t \qquad f(t) = \sin\omega t$

  We could do this from the definition and IBP, but instead we use Euler's formula, exponential transforms, and the linearity of the Laplace transform

$$F(s) = \int_0^\infty \cos\omega t\, e^{-st} dt = \frac{1}{2}\int_0^\infty \left(e^{j\omega t} + e^{-j\omega t}\right)e^{-st} dt$$

$$= \frac{1}{2}\left(\int_0^\infty e^{-(s-j\omega)t} dt + \int_0^\infty e^{-(s+j\omega)t} dt\right) = \frac{1}{2}\left(\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right)$$

$$F(s) = \frac{s}{s^2+\omega^2}$$

$$F(s) = \int_0^\infty \sin\omega t e^{-st} dt = \frac{\omega}{s^2+\omega^2}$$

- ***Transform of a derivative:*** $f(t) = \frac{dg(t)}{dt}$

$$F(s) = \int_0^\infty \frac{dg(t)}{dt} e^{-st} dt$$

$$= g(t)e^{-st}\Big|_0^\infty + \int_0^\infty sg(t)e^{-st} dt$$

$$= g(t)e^{-st}\Big|_0^\infty + s\int_0^\infty g(t)e^{-st} dt$$

$$F(s) = sG(s) - g(0)$$

where we have used integration by parts and $G(s) = \int_0^\infty g(t)e^{-st} dt$

- ***Transform of an integral:*** $f(t) = \int_0^t g(\tau)\, d\tau$

$$F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^\infty \left[\int_0^t g(\tau)\, d\tau\right] e^{-st}dt$$

$$= e^{-st}\int_0^t g(\tau)\, d\tau\Big|_0^\infty + \frac{1}{s}\int_0^\infty g(t)e^{-st}dt$$

$$= \frac{1}{s}\int_0^\infty g(t)e^{-st}dt$$

$$F(s) = \frac{1}{s}G(s)$$

where we have used integration by parts and $G(s) = \int_0^\infty g(t)e^{-st}dt$

$\boldsymbol{f(t)} = \mathcal{L}^{-1}(F(s))$	$F(s) = \mathcal{L}(f(t))$
$\delta(t)$	1
$U(t)$	$1/s$
t	$1/s^2$
e^{-at}	$1/(s+a)$
$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\cos\omega t\,,\sin\omega t$	$\frac{s}{s^2+\omega^2},\frac{\omega}{s^2+\omega^2}$
$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$\frac{dg(t)}{dt}$	$sG(s) - g(0)$
$\int_0^t g(\tau)\,d\tau$	$\frac{1}{s}G(s)$
$f(t-t_0)$	$e^{-st_0}F(s)$

- *Laplace Transforms simplify the calculations of system behavior, but these calculations are performed in the complex frequency (s) domain.*
- *In order to return to a time domain function, the s domain function must be inverted.*
- *Inversion of these functions can be performed via complex variable techniques.*
- *Much more commonly, one uses readily available tables of functions and their Laplace transform pairs*
- *There also exist such transform tables for Fourier transforms.*

http://www.vibrationdata.com/Laplace.htm
http://en.wikipedia.org/wiki/Laplace_transform
http://mathworld.wolfram.com/FourierTransform.html
http://en.wikipedia.org/wiki/Fourier_transform

- ***Response of a single pole low pass filter to an impulse***

Want unity gain at DC $(s = 0)$ *and* 3 dB *rolloff at* $\omega = a$

$$H(s) = \frac{a}{s+a}$$

$$\left(H(j\omega) = \frac{a}{j\omega + a}\right)$$

$$(H(0) = 1;\ \ 3\text{ dB rolloff at } \omega = a)$$

$$X(s) = 1 \quad \textit{Impulse input}$$

$$Y(s) = H(s)X(s) = \frac{a}{s+a}$$

$$y(t) = \mathcal{L}^{-1}\big(Y(s)\big) = \mathcal{L}^{-1}\left(\frac{a}{s+a}\right) = a\mathcal{L}^{-1}\left(\frac{1}{s+a}\right)$$

$$y(t) = ae^{-at}$$

- ***Response to a step:***

$$Y(s) = H(s)X(s) = \frac{a}{s+a}\frac{1}{s}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{a}{s+a}\frac{1}{s}\right)$$

Use partial fractions to expand argument and then linearity of $\mathcal{L}$

$$\frac{a}{s(s+a)} = \frac{A}{s} + \frac{B}{s+a} \qquad\qquad a = (s+a)A + sB$$

$$s = 0 \Rightarrow A = 1$$

$$s = -a \Rightarrow B = -1$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{a}{s+a}\frac{1}{s}\right) = \mathcal{L}^{-1}\left(\frac{1}{s} - \frac{1}{s+a}\right) = \mathcal{L}^{-1}\left(\frac{1}{s}\right) - \mathcal{L}^{-1}\left(\frac{1}{s+a}\right)$$

$$= 1 - e^{-at}$$

- ***Response to an exponential:***

$$x(t) = e^{-bt}$$

$$X(s) = \mathcal{L}\left(e^{-bt}\right) = \frac{1}{s+b}$$

$$Y(s) = H(s)X(s) = \frac{a}{s+a}\frac{1}{s+b}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{\mathrm{a}}{\mathrm{s+a}}\frac{1}{\mathrm{s+b}}\right) = \frac{a}{b-a}\mathcal{L}^{-1}\left(\frac{1}{s+a} - \frac{1}{s+b}\right)$$

$$= \frac{a}{b-a}\left(e^{-at} - e^{-bt}\right)$$

Many of our circuits will have signals with widely separated frequencies

- *Switching element of a fast frequency will control the amplitude of the output voltage or current*
- *The output voltage/current will be much slower than the switching frequency*
- *We will use filtering elements to separate the frequencies.*

We want to find good approximations for these "fast" signals through the "slow" filter elements

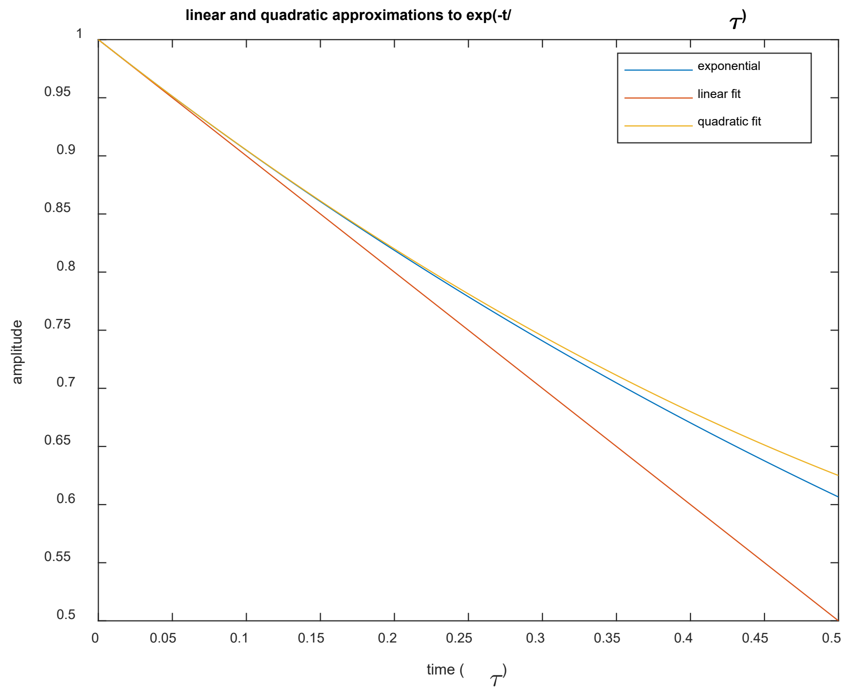
Behavior is exponential

$$x(t) = e^{-\frac{t}{\tau}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\left(\frac{t}{\tau}\right)^n$$

$$x(t) \approx 1 - \frac{t}{\tau} + \frac{1}{2}\left(\frac{t}{\tau}\right)^2$$

Linear approximation accurate to

$1\%\ for\ t < \frac{\tau}{10},\ 2.4\%\ for\ t < \frac{\tau}{5}$



Energy in inductor $\mathcal{E}_L = \frac{1}{2} L I^2$

Cannot supply infinite power to the circuit element ⇒ no step change in I

Voltage-current relation $V_L = L \frac{dI}{dt}$

$$\int_0^t V_L \, d\tau = L \int_0^t dI = L[I(t) - I(0)]$$

Can have a step change in voltage across an inductor

For a constant voltage applied across the inductor

$$I(t) = \frac{V}{L} t$$

the inductor current increases linearly with time

For a system with a periodic current across the inductor $I(t+T) = I(t)$

$$\int_t^{t+T} V_L \, d\tau = L \int_t^{t+T} dI = L[I(t+T) - I(t)] = 0$$

The average voltage across the inductor is zero. $< V_L > = 0$

Energy in capacitor $\mathcal{E}_C = \frac{1}{2} C V^2$

Cannot supply infinite power to the circuit element ⇒ no step change in V

Voltage-current relation $I_C = C \frac{dV}{dt}$

$$\int_0^t I_C \, d\tau = C \int_0^t dV = C[V(t) - V(0)]$$

Can have a step change in current across a capacitor

For a constant current applied across the capacitor

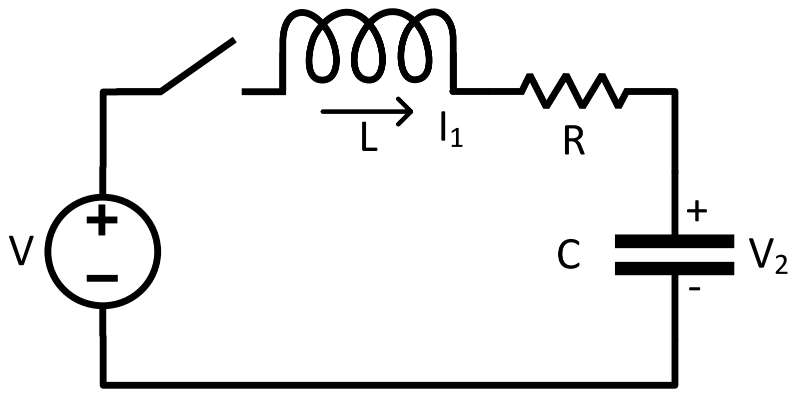
$$V(t) = \frac{I}{C} t$$

the capacitor voltage increases linearly with time

For a system with a periodic voltage across the capacitor $V(t+T) = V(t)$

$$\int_t^{t+T} I_C \, d\tau = C \int_t^{t+T} dV = C[V(t+T) - V(t)] = 0$$

The average current across the capacitor is zero. $< I_C > = 0$



Circuit equations:

$$u(t) = L\frac{di_1}{dt} + Ri_1 + v_2$$

$$C\frac{dv_2}{dt} = i_1$$

$$\begin{bmatrix}\dot{i}_1(t)\\ \dot{v}_2(t)\end{bmatrix} = \begin{bmatrix}-R/L & -1/L\\ 1/C & 0\end{bmatrix}\begin{bmatrix}i_1(t)\\ v_2(t)\end{bmatrix} + \frac{1}{L}\begin{bmatrix}1\\ 0\end{bmatrix}u(t)$$

Take the Laplace transform of both sides

$$s\boldsymbol{I}X(s) - x(0) = \boldsymbol{A}X(x) + \boldsymbol{B}U(s)$$

$$X(s) = (s\boldsymbol{I} - \boldsymbol{A})^{-1}x(0) + (s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}U(s)$$

where $X(s) = \begin{bmatrix}I_1(s)\\ V_2(s)\end{bmatrix}; (s\boldsymbol{I} - \boldsymbol{A}) = \begin{bmatrix}s + R/L & 1/L\\ -1/C & s\end{bmatrix}; \boldsymbol{B} = \begin{bmatrix}1/L\\ 0\end{bmatrix}$

For most cases of interest, there is negligible resistance $R = 0$. *Then*

$$(s\boldsymbol{I} - \boldsymbol{A}) = \begin{bmatrix}s & 1/L\\ -1/C & s\end{bmatrix};\ (s\boldsymbol{I} - \boldsymbol{A})^{-1} = \begin{bmatrix}\frac{s}{s^2+\omega_0^2} & -\frac{1}{L}\frac{1}{s^2+\omega_0^2}\\ \frac{1}{C}\frac{1}{s^2+\omega_0^2} & \frac{s}{s^2+\omega_0^2}\end{bmatrix}$$

where $\omega_0 \equiv 1/\sqrt{LC}$

For a constant voltage source, $u(t) = V_0$ *(series inductor prohibits the use of a current source) we need to make a partial fraction decomposition of*

$$\frac{1}{s^2+\omega_0^2}\frac{1}{s} = \frac{1}{\omega_0^2}\left[\frac{1}{s} - \frac{s}{s^2+\omega_0^2}\right]$$

The equations for the series LC *circuit in the Laplace domain are*

$$I_1(s) = \frac{s}{s^2+\omega_0^2} i_1(0) - \frac{1}{\omega_0 L}\frac{\omega_0}{s^2+\omega_0^2} v_2(0) + \frac{1}{\omega_0 L}\frac{\omega_0}{s^2+\omega_0^2} V_0$$

$$V_2(s) = \frac{1}{\omega_0 C}\frac{\omega_0}{s^2+\omega_0^2} i_1(0) + \frac{s}{s^2+\omega_0^2} v_2(0) + \left(\frac{1}{s} - \frac{s}{s^2+\omega_0^2}\right) V_0$$

Transforming to the time domain, we obtain

$$i_1(t) = \mathrm{i}_1(0)\cos\omega_0 t - \frac{v_2(0)}{\omega_0 L}\sin\omega_0 t + \frac{V_0}{\omega_0 L}\sin\omega_0 t$$

$$v_2(t) = \frac{i_1(0)}{\omega_0 C}\sin\omega_0 t + v_2(0)\cos\omega_0 t + V_0(1-\cos\omega_0 t)$$

We express this equation in its "natural" parameters, rather than in L *and* C.

We have already defined $\omega_0 \equiv 1/\sqrt{LC}$. *This is the frequency of the sinusoidal oscillation.*

The other parameter is $Z_0 \equiv \sqrt{L/C}$, *the coupling between* $i_1(t)$ *and* $v_2(t)$

The series resonant equations, expressed in L *and* C *are*

$$\begin{pmatrix} i_1(t) \\ v_2(t) \end{pmatrix} = \begin{pmatrix} \cos\frac{t}{\sqrt{LC}} & -\frac{\sin\frac{t}{\sqrt{LC}}}{\sqrt{L/C}} \\ \sqrt{\frac{L}{C}}\sin\frac{t}{\sqrt{LC}} & \cos\frac{t}{\sqrt{LC}} \end{pmatrix} \begin{pmatrix} i_1(0) \\ v_2(0) \end{pmatrix} + \begin{pmatrix} \frac{\sin\frac{t}{\sqrt{LC}}}{\sqrt{L/C}} \\ (1-\cos\frac{t}{\sqrt{LC}}) \end{pmatrix} V_0$$

and in natural parameters as

$$\begin{pmatrix} i_1(t) \\ v_2(t) \end{pmatrix} = \begin{pmatrix} \cos\omega_0 t & -\frac{\sin\omega_0 t}{Z_0} \\ Z_0\sin\omega_0 t & \cos\omega_0 t \end{pmatrix} \begin{pmatrix} i_1(0) \\ v_2(0) \end{pmatrix} + \begin{pmatrix} \frac{\sin\omega_0 t}{Z_0} \\ (1-\cos\omega_0 t) \end{pmatrix} V_0$$

As expected, all of the waveforms are sinusoidal with the frequency determined by $\omega_0 = 1/\sqrt{LC}$

The initial current and voltage contribute in-phase, $\cos \omega_0 t$*, terms to the subsequent current and voltage.*

The initial current and voltage contribute quadrature, $\sin \omega_0 t$*, terms to the subsequent voltage and current.*

In many applications, either the initial current or voltage is zero and solutions simplify

For small values of R, $R \ll \omega_0 L, R \ll 1/\omega_0 C$*, the results are about the same as when* $R = 0$

- *Sinusoids damp slightly with time* $(e^{-R/2L\, t})$
- *Frequency decreases slightly* $(\omega_R = \omega_0\sqrt{1-(R/2Z_0)^2}\ ;\ Z_0 = \sqrt{L/C}$
- *Terms experience a slight phase shift*

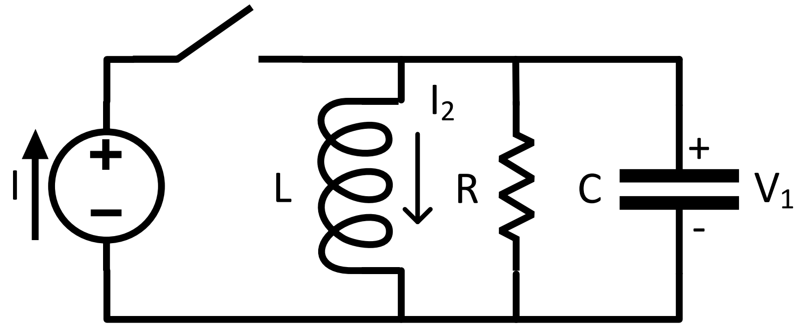
For large values of R $\gg Z_0$ *solutions are damped with time constants* $L/R, RC$

Circuit equations:

$$u(t) = C\frac{dv_1}{dt} + \frac{1}{R}v_1 + i_2$$

$$L\frac{di_2}{dt} = v_1$$

$$\begin{bmatrix}\dot{v}_1(t)\\ i_2(t)\end{bmatrix} = \begin{bmatrix}-1/RC & -1/C\\ 1/L & 0\end{bmatrix}\begin{bmatrix}v_1(t)\\ i_2(t)\end{bmatrix} + \frac{1}{C}\begin{bmatrix}1\\0\end{bmatrix}u(t)$$



These equations are the "dual" of the series circuit.

- *The equations are exactly the same*
- *Only the coefficients change $v(t) \leftrightarrow i(t)$ and $L \leftrightarrow C$*
- *Source is a current source; the shunt capacitor prohibits a voltage source*

By interchanging $v(t) \leftrightarrow i(t)$ and $L \leftrightarrow C$ we write down the circuit equations

$$v_1(t) = \mathrm{v}_1(0)\cos\omega_0 t - \frac{i_2(0)}{\omega_0 C}\sin\omega_0 t + \frac{I_0}{\omega_0 C}\sin\omega_0 t$$

$$i_2(t) = \frac{v_1(0)}{\omega_0 L}\sin\omega_0 t + i_2(0)\cos\omega_0 t + I_0(1 - \cos\omega_0 t)$$

The parallel resonant equations, expressed in terms of ω_0 *and* Z_0 *are*

$$\begin{pmatrix} v_1(t) \\ i_2(t) \end{pmatrix} = \begin{pmatrix} \cos\omega_0 t & -Z_0 \sin\omega_0 t \\ \dfrac{\sin\omega_0 t}{Z_0} & \cos\omega_0 t \end{pmatrix} \begin{pmatrix} v_1(0) \\ i_2(0) \end{pmatrix} + \begin{pmatrix} Z_0 \sin\omega_0 t \\ (1-\cos\omega_0 t) \end{pmatrix} I_0$$

Would have looked even more symmetrical had we used, instead of Z_0*,*

$$Y_0 = Z_0^{-1} = \sqrt{C/L}$$

We point out several features common to the series and parallel resonant equations.

- *The two variables,* (i, v)*, are in quadrature.*
  - *They oscillate with the same frequency but* $\pi/2$ *out of phase*
- *The frequency of oscillation and coupling between voltage and current are the same in both cases*
- *The systems are continuous at* $t = 0$
  - $i(t)$ $(v(t))$ *can only couple to* $v(t)$ $(i(t))$ *in quadrature, via* $\sin\omega_0 t$

Variation of parameters is used to find a particular solution to a diff. eq.

Use this method to solve an equation with a driving term

Homogeneous equation (first order) (no driving term) and solution

$$\frac{dx}{dt} + \alpha x = 0 \Rightarrow x(t) = e^{-\alpha t}$$

Particular equation corresponding to the homogeneous equation

$$\frac{dy}{dt} + \alpha y = u(t)$$

Try as a solution $y(t) = x(t)z(t)$ and solve for $z(t)$

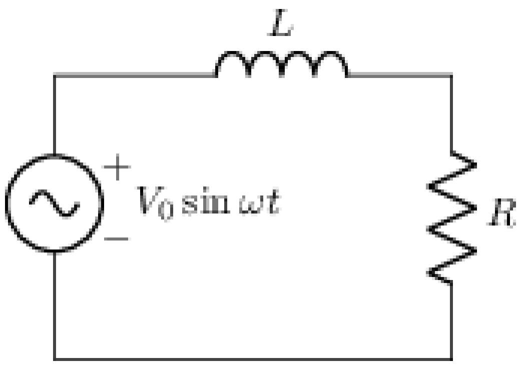
$$\frac{dy}{dt} + \alpha y = \frac{dx}{dt} z + x\frac{dz}{dt} + \alpha xz = \left(\frac{dx}{dt} + \alpha x\right) z + x\frac{dz}{dt} = u$$

$$x\frac{dz}{dt} = u \Rightarrow \frac{dz}{dt} = x^{-1}u$$

$$z(t) = z(t_0) + \int_{t_0}^{t} x^{-1}(t')u(t')\, dt'$$

$$y(t) = x(t)z(t) = x(t)z(t_0) + x(t)\int_{t_0}^{t} x^{-1}(t')u(t')\, dt'$$

Apply variation of parameters to an L-R circuit



$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V_0}{L} \sin \omega t$$

$$\alpha = \frac{1}{\tau} = \frac{R}{L};\ x(t) = e^{-\frac{R}{L}t} = e^{-\alpha t}$$

$$u(t) = \frac{\mathrm{V}_0}{\mathrm{L}} \sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$z(t) = z(t_0) + \frac{\mathrm{V}_0}{\mathrm{L}} \int_{t_0}^{t} e^{\alpha t'} \sin \omega t'\ dt'$$

$$= z(t_0) + \frac{\mathrm{V}_0}{\mathrm{L}} \frac{1}{2j} \int_{t_0}^{t} \left(e^{(\alpha + j\omega)t'} - e^{(\alpha - j\omega)t'}\right) dt'$$

$$= z(t_0) + \frac{\mathrm{V}_0}{2j\mathrm{L}} \left(\frac{e^{(\alpha + j\omega)t'}}{\alpha + j\omega} - \frac{e^{(\alpha - j\omega)t'}}{\alpha - j\omega} \right) \Bigg|_{t_0}^{t}$$

$$= z(t_0) + \frac{V_0}{L} \frac{1}{\alpha^2 + \omega^2} \left(e^{\alpha t'} (\alpha \sin \omega t' - \omega \cos \omega t')\right)\Big|_{t_0}^{t}$$

$$= z(t_0) + \frac{V_0}{L} \frac{1}{\sqrt{\alpha^2 + \omega^2}} \left(e^{\alpha t'} (\sin(\omega t' - \arctan(\omega/\alpha))\right)\Big|_{t_0}^{t}$$

$$i(t) = e^{-\alpha t} z(t_0) + \frac{V_0}{L}\frac{1}{\sqrt{\alpha^2+\omega^2}}\left(e^{-\alpha(t-t')}(\sin(\omega t' - \arctan(\omega/\alpha)))\right)\Big|_{t_0}^{t}$$

$$= e^{-\alpha(t-t_0)} e^{-\alpha t_0} z(t_0) + \frac{V_0}{L}\frac{1}{\sqrt{\alpha^2+\omega^2}}\left(e^{-\alpha(t-t')}(\sin(\omega t' - \arctan(\omega/\alpha)))\right)\Big|_{t_0}^{t}$$

$$= i(t_0) e^{-\alpha(t-t_0)} + \frac{V_0}{L}\frac{1}{\sqrt{\alpha^2+\omega^2}}\left(e^{-\alpha(t-t')}(\sin(\omega t' - \arctan(\omega/\alpha)))\right)\Big|_{t_0}^{t}$$

$$= i(t_0) e^{-\frac{R}{L}(t-t_0)} + \frac{V_0}{\sqrt{R^2+(\omega L)^2}}\left(e^{-\frac{R}{L}(t-t')}(\sin(\omega t' - \arctan(\omega L/R)))\right)\Big|_{t_0}^{t}$$

Check that it holds for the extreme cases

- $L = 0;\ \lim_{L\to 0} e^{-\frac{R}{L}(t-t_0)} = \begin{cases} 1 & t = t_0 \\ 0 & t > t_0 \end{cases};\ i(t) = \frac{V_0}{R}\sin\omega t$, *as expected*
- $R = 0;\ \lim_{R\to 0} \arctan\left(\frac{\omega L}{R}\right) = \frac{\pi}{2};\ i(t) = i(t_0) - \frac{V_0}{\omega L}(\cos\omega t - \cos\omega t_0)$

We see that the current damps with the expected L/R damping time of the circuit

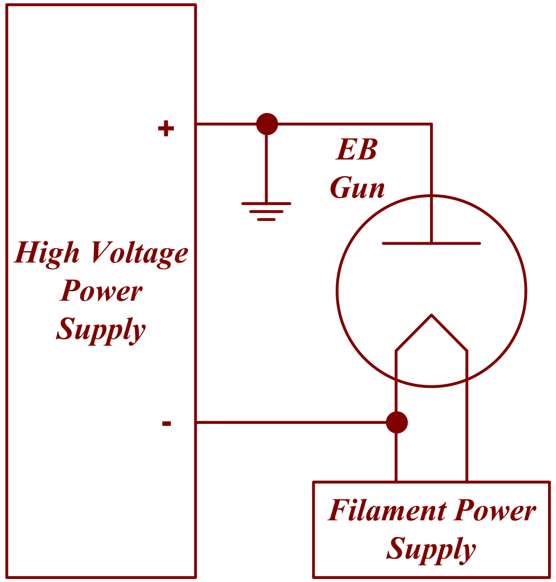
The phase of the current waveform is shifted between $0 \to \pi/2$ as $\frac{\omega L}{R}$ increases

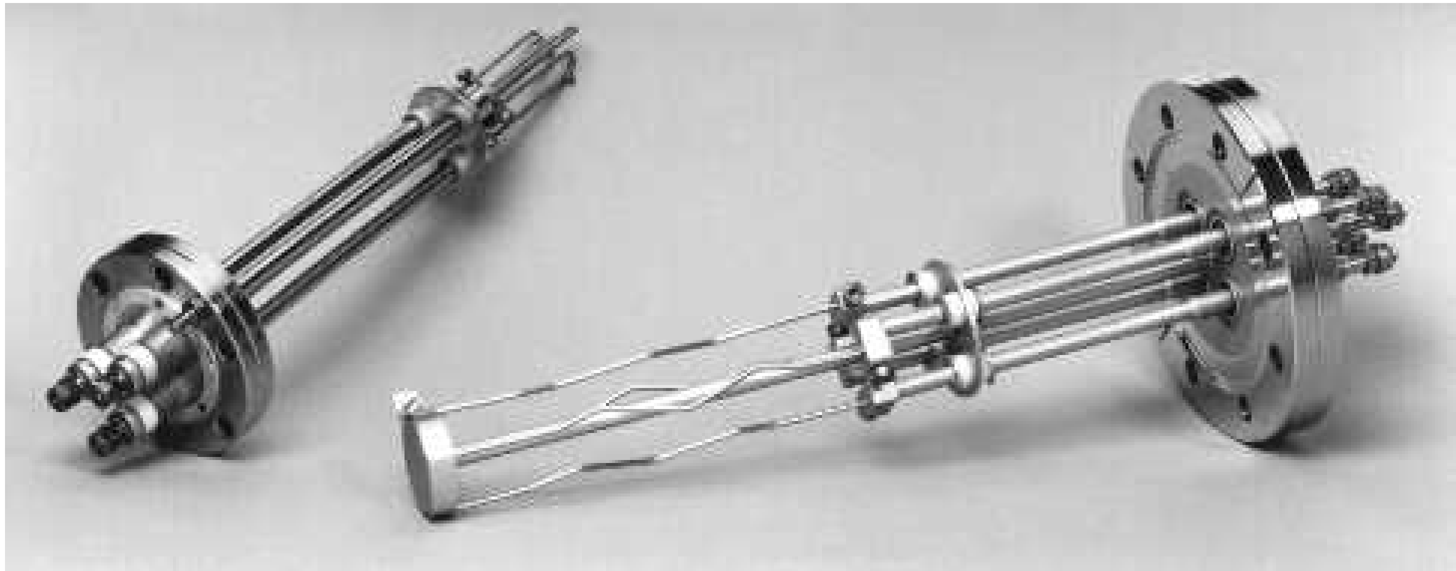
Section 4

- *Typical Load Types*
  - *Resistive – Electron Beam Filament*
  - *Resistive - Titanium Sublimation Pumps (TSPs)*
  - *DC Magnets*
  - *Klystrons*
  - *Electron Beam Gun*
  - *Pulsed Magnets*

Resistive Load Characteristics

Electron Beam Guns (Filament) / Titanium Sublimation Pump Heaters

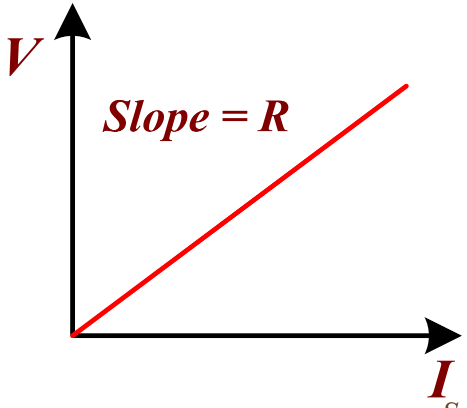


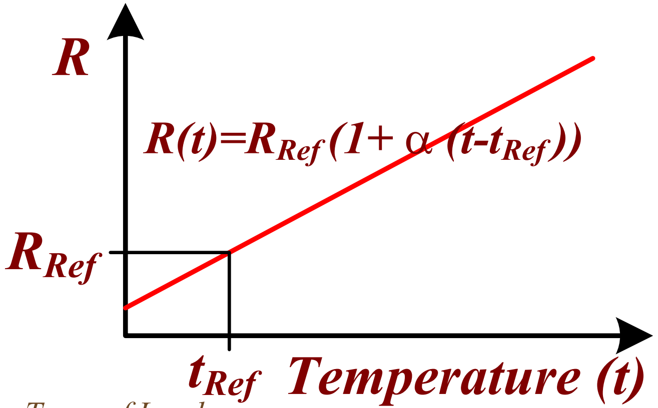


Resistive Load Characteristics

Electron Beam Guns (Filament) / Titanium Sublimation Pump Heaters

- *High temperature – 1,500 ^{O} C not uncommon*
- *High current – 10 to 100s of amperes, low voltage, typically < 50 V*
- *Short thermal time-constants - 100s of milliseconds, power stability needed to keep temperature constant*
- *Resistive with (+) metal or (-) carbon temperature coefficient of resistance*
- *Power with constant voltage, current or power. Might have to avoid DC (more later in AC Controllers) depending upon circumstances*
- *Heat gradually to avoid thermally shocking and breaking brittle loads*
- *Usually linear V-I and R-T characteristics, but sometimes non-linear*





Resistive Load Characteristics

Electron Beam Gun Filaments / Titanium Sublimation Pump Heaters

Ideal Characteristics

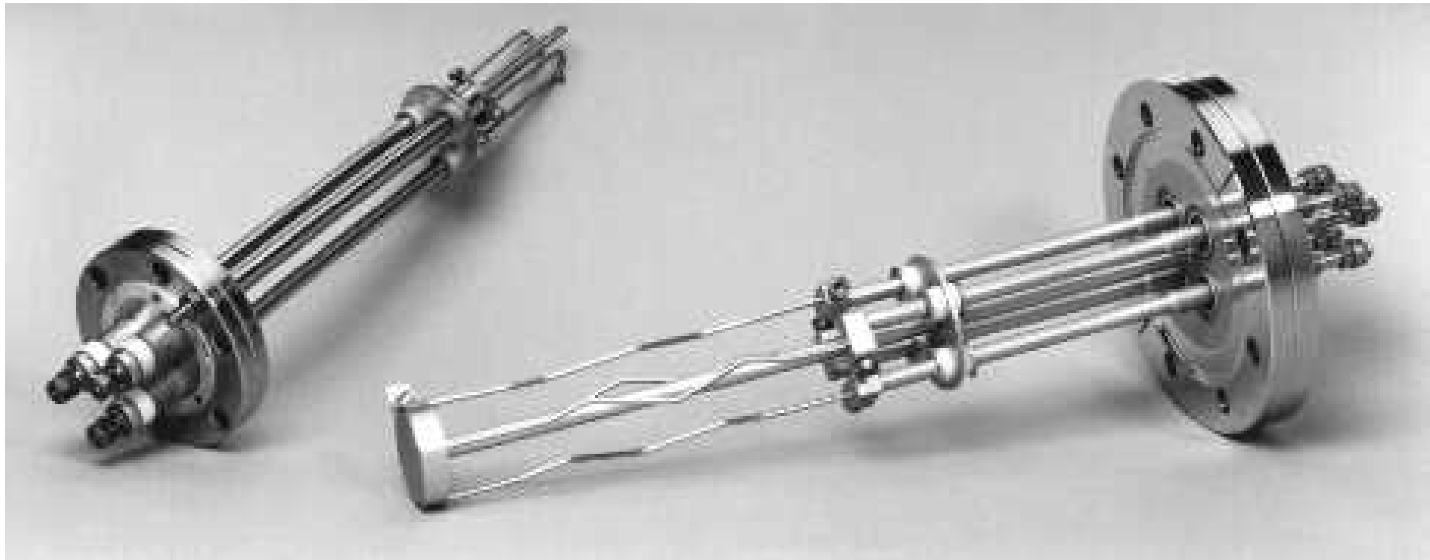
- *Low potential barrier (work function)*
- *High melting point*
- *Chemical stability at high temperatures*
- *Long life*

Work function *- the minimum energy which must be supplied to extract an electron from a solid; symbol ϕ, units J (joule), or more often eV (electron-volt). It is a measure of how tightly electrons are bound to a material. The work function of several metals is given below:*

Material	***Work function (eV)***
Sodium	*2.75*
Silver	*4.26*
Titanium	*4.33*
Silicon	*4.60*
Gold	*5.31*
Graphite	*5.37*
Tungsten	*5.40*

Titanium Sublimation Pumps (TSPs)

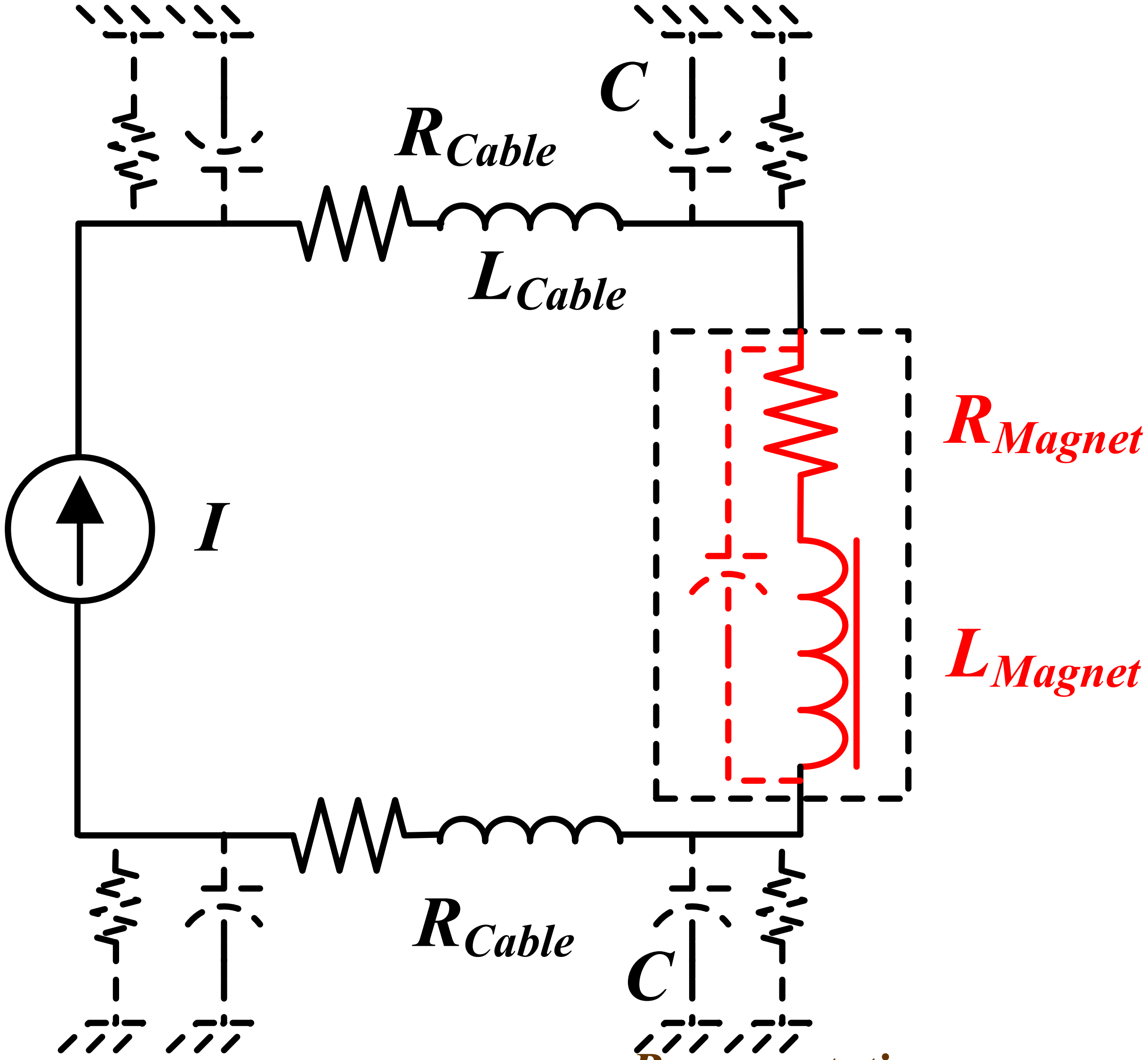
- *Titanium Sublimation Pumps (TSPs) are used to pump chemically reactive, getterable gases, such as H_2, H_2O, CO, N_2, O_2, CO_2 from vacuum vessels. Titanium is effective, easily sublimed, and inexpensive.*
- *TSPs filaments are 85% titanium and 15% molybdenum, a combination which prevents premature filament "burnout" and have high pumping speeds, typically 10 l / sec / cm^2*



Sublimate *- To transform directly from the solid to the gaseous state. Deposition is the passing from the gaseous to the solid state without becoming a liquid.*

DC Magnet Loads – Characteristics

- *Linear and inductive with long (mS to sec) electrical time-constants ($\tau = L/R$)*
- *Families include dipole steering, quadrupole and sextupole focusing / defocusing, corrector / trims*
- *Driven by constant current and require high current stability (ΔI in PPM)*
- *Correctors / trims frequently require current modulation for beam-based alignment / diagnostic systems, orbit correction and stabilization*
- *Air-cooled or water-cooled (temperature or flow interlocks to power supply)*
- *Occasionally series-connected in strings and powered from a common power supply to reduce power system cost*



Representation

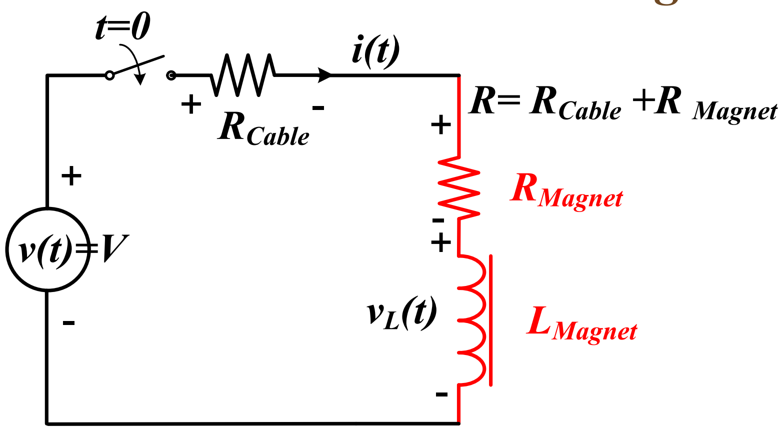
Ampere's Law

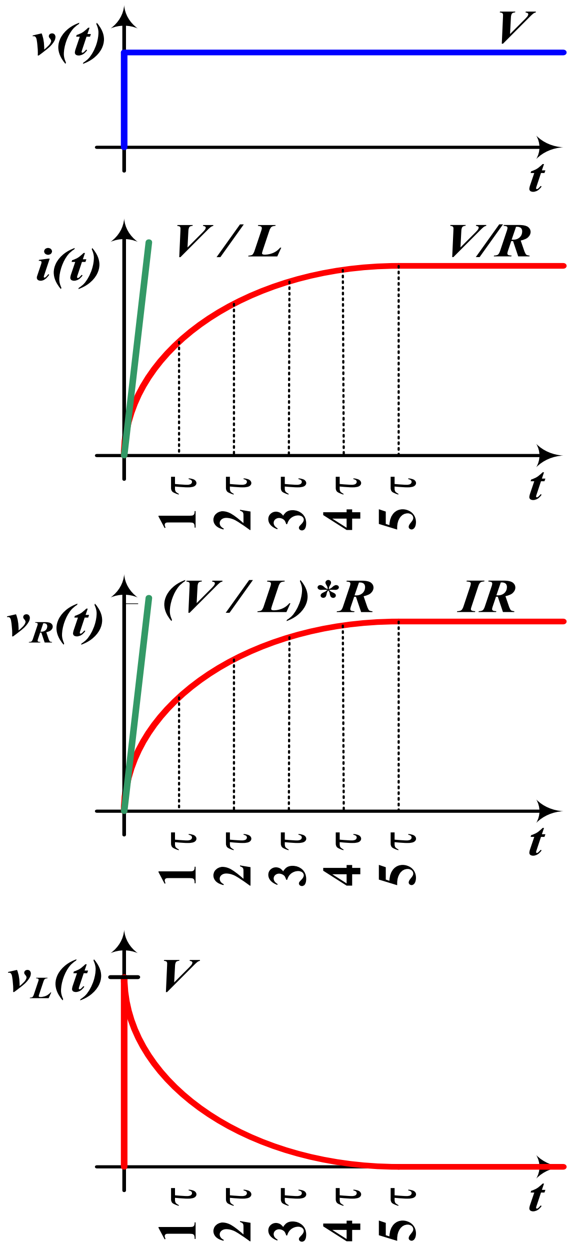
$$B = \mu H = \frac{\mu_r \mu_0 N I}{l}$$

Lorentz Force Law

$$F = q(E + (v \times B))$$

DC Magnet Loads – Characteristics





Using Kirchoff's voltage law (KVL):

$$-v(t) + (R_{cable} + R_{magnet})\, i(t) + L\frac{di(t)}{dt} = 0$$

$$R\, i(t) + L\frac{di(t)}{dt} = v(t)$$

Converting to the s domain

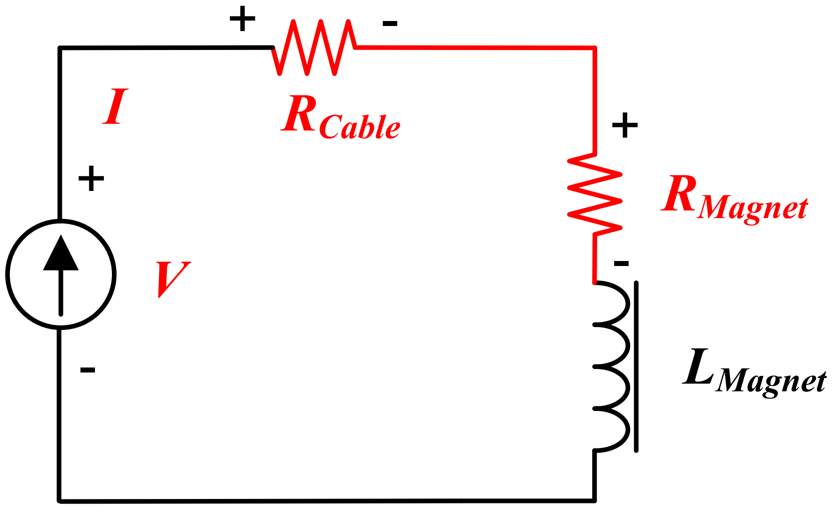
$$R\, I(s) + LsI(s) - L\, i(0) = V(s), \qquad \text{But } i(0) = 0 \text{ and } V(s) = \frac{V}{s}$$

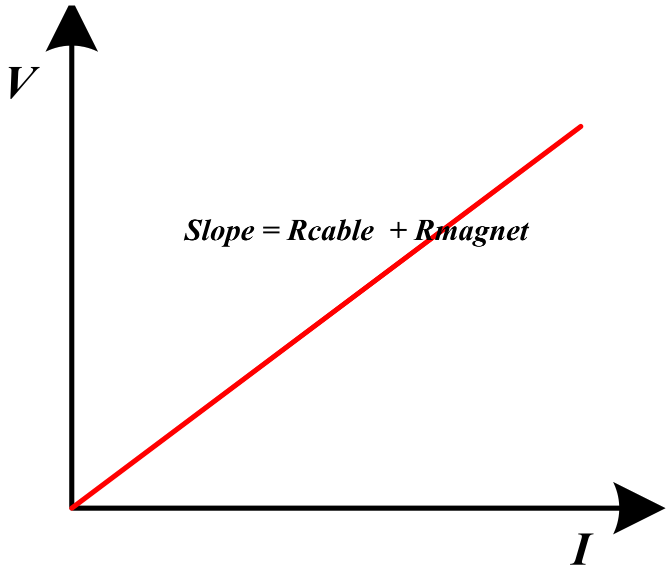
Rearranging gives

$$I(s)\frac{L}{R}\left(s + \frac{R}{L}\right) = \frac{V}{R}\frac{1}{s} \qquad \text{let } \frac{R}{L} = \alpha \text{ and } \frac{L}{R} = \frac{1}{\alpha} = \tau$$

$$I(s) = \frac{V}{R}\frac{\alpha}{s(s+\alpha)} \qquad i(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

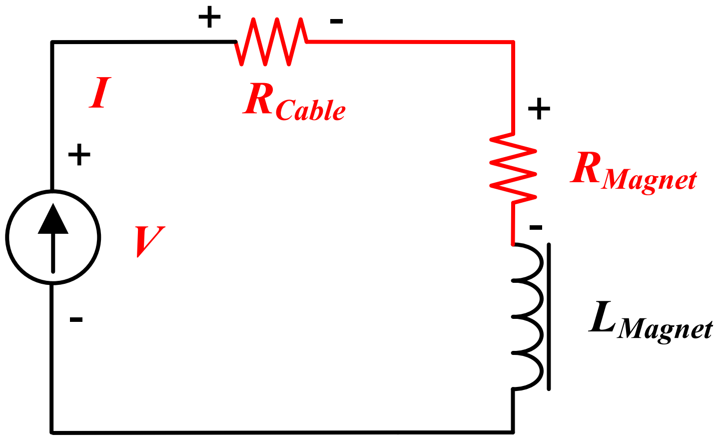
$$v_L(t) = Ve^{-t/\tau}$$

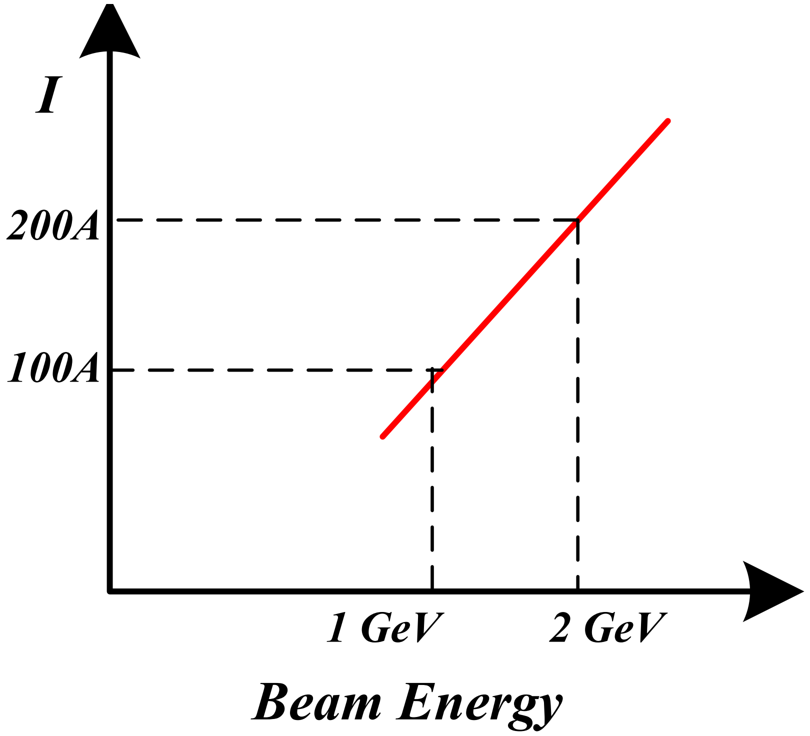


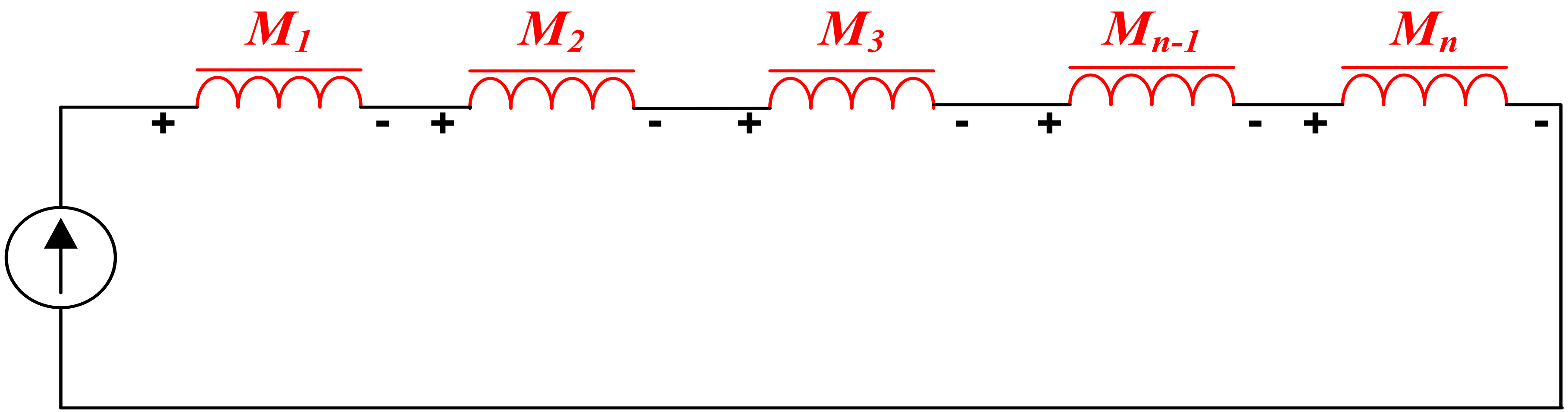


DC V-I Characteristic

DC Magnet Loads – Characteristics



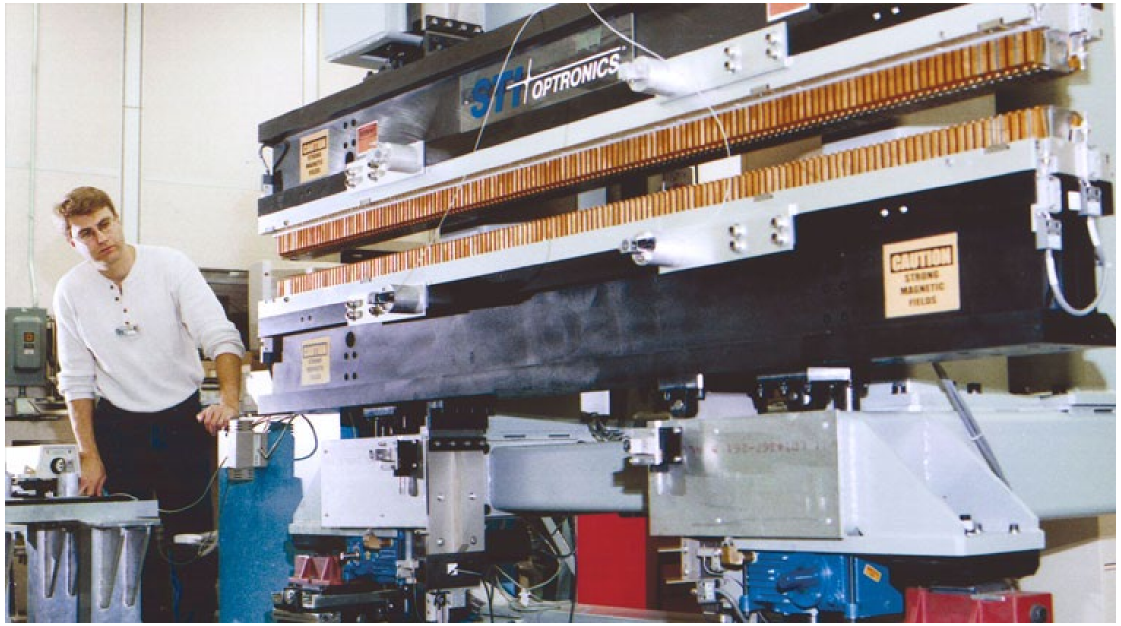


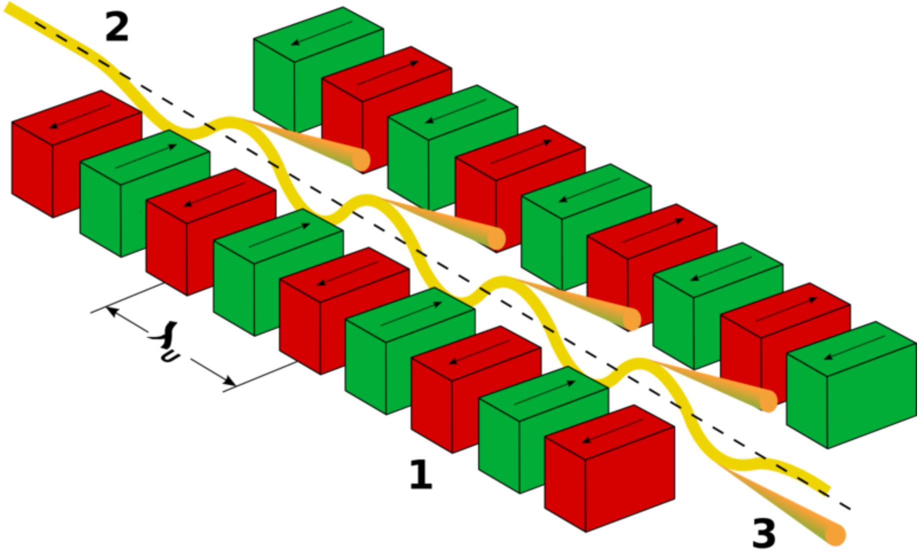


String (series-connect) magnets for economy when there are no special optics requirements. The current in each series-connected magnet is the same.

*An **insertion device (ID)** is a component in modern synchrotron light sources. They are periodic magnetic structures that stimulate highly brilliant, forward-directed synchrotron radiation emission by forcing a stored charged particle beam to perform wiggles, or undulations, as they pass through the device. This motion is caused by the Lorentz force, and it is from this oscillatory motion that we get the names for the two classes of device, which are known as wigglers and undulators*

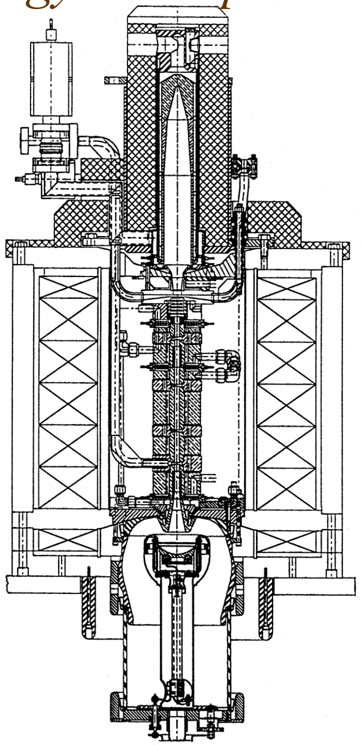
Photograph of an Insertion Device at the APS

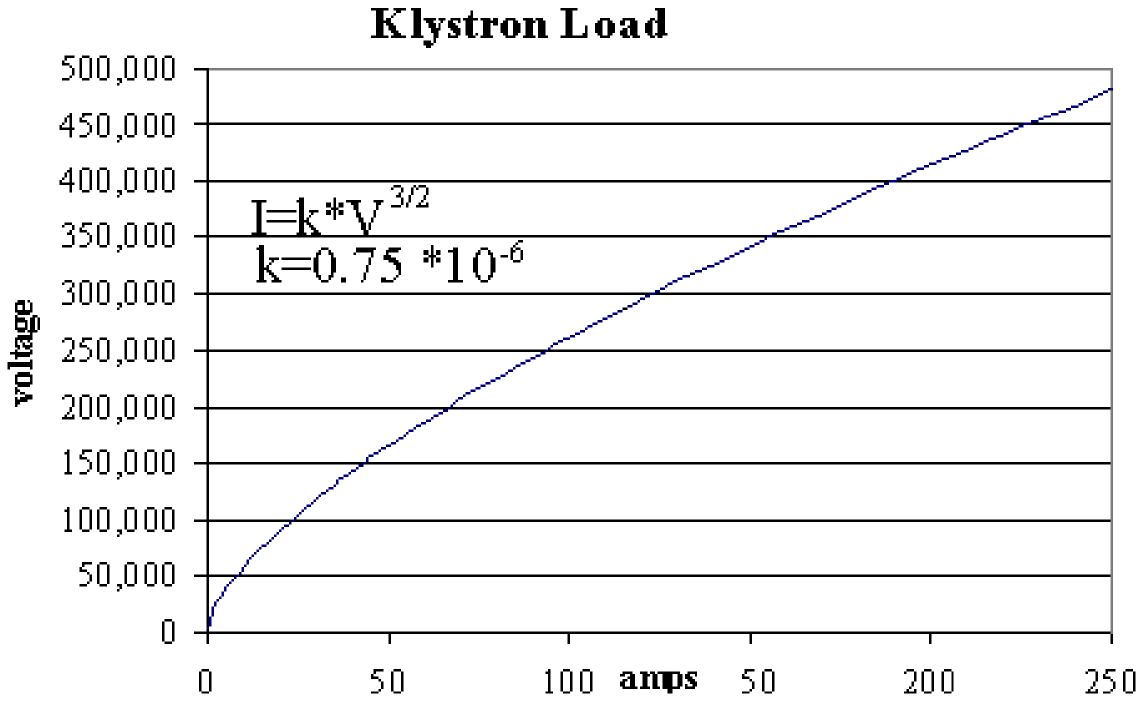




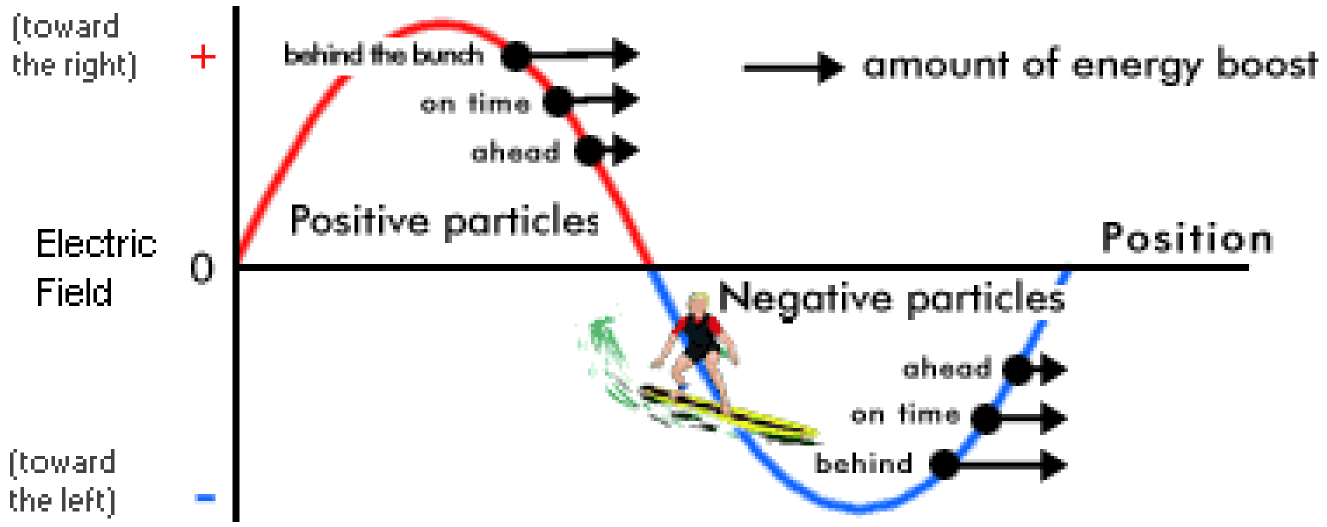
Klystron Load

- *Klystrons in RF and microwave systems accelerate particle beams. They need a power supply and an RF source.*
- *Their transfer function is called perveance (k) which expresses the klystron beam current and accelerating voltage relation. It is usually expressed as µp.*
- *In LINACs they operate in a pulsed mode to accelerate particle beams*
- *In boosters and storage rings they operate in continuous-mode to supply make-up energy to the particle beam to compensate for energy losses or for beam bunching*



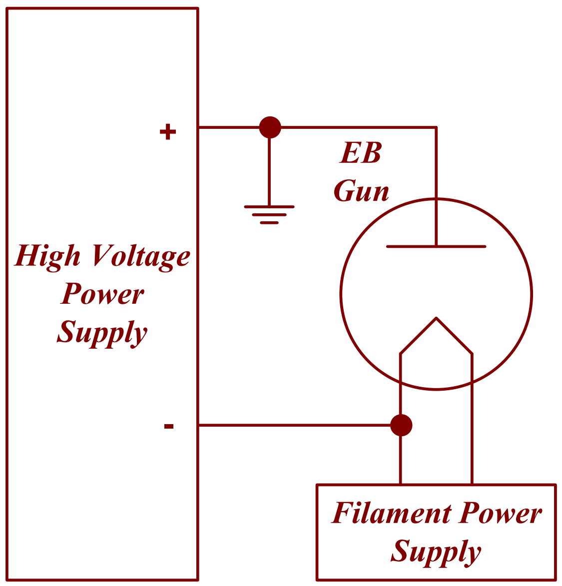


- *Electrons and positrons may be accelerated by injecting them into structures with traveling electromagnetic waves*
- *The microwaves from klystrons are fed into the accelerator structure via waveguides. This creates a pattern of electric and magnetic fields, which form an electromagnetic wave traveling down the accelerator. The beam energy is a function of the energy boost per klystron and the total number of klystrons.*

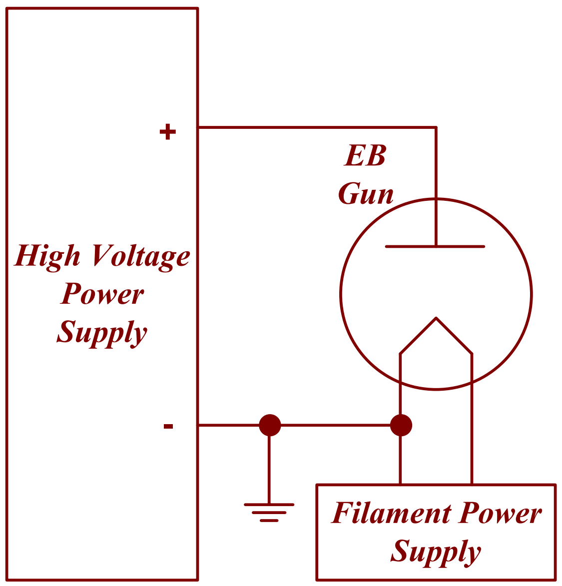


DC Electron Beam Gun Electrical Load Characteristics

- *Electron gun exhibits non-linear V-I characteristics*
- *Capacitive loading*
- *High voltage, low DC current*
- *High peak pulsed current*
- *Subject to arcing*
- *Limited fault energy capability - arc protection (crowbar) needed*



If work surface (anode) is difficult to insulate - put at ground potential. Float filament at HV.



If work surface (anode) is easy to insulate - float at HV. Put filament at ground potential.

Characteristics

- *Capacitive loading*
- *High voltage, low DC current*
- *High peak pulsed current*
- *Subject to arcing*
- *Limited energy capability - arc protection (crowbar) needed*

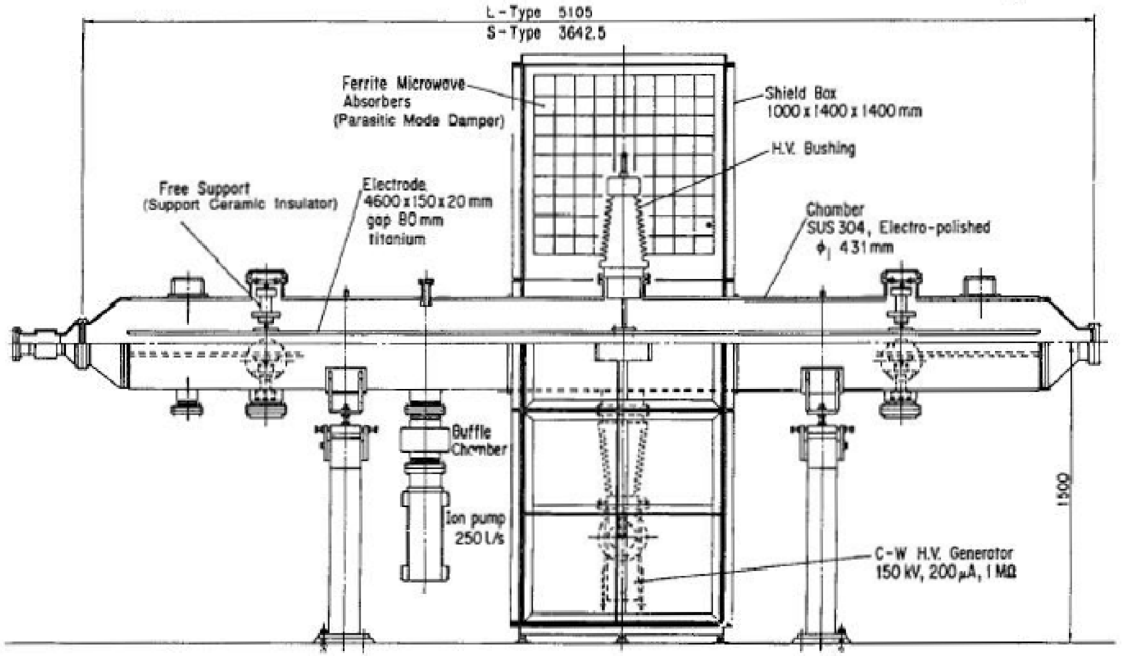
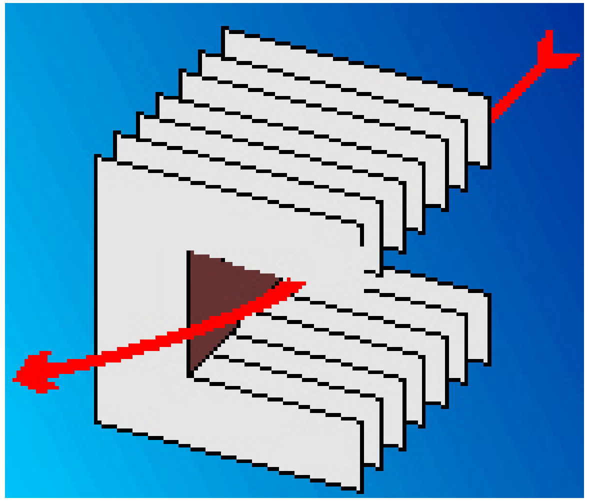


Fig.2 Separator chamber.

- *Kicker magnets interact with positively or negatively charged particle beams which, in most cases, are grouped into bunches*
- *The purpose of an injection kicker is to fully deflect (kick) bunches, without disturbance to the preceding or following bunches, from a beamline into a storage ring*
- *An ejection kicker will do the inverse, that is, kick a particle beam from a storage ring into a working beamline.*



Pulsed Magnet Loads – Kickers, Pulsed Deflectors

- *Short time constants ($\tau = L/R$) << 1 mS*
- *Characteristic impedance is like a transmission line*
- *High voltage, low impedance*
- *Fast pulse, match or terminating resistors*
- *Subject to reflection and breakdown*

Fig. 5. SLAC-designed kicker magnet.

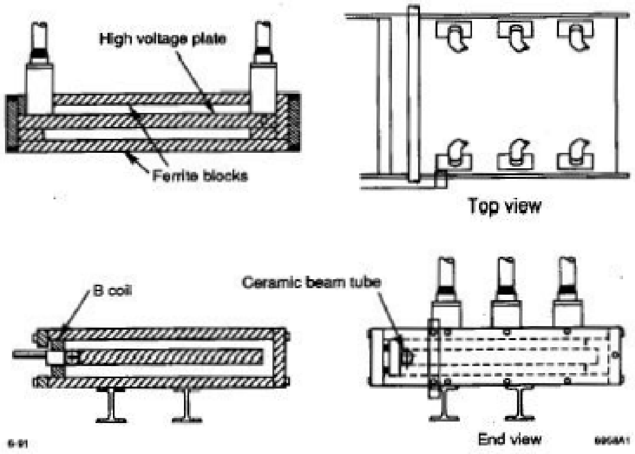
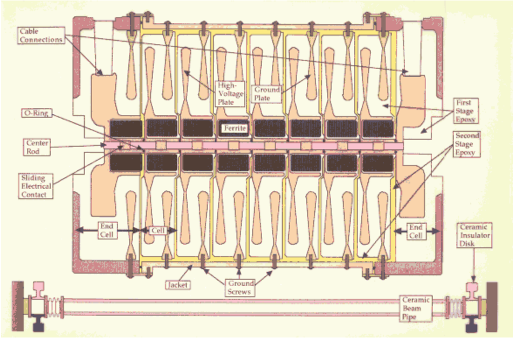
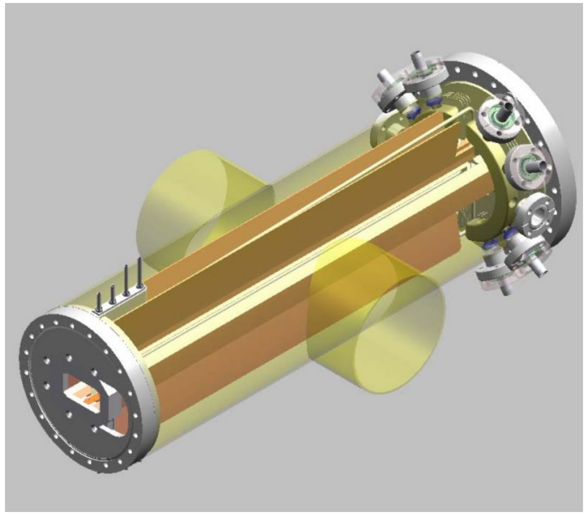
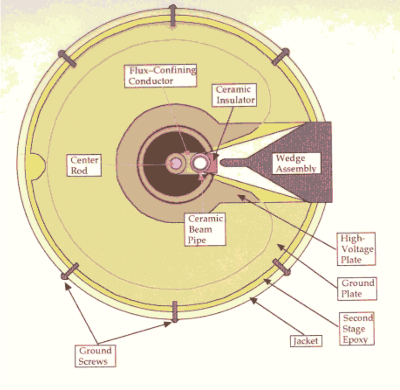
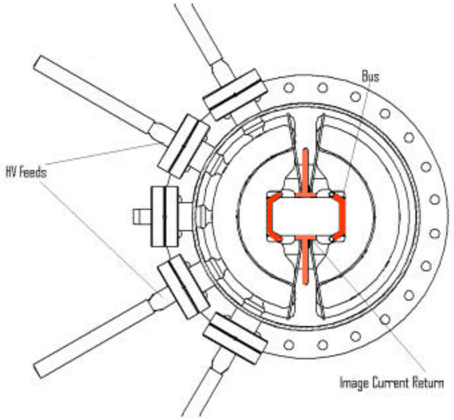


Fig. 6. SLAC-style kicker magnet.





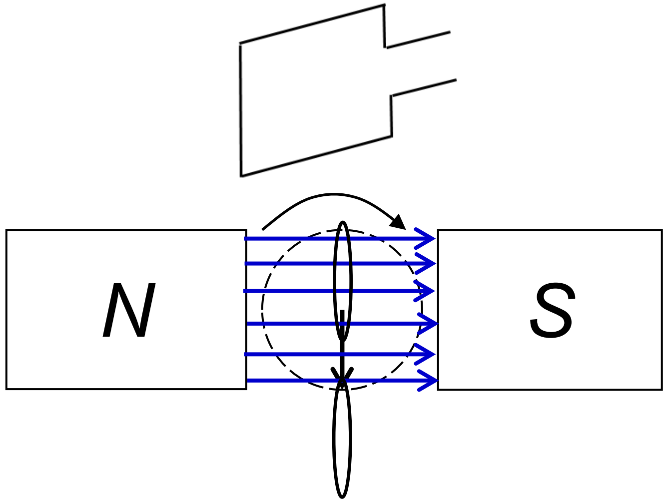




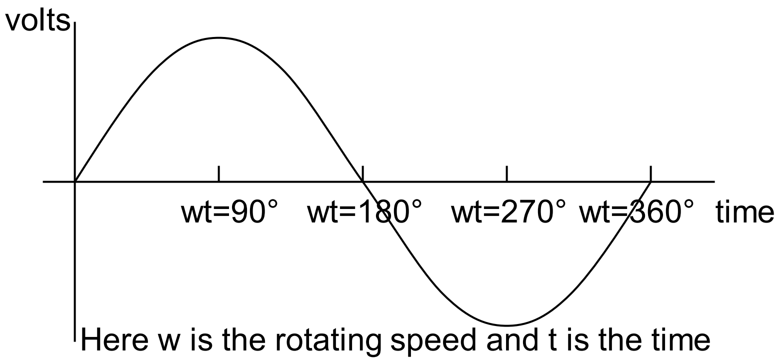
Section 5

- *Power Line and Other Considerations*
  - *Fundamental Quantities*
  - *Single Phase Systems*
  - *Three Phase Systems*
  - *Transformer Primer*
  - *The Per Unit Calculation System*
  - *Harmonics, Complex Waveforms and Fourier Series*
  - *SCR Commutation as Distortion Cause*
  - *Electromagnetic Compatibility and Interference (EMC/EMI)*
  - *Power Factor*

- *Generation of sine waves*



- *Plotting of sine waves*



- *Sine wave equation*

$$v(t) = V_{max} \sin(\omega t)$$

$$\omega = 2\pi f$$

- *Average value:*

$$V_{ave} = \frac{1}{T}\int_0^T v(t)\,dt$$

for AC sine system

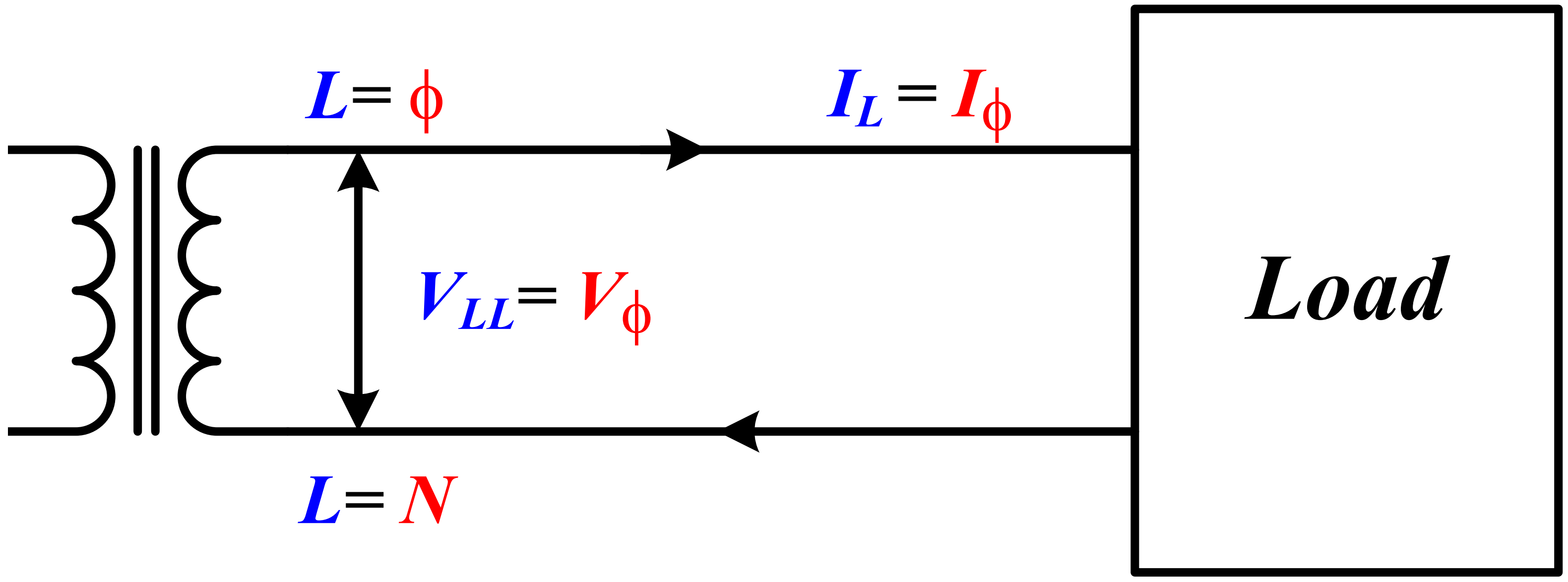
$$v(t) = V_m \sin(\omega t),\ \ then \quad V_{ave} = \frac{2}{\omega T}\int_0^{\pi} V_m \sin(\omega t)\,d\omega t = 0.636V_m$$

- *RMS value:*

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$$

for AC sine system

$$V_{rms} = \sqrt{\frac{1}{\omega T}\int_0^{2\pi} (V_m \sin(\omega t))^2 d\omega t} = \frac{V_m}{\sqrt{2}} = 0.707 * V_m$$

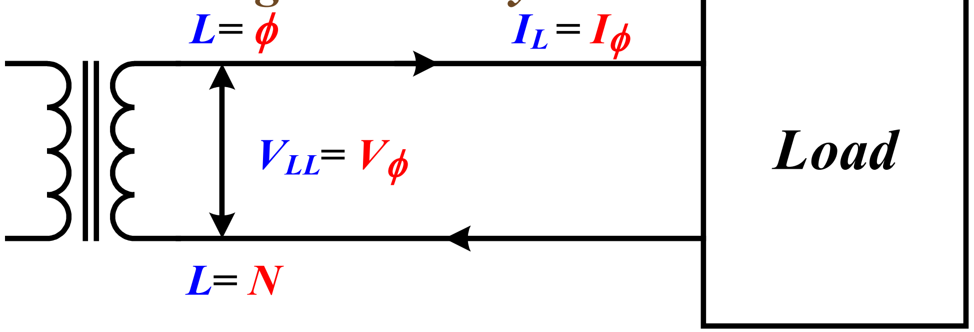


For 1ϕ AC input

$V_\emptyset = V_{LL}$
$I_\emptyset = I_L$ *where* $V_\emptyset$ *and* $I_\emptyset$ *are RMS values*

Example: the common 120V in homes is the RMS value. The peak (max) value is $120V * \sqrt{2} = 169.7V$

Single Phase Systems



Power is, in general, complex $S = VI^*$ (I^* *is complex conjugate of* I)

If the load is not a pure resistor, V and I are not in phase

The Apparent, Real, and Reactive "Powers" are:

Apparent:	$S_{1\phi} = V_{LL} \cdot I_L^* = P_{1\phi} + jQ_{1\phi}$	*(VA)*
Real (active):	$P_{1\phi} = V_{LL} \cdot I_L \cdot \cos\alpha$	*(Watt)*
Reactive:	$Q_{1\phi} = V_{LL} \cdot I_L \cdot \sin\alpha$	*(VAR)*

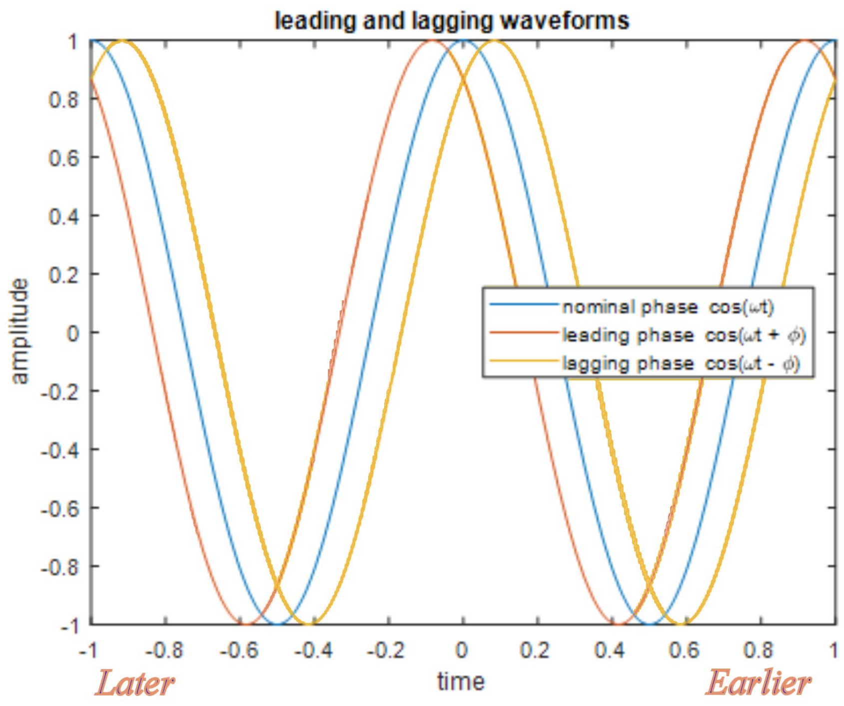
α *is the phase angle between* V_{LL} *and* I_L *with voltage as the reference*

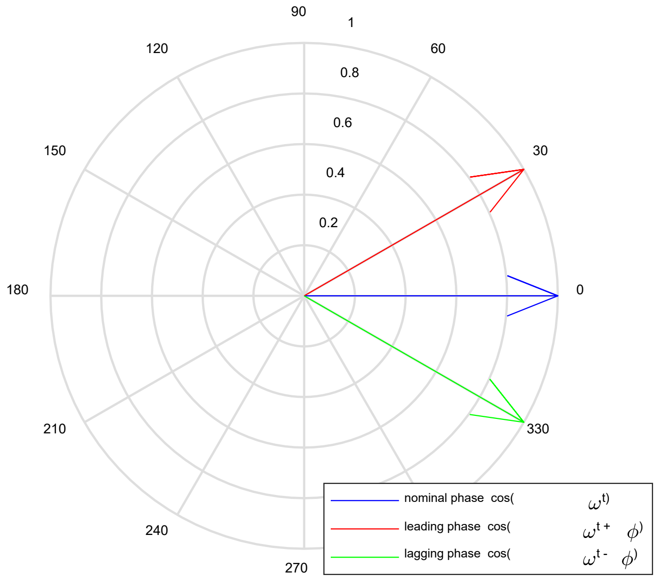
When current lags (inductive load), $Q_{1\phi} > 0$ $\quad S_{1\phi} = \frac{1}{T}\int_0^T v_{LL}(t) \cdot i_{LL}^*(t)\,dt$

All "powers" are average "powers"

$$S_{1\phi} = \sqrt{\frac{1}{T}\int_0^T v_{LL}^2(t)dt} * \sqrt{\frac{1}{T}\int_0^T i_L^2(t)dt} = \frac{1}{T}\int_0^T v_{LL}(t)i_L(t)dt$$

Single Phase Systems





Lead and lag refer to the order of the waveforms

In a rectangular plot, (left figure), the waveform leads if it arrives first

- *Orange leads Blue leads Yellow*

In a polar plot of phasors, phasors rotate CCW with time

- *Red leads Blue leads Green*

The rectangular plot is the projection of the phasor on the x-axis as it rotates

Instantaneous power real $p(t)$ is the product of $v(t)$ and $i(t)$, both real functions

Derivation: $p(t) = v(t) \cdot i(t)$

Let $v(t) = \sqrt{2}V\cos(\omega t)\,;\; i(t) = \sqrt{2}I\cos(\omega t - \phi)$

then $p(t) = 2VI\cos(\omega t)\cos(\omega t - \phi)$

Using the identity $\cos(a)\cos(b) = 1/2[\cos(a-b) + \cos(a+b)]$

$$p(t) = VI[\cos(\phi) + \cos(2\omega t - \phi)]$$
$$p(t) = VI\cos\phi + VI\cos(2\omega t - \phi)$$

Using the identity $\cos(u \pm v) = \cos(u)\cos(v) \mp \sin(u)\sin(v)$

$$p(t) = VI\cos\phi + VI[\cos 2\omega t\cos\phi + \sin 2\omega t\sin\phi]$$

Note that:

- *$p(t)$ has a DC component and an AC component, at twice the frequency ω*
- *DC component is a maximum when voltage and current are in phase* $(\phi = 0)$
- *Power is the product of the RMS, not peak, values of V_{LL} and I_L*
- *Reactive power term not obvious*

Instantaneous power $S(t) = V(t) \cdot I^*(t)$ *using phasors*

Derivation: $S(t) = V(t) \cdot I^*(t)$

Let $V(t) = V_0 e^{j\omega t}; I(t) = I_0 e^{j(\omega t - \phi)}$

then $S(t) = V(t) \cdot I^*(t) = V_0 I_0 e^{j\omega t} e^{-j(\omega t - \phi)} = V_0 I_0 e^{j\phi}$

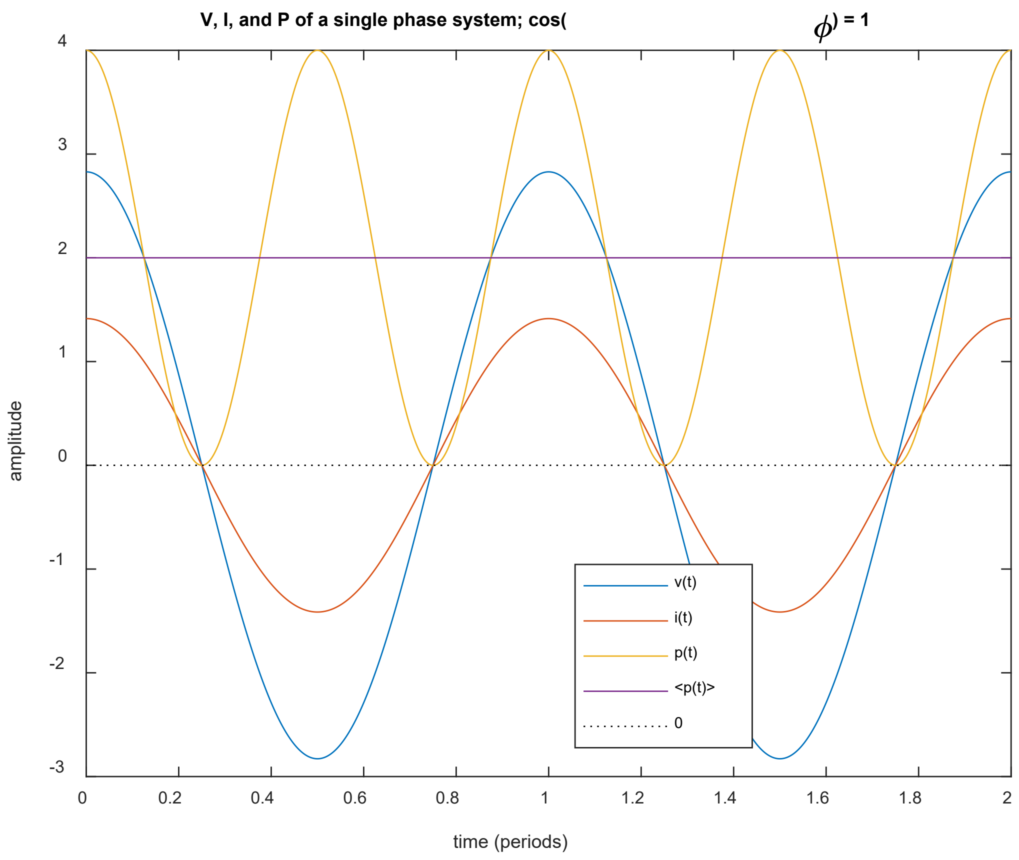
$$S = V_0 I_0 (\cos\phi + j\sin\phi) = V_0 I_0 \cos\phi + jV_0 I_0 \sin\phi = P + jQ$$

$$P = V_0 I_0 \cos\phi\,; \quad Q = V_0 I_0 \sin\phi$$

Note that:

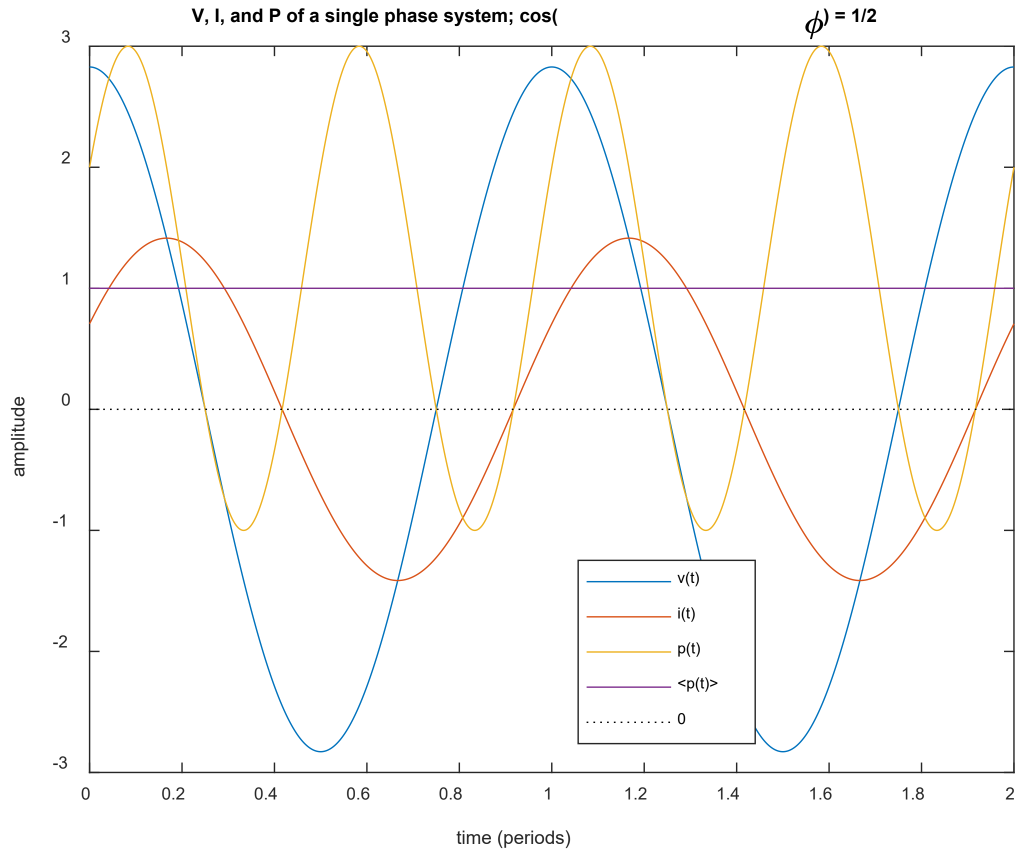
- S, P, Q *have no time dependence, due to* $S = V \cdot I^*$
  - *Only have DC components; AC components have multiplied out*
  - *Real and reactive power calculations both easily handled*
- *DC component is a maximum when voltage and current are in phase* $(\phi = 0)$
- *Phasor amplitude now uses RMS values to get proper power*

Single Phase Systems

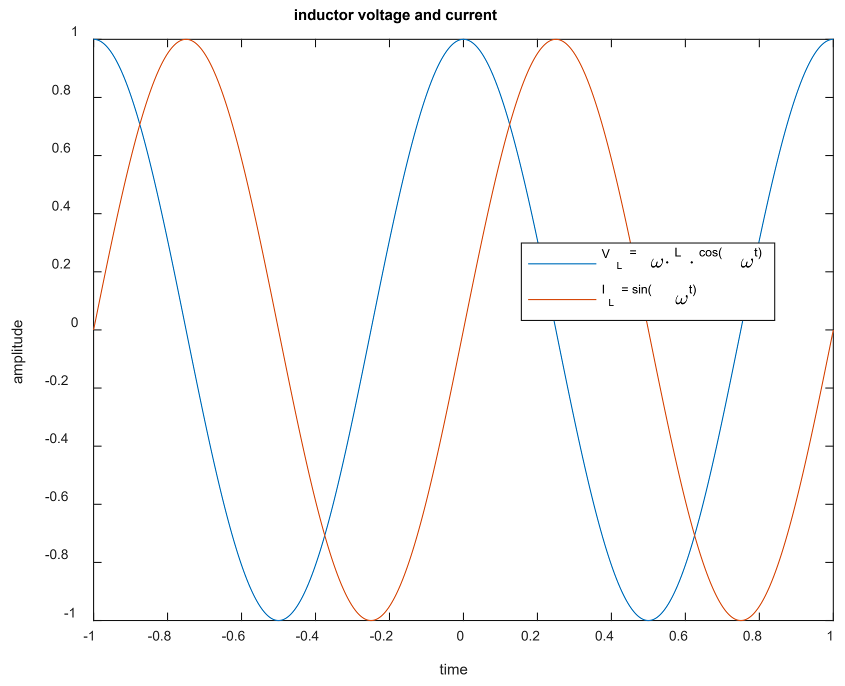


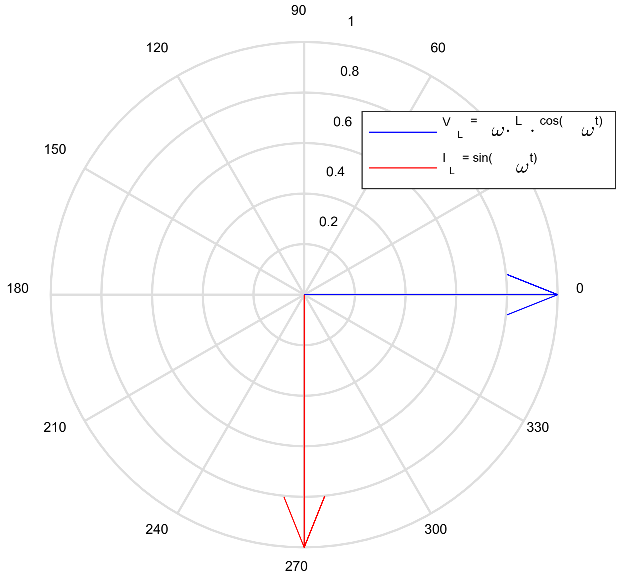
- *Voltage and current are in phase at* 60 Hz *(resistive load)*
- *Power = DC +* 120 Hz *terms, both equal in amplitude*

Single Phase Systems



- *Voltage leads current by* 60° *at* 60 Hz *(partially inductive load)*
- *Power = DC +* 120 Hz *terms, but now unequal in amplitude*
- *Power is + (delivered to load) and – (returned to the AC line) at* 120 Hz
- *+ and – power are equal when current – voltage are* 90° *out of phase*
- *No net power delivered to the load*





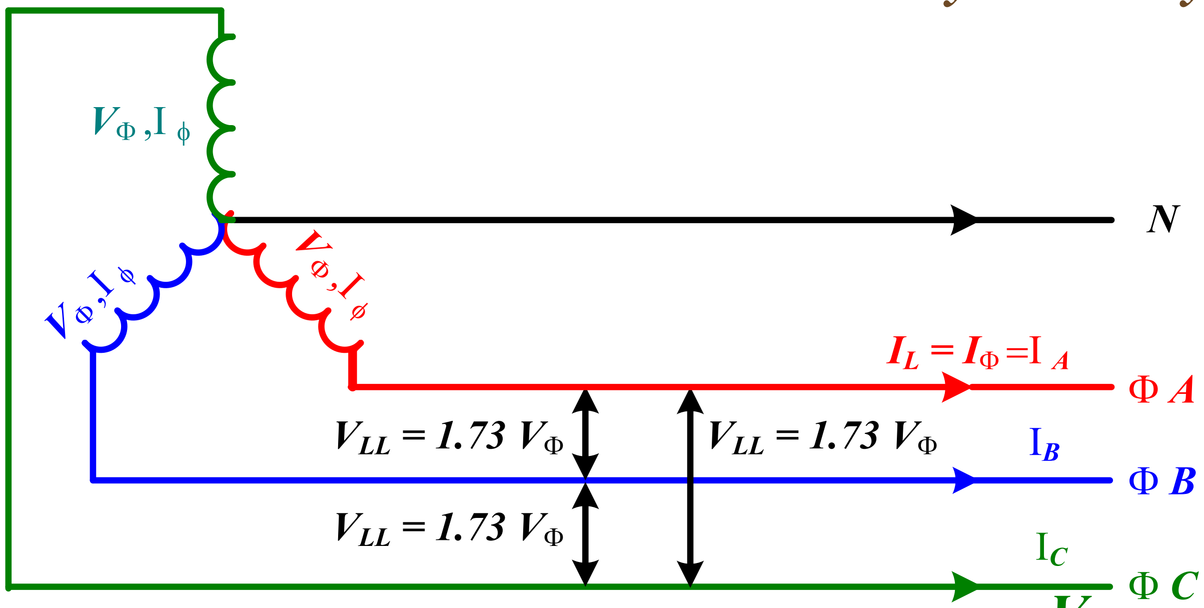
Example: Voltage, current across an inductor

$$I_L = \frac{V_0}{\omega L}\sin\omega t\,; \qquad V_L = L\frac{dI_L}{dt} = V_0\cos\omega t\,; \qquad V_L I_L = \frac{V_0^2}{2\omega L}\sin 2\omega t$$

$\cos\omega t$ *leads* $\sin\omega t$*, inductor current lags the inductor voltage. No DC term*

Example using phasors: $e^{j\omega t} = \cos\omega t + j\sin\omega t$

$$V_L = V_0 e^{j\omega t};\;\; I_L = \frac{V_0}{j\omega L}e^{j\omega t};\;\; S_L = V_L I_L^* = V_0 e^{j\omega t}\frac{jV_0}{\omega L}e^{-j\omega t} = j\frac{V_0^2}{\omega L} = jQ$$



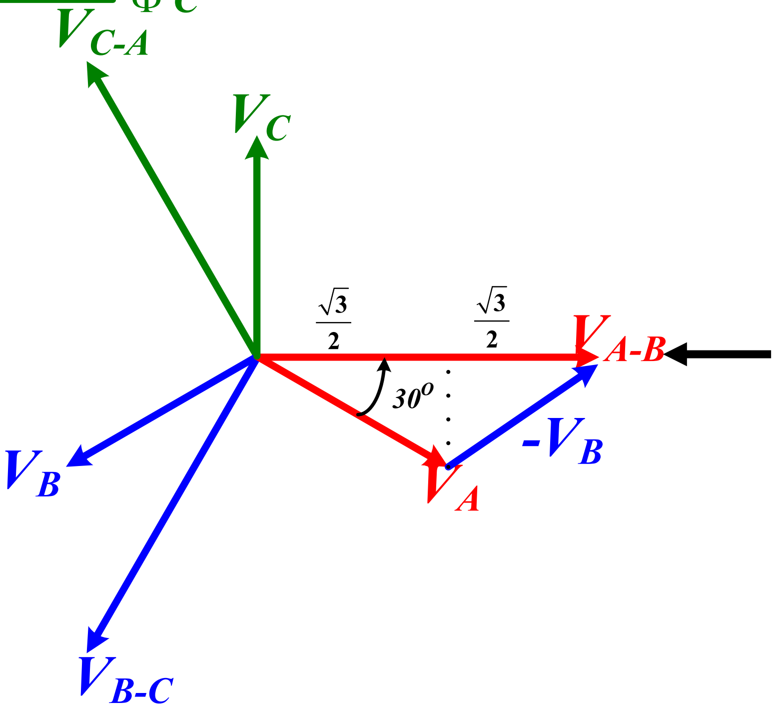
Line current is just the phase current

Magnitudes of V_ϕ add vectorially

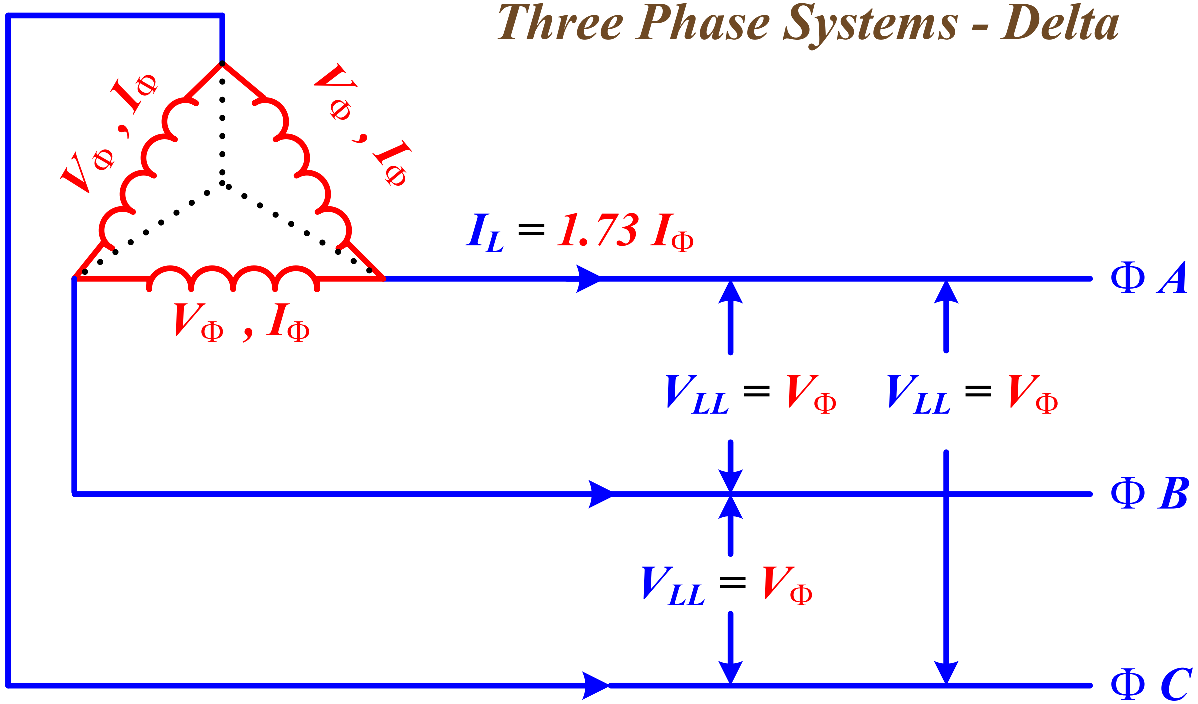
$$V_{LL} = \sqrt{3}\, V_\phi; \;\; I_L = I_\phi$$

$$S_{3\phi} = 3\, V_\phi I_\phi$$

$$S_{3\phi} = \sqrt{3}\, V_{LL} I_L$$



Three Phase Systems - Delta



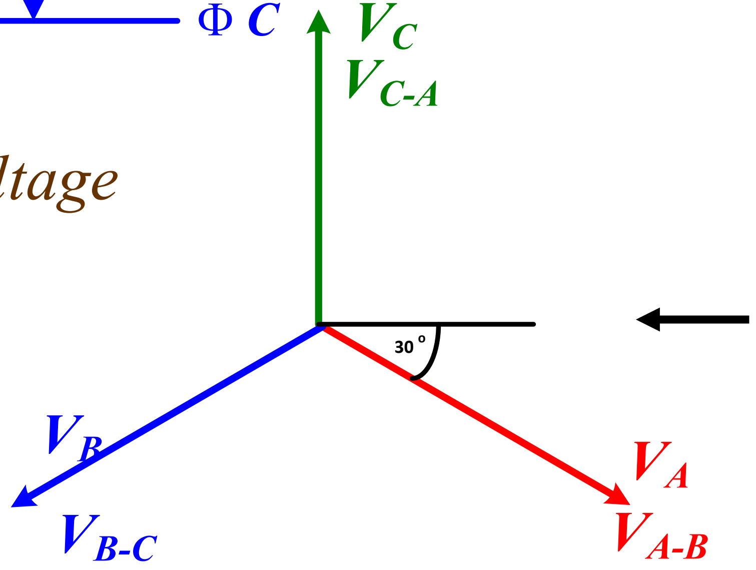
Line-line voltage is just the phase voltage

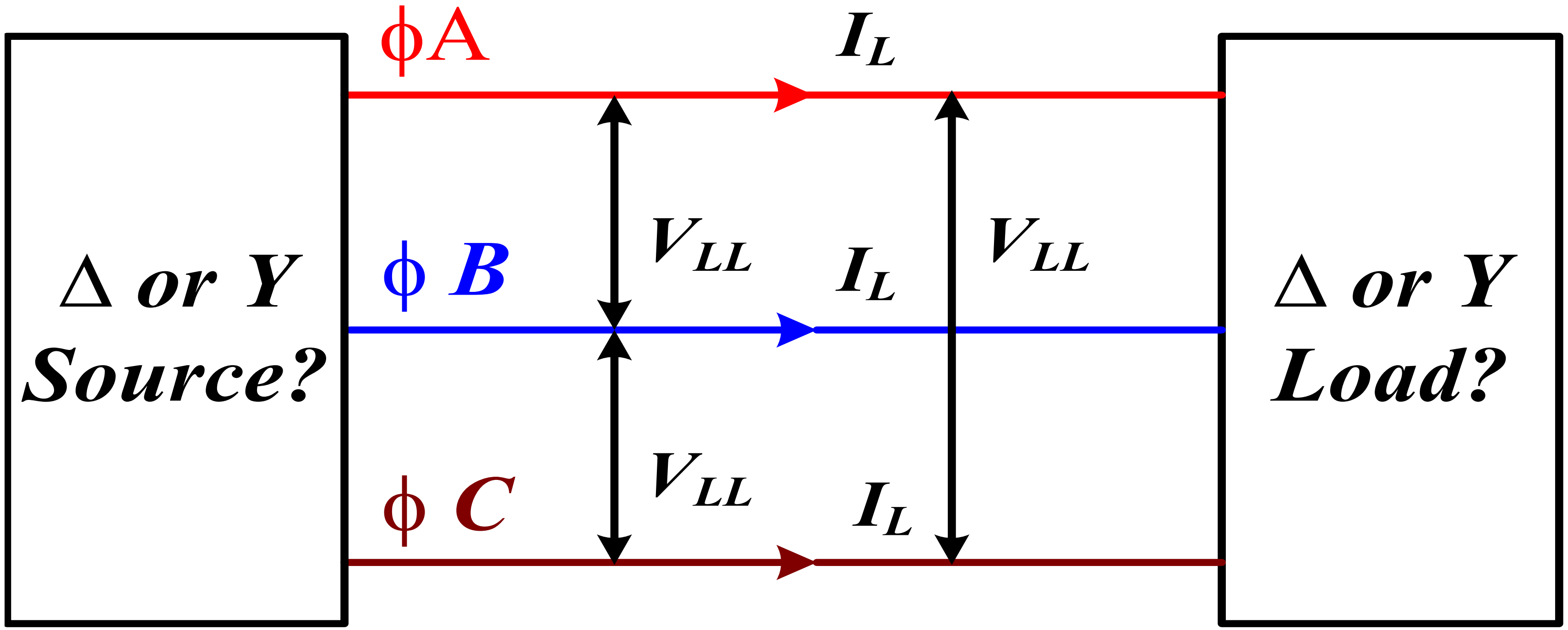
Magnitudes of I_ϕ *add vectorially*

$$V_{LL} = V_\phi;\ \ I_L = \sqrt{3}\, I_\phi$$

$$S_{3\phi} = 3\, V_\phi I_\phi$$

$$S_{3\phi} = \sqrt{3}\, V_{LL} I_L$$





For both Wye and Delta configurations: $S_{3\phi} = \sqrt{3}\, V_{LL} I_L$

- $V_{AB} = |V_{AB}|e^{j0}$
- $V_{BC} = |V_{BC}|e^{-j2\pi/3}$ *phasor notation of ϕ to ϕ voltages*
- $V_{CA} = |V_{CA}|e^{-j4\pi/3}$

Three Phase Systems – Constant Power

$$p(t) = v_{AB}(t)\cdot i_A(t) + v_{BC}(t)\cdot i_B(t) + v_{CA}(t)\cdot i_C(t)$$

$$= \frac{|V_{AB}|}{\sqrt{2}}\cos(\omega t)\frac{|I_A|}{\sqrt{2}}\cos(\omega t-\phi)$$

$$+\frac{|V_{BC}|}{\sqrt{2}}\cos(\omega t-2\pi/3)\frac{|I_B|}{\sqrt{2}}\cos(\omega t-2\pi/3-\phi)$$

$$+\frac{|V_{CA}|}{\sqrt{2}}\cos(\omega t-4\pi/3)\frac{|I_C|}{\sqrt{2}}\cos(\omega t-4\pi/3-\phi)$$

For balanced source $|V_{AB}| = |V_{BC}| = |V_{CA}| = V$ *and load* $|I_A| = |I_B| = |I_C| = I$

Using $\cos A\cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$ *we express* $p(t)$ *as*

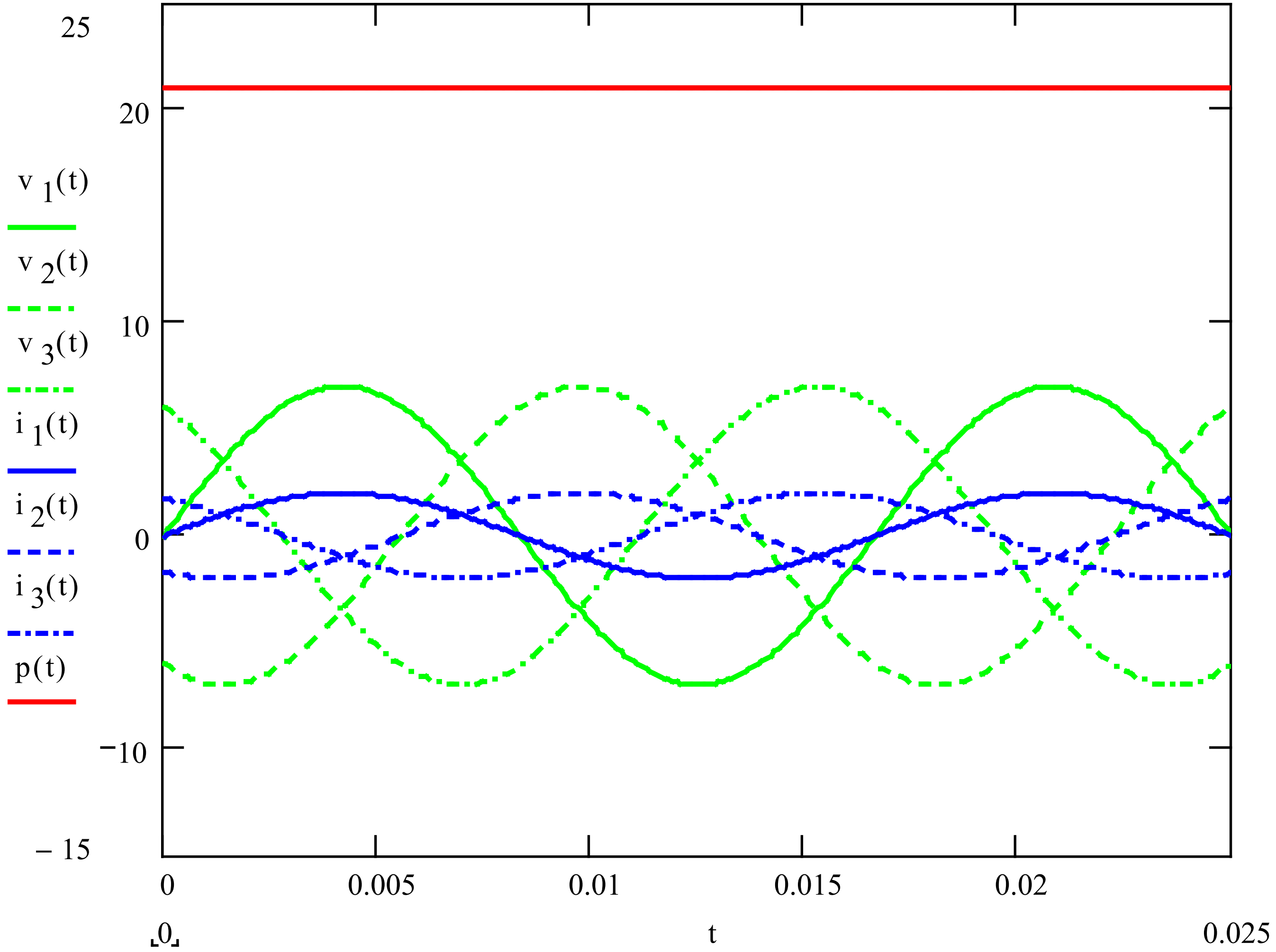
$$p(t) = VI[\cos(2\omega t-\phi)+\cos\phi] + VI[\cos(2\omega t-4\pi/3-\phi)+\cos\phi]$$
$$+VI[\cos(2\omega t-8\pi/3-\phi)+\cos\phi]$$

We can show by symmetry that

$$\cos(2\omega t-\phi)+\cos(2\omega t-4\pi/3-\phi)+\cos(2\omega t-8\pi/3-\phi) = 0$$

So $p(t) = 3VI\cos\phi$

Power delivered in this balanced system is constant, maximum when $\phi = 0$



- $p(t) = v_A(t) * i_A(t) + v_B(t) * i_B(t) + v_C(t) * i_C(t)$
- *3 times the single phase power with only 3 conductors, not 6*
- *For balanced load, p (t) is constant*

$$s(t) = v_{AB}(t) \cdot i_A^*(t) + v_{BC}(t) \cdot i_B^*(t) + v_{BC}(t) \cdot i_C^*(t)$$
$$= |V_{AB}|e^{j\omega t}|I_A|e^{-j(\omega t-\phi)} + |V_{BC}|e^{j(\omega t-2\pi/3)}|I_B|e^{-j(\omega t-2\pi/3-\phi)}$$
$$+|V_{CA}|e^{j(\omega t-4\pi/3)}|I_C|e^{-j(\omega t-4\pi/3-\phi)}$$

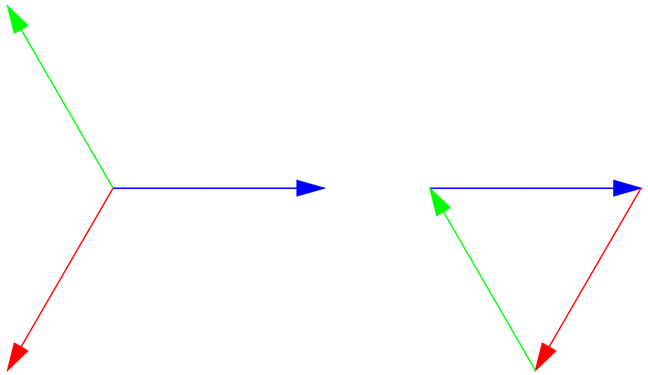
For balanced source $|V_{AB}| = |V_{BC}| = |V_{CA}| = V$ *and load* $|I_A| = |I_B| = |I_C| = I$

All common phase terms in the exponentials multiply out, leaving

$$S = 3VIe^{j\phi} = P + jQ = 3VI\cos\phi + j3VI\sin\phi$$

Note, from the figure below, that the three symmetric phasors add to zero.

Since $e^{j\theta} = \cos\theta + j\,\sin\theta$*, if the sum of the complex exponentials vanishes, so do the sums of the cosines and sines.*



Transformer Primer - Why Needed

- *Needed to transform the load voltage to the line voltage*
  - *Utility power is efficiently transported at high voltage and low current*
    - *Transmission loss due to I^2R losses in the conductors*
    - *Transmission lines have large distances between lines to support high voltage isolation*
  - *High voltage may be difficult to handle at the load side*
    - *Clearances*
    - *Devices – semiconductors, resistors, capacitors*
    - *Insulation*
    - *Personnel safety*

- *Needed to isolate the load from the line for better ground fault immunity and to reduce the magnitude of fault currents*

- *We want a "perfect" transformer*
  - *Transform line voltage to load voltage*
  - *All input power is transformed to be output power – no losses*
- *Use magnetic coupling*

- *Ampere's law: a current, I, generates a magnetic induction,* ***B***

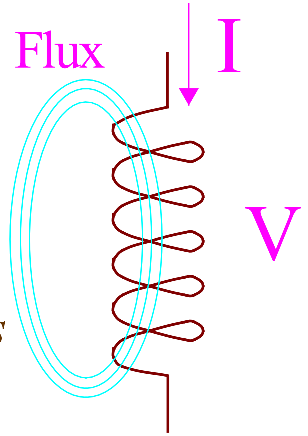
$$\nabla \times \boldsymbol{H} = \boldsymbol{j} + \epsilon_0 \frac{\partial \boldsymbol{D}}{\partial t}; \ \oint \boldsymbol{H} \cdot \boldsymbol{dl} = \iint \boldsymbol{j} \cdot \boldsymbol{dA} = I;$$

(For this discussion, the last term in the first eqn is small and can be neglected)

- *Faraday's law of induction: the change in* ***B*** *generates an electric field*

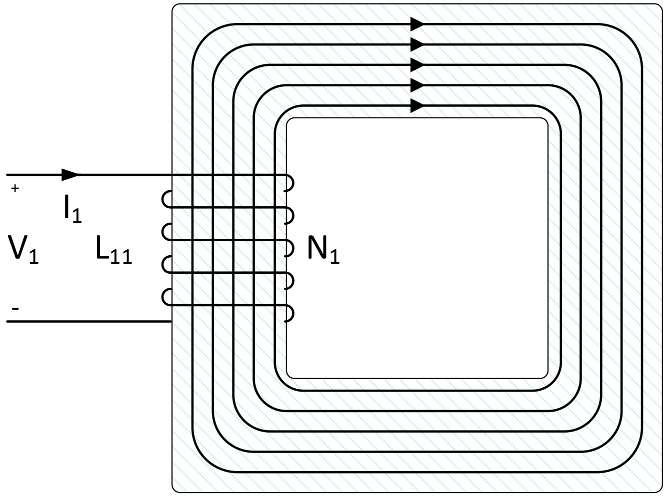
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}; \ -V_0 = \oint \boldsymbol{E} \cdot \boldsymbol{dl} = -\frac{d}{dt} \iint \boldsymbol{B} \cdot \boldsymbol{dA} = -\frac{d\Phi}{dt}$$

- *If we put a loop around the changing* ***B*** *we generate a voltage* V_0 *(volt/turn)*
  - *N turns generates* NV_0; $V = N\frac{d\Phi}{dt}$
  - $I \Rightarrow \boldsymbol{H}$; $\boldsymbol{B} = \mu \boldsymbol{H}$; $d\boldsymbol{B}/dt \Rightarrow V$
  - *We relate V to I with a quantity, L, called the inductance*
  - $V = L\frac{dI}{dt}$
  - *L depends on the system geometrical and material properties*



- *In objects made from material with large magnet moments*
  - *An external induction field,* ***B****, causes the moments to align, generates* ***H***
  - *Energetically favorable for the flux lines to be contained in the object*
  - $\mu = \mu_R \mu_0$; $\mu_R \gg 1$ $(\approx 10^4 - 10^5)$ *in iron*
- *This principle is used in the design of "iron-dominated" magnets to shape the fields generated by the magnets*

- ***B*** *field is always in loops; it has to close on itself* $(\nabla \cdot \boldsymbol{B} = 0)$
- *We want an inductor that contains all of the field loops*
- *In an iron core, picture frame structure*
  - ***H*** *is heavily concentrated and uniform in the core*
  - $\oint \boldsymbol{H} \cdot \boldsymbol{dl} \simeq Hl$ *where* l *is the average core circumference*
  - $\Phi = \iint \boldsymbol{B} \cdot \boldsymbol{dA} \simeq \mu HA$ *where* A *the is typical core cross-section*
  - I_1 *is the input current and* N_1 *is number of turns*
  - $I = N_1 I_1$ *where* I *is total current enclosed in core*
- *The voltage generated across each turn is*
  - $V_0 = \frac{d\Phi}{dt} = \mu A \frac{dH}{dt} = \frac{\mu A}{l} \frac{dI}{dt} = \frac{\mu A}{l} N_1 \frac{dI_1}{dt}$
- *The voltage generated across the* N_1 *turn coil is*
  - $V_1 = N_1 V_0 = \frac{\mu A}{l} N_1^2 \frac{dI_1}{dt} = L_{11} \frac{dI_1}{dt}$
  - $L_{11} = \frac{\mu A}{l} N_1^2$



Transformers (xfmrs) are inductors with linked flux Φ

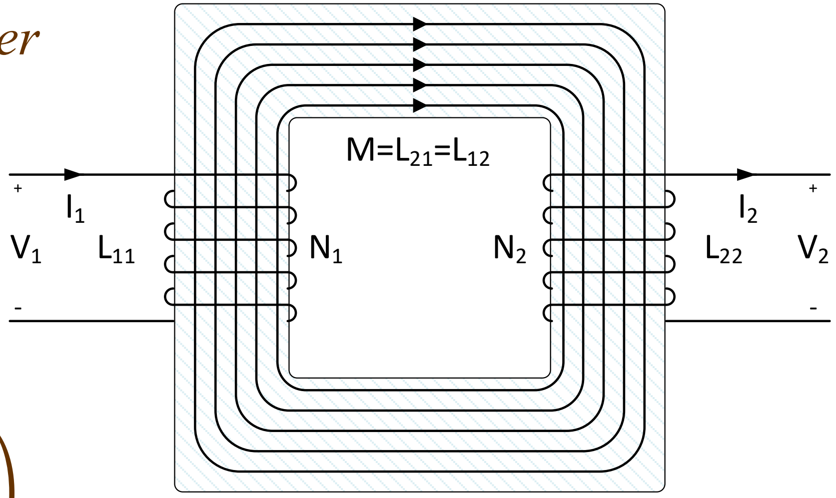
- *The same flux exists in all of the iron*
- *It generates the same voltage across any conductor loop*
- *Add a "secondary" coil of N_2 turns and use that to "transform" the voltage of the system from V_1 to $V_2 = N_2 V_0 = \frac{N_2}{N_1} V_1$ with an output current I_2*

Cannot create power, so loss-less system requires $S_{IN} = V_1 I_1^* = S_{OUT} = V_2 I_2^*$

- $I_2 = \frac{N_1}{N_2} I_1 \quad \Rightarrow \quad N_1 I_1 = N_2 I_2$ *Ampere-turns in equal ampere-turns out*

This is the definition of an "ideal" transformer

- $\begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} N_2/N_1 & 0 \\ 0 & N_1/N_2 \end{pmatrix} \begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$
- *All input power transferred to output*



Expressing the equations differently

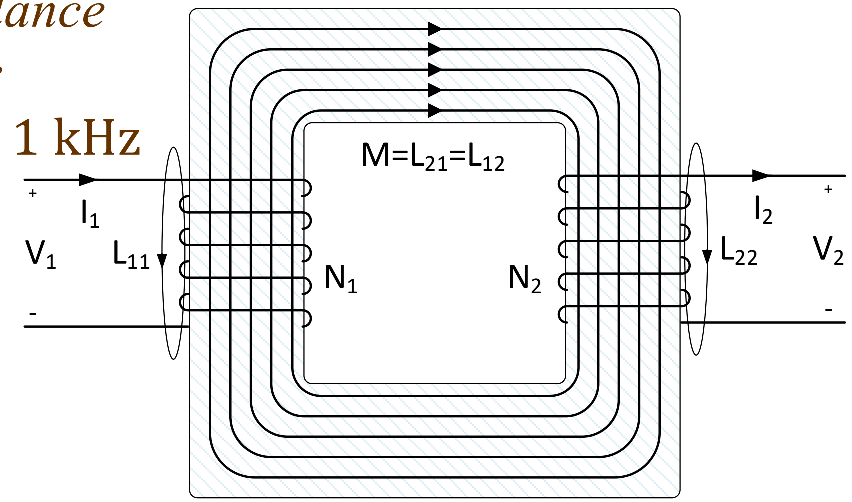
$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \dot{i}_1 \\ \dot{i}_2 \end{pmatrix} = \begin{pmatrix} L_{11} & M \\ M & L_{22} \end{pmatrix} \begin{pmatrix} \dot{i}_1 \\ \dot{i}_2 \end{pmatrix}$$

where M is the mutual inductance between the coils. M is also defined as

$$M = k\sqrt{L_{11} L_{22}}$$

Transformer Primer

- *An ideal transformer is only a mathematical construct*
- *The best we can build is a "perfect" transformer*
  - *The transformer still has the "magnetizing" inductance* $L_{11} = \frac{\mu A}{l} N_1^2$
  - *This inductance is in parallel with the ideal transformer*
    - *Ideal transformer is the limit of the perfect transformer as* $\mu \to \infty$
- *In all practical cases the magnetizing inductance is very large*
  - *Its typical current draw on the system is* $\approx 1\%$ *of that of the rated transformer load and usually can be neglected for most calculations*
- *A perfect transformer requires all of the flux from coil* 1 *couple to coil* 2
  - *But space exists between coils and core and* $\mu \neq \infty$
  - *"Leakage" inductance around each winding;* $\Rightarrow k \neq \pm 1$
  - *Leakage inductance defines an impedance*
  - *Impedance in series with transformer*
- *Iron core transformers typically used* $f \leq 1$ kHz
- *Less lossy ferrites for* $f > 1$ kHz



Equivalent Transformer Circuit

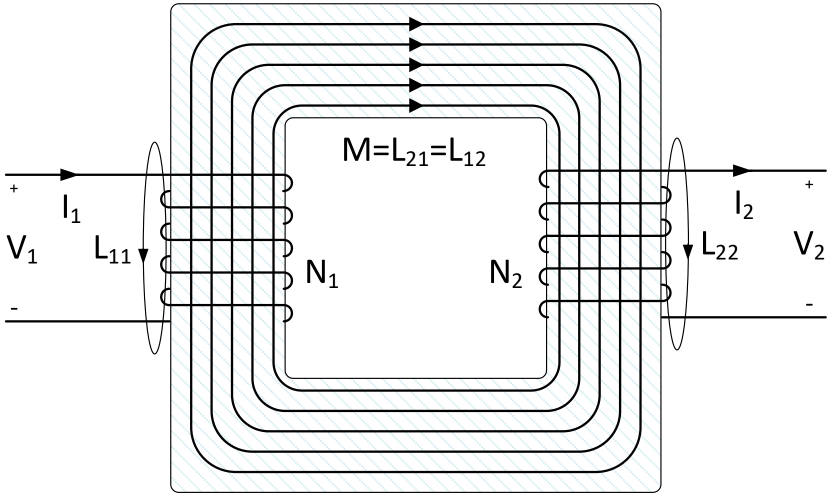
- *The current required to magnetize the core with flux is called the magnetizing current and is made up of two parts:*

  1. *A component out of phase with the induced voltage due to the magnetizing inductance.*

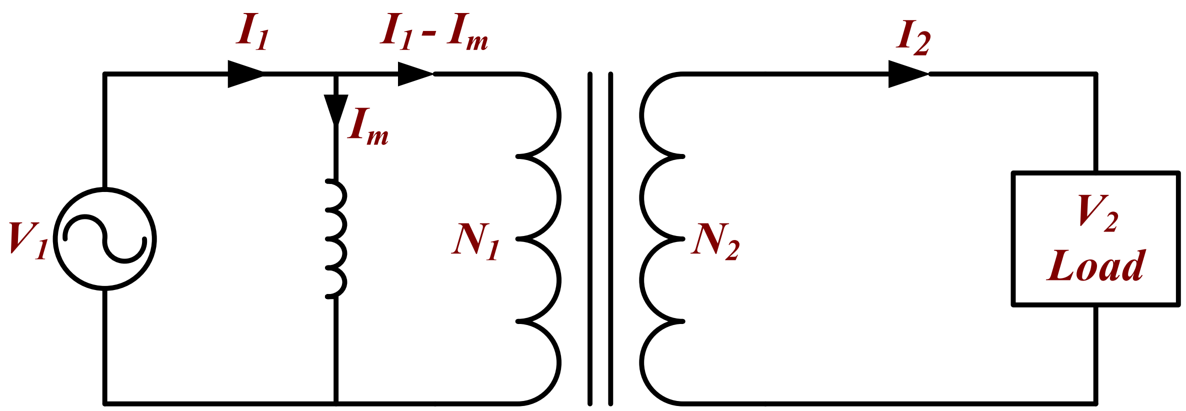
  2. *A component in phase with the induced voltage from losses due to eddy current and hysteresis losses. These losses generate heat in the core.*

- *The magnetizing inductance is obtained by driving the transformer with the secondary open circuited* $(I_2 = 0)$ *and measuring the Primary voltage and current.*

$$L_m = \frac{V_1}{\omega I_1}\Big|_{I_2=0} \quad (L_m \gg L_{11})$$



Transformer Primer – Turns / Voltage / Current Ratios

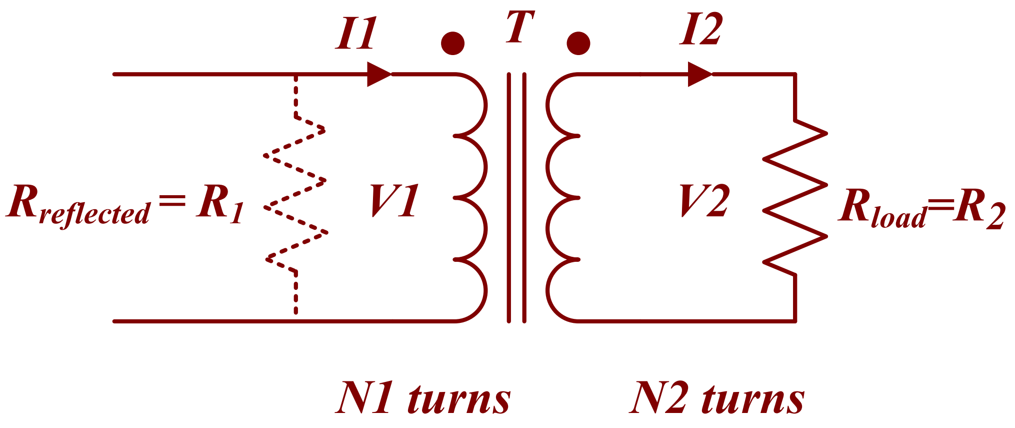


- *As discussed above, the common flux in the transformer core couples the secondary to the primary.*
- *For each turn in each coil, the flux produces a common Volts/turn*

$$\frac{d\Phi}{dt} = \frac{V_1}{N_1} = \frac{V_2}{N_2} \Rightarrow \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

- *Because of the magnetizing current*
  - *The input to our ideal transformer is $I_1 - I_m$ and not I_1, therefore*

$$\frac{I_2}{I_1 - I_m} = \frac{N_1}{N_2}; \text{ but if } I_m \ll I_1, \ \frac{I_2}{I_1} = \frac{N_1}{N_2}$$



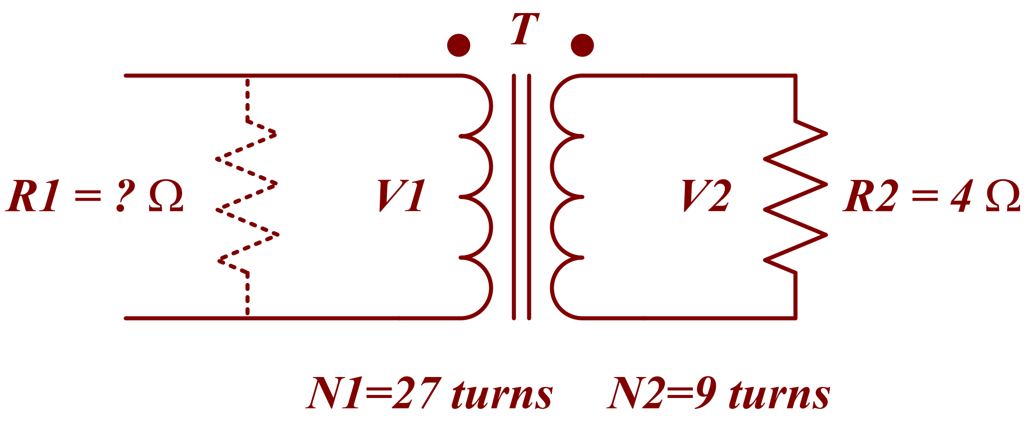
We are usually given the impedance R_2 on the secondary side of the transformer. In order to determine the loading on the source, we want to transform that impedance to the primary, that is, create an equivalent circuit without the transformer. Given

$$R_2 = \frac{V_2}{I_2}$$

$$R_1 = \frac{V_1}{I_1} = \frac{\left(\frac{N_1}{N_2}\right) V_2}{\left(\frac{N_2}{N_1}\right) I_2} = \left(\frac{N_1}{N_2}\right)^2 \frac{V_2}{I_2} = \left(\frac{N_1}{N_2}\right)^2 R_2$$

$$\frac{R_1}{R_2} = \left(\frac{N_1}{N_2}\right)^2$$

Example: If $R_2 = 4\ \Omega$, what is the value of the reflected resistance as seen on the primary side?

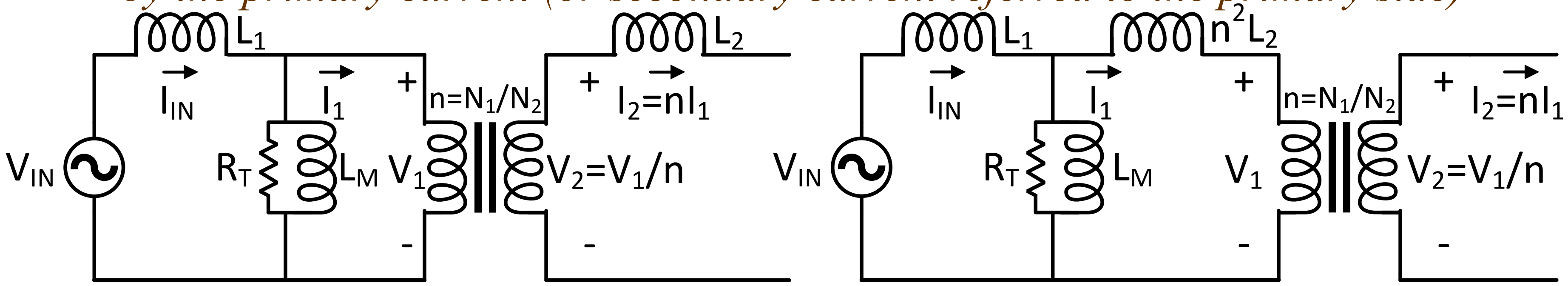


$$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_2$$

$$R_1 = \left(\frac{27}{9}\right)^2 4\ \Omega$$

$$R_1 = 9 \cdot 4\ \Omega = 36\ \Omega$$

- *Flux that does not couple both windings is called the leakage flux and acts like a series inductor called the leakage inductance*
- *If the secondary is shorted and the magnetizing current is small ($I_m \ll I_1$), then the leakage inductance is proportional to the primary voltage divided by the primary current (or secondary current referred to the primary side)*



Leakage inductances on pri. and sec. Inductances reflected to primary

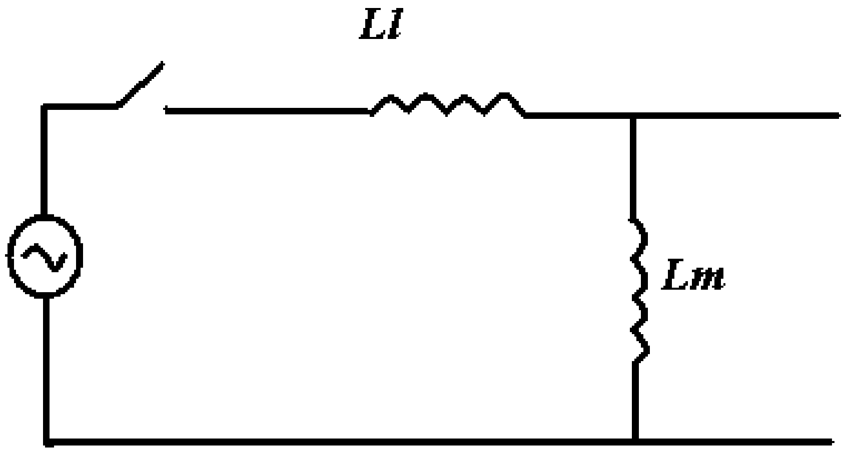
- *Choose appropriate transformer approximation, convert real transformer to ideal transformer plus associated impedances, then use ideal transformer equations to "transform" impedances across transformer.*

- *The transformer "percent impedance" is the ratio of V_{IN}/V_{RATED} required to obtain full load I_{OUT} flowing into a shorted secondary*

$$Z_{\%} = 100 \cdot \frac{V_{IN}}{V_{RATED}}\bigg|_{I_{OUT},\,Z_L=0}$$

- *Transformer is rated for*
  - *Voltage rating*
    - *Turns ratio:* $V_2/V_1 = N_2/N_1$
    - *Voltage isolation requirements*
  - *Power rating:* $S = V_1 I_1 = V_2 I_2$
    - *Current rating, resistance of conductors*
    - *Core size*
    - *Cooling*
  - *Impedance*
    - *Inrush current*
    - *Available short circuit current*
    - *Inductive impedance dominates; resistance typically neglected*
  - *Frequency*
    - *Core material*
    - *Coil winding thickness*

- *Example: Single phase:* 12470: 480; 500 kVA; 6.00%; 60 Hz
  - *Voltage rating*
    - *Turns ratio:* $V_1/V_2 = 12470/480 = 25.98 \approx 26/1$
    - *Voltage isolation requirements; Primary must hold off* 12.47 kV
  - *Power rating:* $S = V_1 I_1 = V_2 I_2 = 500 \times 10^3$
  - *Full load current:* $I_F = S/V$
    - *Primary:* $I_{F1} = 500/12.47 = 40.10\text{ A}$
    - *Secondary:* $I_{F2} = 500/0.48 = 1042\text{ A}$
  - *Impedance*
    - *Full load (inductive):* $Z_F = V/I_F = V/(S/V) = V^2/S$
      - *Primary referenced:* $Z_{F1} = 12470^2/500 \times 10^3 = j311.0\ \Omega$
      - *Secondary referenced:* $Z_{F2} = 480^2/500 \times 10^3 = j0.4608\ \Omega$
    - *Transformer impedance*
      - *Primary referenced:* $Z_1 = 0.06 \cdot Z_{F1} = j18.66\ \Omega$
      - *Secondary referenced:* $Z_2 = 0.06 \cdot Z_{F2} = j0.0276\ \Omega$

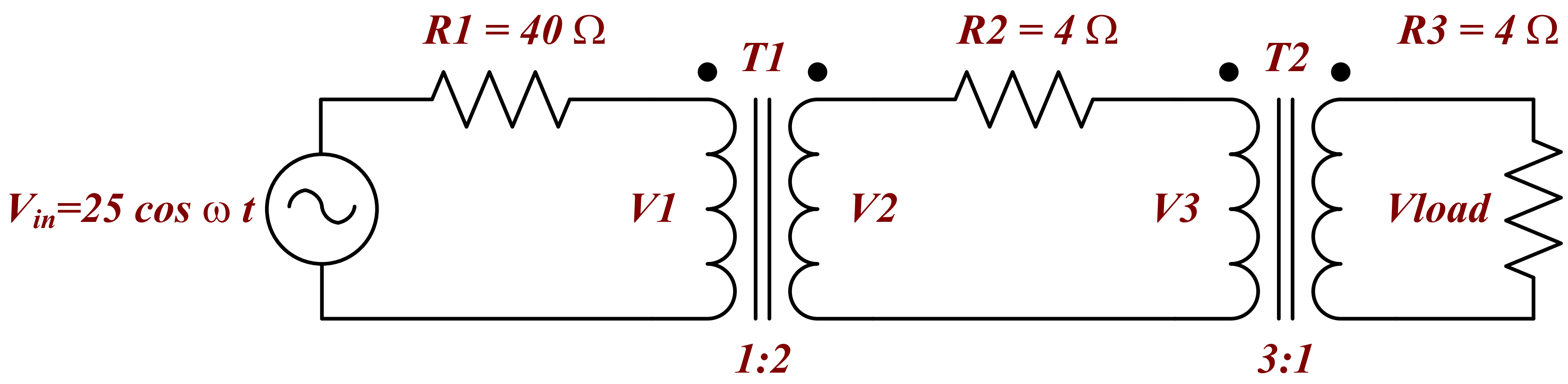


transformer model

An air gap is undesirable in a transformer because:

- *It reduces L_m , and a large L_m is desired to reduce the magnetizing and inrush current*
- *It increases L_l, and a small L_l is desired to lower energy and other losses*

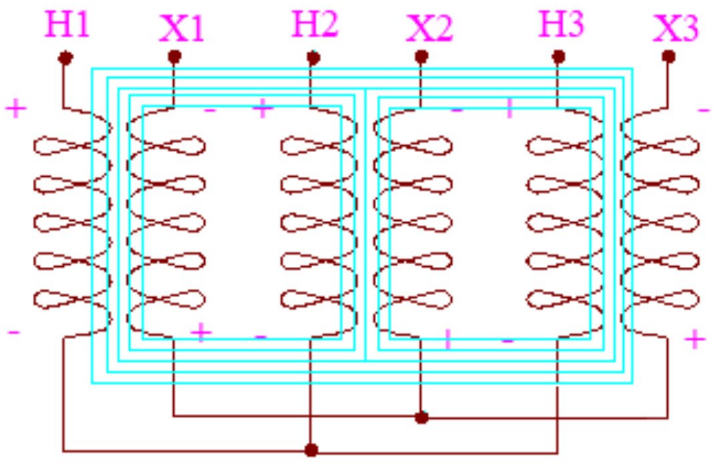
Calculate the output voltage in the circuit shown below.



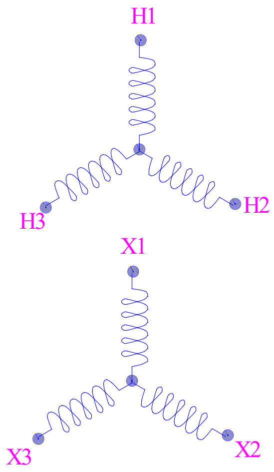
Transformer Primer - Configuration

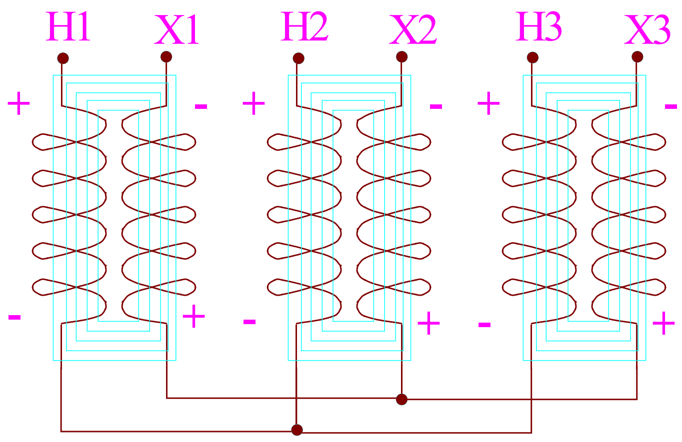
- *Low frequency, 60 Hz, transformers almost always use laminated iron cores to reduce Eddy Current and hysteresis losses*
- *For low power applications < 2.5 kW single phase transformers are used to eliminate the need for costly 3 phase input power lines.*
- *3 phase lines and transformers are used to reduce the cost of higher power systems (usually >2.5 kW)*
- *3 phase lines allow the use of phase shifting transformers to generate any number of output phases*

re three phase transformers

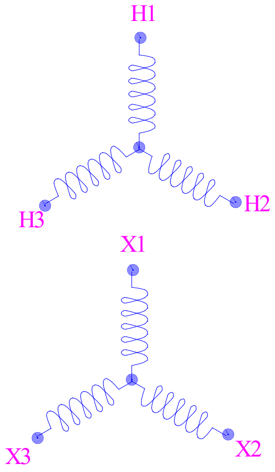


Wye-Wye Transformer



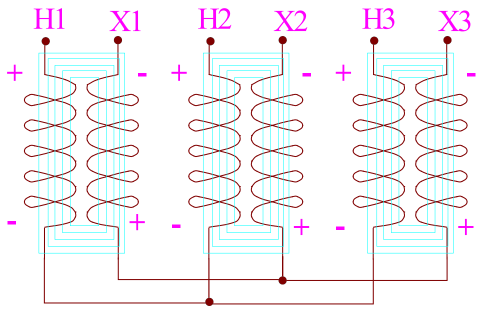


Wye-Wye Transformer

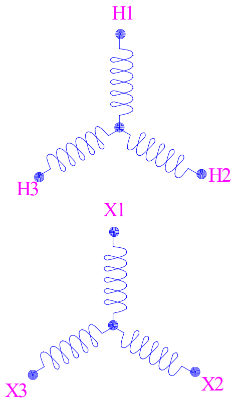


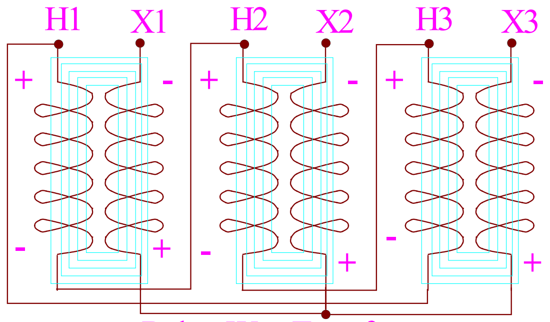
Three phase Transformers

- *A three phase transformer can be constructed with 1 core or 2 or 3 independent cores*
- *Independent core transformers are more expensive (use more steel) and can result in line imbalances*

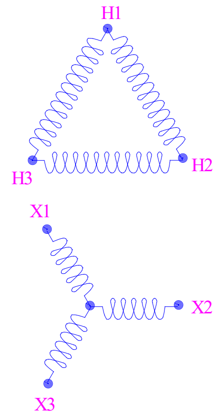


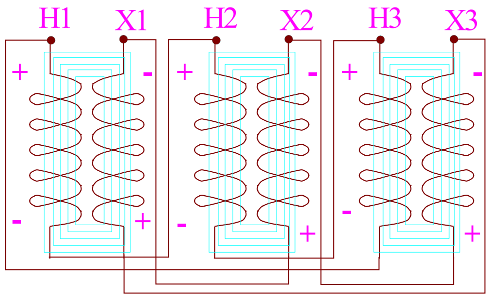
Wye-Wye Transformer



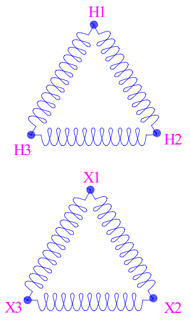


Delta -Wye Transformer

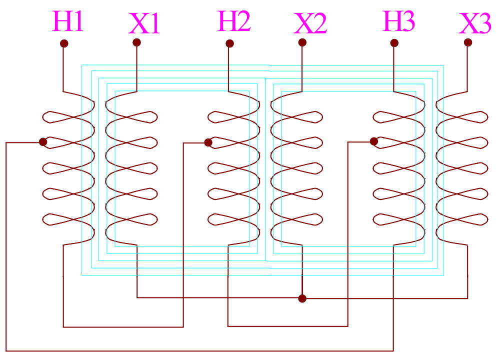




Delta - Delta Transformer

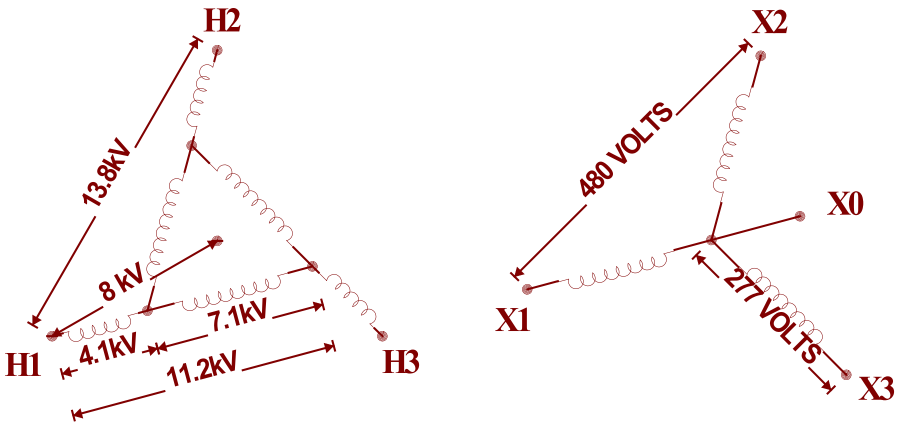


Extended Delta Phase shifting transformer



Extended Delta-Wye Transformer 3 core

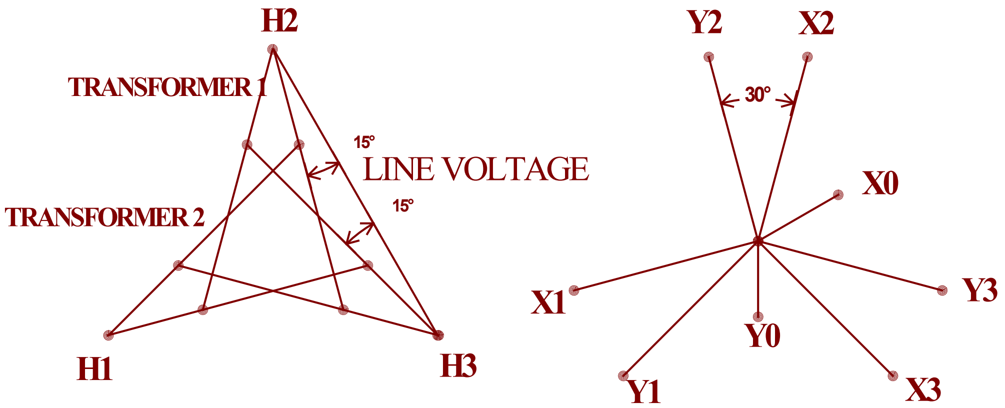
EXTENDED DELTA 13.8kV to 480 V 7.5°



PRIMARY VOLTAGE RELATIONSHIP

SECONDARY VOLTAGE RELATIONSHIP

PHASE RELATIONSHIP

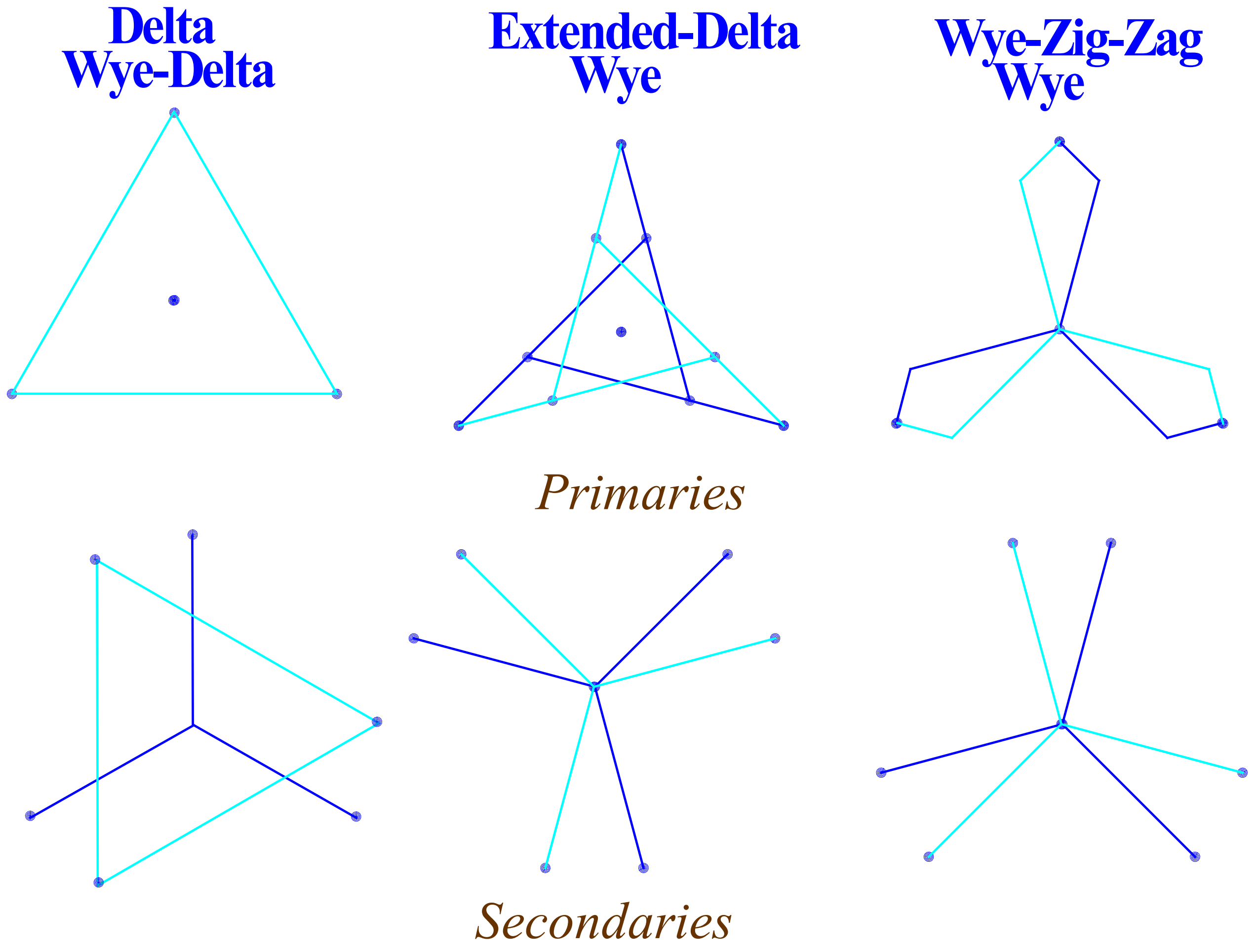


PRIMARY PHASE RELATIONSHIP

SECONDARY PHASE RELATIONSHIP

Transformer Primer - Three Phase Phase Shifting Transformer

Phase shifting transformer for 12 Pulse operation



Standards for Power Rectifier Transformers

1) *Practice for Semiconductor Power Rectifiers ANSI C34.2-1973*
2) *IEEE standards for Transformer and Inductors for Electronic Power Conversion Equipment ANSI/IEEE std 388-1992*

Insulation Class Recommendations for Rectifier Transformers

1) *Oil filled, 65℃ rise over ambient (paper oil insulation)*
2) *Dry type, Class B 80°C rise over ambient, (paper, varnish)*
3) *Dry type, Class H 150°C over ambient (fiberglass, epoxy)*

Phase Relationship and labeling

1) *General Requirements for Distribution Power and Regulating Transformers ANSI C57.12.00-1973*

Low Frequency Transformers have been around a long time and designs are well established. There are a few problems related to rectifier operation that should be considered when using transformers;

*1) **Harmonic currents** in the core and coils can result in excessive losses.*

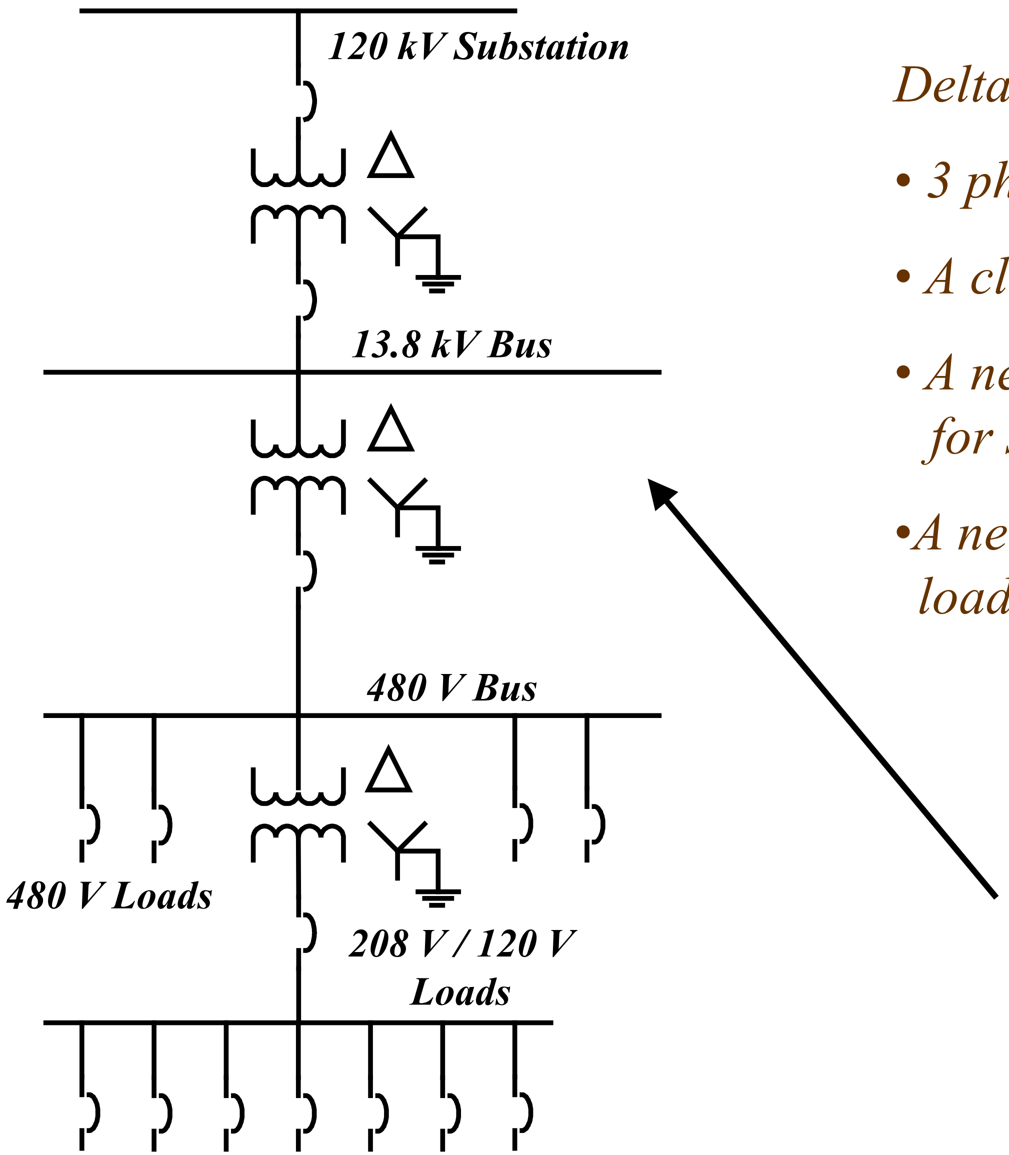
*2) Presence of DC and/or **second harmonic** currents/ voltage can saturate the core resulting in more harmonics and excessive core hysteresis loss.*

*3) **Short circuits** are common in rectifiers resulting in high forces on the coils and the coil bracing resulting in coil faults.*

*4) Connection to the center of a wye can generate excessive **third harmonic** current resulting in voltage distortion and overheating.*

*5) The **fast switching voltages** of rectifiers under commutation can produce non-uniform voltage distribution on coil windings resulting in insulation failure.*

Three Phase Systems - Delta - Wye Configuration – The Preferred Choice

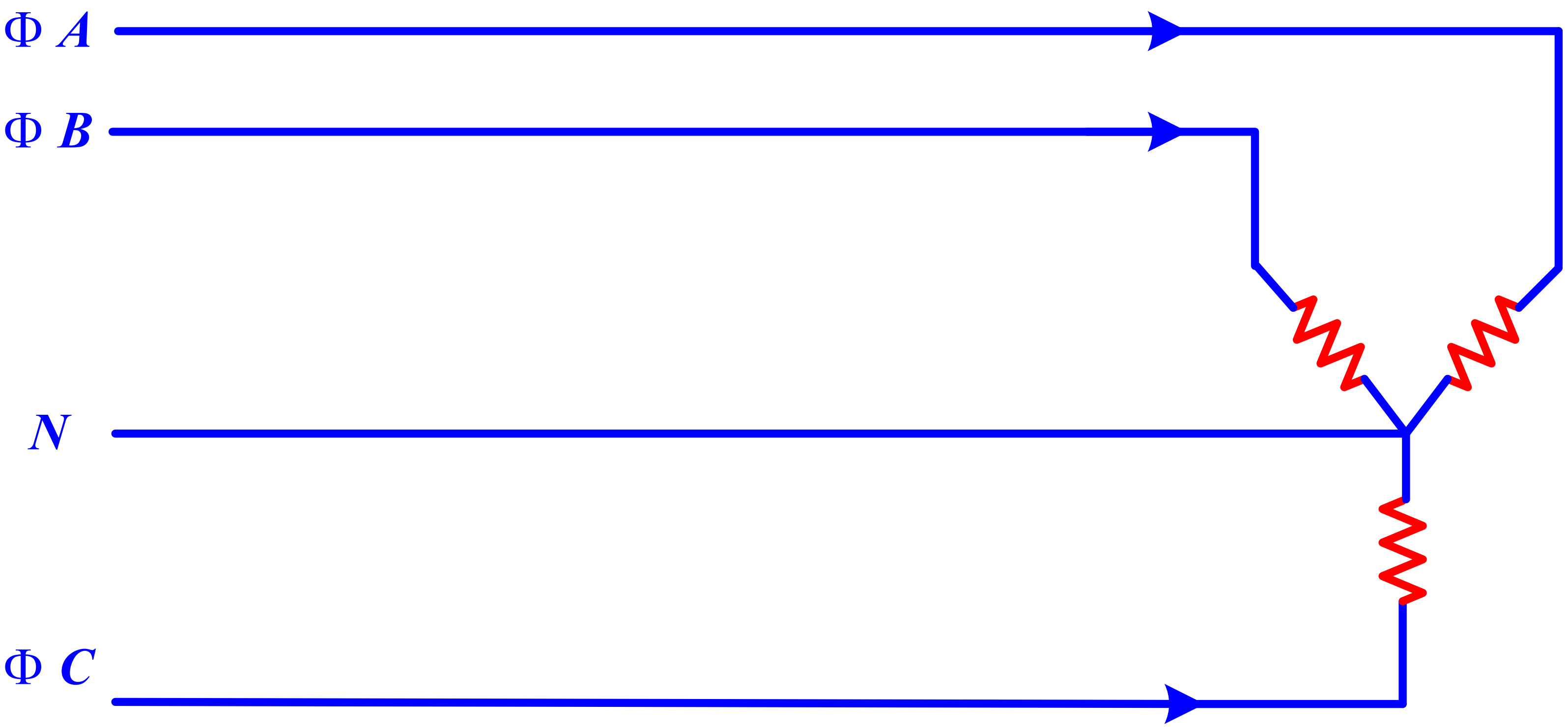


Delta - Wye Provides

- *3 phase output*
- *A closed path for harmonic currents*
- *A neutral for connection to ground for safety*
- *A neutral for deriving single phase loads*

Single or One-Line Diagram

Three Phase Systems - Neutral Wire Size – Balanced, Linear Load



$I_A = |I_A|e^{-j30}$ $I_B = |I_B|e^{-j150}$ $I_C = |I_C|e^{-j270}$

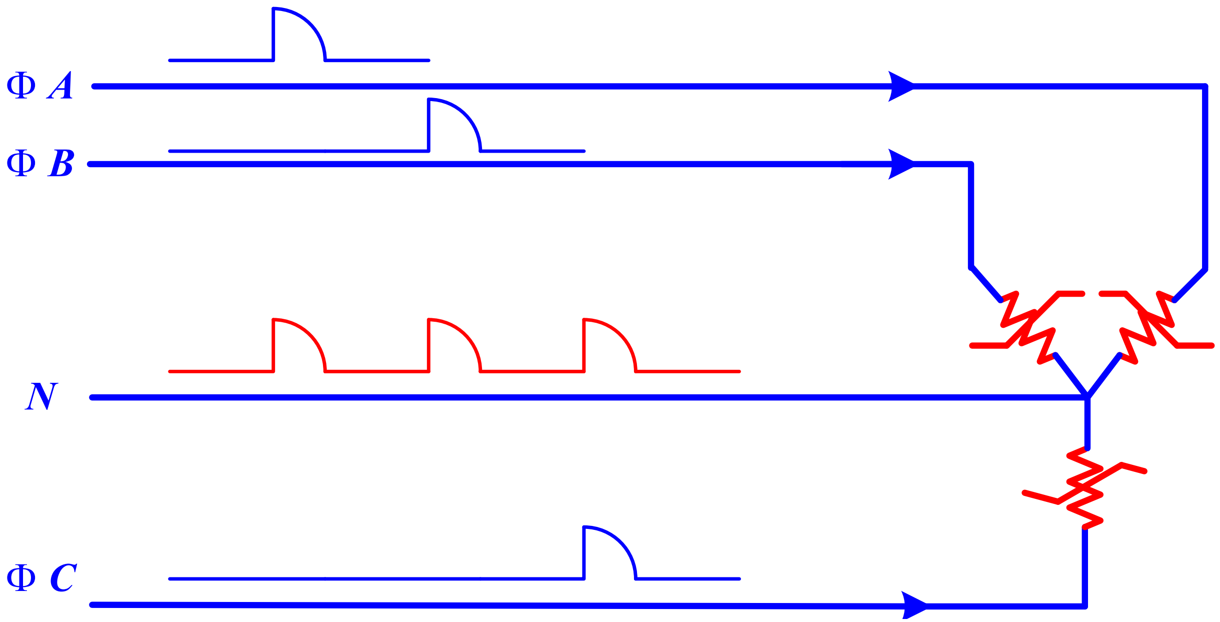
$|I_A| = |I_B| = |I_C|$

$I_N = I_A + I_B + I_C$

$I_N = |I_A|[(0.87 - j0.5) + (-0.87 - j0.5) + (0 + j1)] = 0$

There is no neutral current flow if load is balanced and linear

Three Phase Systems - Neutral Wire Size – Unbalanced and/or Non-linear Loads



For balanced non-linear loads $|I_A| = |I_B| = |I_C| = |I_L|$

$$|I_N| = \sqrt{|I_A|^2 + |I_B|^2 + |I_C|^2} = \sqrt{3}\, I_L$$

For unbalanced linear or non-linear loads $|I_A| \neq |I_B| \neq |I_C|$

$$|I_N| = \sqrt{|I_A|^2 + |I_B|^2 + |I_C|^2}$$

The neutral conductor can safely be sized for $\sqrt{3} * MAX(I_A, I_B, I_C)$

Fundamental Quantities American Commercial and Residential AC Voltages

Class	Voltage	Type	Derivatives
High Voltage	138 kV	3ϕ	None
	69 kV	3ϕ	None
Medium Voltage	13.8 kV	3ϕ	None
	12.47 kV	3ϕ	None
	4.16 kV	3ϕ	None
Low Voltage	480 V	3ϕ	277 V, 1ϕ
	240V	1ϕ	120 V, 1ϕ
	208 V	3ϕ	120 V, 1ϕ
	120 V	1ϕ	None

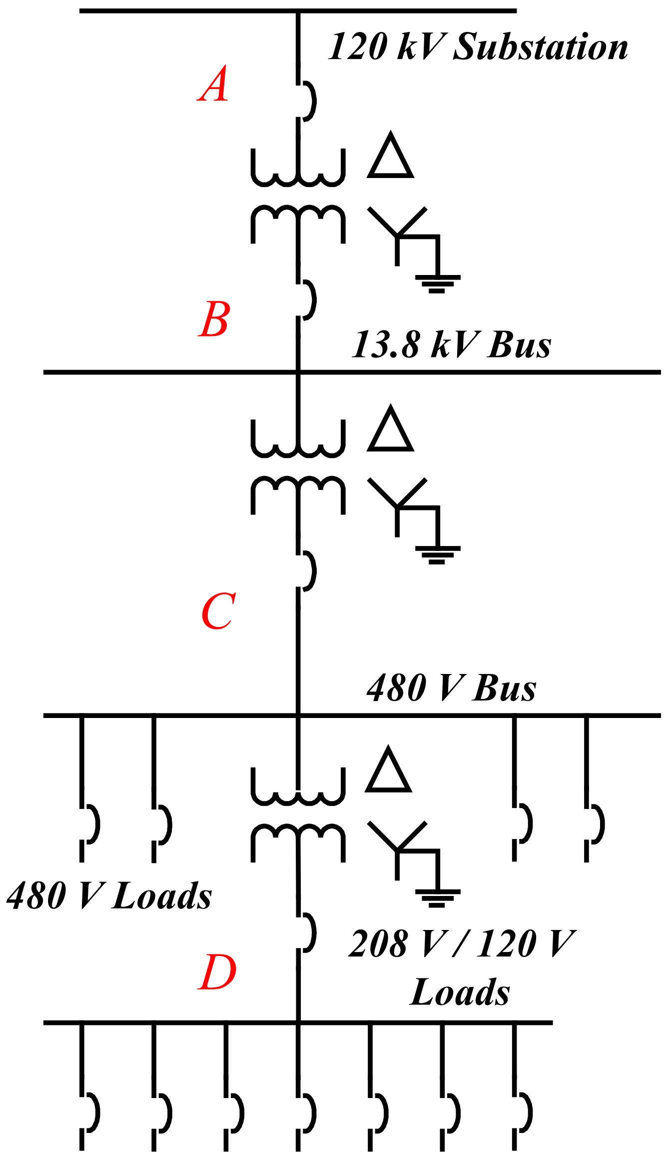
$$V_{LL}(RMS) = \sqrt{\frac{1}{T}\int_0^T v_{LL}^2(t)dt}$$

Why Mentioned Here

- Because the power supplies will interface to the AC line
- Because all AC power equipment (generators, motors, transformers and chokes) impedances are expressed in %
- Because line limitations (short-circuit currents, arc flash, V droop, transients, harmonics) must be considered. These effects are usually calculated in the per unit system

Why Used

- To make quantities and values convenient and manageable
- To put quantities on a single per phase or 3-phase basis
- To avoid having to remember to correct for transformer turns ratios, reflected voltages, current and impedances
- No worries about delta or wye configurations



Example - various locations on one-line diagram

Single phase:

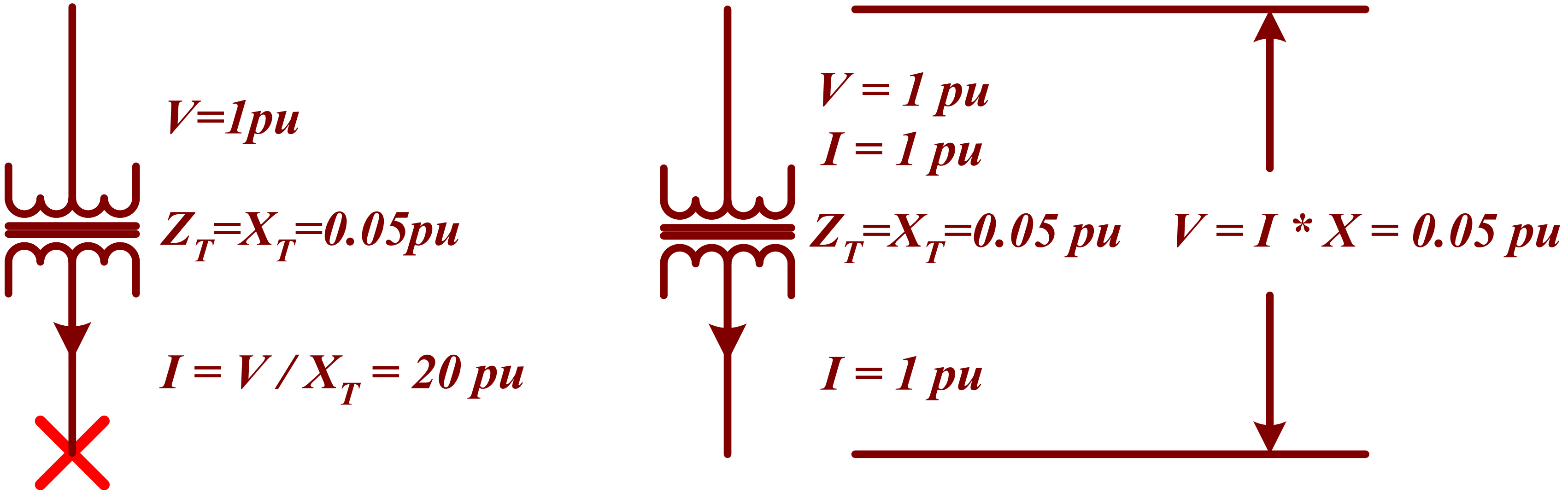
- *Power base:* $(S_{1\phi}, P_{1\phi}, Q_{1\phi})$ ~kVA
- *Voltage base:* $V_{1\phi} = V_{LN}$ ~kV
- *Current base:* $I_{1\phi} = S_{1\phi}/V_{1\phi}$ ~A
- *Impedance base:* $Z_{1\phi} = V_{1\phi}/I_{1\phi} = V_{1\phi}/\left(S_{1\phi}/V_{1\phi}\right) = V_{1\phi}^2/S_{1\phi}$ ~kΩ

Three phase:

- *Power base:* $S_{3\phi} = 3S_{1\phi}$
- *Voltage base:* $V_{3\phi} = V_{LL} = \sqrt{3}V_{LN} = \sqrt{3}V_{1\phi}$ ~kV
- *Current base:* $I_{3\phi} = S_{3\phi}/3V_{LN} = S_{3\phi}/\left(\sqrt{3}V_{LL}\right) = S_{3\phi}/(\sqrt{3}V_{LL}) = I_{1\phi}$
- *Impedance base:* $Z_{3\phi} = V_{3\phi}^2/S_{3\phi} = \left(\sqrt{3}V_{1\phi}\right)^2/\left(3S_{1\phi}\right) = V_{1\phi}^2/S_{1\phi} = Z_{1\phi}$

A transformer impedance of 5% means:

- The short circuit current is 20X rated full load input / output
- The voltage drop across the transformer at full load is 5% of rated



The Per Unit Calculation System

- *The bases of all devices in a system may not be all the same*
  - *If you design a new system, they likely will be*
  - *However, if a* 124.7 kV: 12.47 kV, 10000 kVA *transformer fails you can replace it with a spare* 139 kV: 13.9 kV, 15000 kVA *transformer*
    - *Has the same turns ratio*
    - *Will support the required voltage and power requirements*
- *If the bases of the devices change, one needs to transform the given* p.u. *in the original basis to the* p.u. *in the new basis.*
  - p.u.= *actual value / Base value*
- *Requirements on bases:*
  - *Turns ratios across transformers must be preserved*
  - *Actual impedances must be preserved*

The Per Unit Calculation System

- *Voltage, current relation in Per Unit:* $I = S/V$

  - *Since we need to maintain the turns ratio for both* V *and* I *at each transformer, and they are inverses of each other, we need a single, uniform power base* (S) *throughout the system.*

- *Impedance transformation:*

$$Z_{pu} = \frac{Z_{actual}}{Z_{base}} \Rightarrow Z_{actual} = Z_{pu} Z_{base}$$

$$Z_{pu-new} Z_{base-new} = Z_{pu-given} Z_{base-given}$$

$$Z_{pu-new} = Z_{pu-given} \frac{Z_{base-given}}{Z_{base-new}}$$

$$Z_{pu-new} = Z_{pu-given} \frac{\left(\text{Base kV}_{\text{given}}\right)^2}{\text{Base kVA}_{\text{given}}} \frac{\text{Base kVA}_{\text{new}}}{(\text{Base kV}_{\text{new}})^2}$$

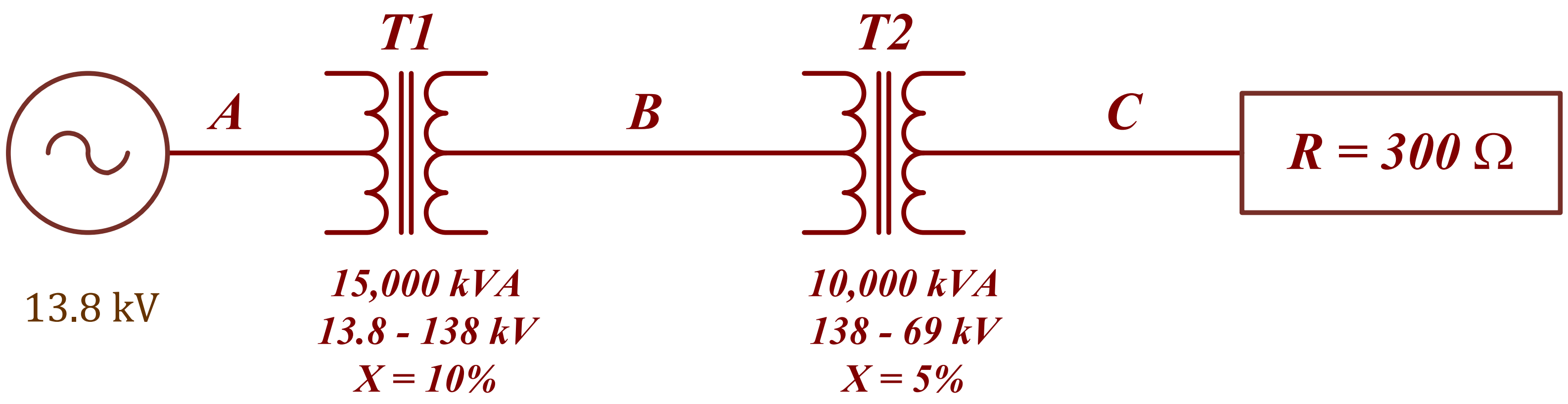
$$Z_{pu-new} = Z_{pu-given} \left(\frac{\text{Base kV}_{\text{given}}}{\text{Base kV}_{\text{new}}}\right)^2 \frac{\text{Base kVA}_{\text{new}}}{\text{Base kVA}_{\text{given}}}$$

Choose the system and base that yield the most convenient numbers and calculations!

The Per Unit Calculation System

Establish Configuration, then Power, Voltage, Current and Impedance Bases			
Base	**Per ϕ Phase**	**3 Phase**	**Notes**
S,P,Q	= Base kVA	= Base kVA = 3* per ϕ Base kVA	One power base must be used throughout
V	= Base kV (L-N)	= Base kV (L-L)	V Base location dependent
I	= Base kVA / Base kV	= Base kVA / √3Base kV	I Base location dependent
Z	= (Base kV) ^{2} / Base kVA	= (Base kV) ^{2} / Base kVA	Z Base location dependent Z Base phase independent perϕ Z Base = 3ϕ Z Base

Impedance Transformations – 1ϕ Example to Calculate Line Currents



Calculate the impedances of each transformer (referred to their primaries)

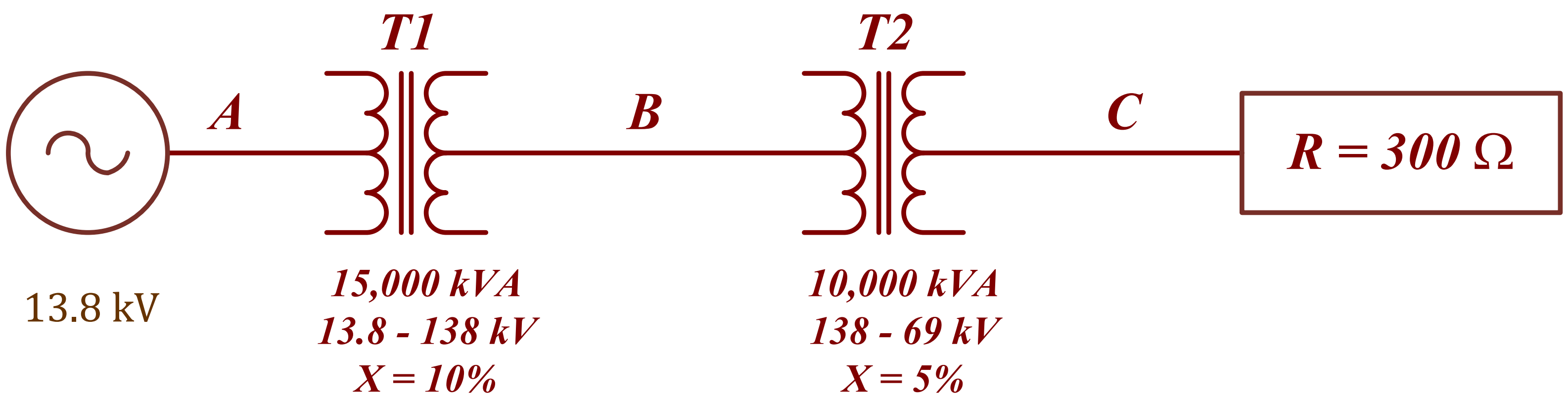
- *T1:* $X_{1P} = 0.10 \cdot V_{1P}^2 / S_{1P} = 0.10 \cdot 13.8^2/15 = j1.270\ \Omega$
- *T2:* $X_{2P} = 0.05 \cdot V_{2P}^2 / S_{2P} = 0.05 \cdot 138^2/10 = j95.22\ \Omega$

Transform all of the impedances upstream to section A

$$Z_A = j1.270 + j\left(\frac{13.8}{138}\right)^2 95.22 + \left(\frac{13.8}{138}\frac{138}{69}\right)^2 300 = (12 + j2.222)\ \Omega$$

Calculate currents

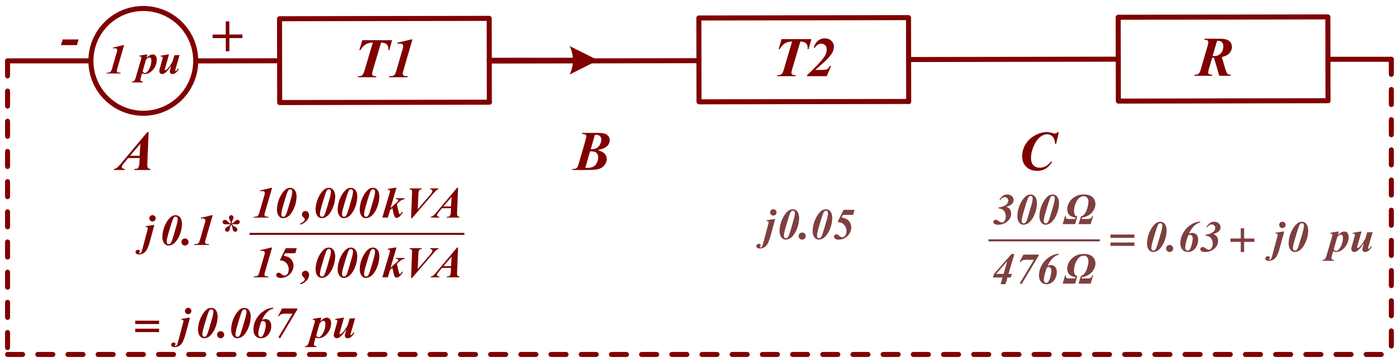
$$I_A = \frac{13800}{12 + j2.222} = 1131\angle -10.5°;\ \ |I_B| = 113.1;\ \ |I_C| = 226.2$$



Establish Bases – S constant throughout; V's preserve turns ratios

Section A	Section B	Section C
$Base\ S = 10000\ \text{kVA}$	$Base\ S = 10000\ \text{kVA}$	$Base\ S = 10000\ \text{kVA}$
$Base\ V = 13.8\ \text{kV}$	$Base\ V = 138\ \text{kV}$	$Base\ V = 69\ \text{kV}$
$Base\ I = \frac{S}{V} = \frac{10000\ \text{kVA}}{13.8\ \text{kV}} = 725\ \text{A}$	$Base\ I = \frac{S}{V} = \frac{10000\ \text{kVA}}{138\ \text{kV}} = 72.5\ \text{A}$	$Base\ I = \frac{S}{V} = \frac{10000\ \text{kVA}}{69\ \text{kV}} = 145\ \text{A}$
$Base\ Z = \frac{V^2}{S} = \frac{(13.8\ \text{kV})^2}{10000\ \text{kVA}} = 19\ \Omega$	$Base\ Z = \frac{V^2}{S} = \frac{(138\ \text{kV})^2}{10000\ \text{kVA}} = 1900\ \Omega$	$Base\ Z = \frac{V^2}{S} = \frac{(69\ \text{kV})^2}{10000\ \text{kVA}} = 476\ \Omega$

Obtain pu values

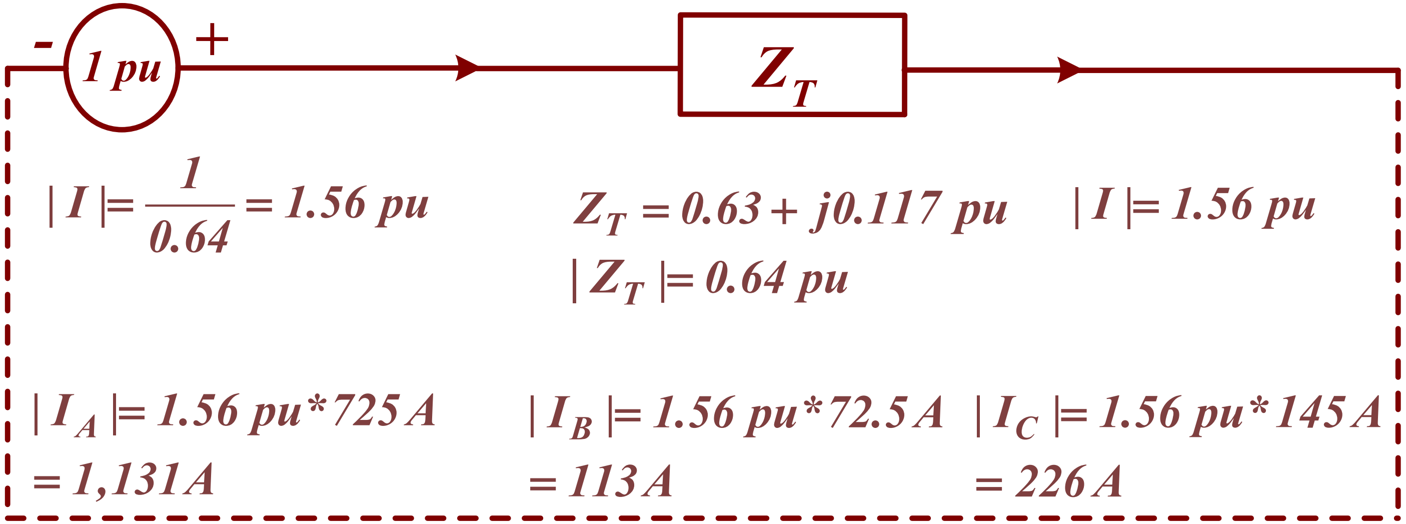


$$j0.1 * \frac{10{,}000kVA}{15{,}000kVA} = j0.067\ pu$$

$$j0.05$$

$$\frac{300\Omega}{476\Omega} = 0.63 + j0\ pu$$

Combine impedances – Solve for I



$$|I| = \frac{1}{0.64} = 1.56\ pu$$

$$Z_T = 0.63 + j0.117\ pu$$

$$|Z_T| = 0.64\ pu$$

$$|I| = 1.56\ pu$$

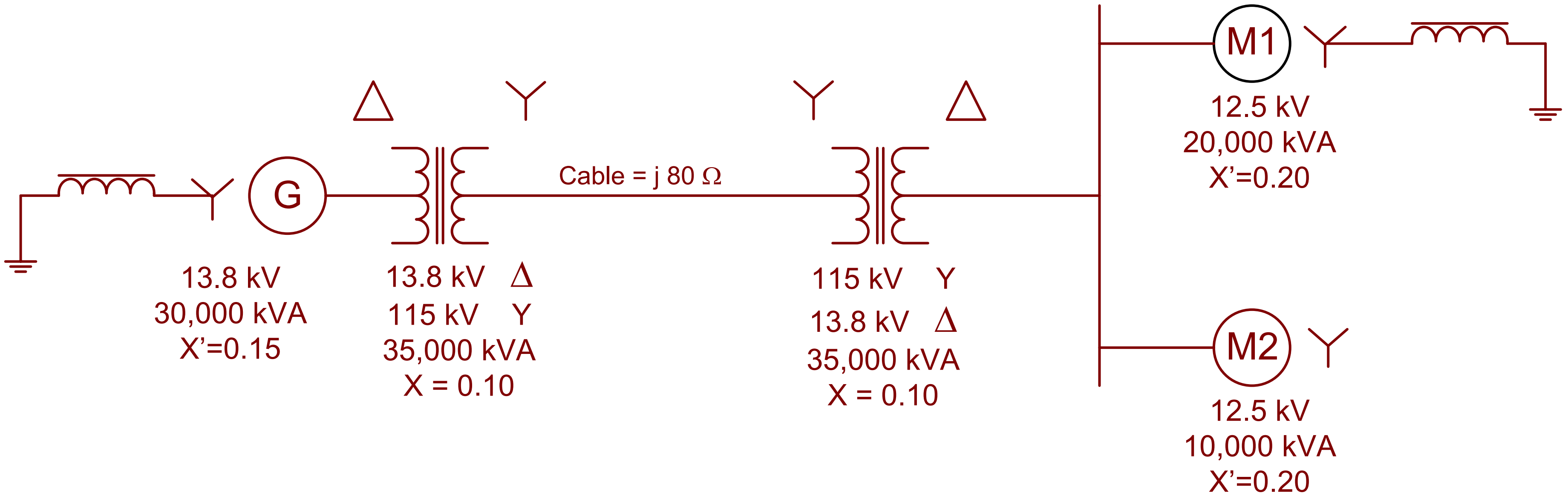
$$|I_A| = 1.56\ pu * 725A = 1{,}131A$$

$$|I_B| = 1.56\ pu * 72.5A = 113A$$

$$|I_C| = 1.56\ pu * 145A = 226A$$

Referring to the one-line diagram below, determine the line currents in the:

A. Generator B. Transmission Line C. M1 D. M2



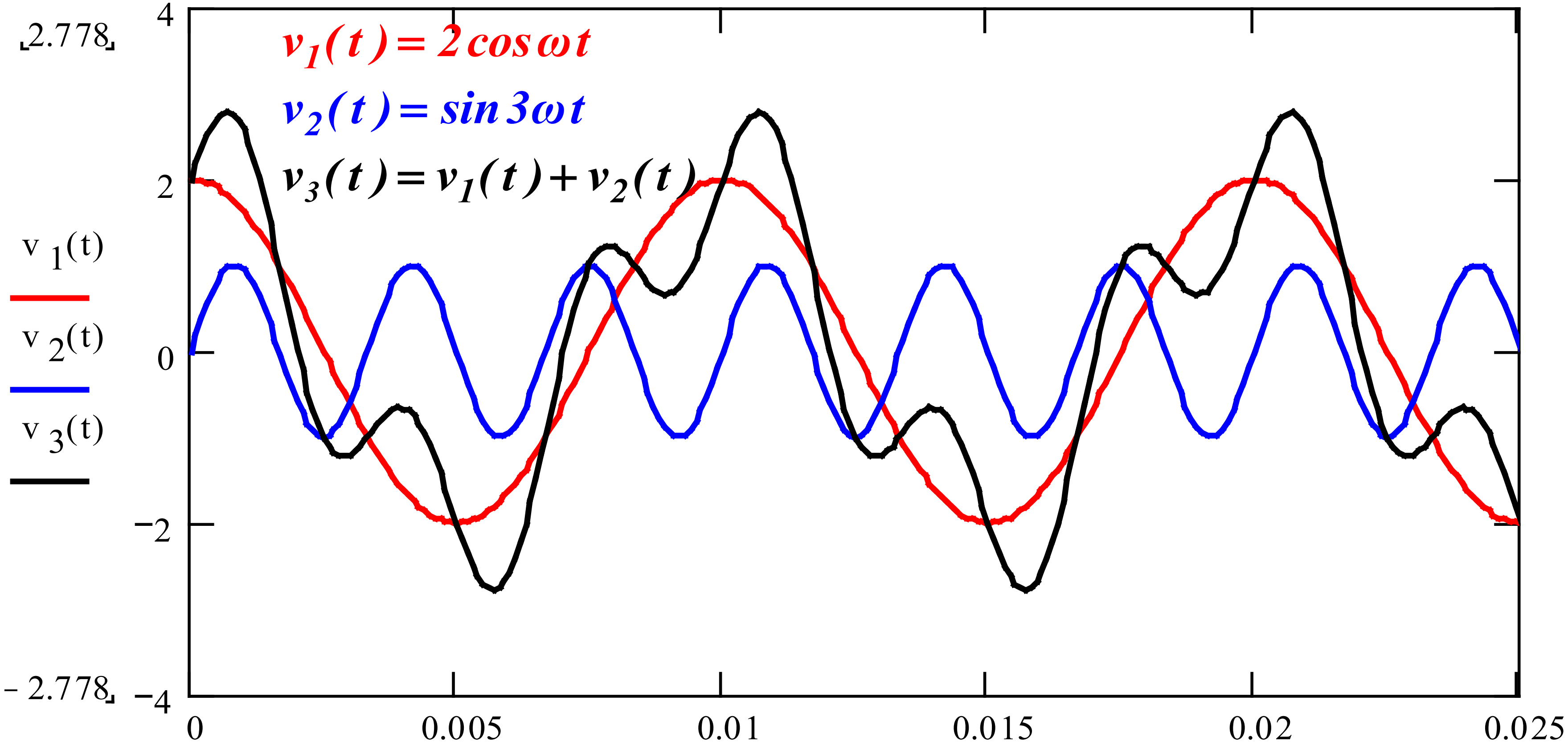
A 1000kVA, 12.47kV to 480V, 60Hz three-phase transformer has an impedance of 5%. Calculate:

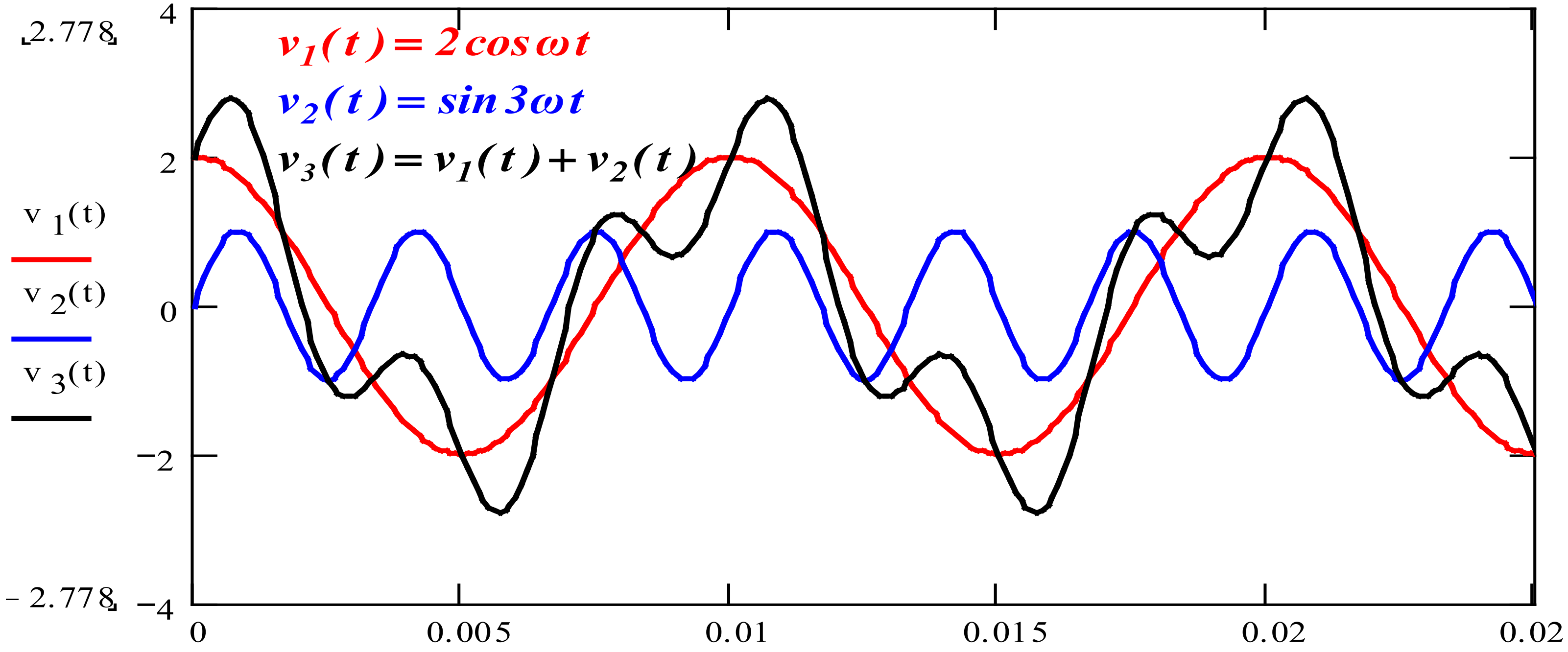
a. The actual impedance and leakage inductance referred to the primary winding

b. The actual impedance and leakage inductance referred to the secondary winding

c. The magnetizing inductance referred to the primary winding

- *Non-sinusoidal waves are complex and are composed of sine and cosine harmonics*
- *The harmonics are integral multiples of the fundamental frequency (1st harmonic) of the wave. The second harmonic is twice the fundamental frequency, the third harmonic is 3 X the fundamental frequency, etc.*

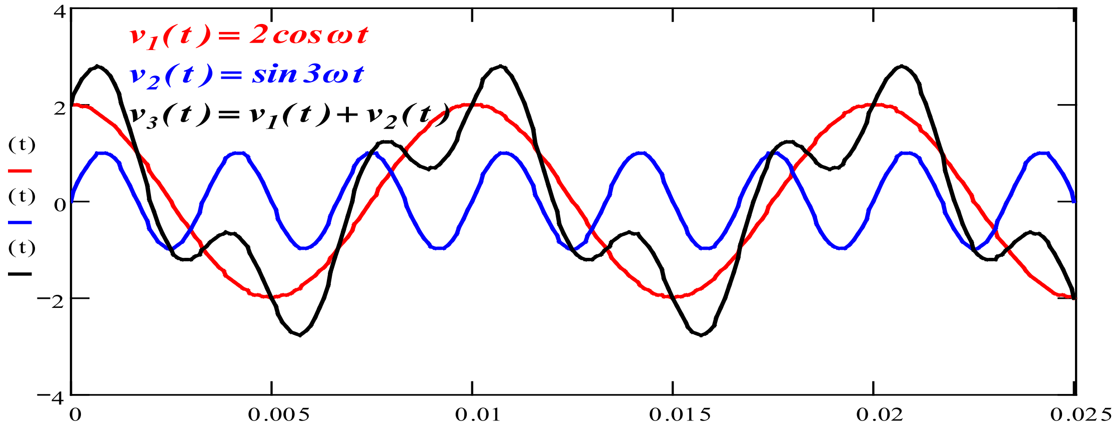




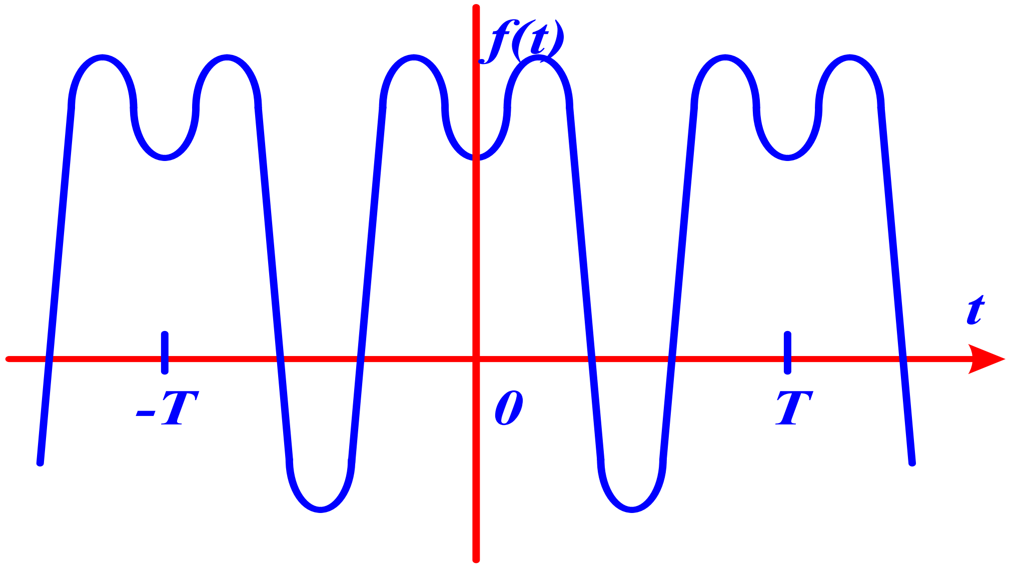
Trigonometric forms of the Fourier Series

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt \qquad a_k = \frac{2}{T}\int_0^T f(t)\cos k\omega t\,dt \qquad b_k = \frac{2}{T}\int_0^T f(t)\sin k\omega t\,dt$$

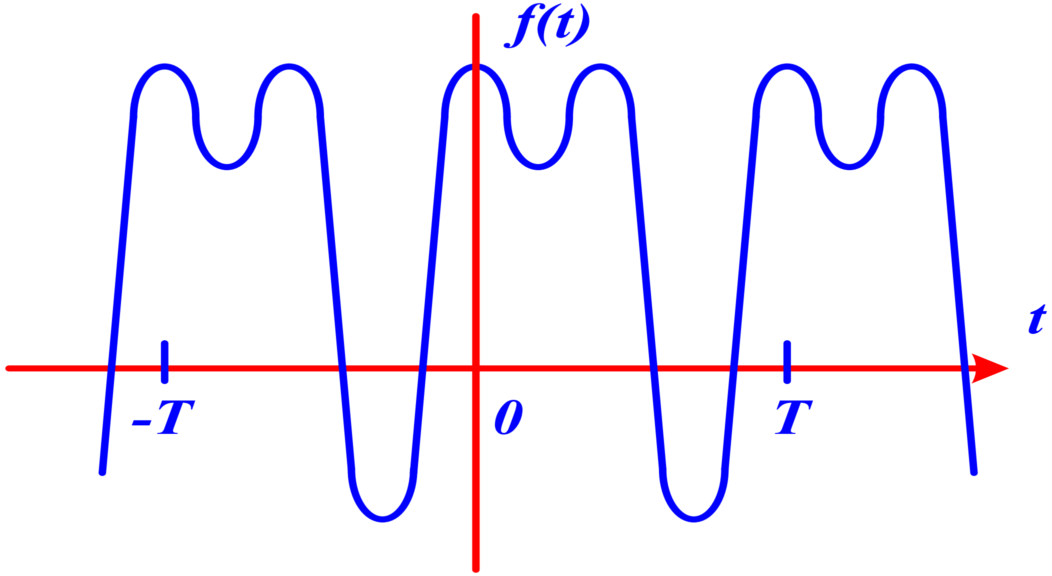
$$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos\frac{2\pi kt}{T} + b_k \sin\frac{2\pi kt}{T}$$



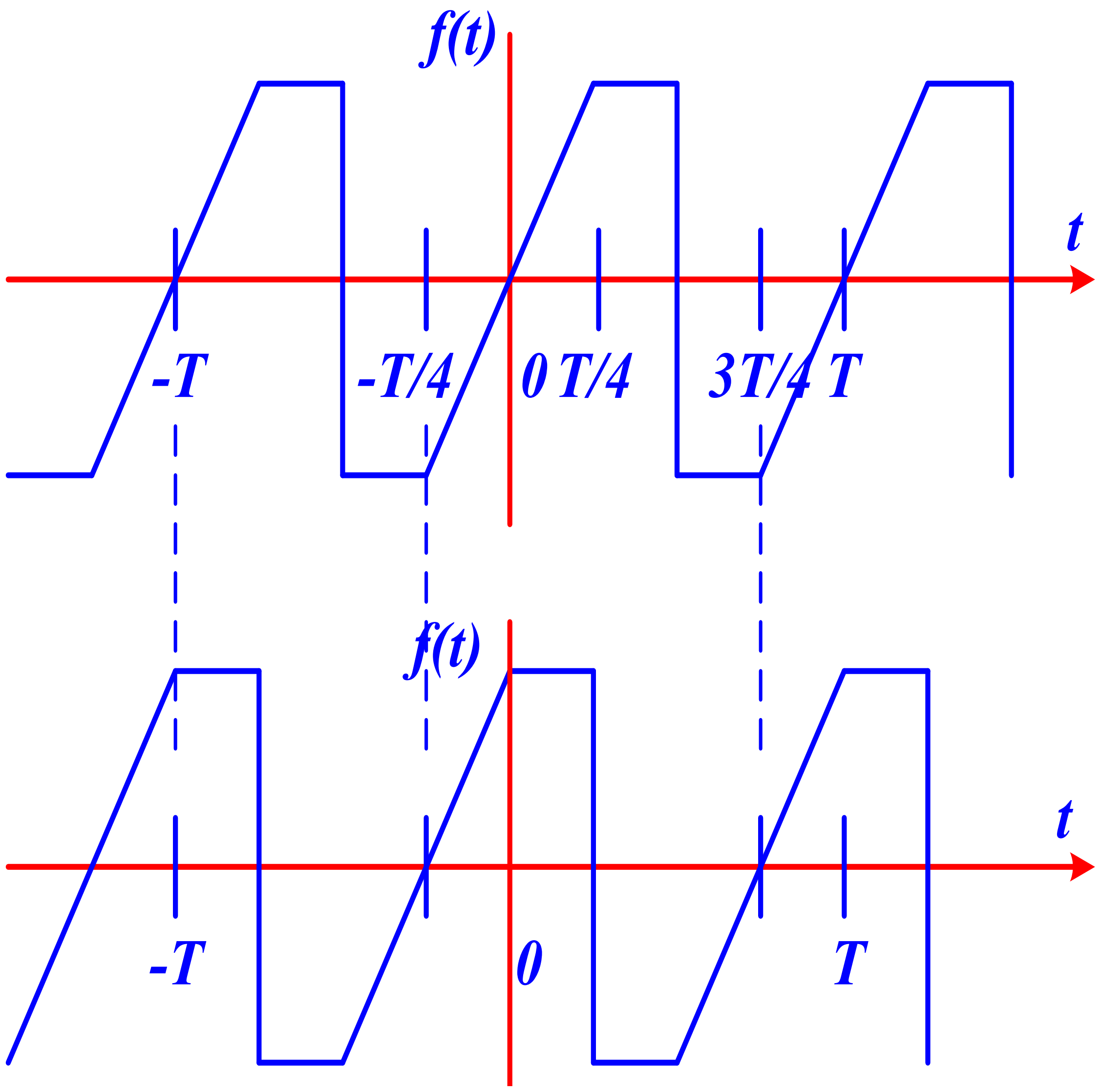
No symmetries		a_k *cosines,* b_k *sines, for all k*	*May or may not have DC component*
Even function symmetry	$f(t) = f(-t)$	*Only* a_k *cosines for all k* $(b_k = 0)$	*Has DC component if no half-wave symmetry*
Odd function symmetry	$f(t) = -f(-t)$	*Only* b_k *sines for all k* $(a_k = 0)$	*No DC component*
Half-wave symmetry	$f(t) = -f\left(t - \frac{T}{2}\right)$	a_k *cosines,* b_k *sines, for odd k*	*No DC component*
Half-wave, even function symmetry	$f(t) = -f\left(t - \frac{T}{2}\right)$ $f(t) = f(-t)$	*Only* a_k *cosines for odd k* $(b_k = 0)$	*No DC component*
Half-wave, odd function symmetry	$f(t) = -f\left(t - \frac{T}{2}\right)$ $f(t) = -f(-t)$	*Only* b_k *sines for odd k* $(a_k = 0)$	*No DC component*



- $f(t) = f(-t)$ even function
- $f(t) \neq -f(t-T/2)$
- No half-wave symmetry
- DC component, a_o
- No sine terms, only cosines, all ks
- $a_k = \frac{2}{T}\int_0^T f(t)\cos k\omega_o t\,dt$



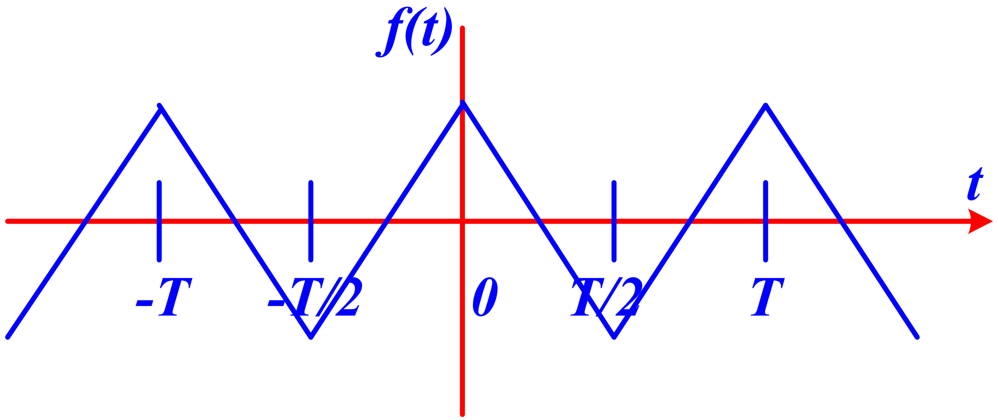
- No even or odd function symmetry
- No half-wave symmetry
- Have sine and cosine terms, all k
- DC component, a_o
- a_o, a_k , b_k terms



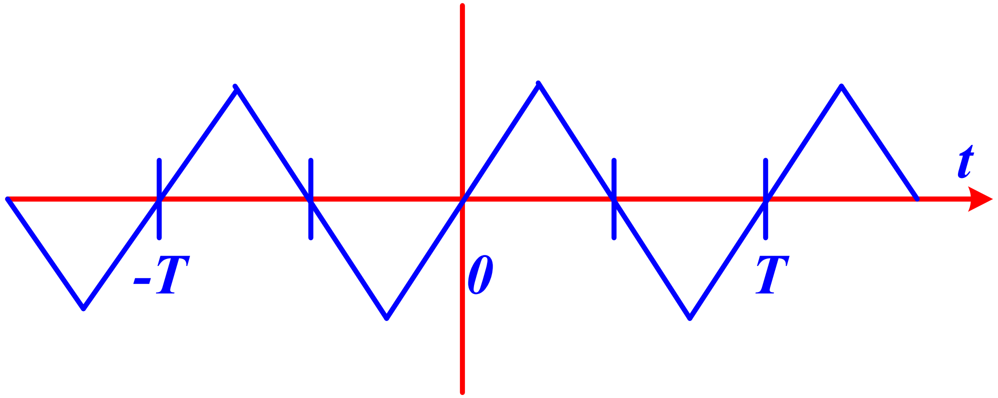
- $f(t)=-f(-t)$ odd function
- No half-wave symmetry
- No DC component
- Just b_k sines, all k
- $b_k=\frac{2}{T}\int_0^T f(t)\sin k\omega_o t\,dt$

- No symmetries
- No DC component
- Have a_k cosine and b_k sine terms, all k

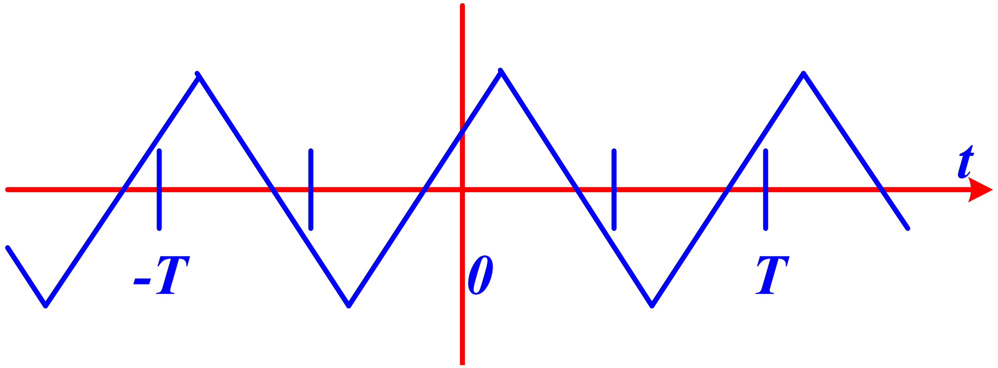
Fourier Series - Examples of Periodic Waveforms



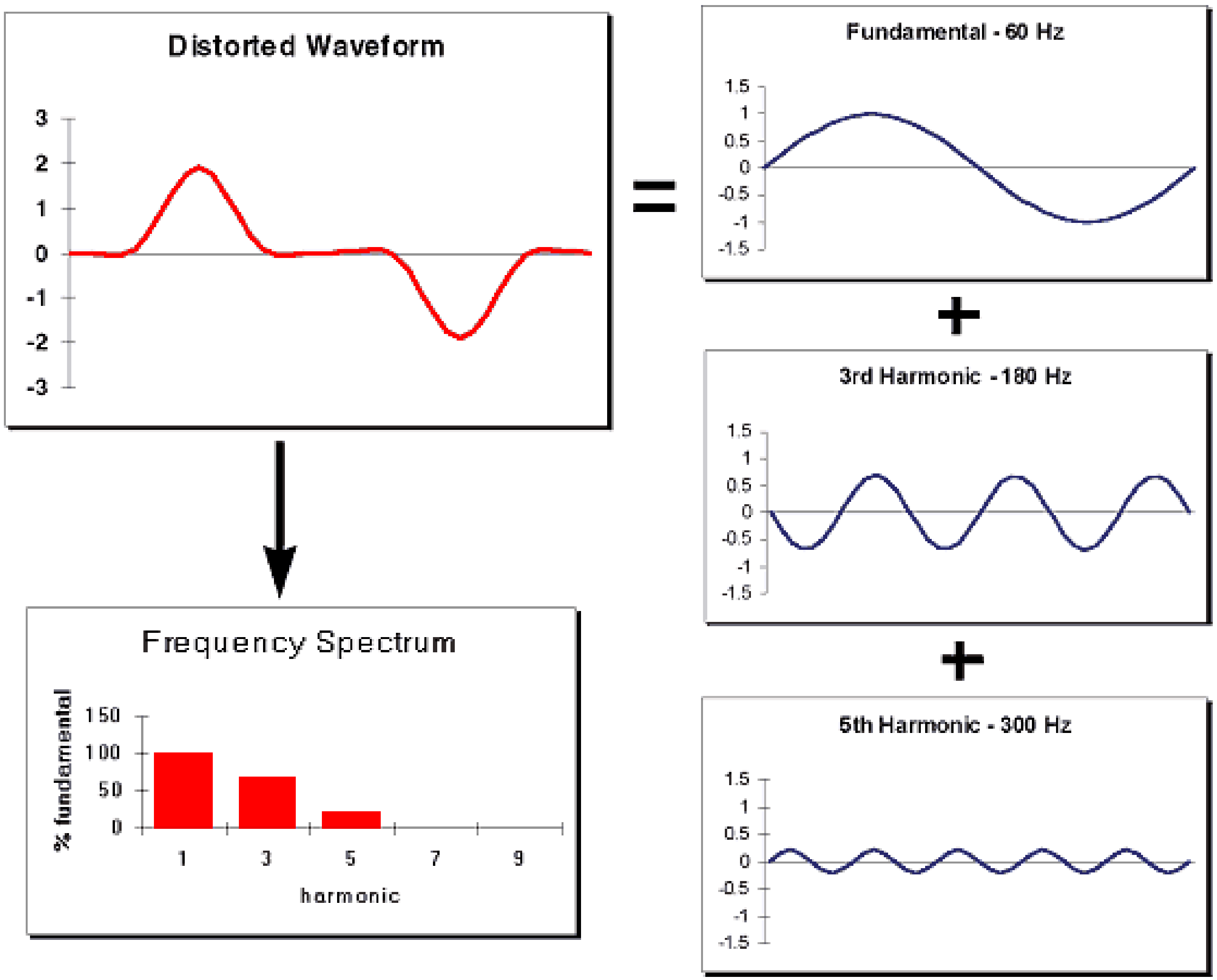
- $f(t)=f(-t)$ even function
- Half-wave symmetry
- No DC component
- Have a_k for odd ks



- $f(t)=-f(-t)$
- Half wave symmetry
- No DC component
- Have b_k for odd ks



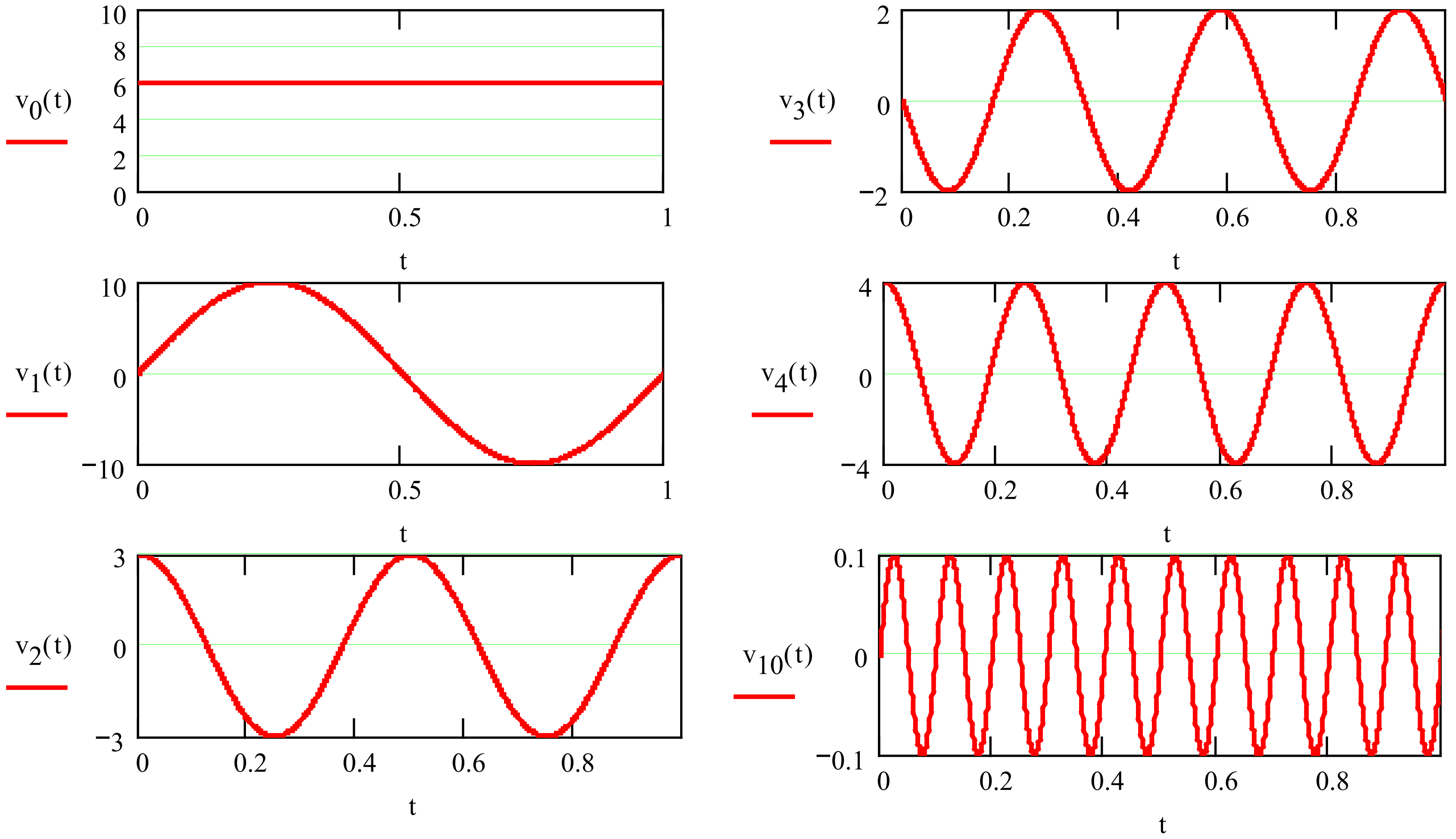
- No even or odd symmetry
- Half wave symmetry
- No DC component
- Have a_k , b_k for odd ks



Distorted Waveform
3
2
1
0
-1
-2
-3
=
Fundamental - 60 Hz
1.5
1
0.5
0
-0.5
-1
-1.5
+
3rd Harmonic - 180 Hz
1.5
1
0.5
0
-0.5
-1
-1.5
+
5th Harmonic - 300 Hz
1.5
1
0.5
0
-0.5
-1
-1.5
Frequency Spectrum
% fundamental
150
100
50
0
1
3
5
7
9
harmonic

Fourier Series - Homework Problem #4

A waveform v(t) was analyzed and found to consist of 6 components as shown here.



*a. Write the mathematical expression for each component in terms of ω=(2*π)/T*

b. Show the harmonic content graphically by plotting the frequency spectrum

c. Give the numerical result of

$$b_3 = \frac{2}{T}\int_0^T v(t)\sin 3\omega t\, dt;$$

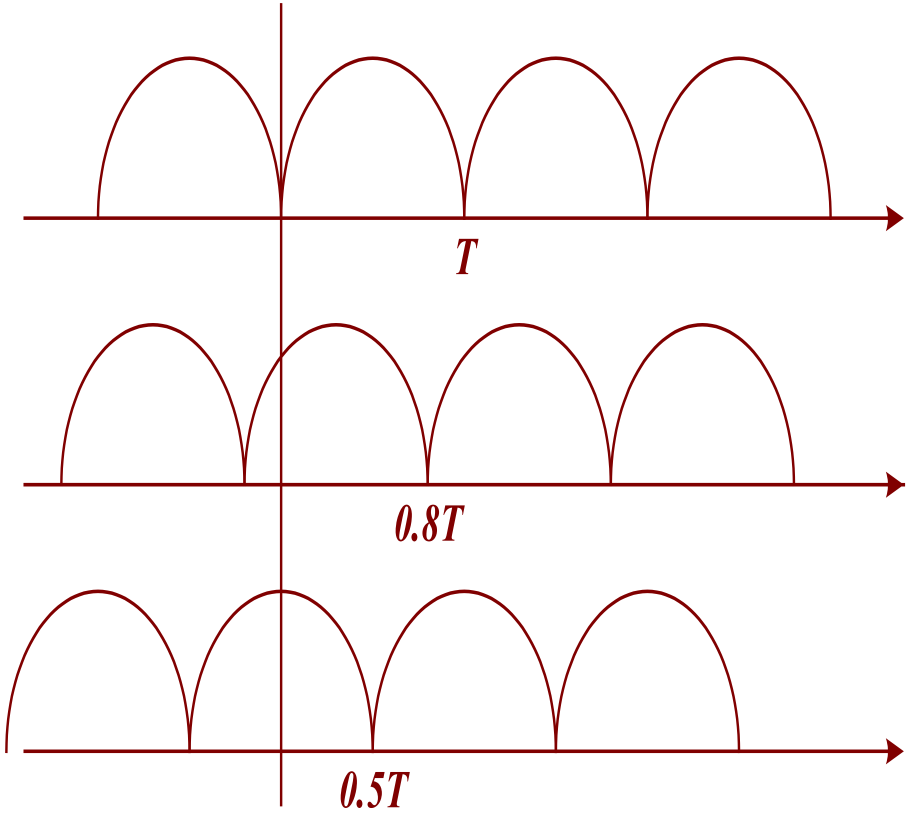
Hint: $\int \sin^2(3\omega t)\, dt = \frac{t}{2} - \frac{\sin(6\omega t)}{12\omega}$

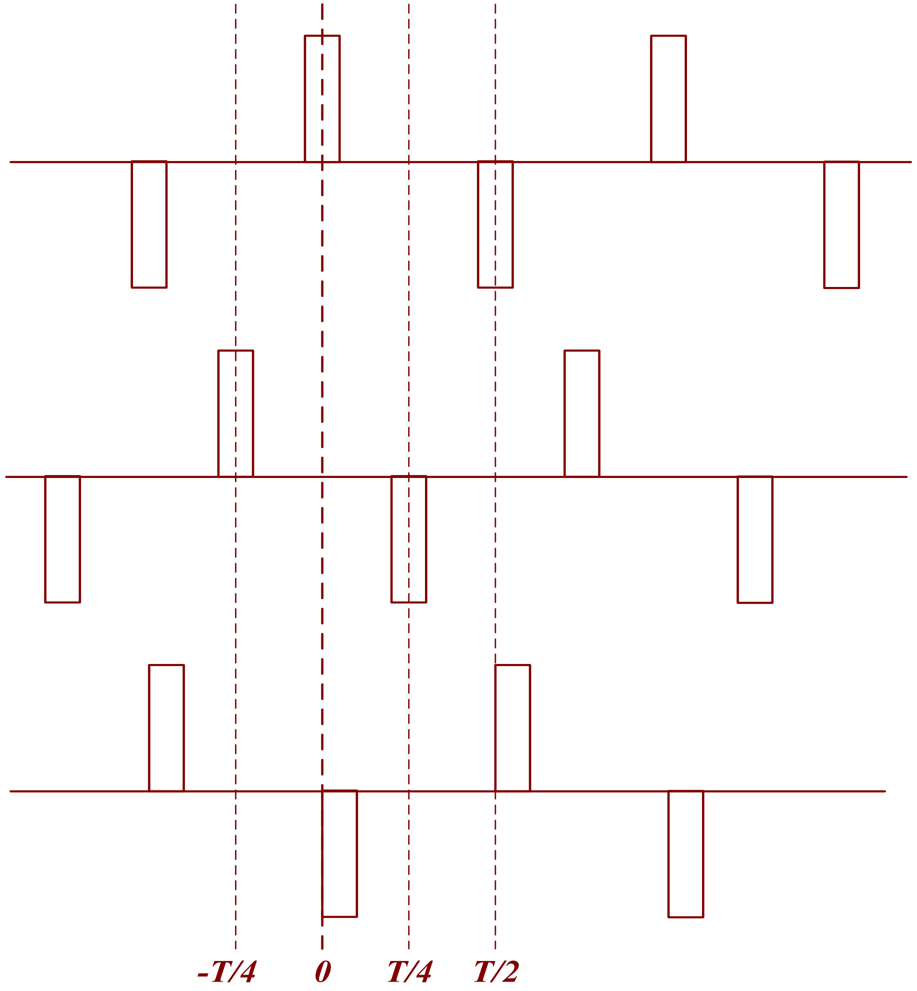
$$b_4 = \frac{2}{T}\int_0^T v(t)\sin 4\omega t\, dt;$$

Hint: $\int \cos(4\omega t)\sin(4\omega t)\, dt = \frac{\sin^2(4\omega t)}{8\omega}$

Where b_3 *and* b_4 *are from the results of Part a, above.*

Each waveform below can be written as a Fourier series. The result depends upon the choice of origin. For each of the 6 cases, state the type of symmetry present, non-zero coefficients and the expected harmonics.



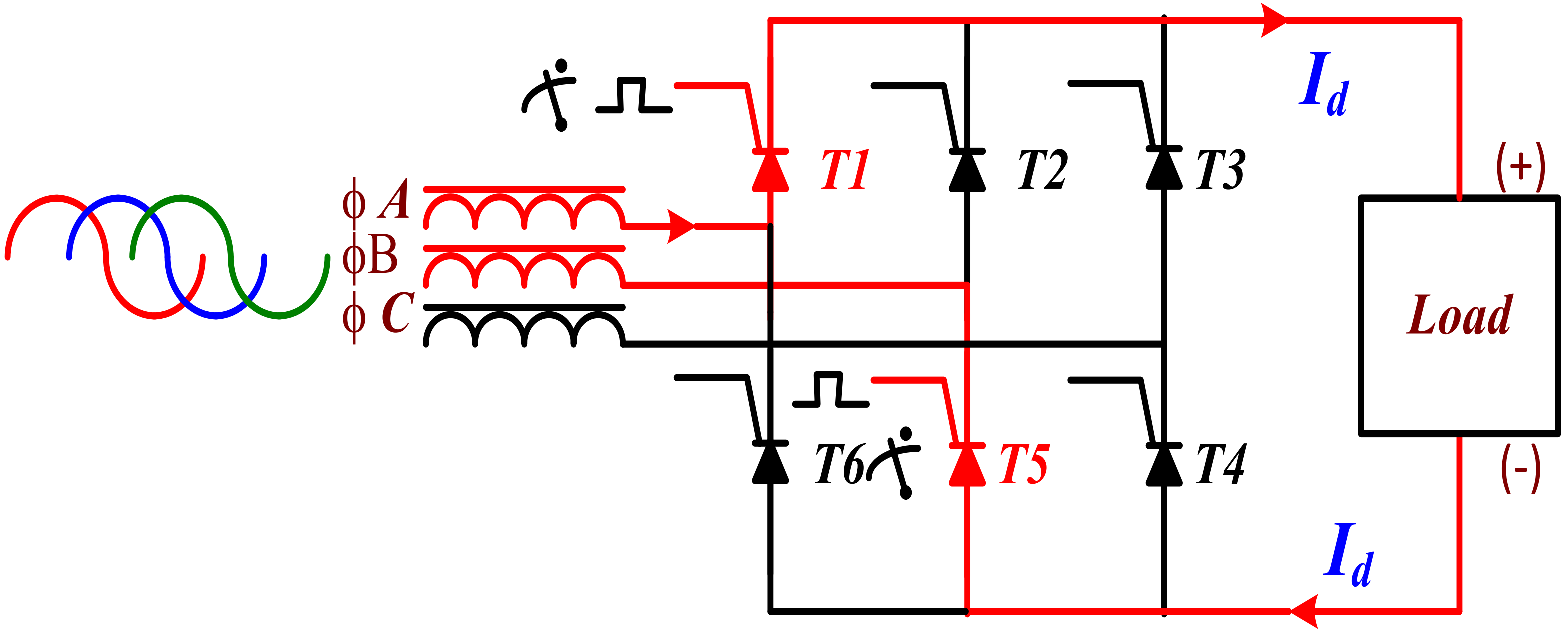


Signal Total Harmonic Distortion (THD): *The ratio of the square root of the summed squares of the amplitudes of all harmonic frequencies above the fundamental frequency to the fundamental frequency for voltage and/or current*

$$\mathrm{THD}_V = \frac{\sqrt{\sum_{i=2}^{\infty} V_i^2}}{V_1} \times 100\%$$

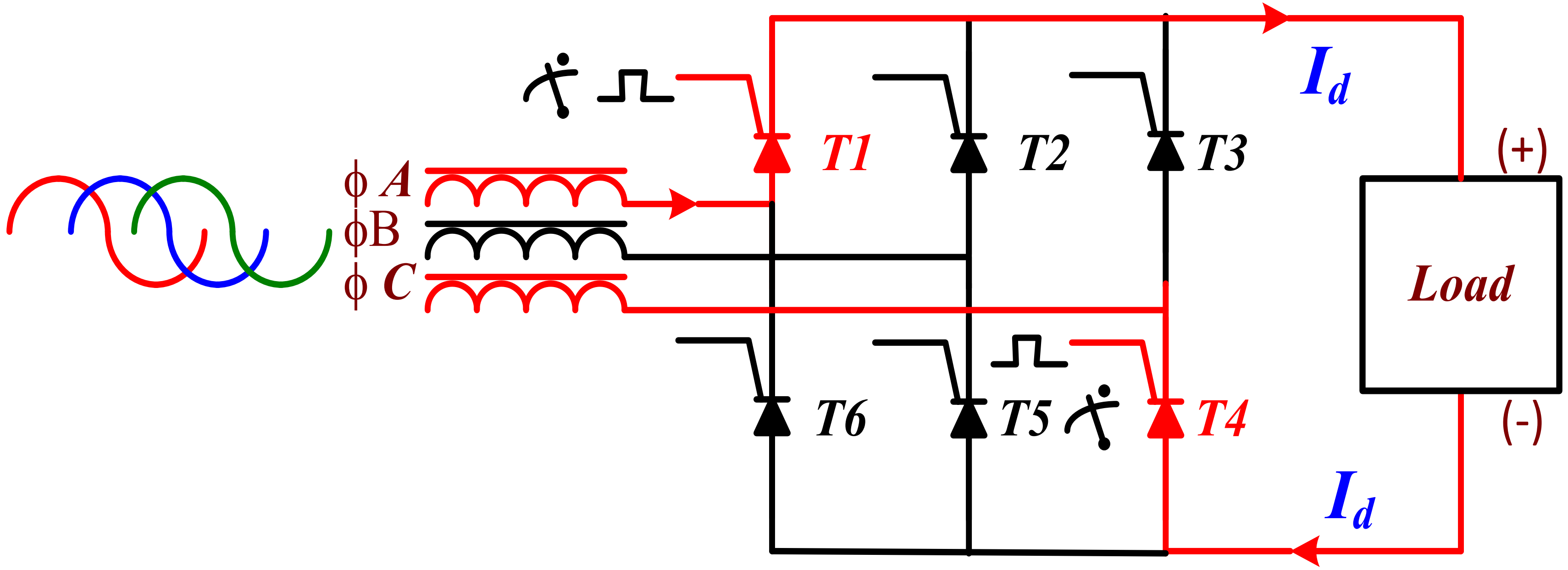
$$\mathrm{THD}_I = \frac{\sqrt{\sum_{i=2}^{\infty} I_i^2}}{I_1} \times 100\%$$

- *SCR or diode commutation*
- *Unbalanced 3-phase, non-linear loads*



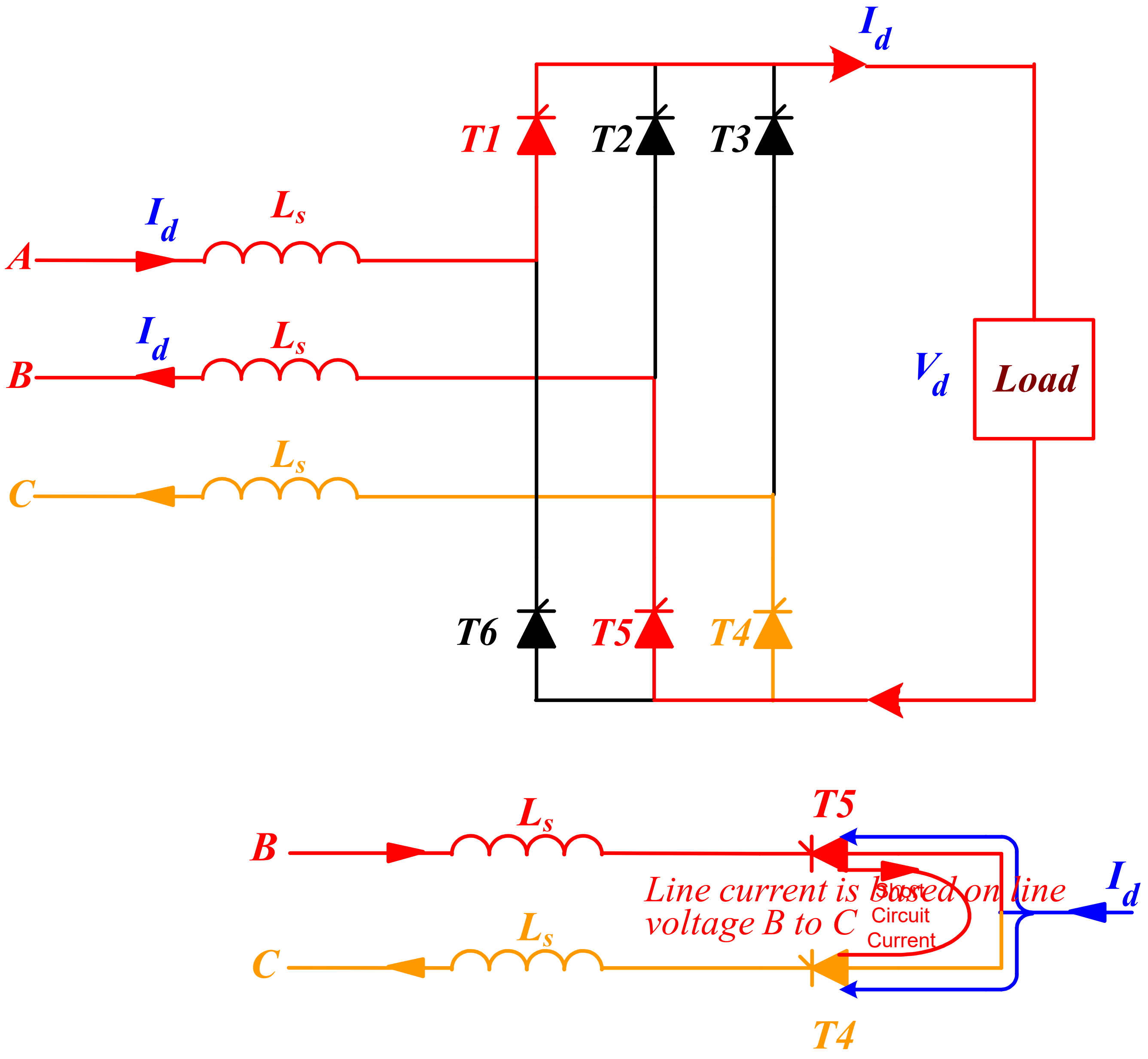
State 1: A-B (+) SCR s 1 – 5 On

SCR Commutation as Distortion Cause



State 2 : A-C (+), 5 off, 4 on, SCR s 1 – 4 On

SCR Commutation as Distortion Cause



SCR Commutation Voltage Drop

$$V_d = V_{do} - V_u$$

$$V_{Ls} = L_s \frac{di}{dt}$$

$$V_u = \frac{q}{\omega T}\int_{\alpha}^{\alpha+\mu} V_{Ls}\, d(\omega t) = \frac{q}{2\pi}\omega L_s \int_0^{I_d} di = \frac{q}{2\pi}\omega L_s I_d = q\, f\, L_s I_d$$

$$V_d = \frac{q\sqrt{2}}{2\pi} V_{LL} \cos\alpha - q\, f\, L_s I_d$$

Commutation voltage drop

V_d = reduced output, V_{do} = Theoretical output, V_u = commutation drop

V_{Ls} = Voltage drop due to line impedance, i = phase current

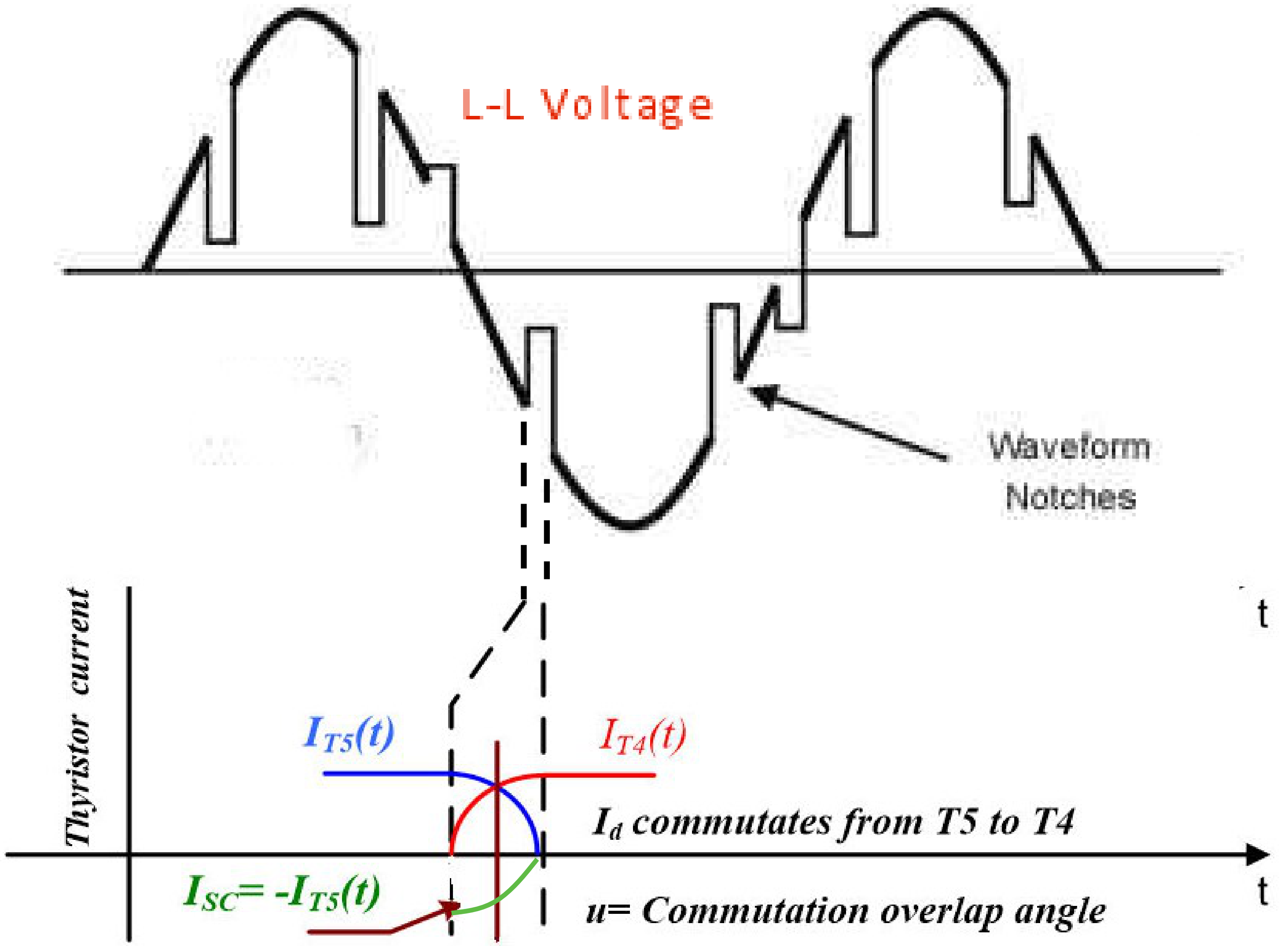
q = number of rectifier states, α = SCR gate trigger retard angle, μ = commutation overlap angle

ω = operating frequency in radians, f = frequency in Hz, I_d = Load current

Conclusions

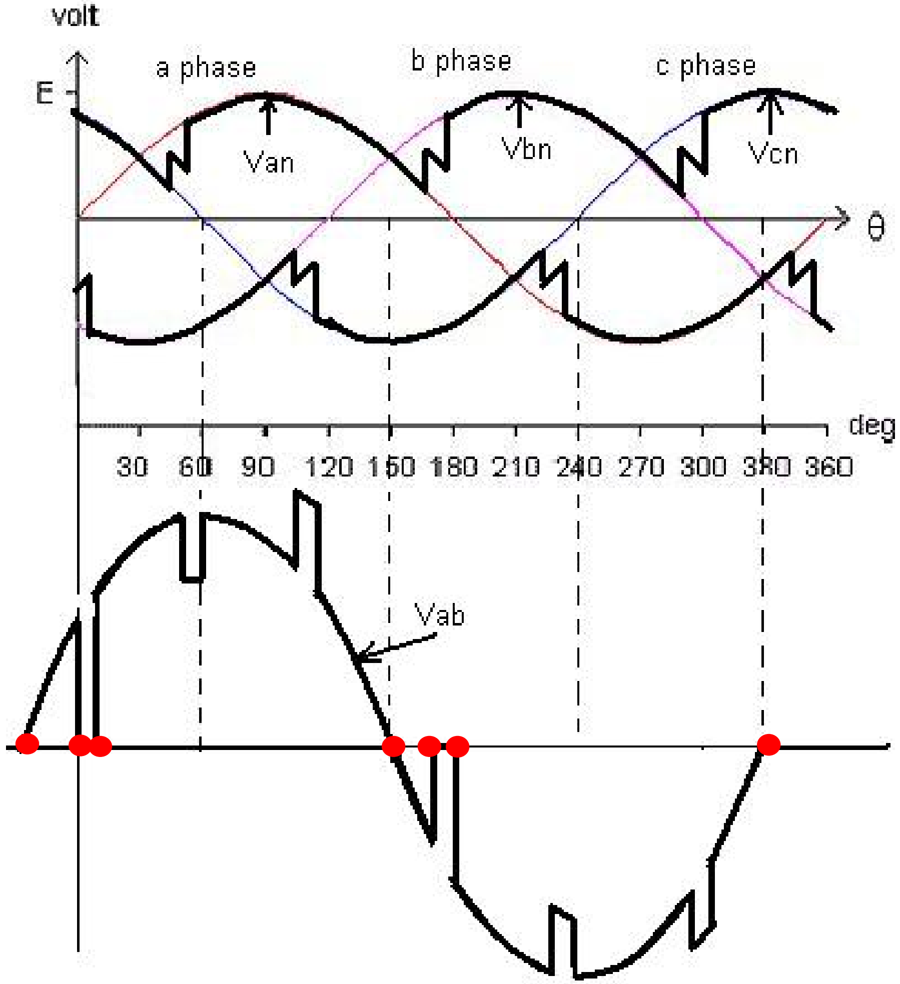
- The current commutation takes a finite commutation interval u.
- During the commutation interval, three SCRs conduct.
- Vu (and line voltage distortion) is directly proportional to the inductance of the input AC line or transformer and the DC current flowing in the load

SCR Commutation as Distortion Cause



SCR / diode commutation line notches:

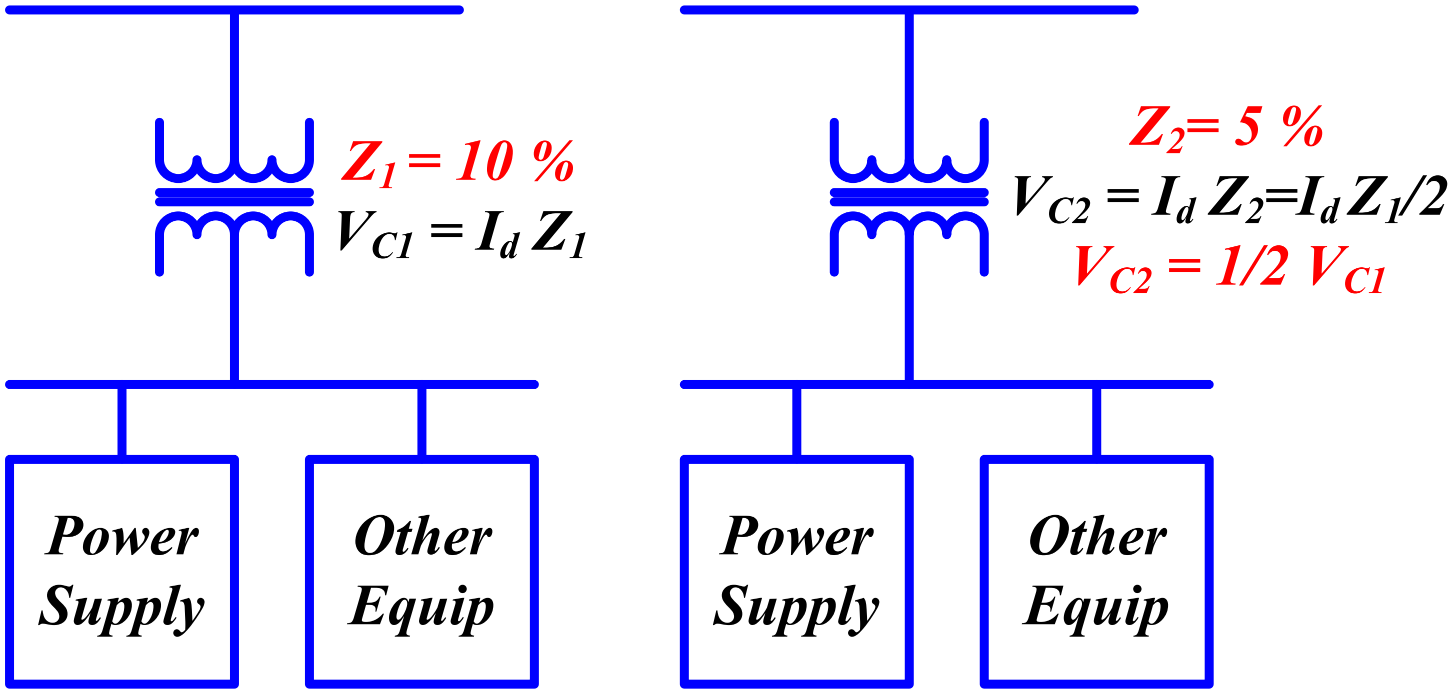
- *Are a source of line voltage distortion*
- *If deep enough, they cause extra zero crossovers in the line voltage. In 3 phase systems, instead of 2 zero crossovers per cycle, 6 zero crossovers can be experienced*
- *The extra zero crossovers can upset equipment timing. This can cause SCRs to trigger at the wrong time, damaging the power supply or cause false turn-on and damage to other equipment.*



line notch

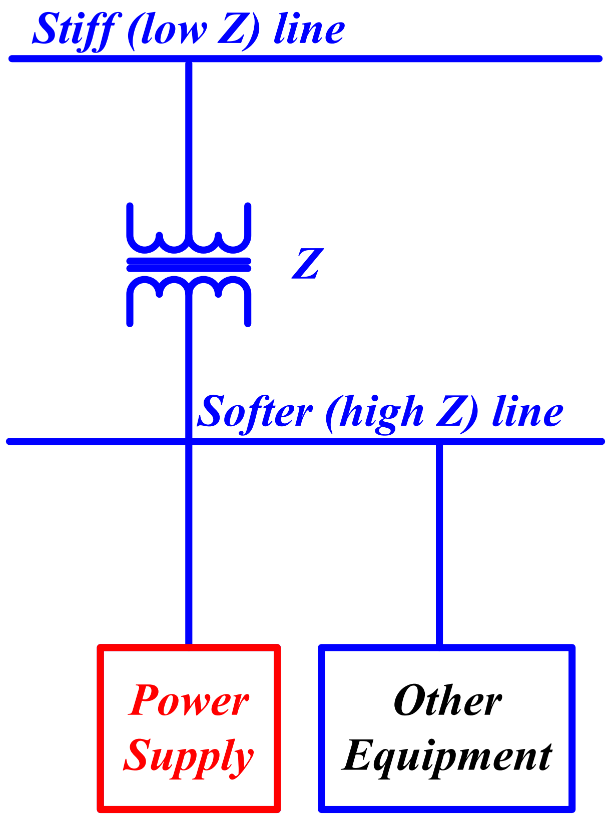
Reducing SCR commutation effects

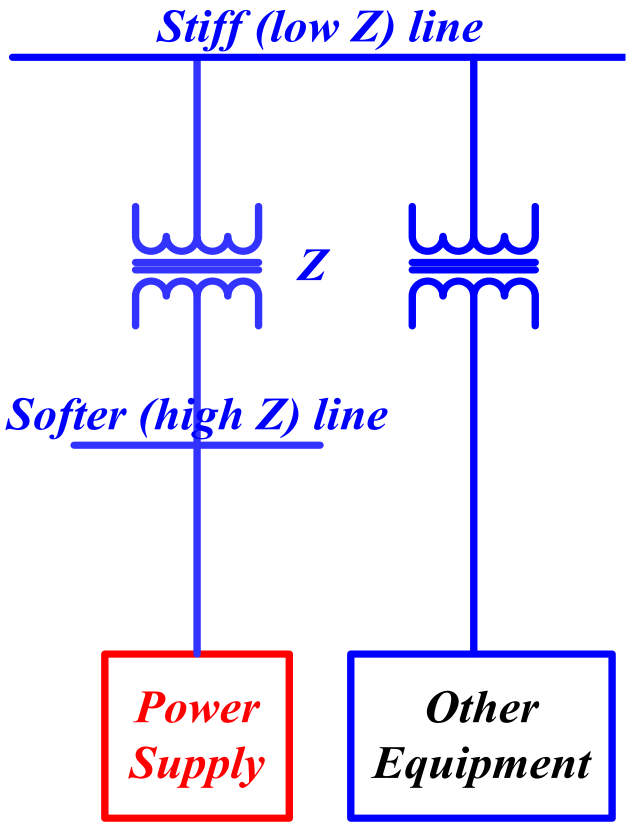
- Commutation notches (voltage drops) are directly proportional to system Z and DC load current. To reduce commutation notch depth, use a stiff (large, low Z) line.



Reducing SCR commutation effects on other equipment

- Isolate other equipment by placing them on another line





Australia	*AS/NZS 61000.3.6 , replaces AS 2279 - "Disturbances in Mains Supply Networks" and is compatible with IEEE 519 recommendations*
Britain	*G5/4 – 1 "Standard for Harmonic Control in Power Systems" which is compatible with IEEE 519 – 1992*
Europe	*International Electrotechnical Commission* *IEC 555 Series for harmonic current distortion limits for small devices (extended by IEC 1000 standards)* *Larger devices IEC61000-3-2, EN61000-3-2*
United States	*IEEE 519 – 1992 "Standard Practices and Requirements for Harmonic Control in Electrical Power Systems" .*

Table 10.2 Low Voltage System Classification And Distortion Limits

	Special Applications [1]	General Systems	Dedicated Systems [2]
THD (Voltage)	3%	5%	10%
Notch Depth	10%	20%	50%
Notch Area [3]	16,400 V - μ S	22,800 V - μ S	36,500 V - μ S

1. Airports and hospitals
2. Exclusive use converters
3. Multiply by V / 480 for other than 480 V systems

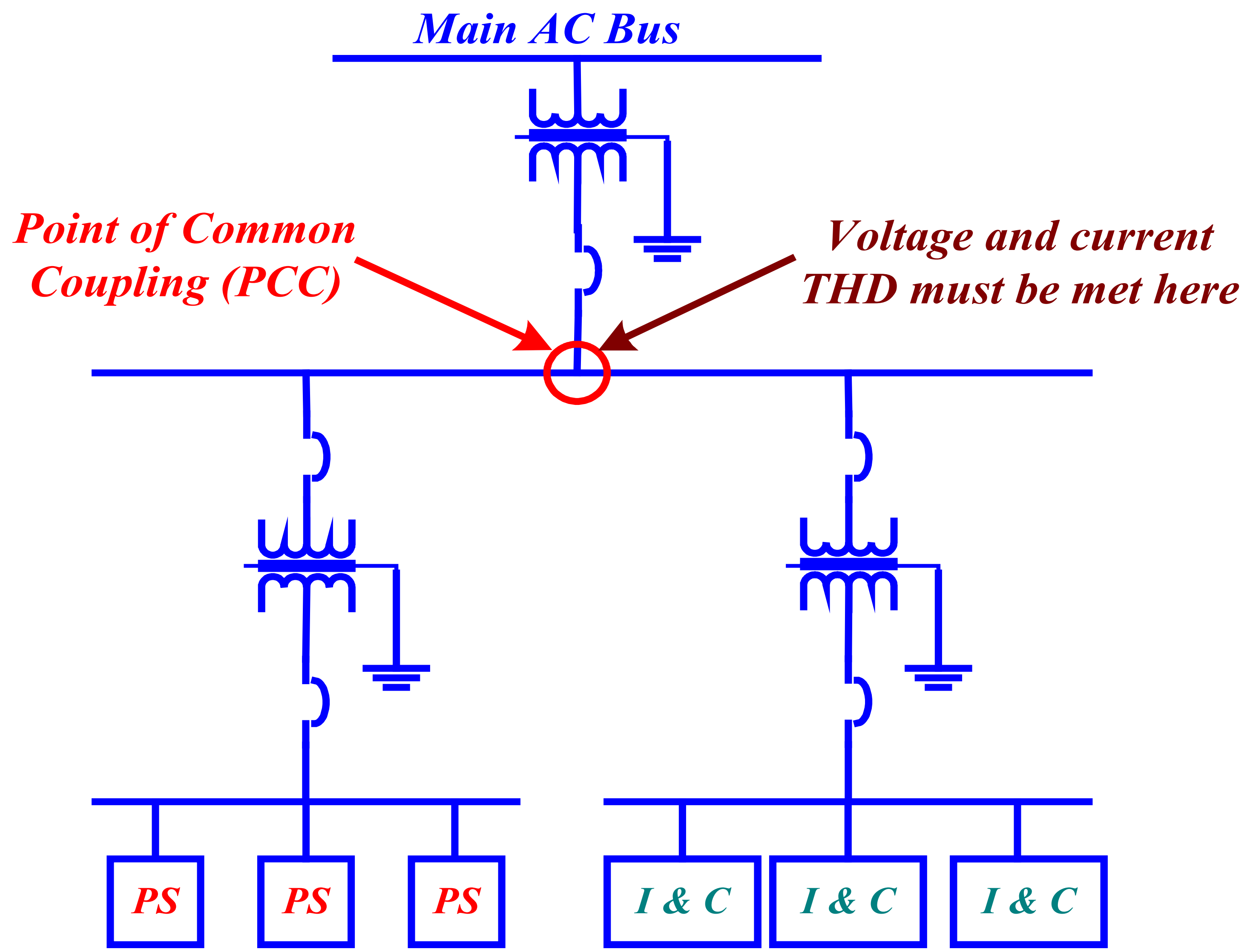
$$Example: \; 480V * \sqrt{2} = 678.8V \qquad 20\% \text{ notch depth} = 135.8V$$

$$\frac{22{,}800V * \mu S}{135.8V} = 168\mu S \qquad \frac{168\mu S}{16.6mS} \sim 1\% \text{ of 60Hz period}$$

SCR Commutation Effects - IEEE 519- 1992 Load Current Distortion Limits

General Distribution Systems – 120 V Through 69 kV	
I_{SC} / I_L	Maximum THD
< 20	5
20 < 50	8
50 < 100	12
100 < 1,000	15
> 1,000	20

1. I_{SC} = maximum short-circuit current at Point of Common Coupling (PCC)
2. I_L = maximum load current at PCC
3. I_{SC} / I_L = system short-circuit current capability to load current ratio


Main AC Bus
Point of Common Coupling (PCC)
Voltage and current THD must be met here
PS
PS
PS
I & C
I & C
I & C

Electromagnetic Interference (EMI) *is any electromagnetic disturbance that interrupts, obstructs, or otherwise degrades or limits the effective performance of electronics/electrical devices, equipment or systems. Sometimes also referred to as radio frequency interference (RFI)*

Electromagnetic Compatibility (EMC) *describes how an electronic device will behave in a "real world" setting of EMI*

Broadband Interference *This type of interference usually exhibits energy over a wide frequency range and is generally a result of sudden changes in voltage or current. It is normally measured in decibels above one micro-volt (or micro-ampere) per megahertz* $dB\,\mu V / MHz$ *or* $dB\,\mu A / MHz$

Narrowband Interference *has its spectral energy confined to a specific frequency or frequencies. This type of interference is usually produced by a circuit which contains energy only at the frequency of oscillation and harmonics of that frequency. It is normally measured in "decibels above one micro-volt (or micro-ampere)", e.g.,* $dB\,\mu V$ *or* $dB\,\mu A$*.*

Five Types of EMI

- Conducted Emissions (CE) - the EMI emitted into lines and connections by an electronic device. Of particular interest is the EMI conducted onto the AC input power lines
- Conducted Susceptibility (CS) - the EMI present on lines and connections (e.g. power lines) and its effect on a connected electronic device.
- Radiated Emissions (RE) - the EMI radiated by an electronic device
- Radiated Susceptibility (RS) - radiated EMI effect on an electronic device
- Electromagnetic Pulse (EMP) – radiated EMI by lightning or atomic blast

Culprits and Victims

- Culprits are devices, equipment or systems that emit EMI
- Victims are devices, equipment or systems that are susceptible to EMI

USA

- ***MIL-STD-461E Emissions & Susceptibility Standard for Defense Electronics***
  This standard sets the Emissions & Susceptibility (Immunity) noise limits and test levels for electrical / electronic and electromechanical equipment

- ***MIL-STD-462E*** *is the companion standard that describes the methods and test procedures for certification under MIL-STD-461.*

- *The object of the standards is to maximize safety and reliability and to minimize downtime and breakdowns of equipment essential for defense.*

- *The worldwide defense electronics and aerospace community recognizes and generally accepts MIL-STD-461.*

USA

Federal Communications Commission (FCC) under the Code of Federal Regulations CFR, Part 15, Sub-Part J, for Class A and B devices and equipment.

Germany

Verband Deutscher Elektrotechniker (VDE) has developed VDE 0871 for Level A and Level B.

European Community

EMC Directives of 1996

The FCC and VDE specifications are similar in that Class A and Level A describe industrial equipment, while Class B and Level B are applicable to consumer equipment.

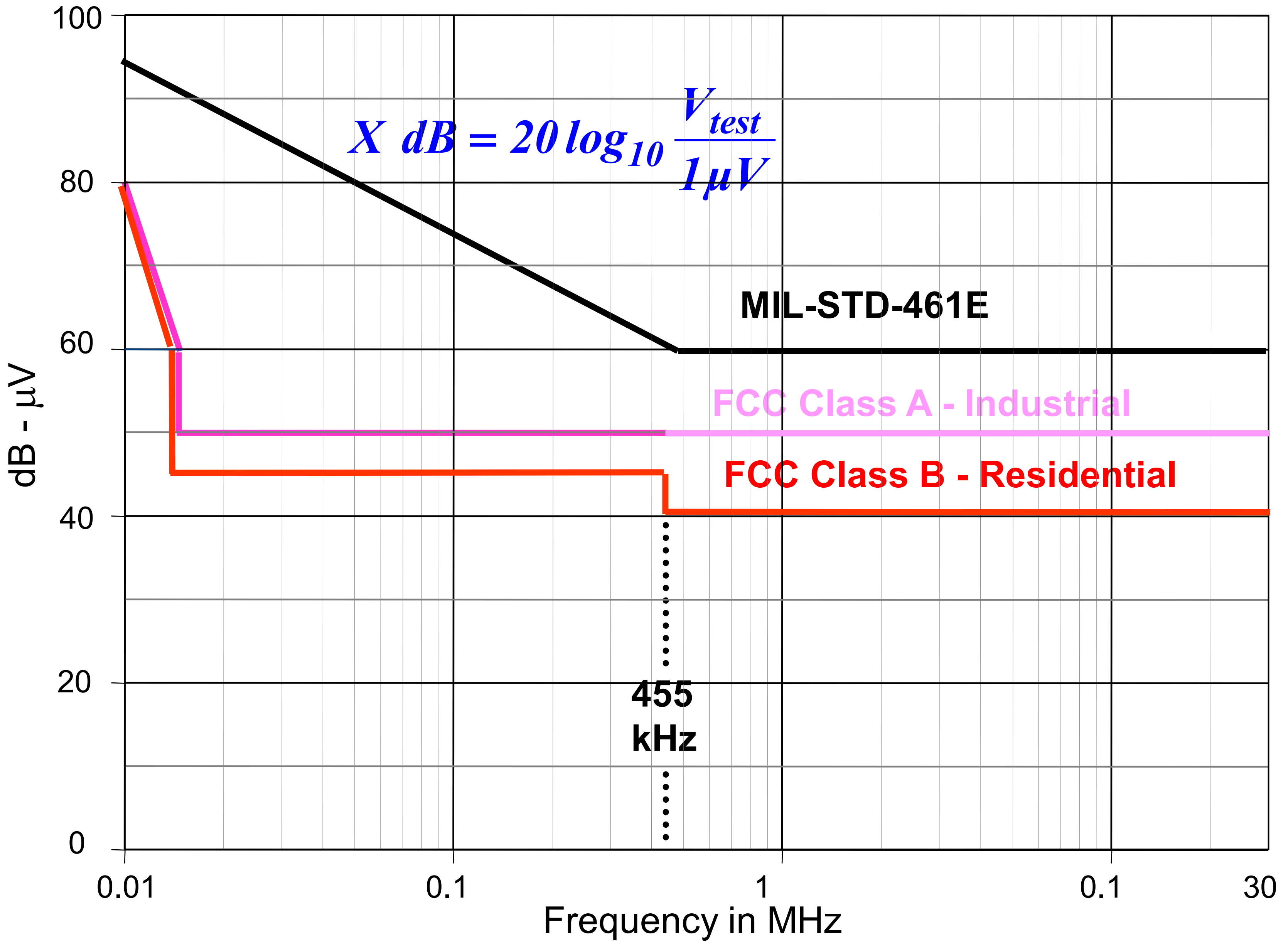
Conducted emissions

- *EMI conducted onto AC Lines by the power supply.*
- *Typically* 10 kHz *to* 30 MHz
- *Measured in* μV *or* dB$-\mu$V *(Reference value:* $1\ \mu\text{V} = 0\ \text{dB}$*)*

$$\text{dB} = 20 \cdot \log_{10} \frac{\text{measured}\ \mu\text{V}}{1\ \mu\text{V}}$$

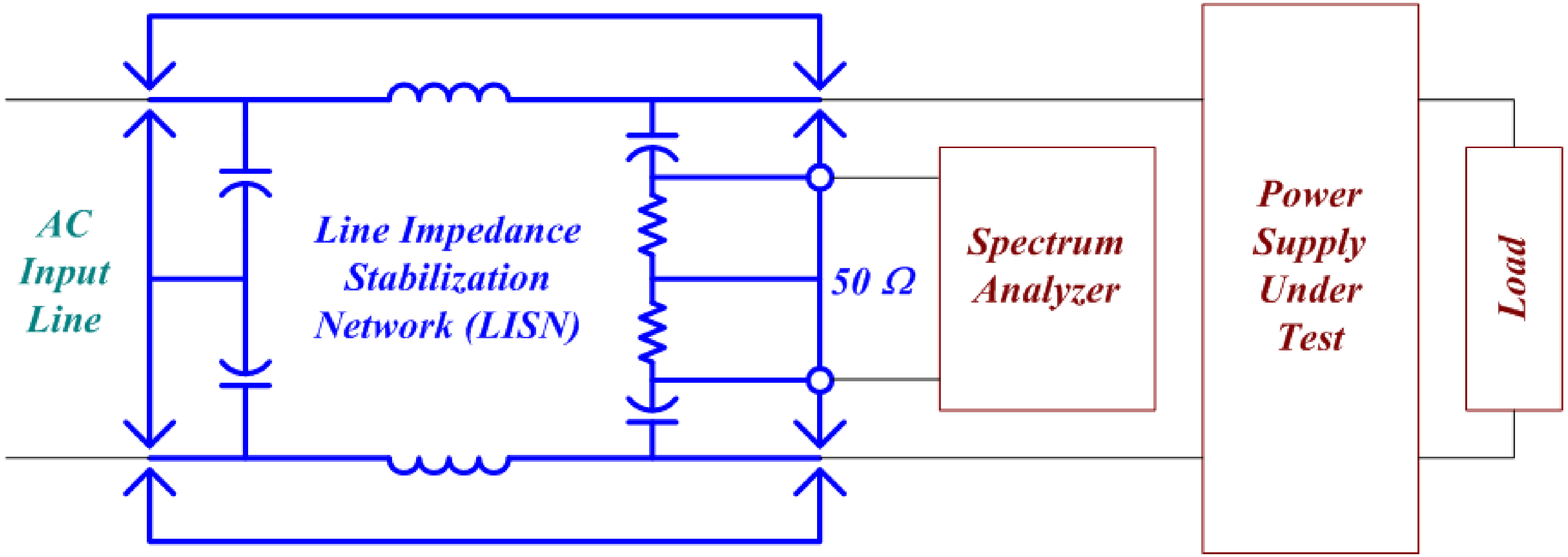
Example: Measured noise = $100\ \mu\text{V}$

$$\text{dB} = 20 \cdot \log_{10} \left(\frac{100\ \mu\text{V}}{1\ \mu\text{V}} \right) = 40\ \text{dB}$$


100
80
60
40
20
0
dB - µV
0.01
0.1
1
0.1
30
Frequency in MHz
$X\ dB = 20 \log_{10} \frac{V_{test}}{1 \mu V}$
MIL-STD-461E
FCC Class A - Industrial
FCC Class B - Residential
455
kHz

***Test equipment used** – Spectrum analyzers with Line Impedance Stabilization Networks (LISNs) that*

- *Filter and divert external AC line intrinsic noise from the EMI measurements*
- *Isolate and decouple the AC line high voltage and prevent line transients from damaging spectrum analyzers and other sensitive test equipment*
- *Present a known, fixed impedance at RF frequencies to the power supply undergoing test*



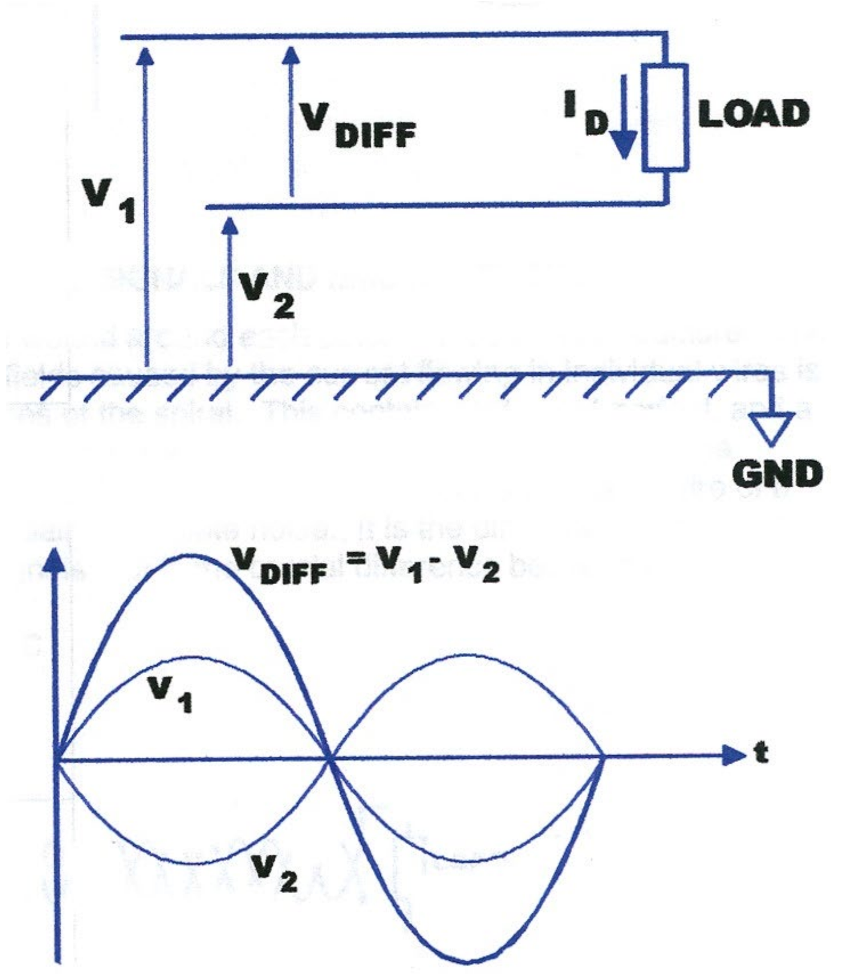
LISN considerations:

- Desired impedance (typically 50 Ω)
- Bandwidth (typically victims are susceptible to 10 kHz to 30 MHz)
- Line type (DC, Single phase, 3 ϕ delta, 3 phase wye)
- Line voltage (120 V, 208 V, 480 V, etc)
- Power supply input current when under load

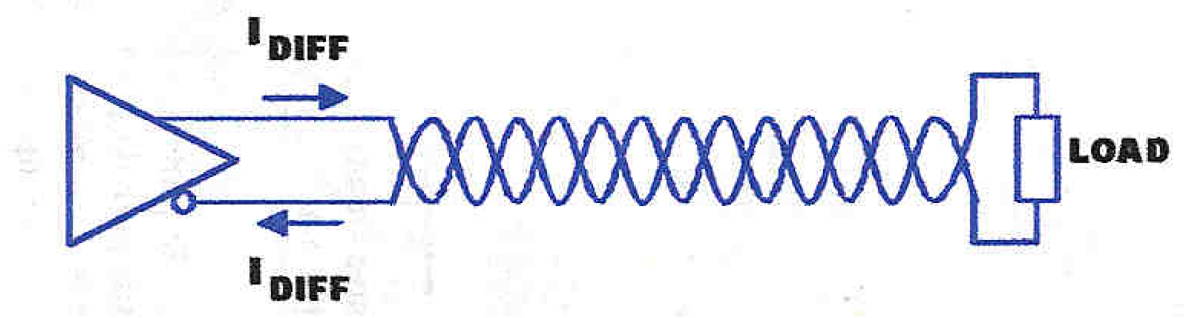
Spectrum Analyzers

Anritsu, Keysight, Rigol,

Rohde and Schwarz

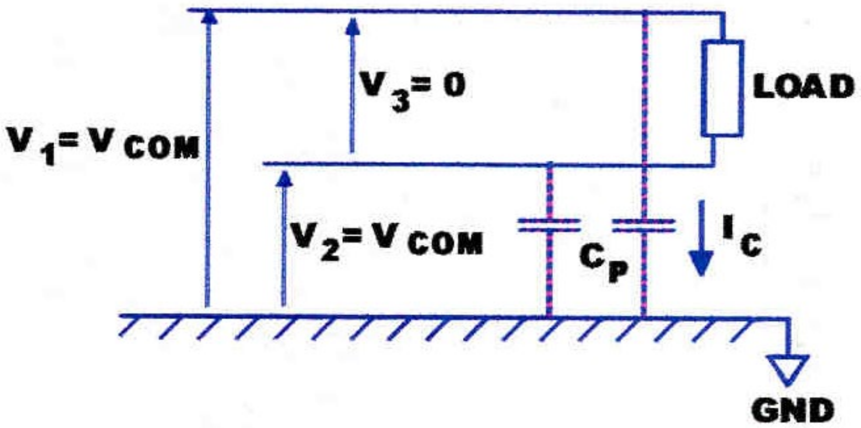


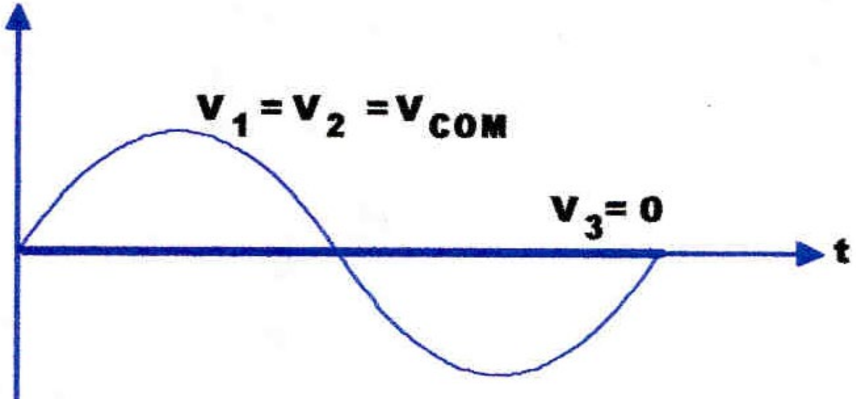
- *Produced as a natural result of complex, high frequency switching V and I*
- $V_1 = - V_2$
- *Magnitudes are equal*
- *Phase difference is* 180^O
- $V_{Load} = V_1 – V_2$ = *KVL unwanted signal*
- $I_D = (|V_1| + |V_2|)/ R_{Load}$



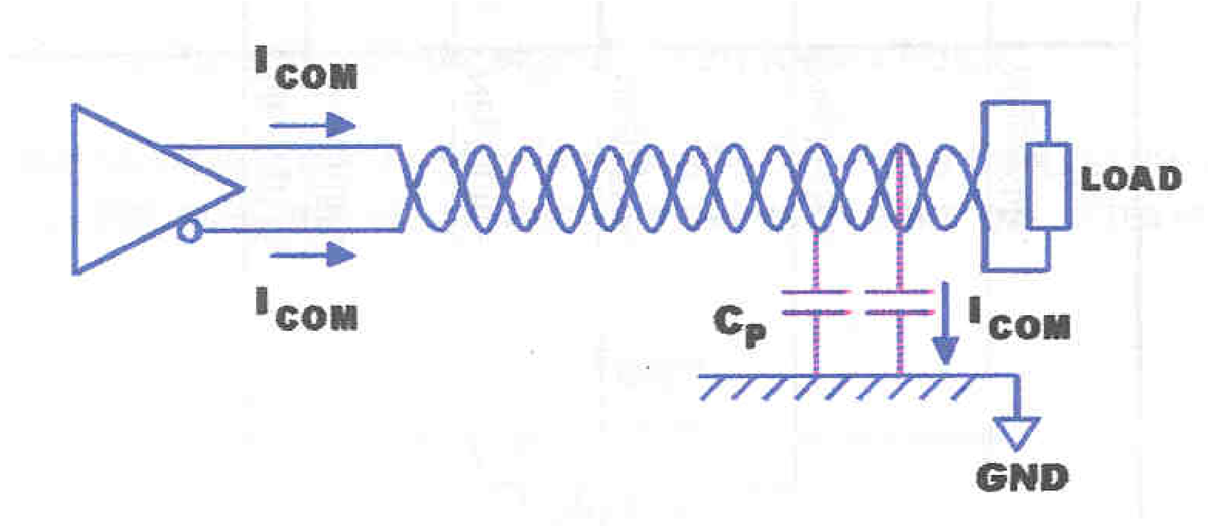
- *Current flow in opposite directions so that the magnetic field is contained within the spirals*
- *The tighter the cable twist the greater the containment and noise attenuation*
- *Shielding the pair (and tying the shield to ground in one or more places) will also increase noise attenuation*

EMC/EMI - Common Mode Noise





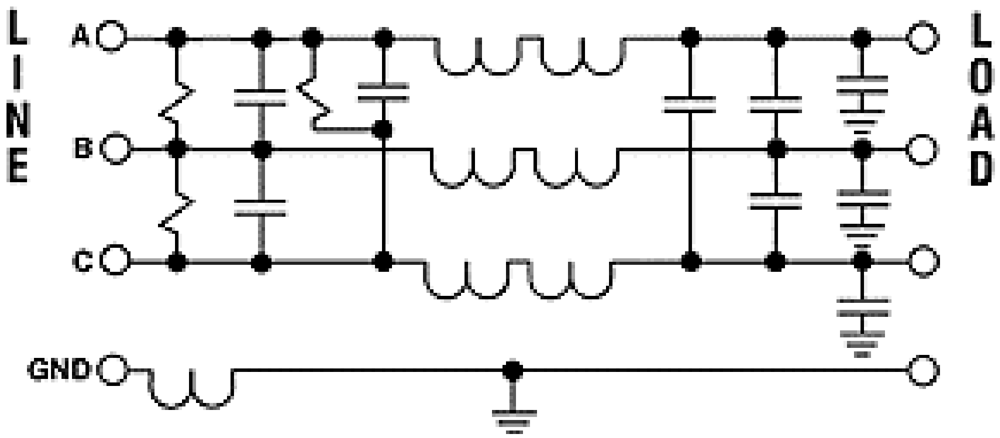
- Produced as a result of circuit imbalances, currents produced by simultaneous high frequency voltages on (+) and (-) lines capacitively coupled to ground
- $V_1 = V_2 = V_{COM}$
- Magnitudes are equal
- Phase difference is 0^O
- $I_{Load} = (V_1 - V_2) / R_{Load}$
- $V_{SUM} = V_1 + V_2 = 0$

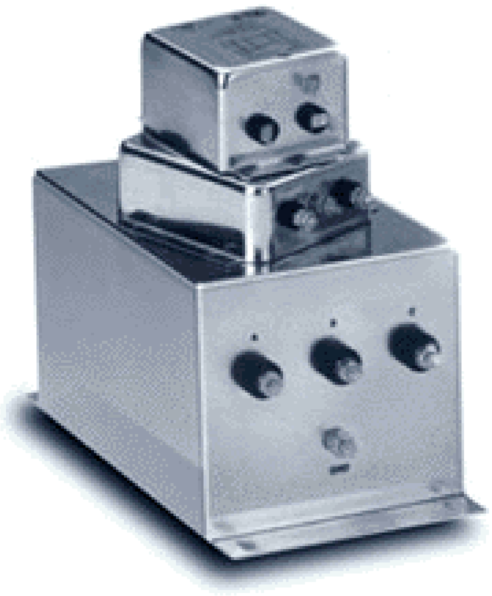


- *Common mode current generated by common mode voltages impressed across parasitic capacitances to ground*
- *Current flows are the same magnitude and in the same direction so that the spirals have no effect on containing the magnetic fields*
- *The pair must be shielded and the shield tied to ground in one or more places for noise attenuation*

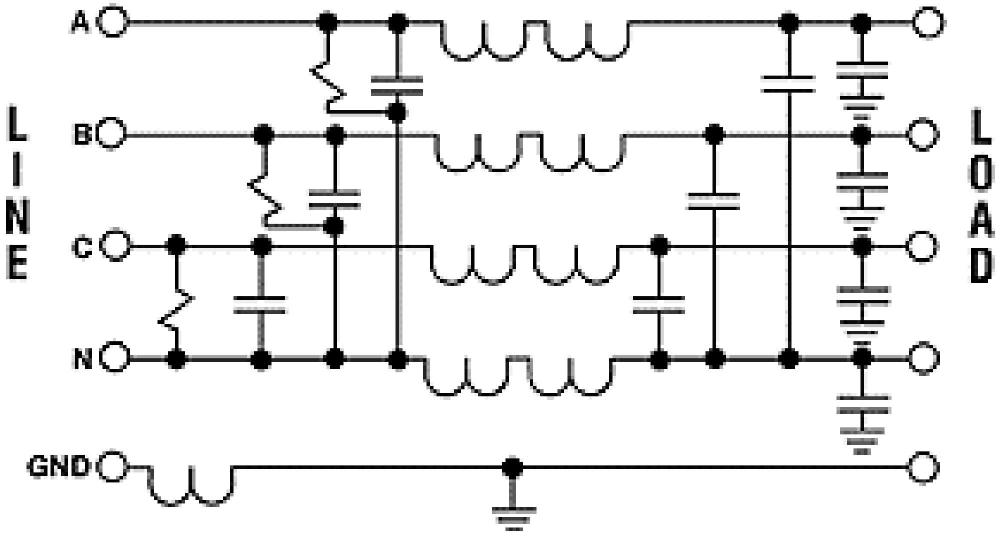
EMC/EMI - Input Conducted Line Noise Filters

Delta input



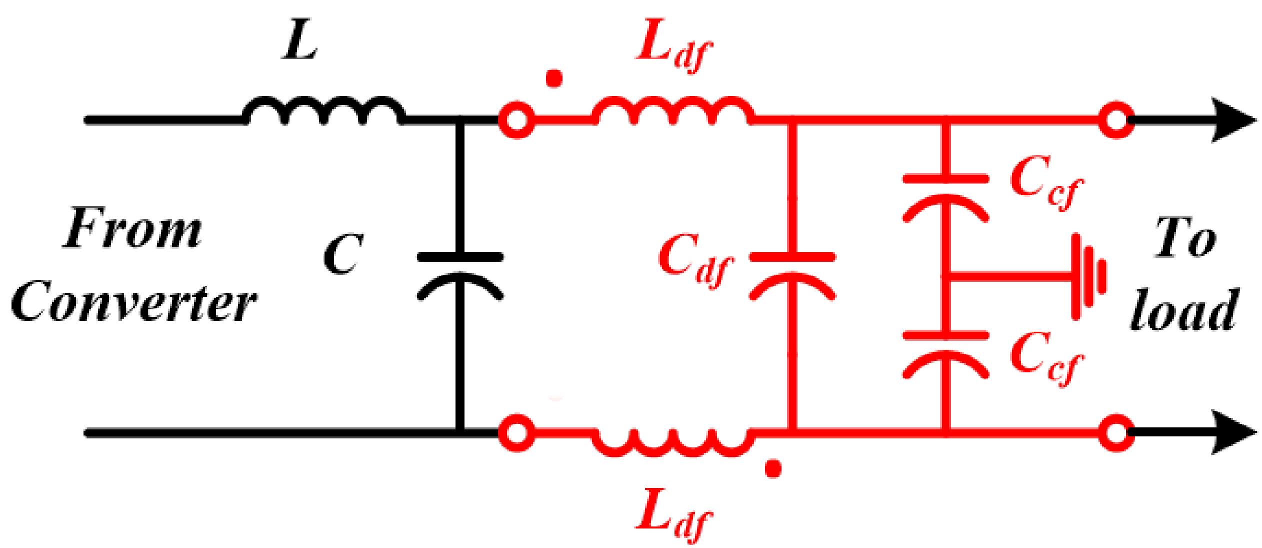


Wye input

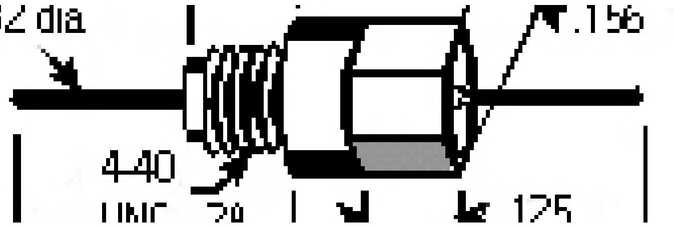


- *Configurations C, L, Pi, T*
- *Attenuation 20 to 70dB*
- *Filters both differential and common mode noise*

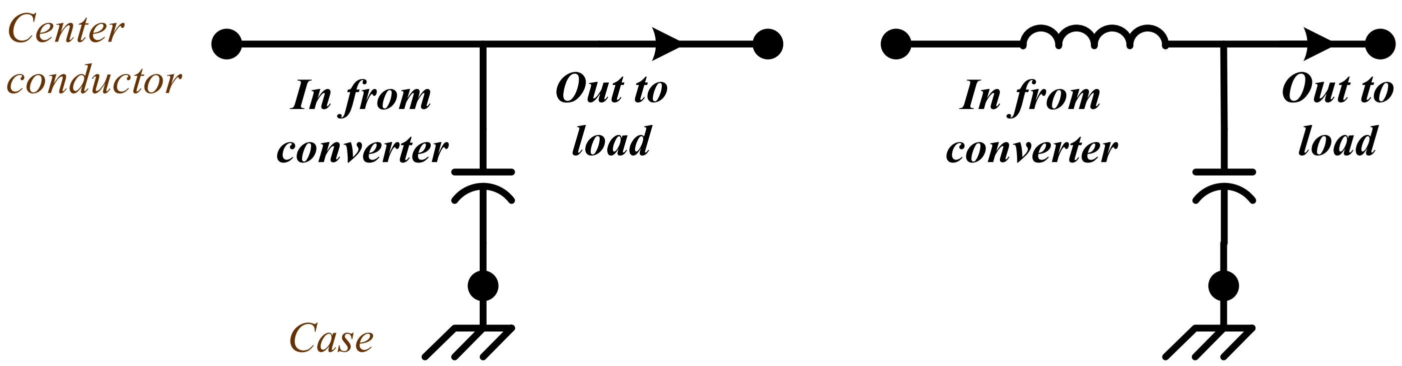
http://www.filterconcepts.com/three_phase/3v_series.html



- *L and C are not good noise ($f > f_{sw}$) filters*
- *L looks capacitive at $f > f_{sw}$, C looks inductive at $f > f_{sw}$*
- *L_{df} is a differential / common mode noise filter inductor and might be a real inductance or the intrinsic inductance of the bus*
- *C_{df} is a differential mode noise filter capacitor*
- *C_{cf} are common mode noise filter capacitors*

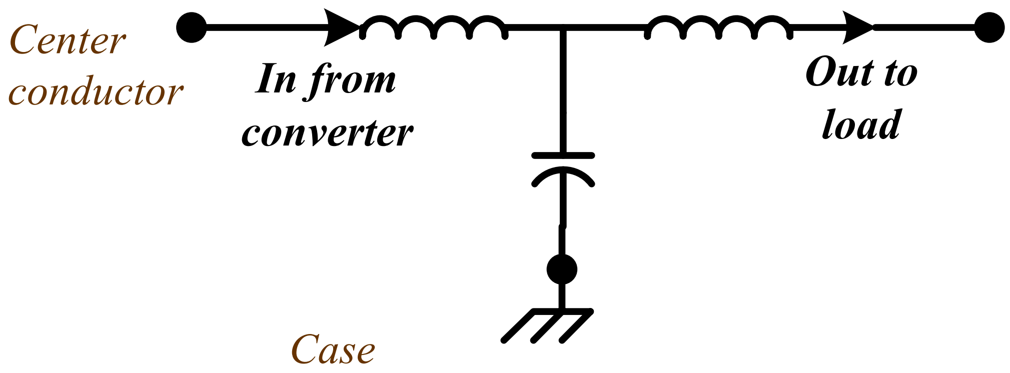


• ***C filters*** *are the most common EMI filter, consisting of a 3 terminal feed-thru capacitor, used to attenuate high frequency signals*

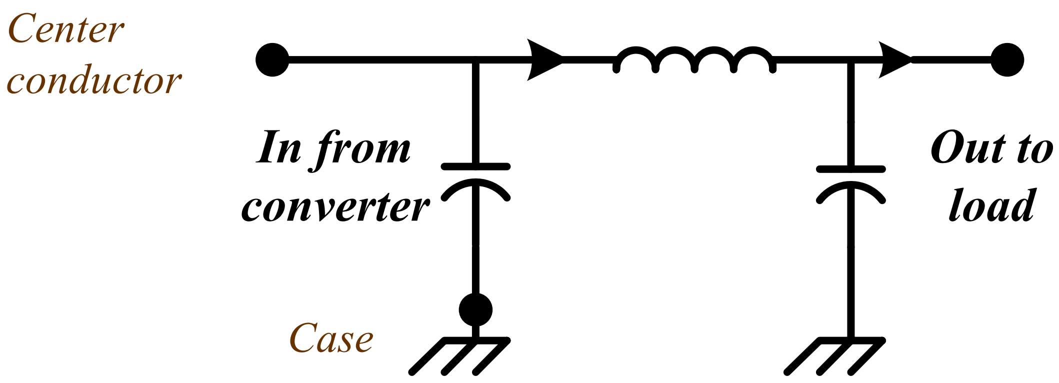


• ***L filters*** *consist of one inductive element and one capacitor. One disadvantage is that the inductor element in smaller filters consists of a ferrite bead that will saturate and lose effectiveness at larger load currents*

- ***T filters*** *consist of two inductive elements and one capacitor. This filter presents a high impedance to both the source and load of the circuit*

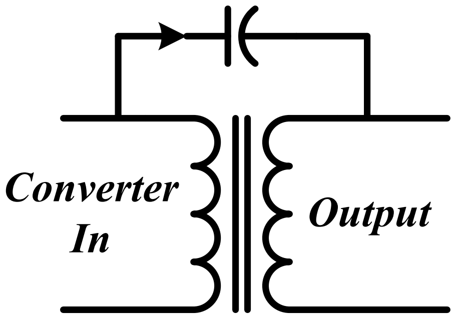


- ***Pi*** *filters consist of two capacitors and one inductor. They present a low impedance to both source and load. The additional capacitor element, provides better high frequency attenuation than the C or L filters*

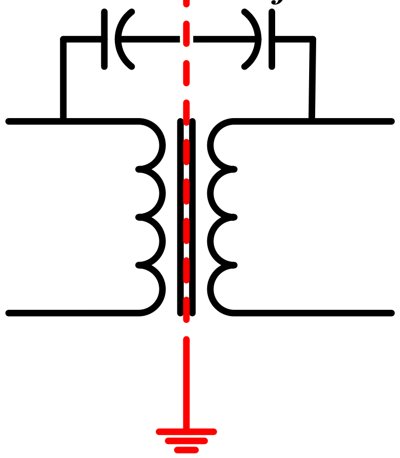


EMC/EMI - Other Conducted Noise Filters

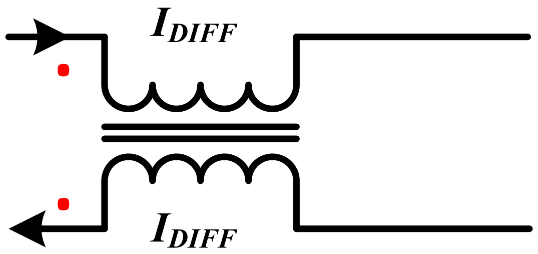
Plain Transformer

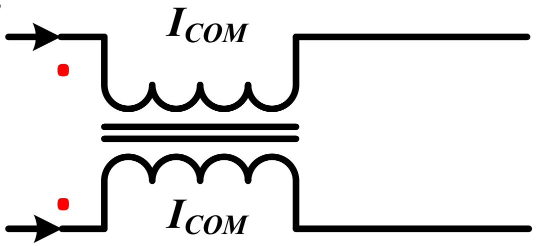


Electrostatic (Faraday) Shielded Transformer



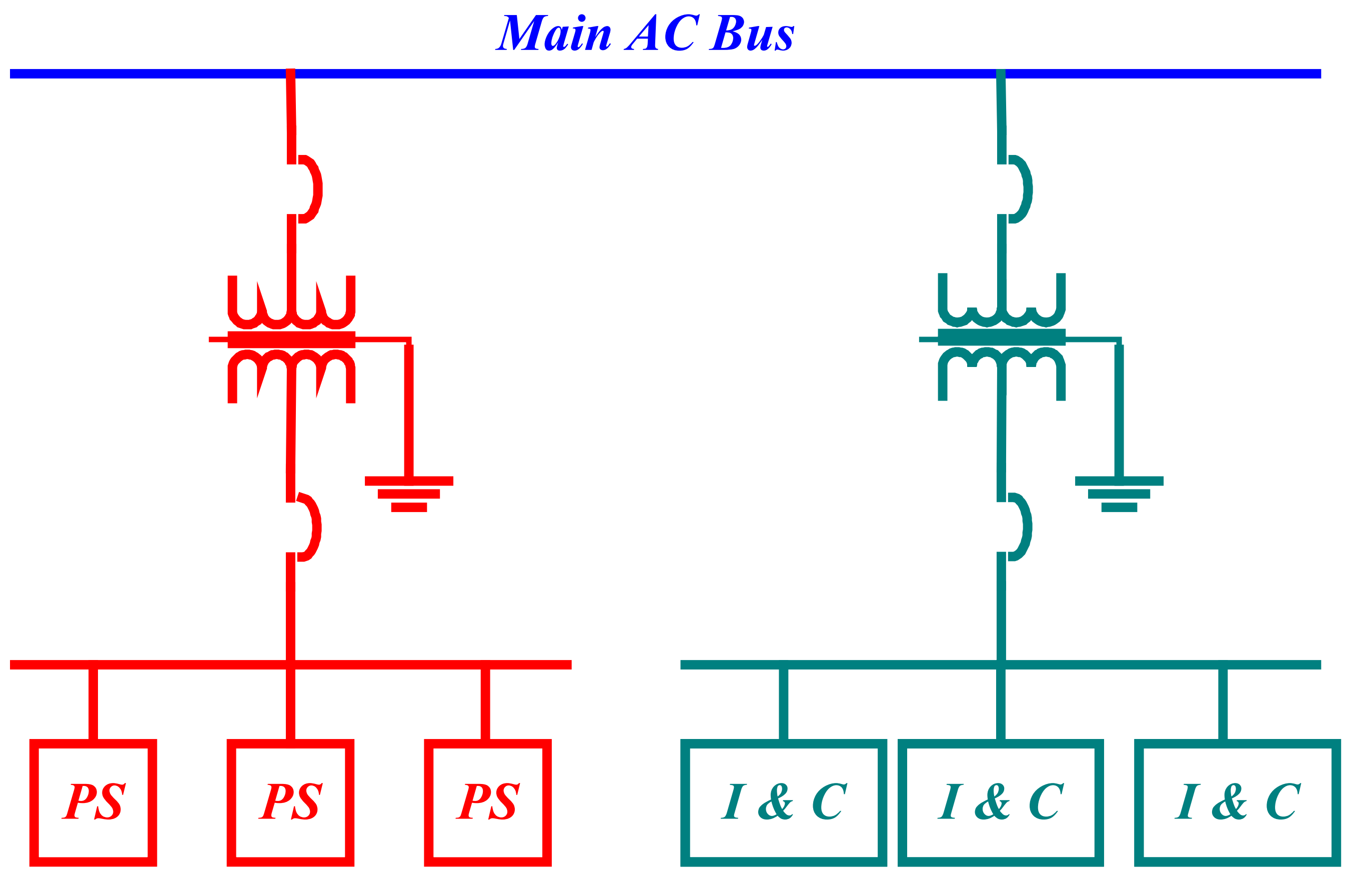
Common Mode Choke





Differential mode currents flow in opposite directions. Magnetic fields cancel, choke presents low impedance, low attenuation to noise

Common mode currents flow in same direction. Magnetic fields add, choke presents high impedance, high attenuation to noise



- *Separate noisy power supplies from sensitive I & C loads by Faraday-shielded transformers to attenuate common mode noise*

Radiated emissions

- EMI radiated from cables, transformers, other components.
- Typically 30 MHz to > 1GHz. 30 MHz start because cables and other equipment are effective radiators of frequencies above 30 MHz
- Measured in $\mu V/m$ or dB - $\mu V/m$ (Reference: 1 $\mu V/m$ = 0 dB)
- Measured 3 m (residential) or 30 m (industrial) from the emitting equipment. TVs located within 3 m of computers in the home and within 30 m in the industrial setting. Limits 100 to 200 $\mu V/m$ are 1/10 of TV reception signal
- Industrial FCC Class A limits of 200 $\mu V/m$ are higher (less severe) than residential Class B because it is assumed that there will be an intervening wall between culprit and victim that will provide some shielding

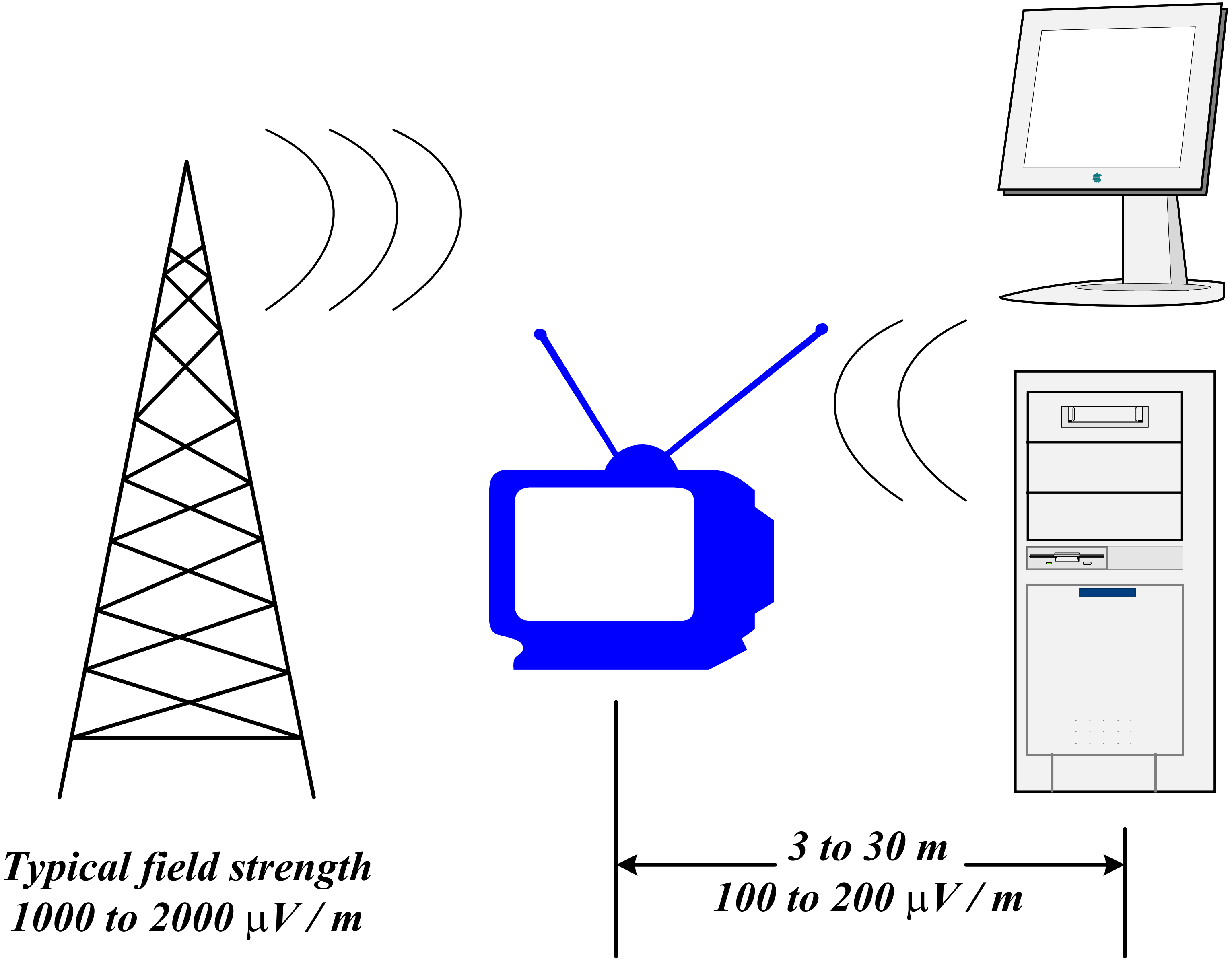
Test equipment used

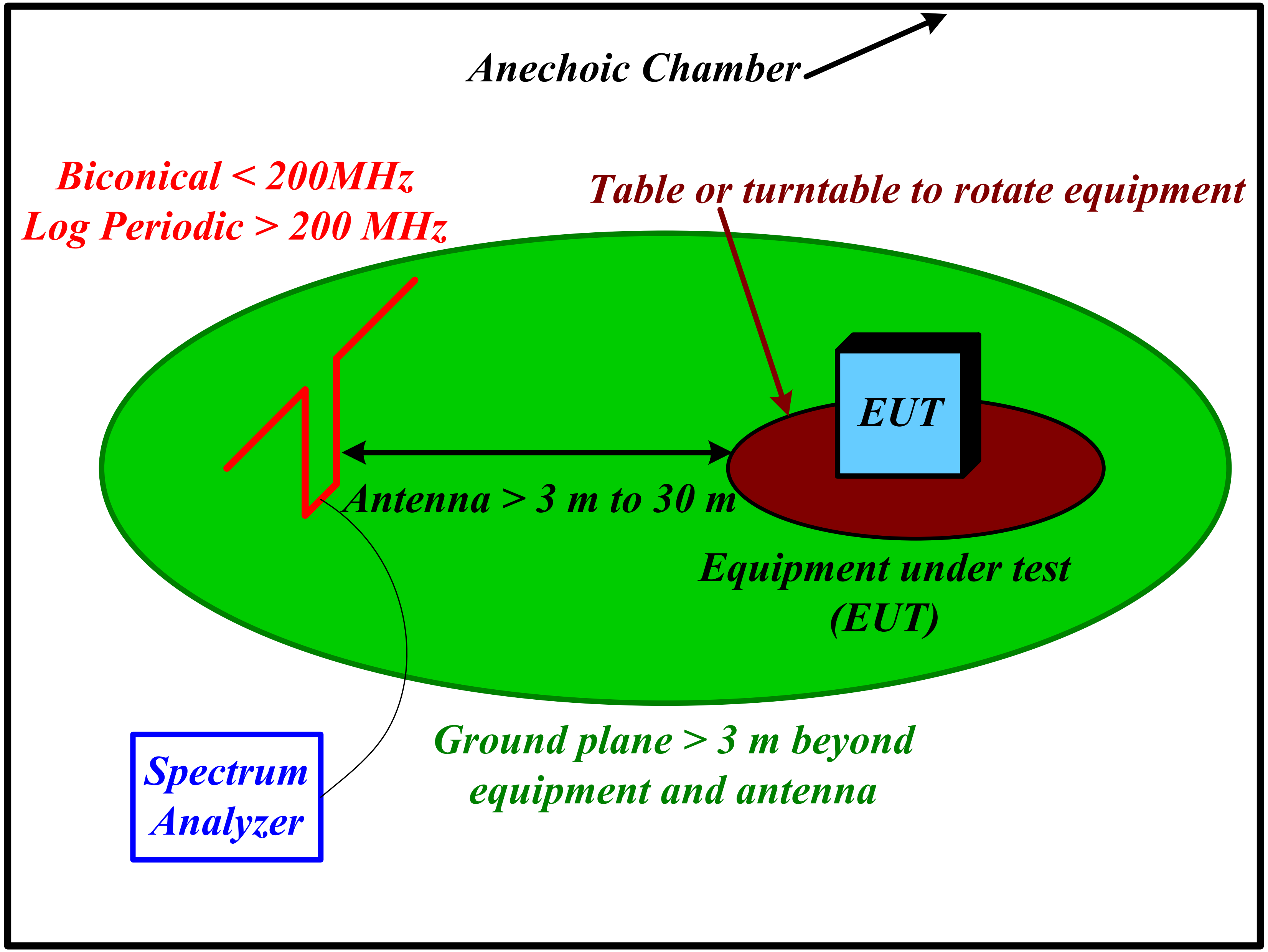
- Spectrum Analyzers, rotating tables, conical and/or log periodic antennas and anechoic chambers designed to minimize reflections and absorb external EMI

Any component or cable > 1/2 wavelength (λ) will be an efficient radiating or receiving antenna

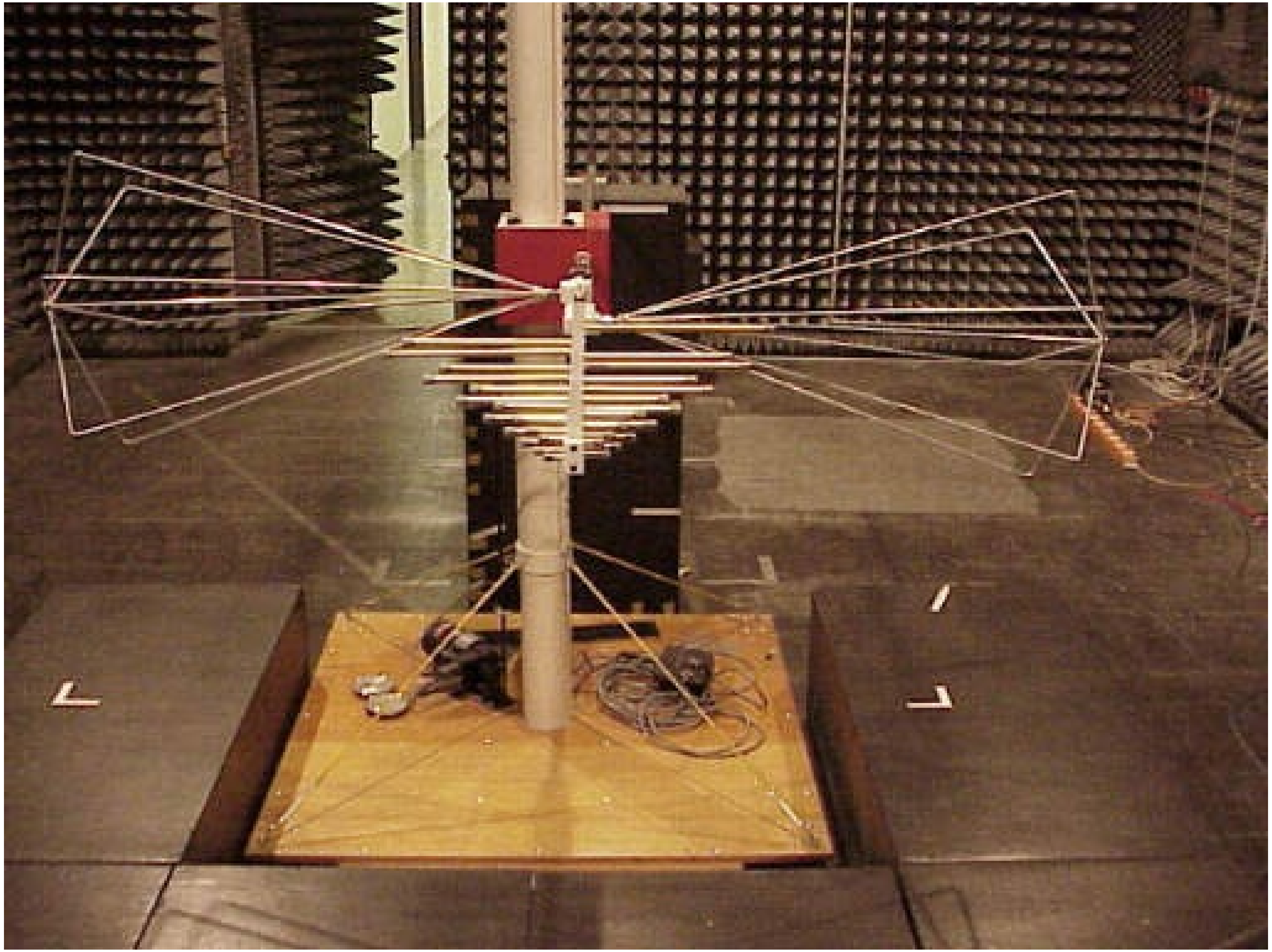
Cable Lengths Vs Wavelength			
Frequency	**λ**	**1/2 λ**	**1/4 λ**
10 kHz	30 km	15000 m	7500 m
100 kHz	3 km	1500 m	750 m
1 MHz	300 m	150 m	75 m
10 MHz	30 m	15 m = 50 ft	7.5 m = 25 ft
30 MHz	10 m	500 cm = 16 ft	2.5 m = 8 ft
100 MHz	3 m	150 cm = 5 ft	75 cm = 2.5 ft
1 GHz	30 cm	15 cm = 6 in	7.5 cm = 3 in

EMC/EMI - Basis For Industrial – Residential Emission Limits




Anechoic Chamber
Biconical < 200MHz
Log Periodic > 200 MHz
Table or turntable to rotate equipment
EUT
Antenna > 3 m to 30 m
Equipment under test
(EUT)
Spectrum
Analyzer
Ground plane > 3 m beyond
equipment and antenna

EMC/EMI - Bi-Conical Antenna



EMC/EMI - Log-Periodic Antenna

- $B = T = 10{,}000$ gauss
- $A = m^2$
- $(T/s)*m^2 = V$

Faraday's Induced Voltage Law

$$V = \oint E \bullet dl = -\frac{d\varphi}{dt} = -\frac{dB}{dt} A$$

Hint: Homework problem

$V \propto \frac{dB}{dt}$ *the magnitude and rate of change of flux density with time*

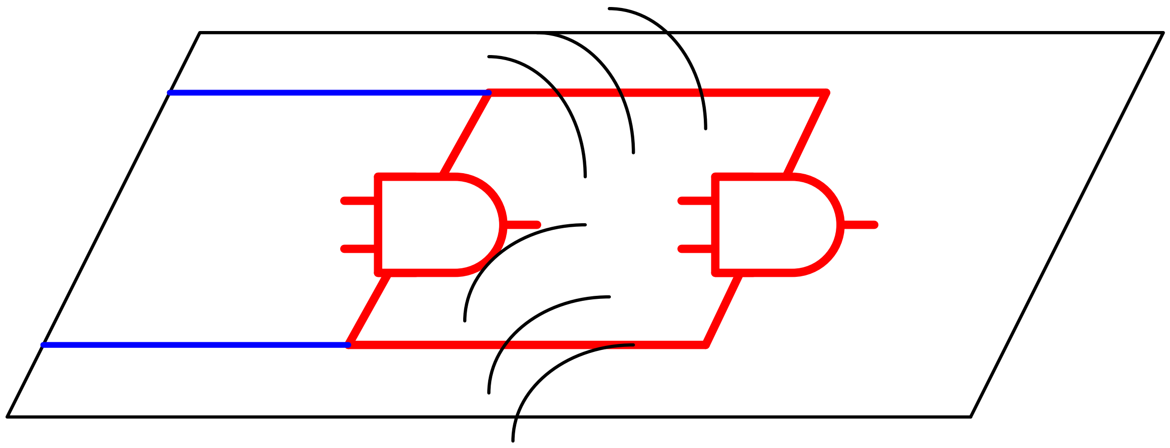
$V \propto A$ *the area of the loop cut by flux*

Moral - minimize loop areas by:

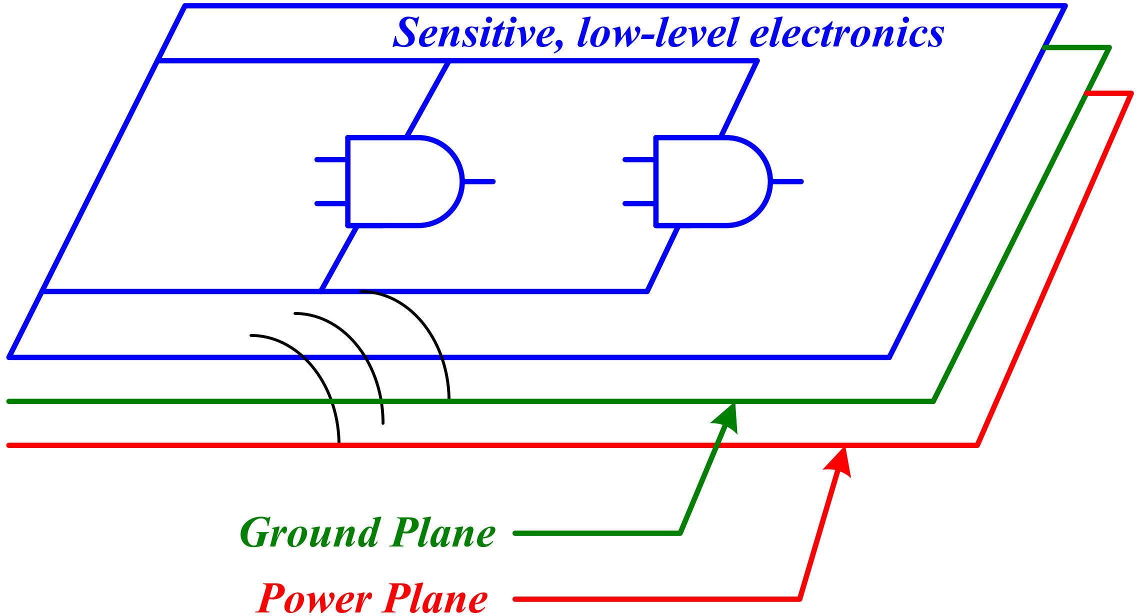
running supply and return bus or cable conductors together

twisting cables whenever possible

EMC/EMI - Radiated Noise Reduction By PCB Small Loops

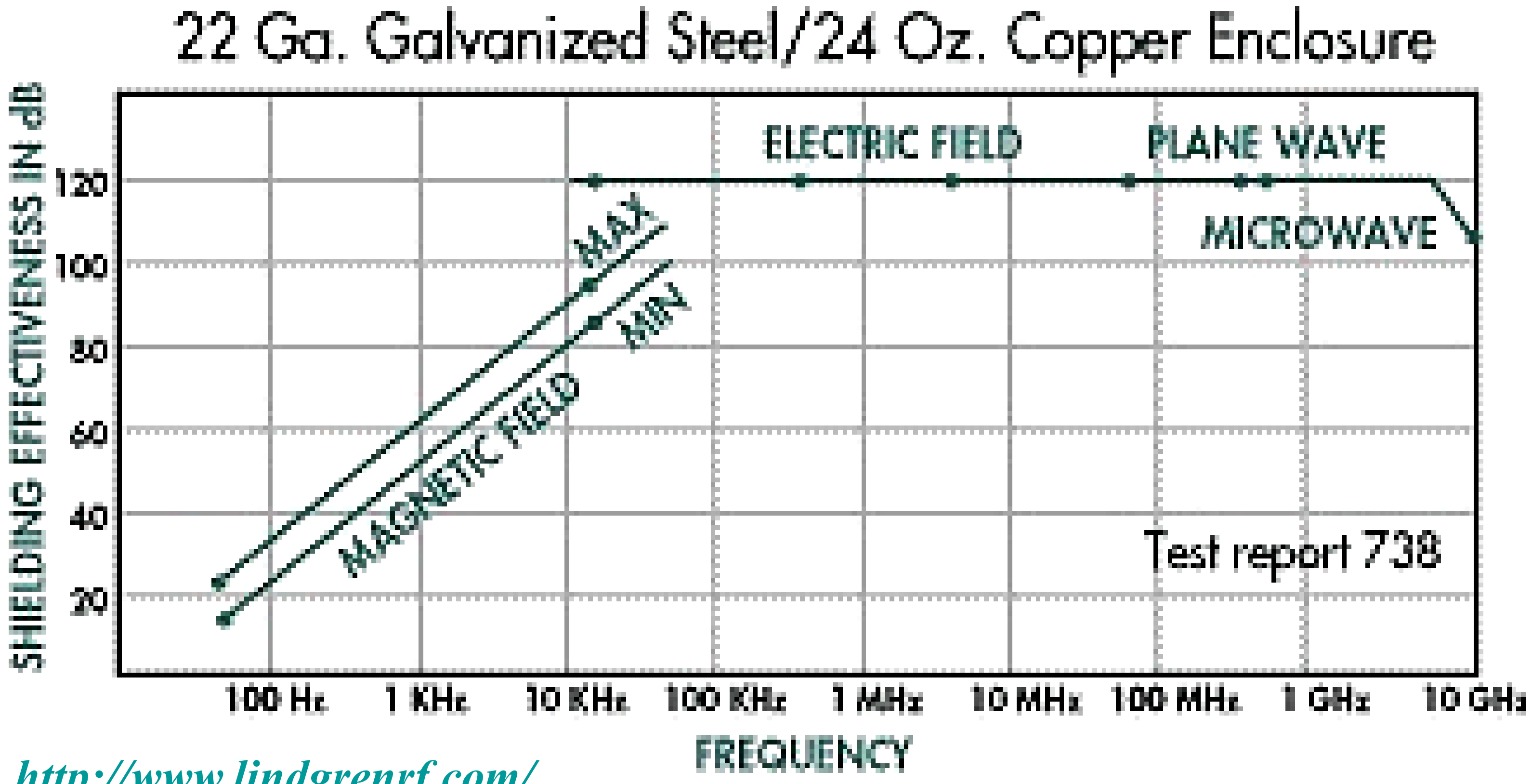


Radiated Noise Reduction By PCB Ground Planes



Use shielded cables

Use shielded enclosures (if necessary for interior controls)



http://www.lindgrenrf.com/

note – link no longer valid

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

Shielding

- *Use ground planes extensively to minimize E and H fields*
- *If ribbon cable is used, employ and spread ground conductors throughout to minimize loop areas*
- *Avoid air gaps in transformer/inductor cores.*
- *Use toroid windings for air core inductors*
- *If shielding is impractical, then filter*

Filtering

- *Use common mode chokes whenever practical*
- *Use EMI ferrites, not low-loss ferrites – useful frequency range 50 to 500 MHz. Be careful of DC or low-frequency current saturation*
- *Use capacitors and feed-through capacitors, separately or in conjunction with chokes/ferrites. Be mindful of capacitor ESR and inductance*

Homework Problem # 6

A uniform magnetic field B is normal to the plane of a circular ring 10 cm in diameter made of #10 AWG copper wire having a diameter of 0.10 inches. At what rate must B change with time if an induced current of 10 A is to appear in the ring? The resistivity of copper is about 1.67 $\mu\,\Omega * cm$.

Hints: $R = \frac{\rho * L}{A}$ *and use the 10 cm dimension as the ring diameter*

Single Phase System

$$S_{1\phi} = V_\phi \, I^*_\phi = P_{1\phi} + j\, Q_{1\phi}$$

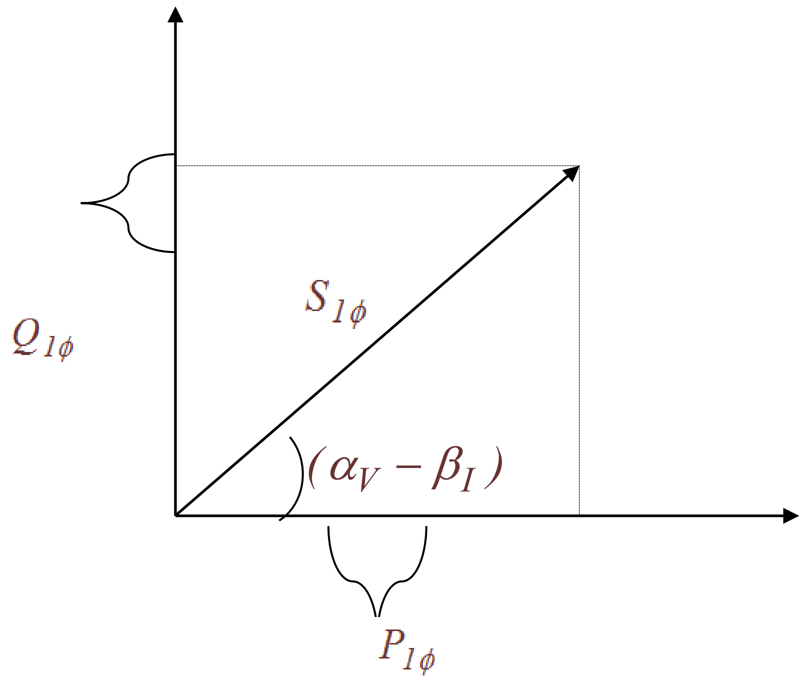
$$|S_{1\phi}| = |V_\phi||I_\phi| e^{j\alpha_V} e^{-j\beta_I}$$

$$|S_{1\phi}| = |V_\phi||I_\phi|[cos(\alpha_V - \beta_I) + j\, sin(\alpha_V - \beta_I)]$$

$$P_{1\phi} = |V_\phi||I_\phi| cos(\alpha_V - \beta_I)$$

$$Q_{1\phi} = |V_\phi||I_\phi| sin(\alpha_V - \beta_I)$$

$$PF = \frac{|P_{1\phi}|}{|S_{1\phi}|} = cos(\alpha_V - \beta_I)$$



$0 \le PF \le 1$, leading or lagging, curr , voltage is reference

PF is not efficiency $\quad Eff = \frac{P_o}{P_i}$

Power Factor - Calculation and Importance

$Balanced$ three Phase

$S_{3\varphi} = 3V_{\varphi}I_{\varphi} = \sqrt{3}V_{LL}I_{L}$

$P_{3\varphi} = 3V_{\varphi}I_{\varphi}\cos(\alpha_{V\varphi} - \beta_{I_{\varphi}})$

$$PF_{3\varphi} = \frac{P_{3\varphi}}{S_{3\varphi}} = \cos(\alpha_{V_{\varphi}} - \beta_{I_{\varphi}})$$

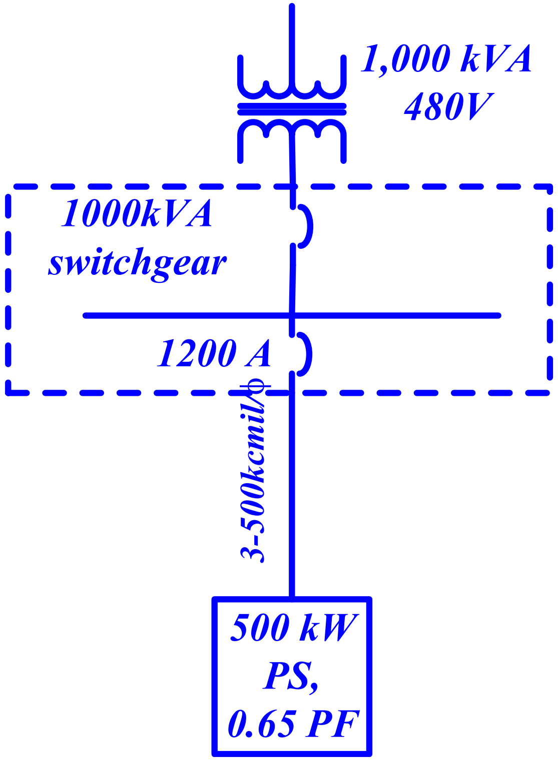
$Unbalanced$ three phase power

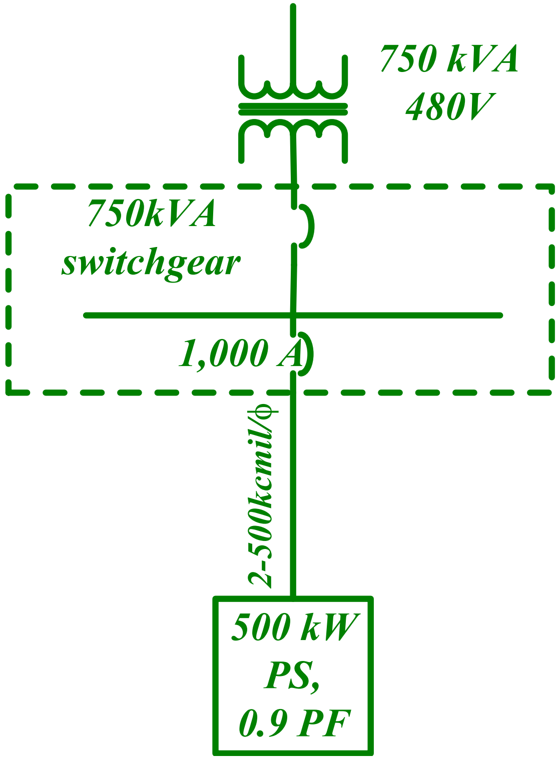
$S_{3\varphi} = V_{\varphi A}I_{\varphi A} + V_{\varphi B}I_{\varphi B} + V_{\varphi C}I_{\varphi C}$

$$P_{3\varphi} = V_{\varphi A}I_{\varphi A}\cos(\alpha_{V_{\varphi A}} - \beta_{I_{\varphi}A}) + V_{\varphi B}I_{\varphi B}cos(\alpha_{V_{\varphi B}} - \beta_{I_{\varphi}B}) + V_{\varphi C}I_{\varphi C}cos(\alpha_{V_{\varphi}C} - \beta_{I_{\varphi}C})$$

$$PF_{3\varphi} = \frac{P_{3\varphi}}{S_{3\varphi}}$$

Power Factor is Important - Capital Equipment Cost





$$S = \frac{P}{PF} = \frac{500kW}{0.65} = 769kVA$$

$$I = \frac{769kVA}{\sqrt{3} * 480V} = 925A$$

$$I_{CB} = 925A * 1.25 = 1,156A, \text{ buy } 1200A$$

Buy 1000kVA switchgear/transformer

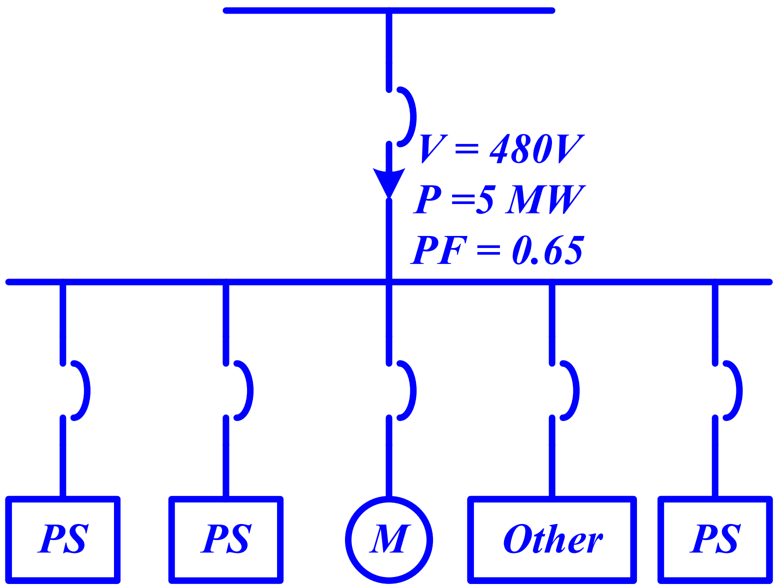
$$S = \frac{P}{PF} = \frac{500kW}{0.9} = 555kVA$$

$$I = \frac{555kVA}{\sqrt{3} * 480V} = 667A$$

$$I_{CB} = 667A * 1.25 = 834A, \text{ buy } 1000A$$

Buy 750kVA switchgear/transformer

Power Factor is Important – Energy Cost



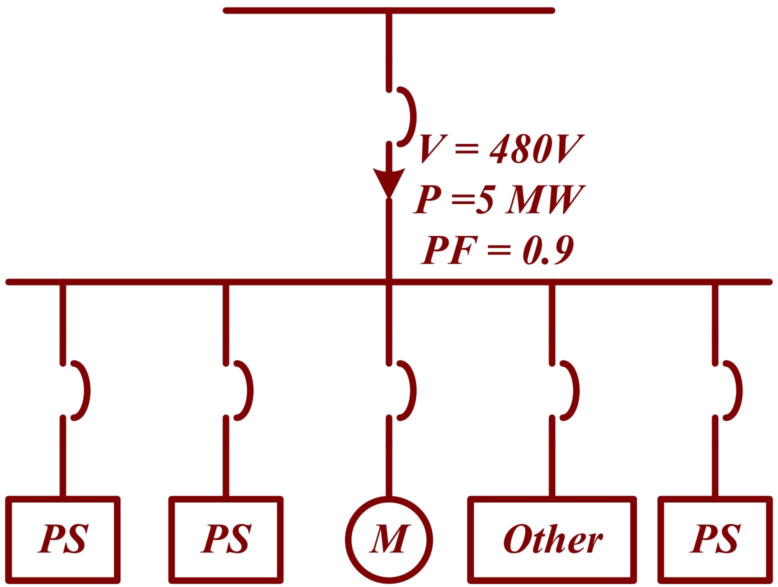
$$S = \frac{P}{PF} = \frac{5MW}{0.65} = 7.7MVA$$

$$Electric\ rate = \frac{\$0.06}{kW - Hr}$$

$$9\,months * \frac{30days}{month} * \frac{24hr}{day} = \frac{6480hr}{yr}$$

$$7.7MVA * \frac{\$0.06}{kW - Hr} * \frac{6480hr}{yr} = \frac{\$3.0M}{yr}$$

$$\frac{\$3M}{yr} * 20yr = \$60M$$



$$S = \frac{P}{PF} = \frac{5MW}{0.9} = 5.6MVA$$

$$Electric\ rate = \frac{\$0.06}{kW - Hr}$$

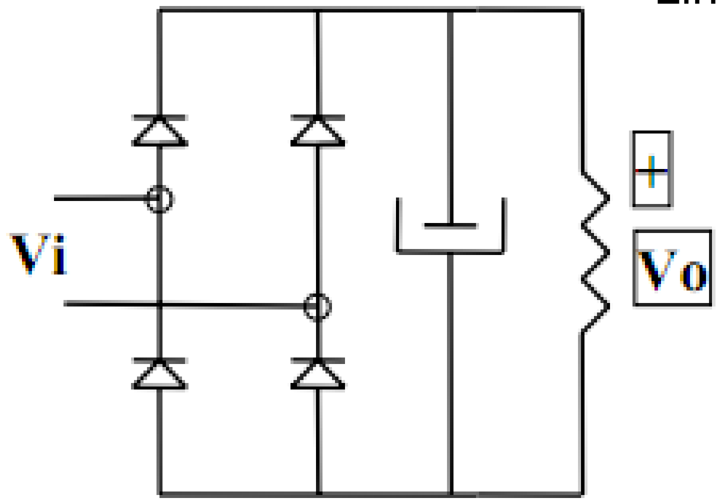
$$9\,months * \frac{30days}{month} * \frac{24hr}{day} = \frac{6480hr}{yr}$$

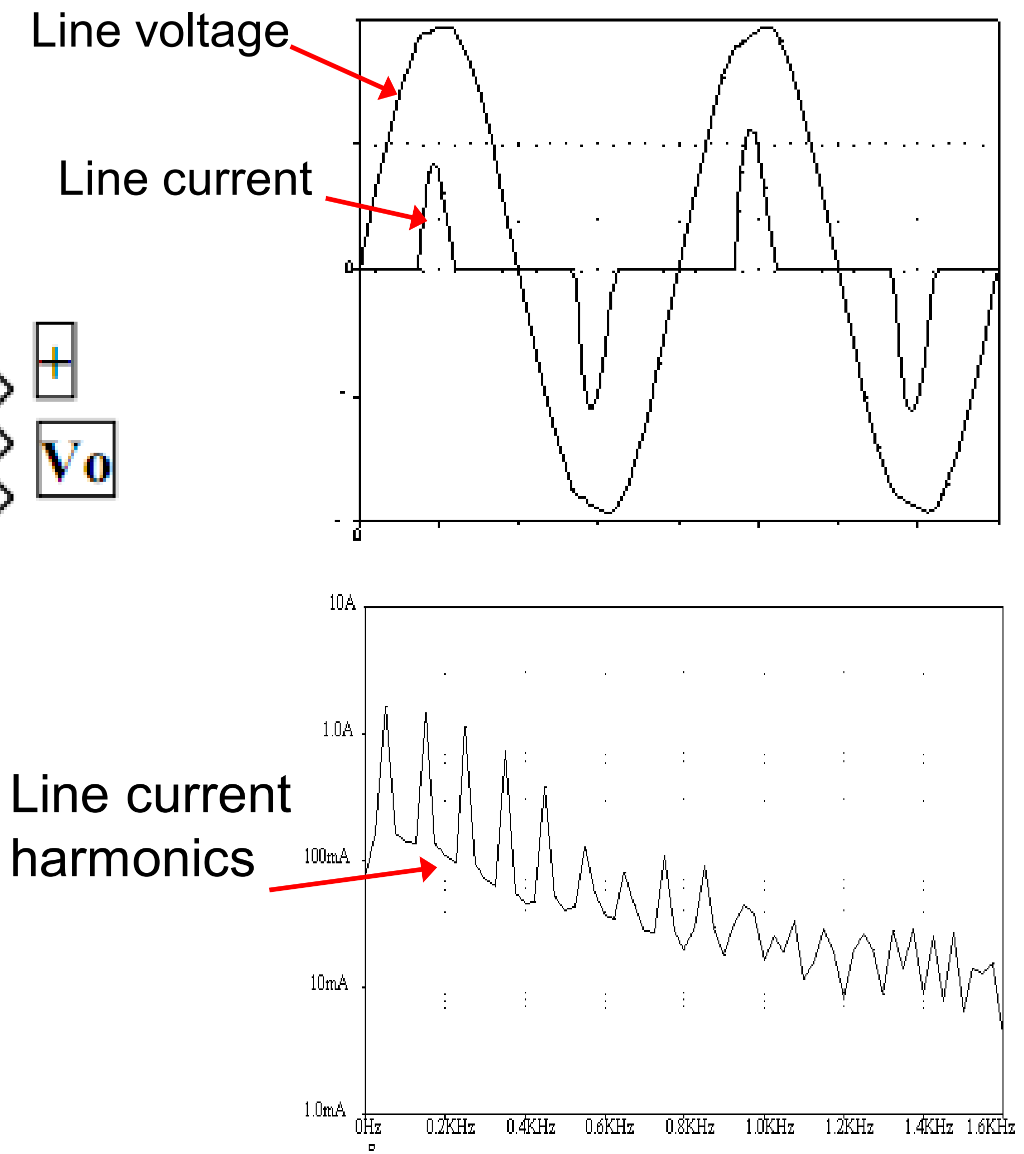
$$5.6MVA * \frac{\$0.06}{kW - Hr} * \frac{6480hr}{yr} = \frac{\$2.2M}{yr}$$

$$\frac{\$2.2M}{yr} * 20yr = \$44M$$

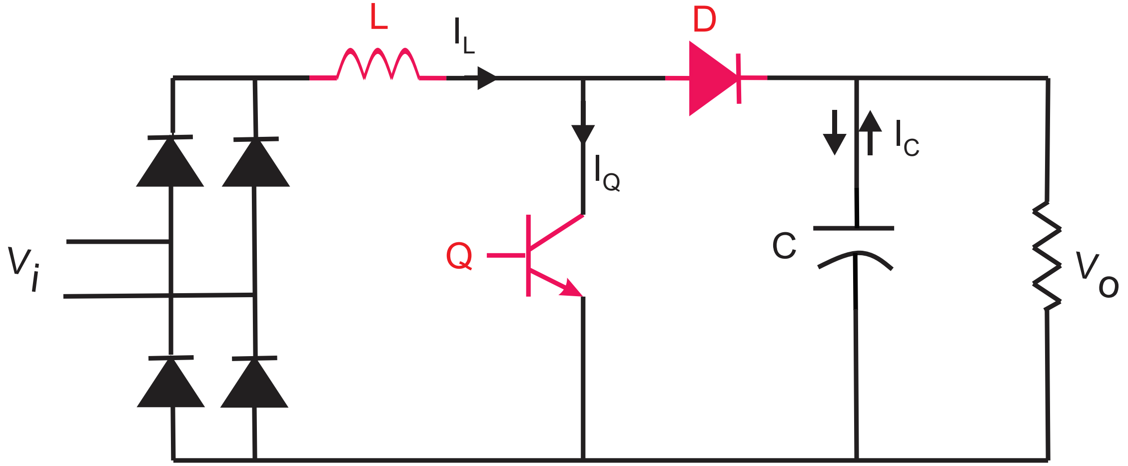
Higher Power Factor Translates to:

- Lower apparent power consumption
- Lower equipment electrical losses
- Electrically/physically smaller equipment
- Less expensive equipment
- Lower electric bill
- Implies lower distortion of the line voltage and current

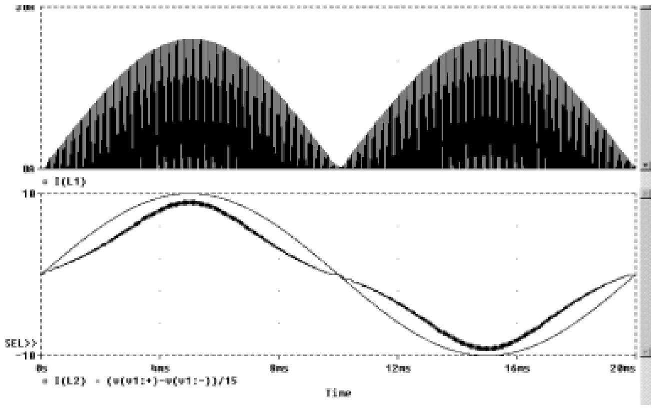

Vi
+
Vo


Line voltage
Line current
Line current harmonics
10A
1.0A
100mA
10mA
1.0mA
0Hz
0.2KHz
0.4KHz
0.6KHz
0.8KHz
1.0KHz
1.2KHz
1.4KHz
1.6KHz

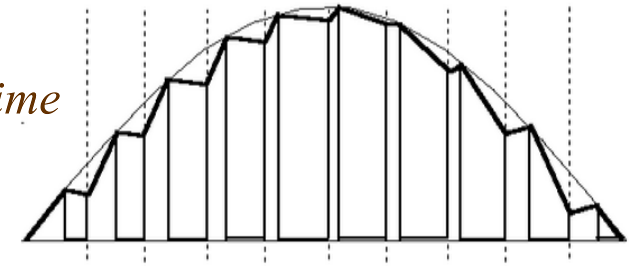
Active Power Factor Correction, AC – DC Converter with PF Control



I_c v. time



V_i, I_L, I_Q v. time



L V-s charging = L V-s discharging

$$\Delta i_{LQon} = \Delta i_{LQoff}$$

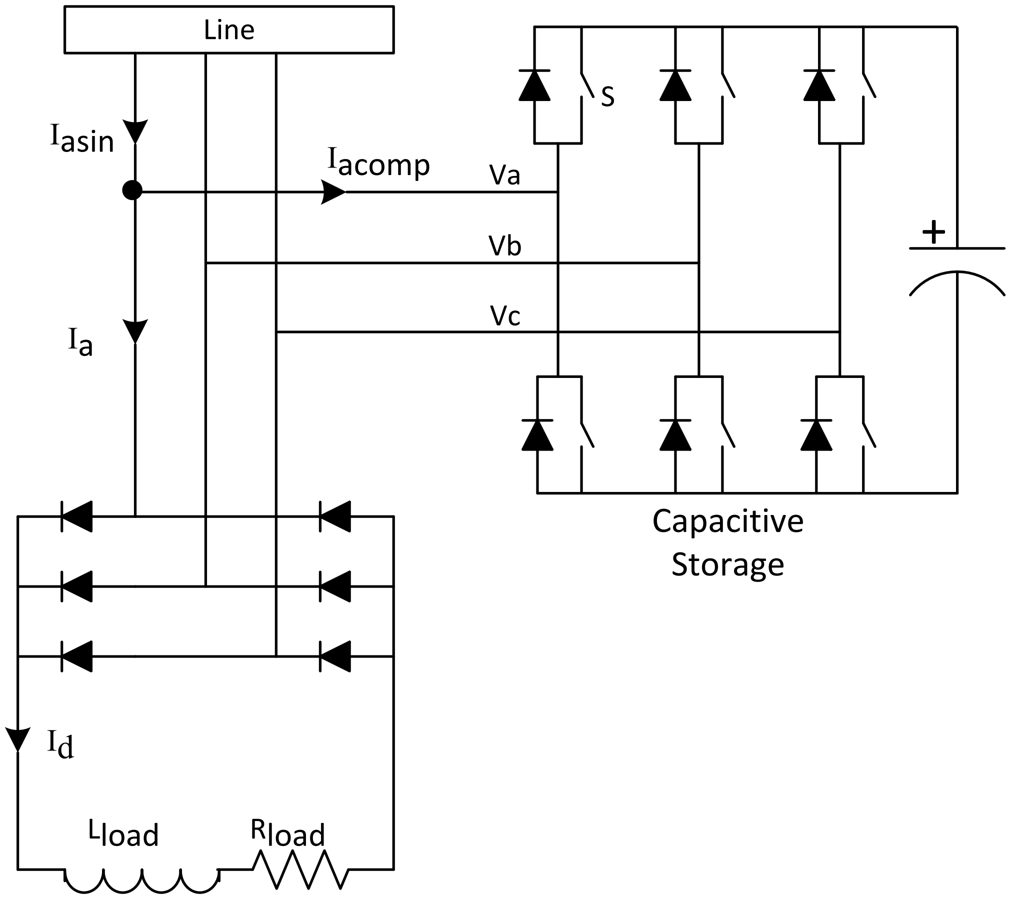
$$\Delta i_{LQon} = \frac{V_i * t_{on}}{L} \qquad \Delta i_{LQoff} = \frac{(V_o - V_i) * t_{off}}{L}$$

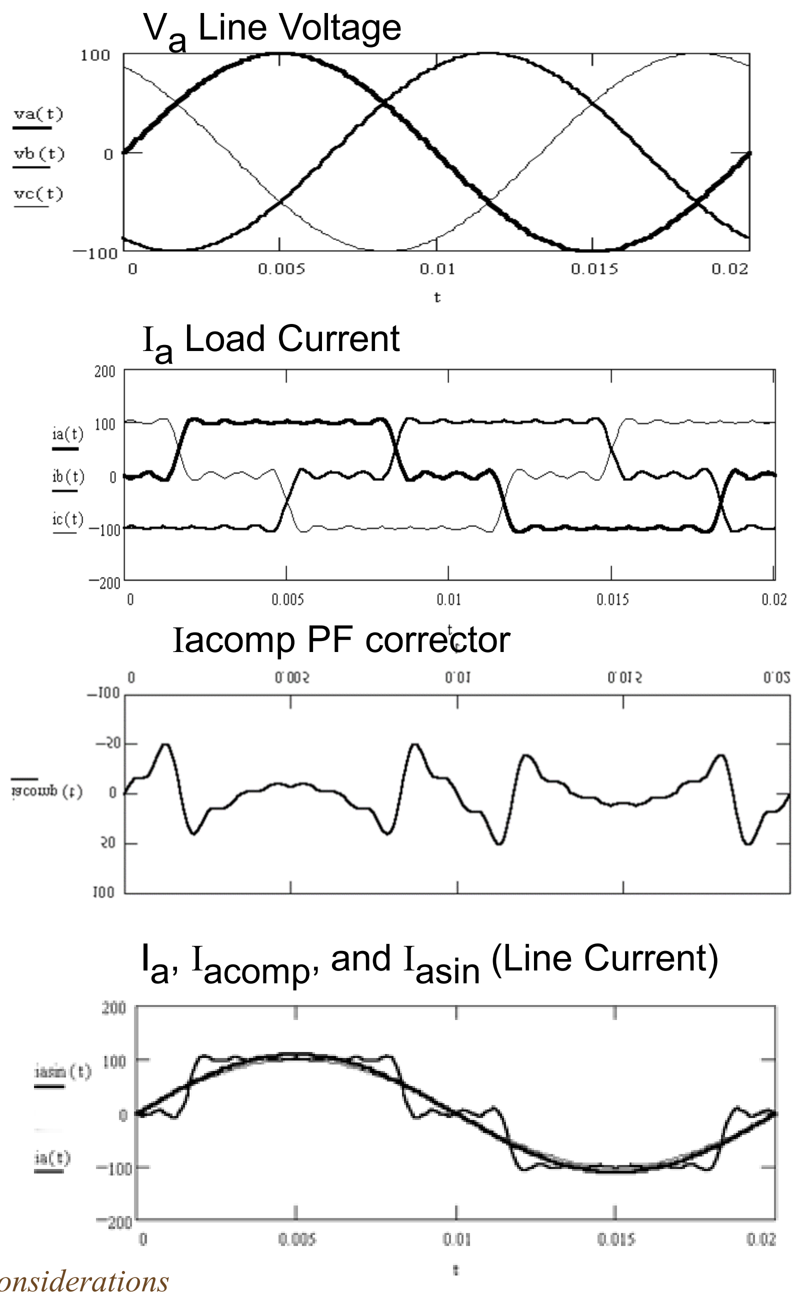
$$T = \frac{1}{f} = t_{on} + t_{off}$$

$$\Delta i_{LQon} = \frac{T}{L}\left(\frac{V_i t_{on}}{t_{on} + t_{off}}\right) = \frac{V_i}{L * f} D$$

$$\Delta i_{LQoff} = \frac{V_o - V_i}{L * f} * (1 - D)$$

We simulate a sine wave by adjusting L, f, and D

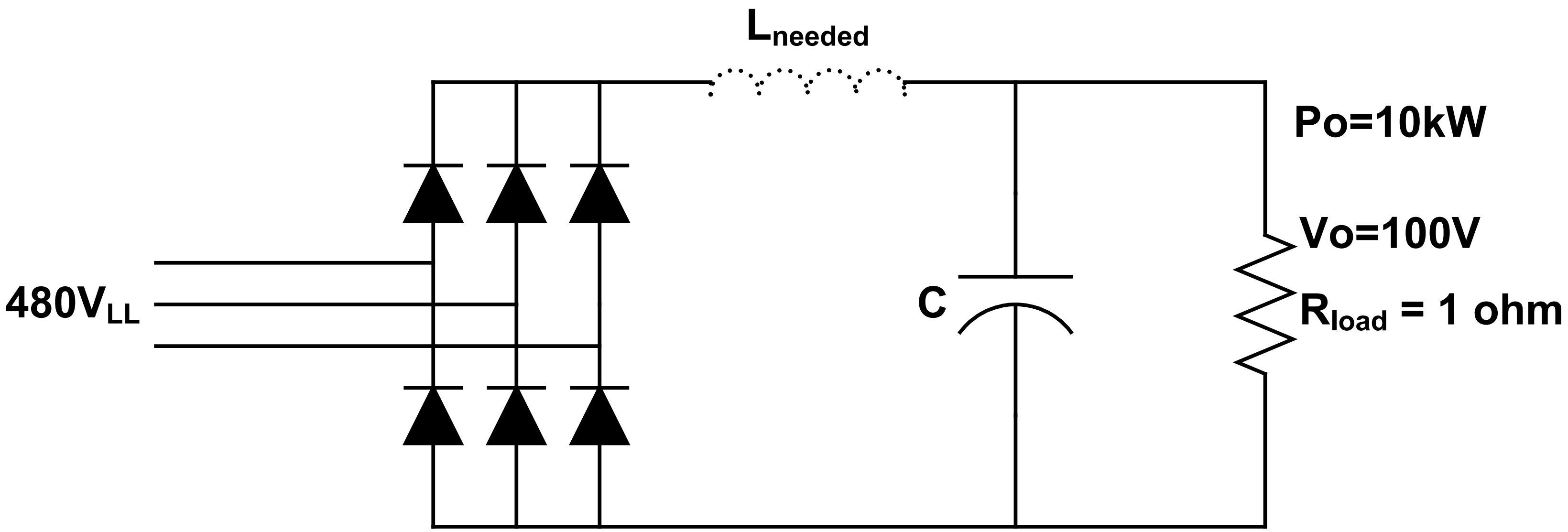




- *Appropriate switches (s) are rapidly opened and closed to control charging and discharging of the capacitor (I_{acomp})*
- *From KCL, $I_{asin}=I_a + I_{acomp}$*

Homework Problem # 7

A 10kW, 3 phase power supply has an efficiency of 90% and operates with a leading power factor of 0.8. Determine the size of the inductor needed to improve the power factor to 1.0.

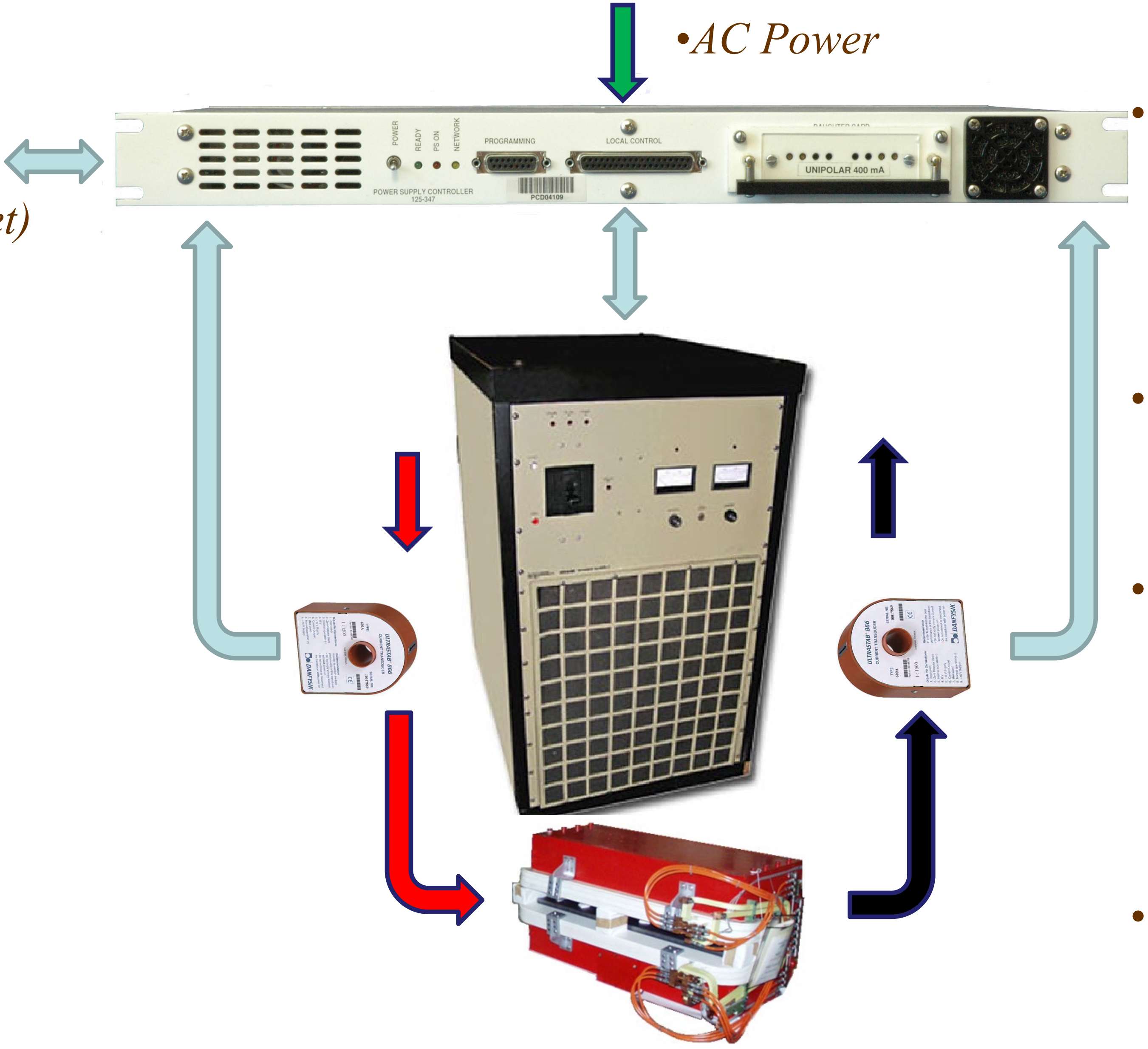


Section 6 – DC Power Supplies

- *Power Supply Definition, Purpose, and Scope*
- *Rectifiers*
- *AC Controllers*
- *Voltage and Current Sources*
- *Linear Systems Disadvantage*
- *Switchmode DC Power Supplies*
  - *Advantages*
  - *Switch Candidates*
  - *Converter Topologies*
  - *Pulse Width Modulation*
  - *Conducting and Switching Losses*
  - *Resonant Switching*
- *High Frequency Transformers and Inductors*
- *Ripple Filters*
- *Other Design Considerations*
- *Power Supplies in Particle Accelerators*

A Typical DC Magnet Power System

- *EPICS*
- *(Ethernet)*



- *AC Power*
- *Power Supply Controller*
- *Power Supply*
- *Feedback and Monitor Transductors*
- *Magnet*

Definition

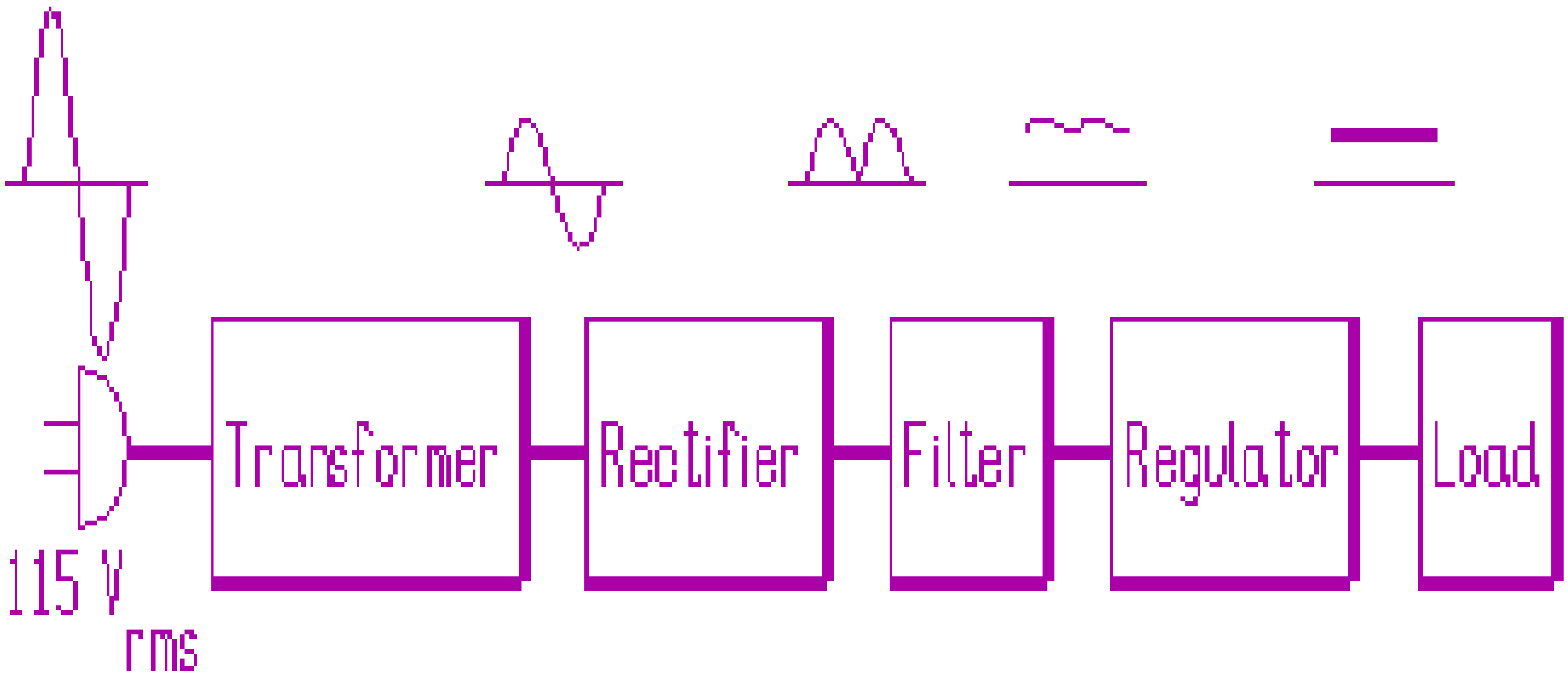
- *A "DC power supply" is a device or system that draws uncontrolled, unregulated input AC or DC power at one voltage level and converts it to controlled and precisely regulated DC power at its output in a form required by the load*

Purpose

- *Change the output to a different level from the input (step-up or step-down)*
- *Rectify AC to DC*
- *Isolate the output from the input*
- *Provide for a means to vary the output*
- *Stabilize the output against input line, load, temperature and time (aging) changes*

Example

- *120 VAC is available. The load is a logic circuit in a personal computer that requires regulated 5V DC power. The power supply makes the 120 V AC power source and 5V DC load compatible*

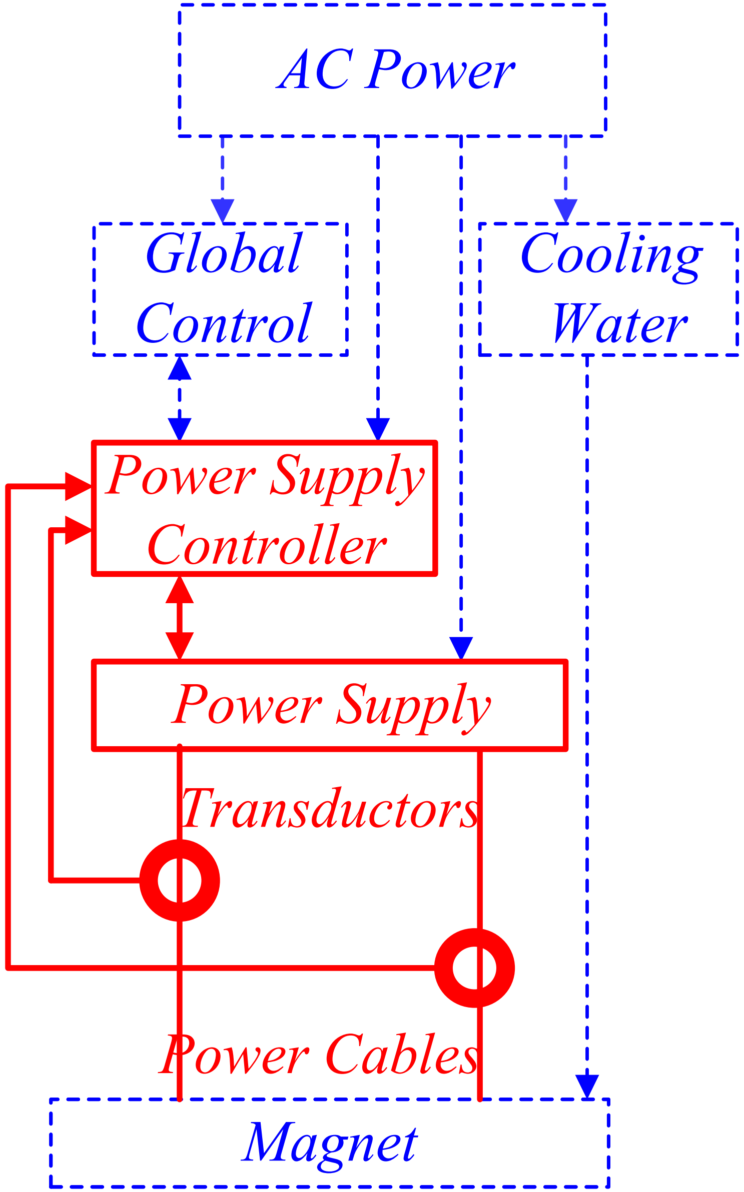


Power Line EMI/EMC

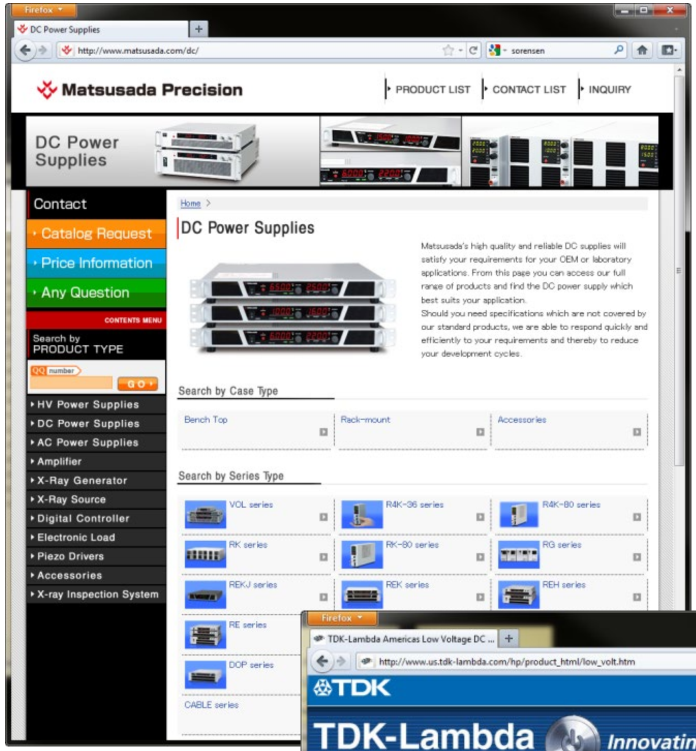
Controls

Interlocks

Reliability


AC Power
Global Control
Cooling Water
Power Supply Controller
Power Supply
Transductors
Power Cables
Magnet

Power Supply Definition, Purpose, and Scope – A DC Magnet Power System



Power Supply Definition, Purpose, and Scope – Characteristics

Some characteristics of the power supplies most often used in particle or synchrotron accelerators are:

• They are voltage or current sources that use the AC mains (off-line) as their source of energy.

•They can be DC-DC converters

•They are not AC controllers.

• They are <u>not</u> computer power supplies or printed circuit board converters

•They have a single output.

• The output voltage or current is not fixed (such as those used by the telephone and communications industry), but are adjustable from zero to the full rating

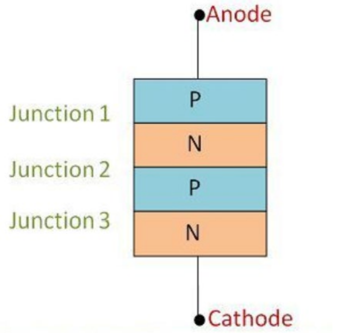
•The DC output power ratings range from a few watts to several megawatts

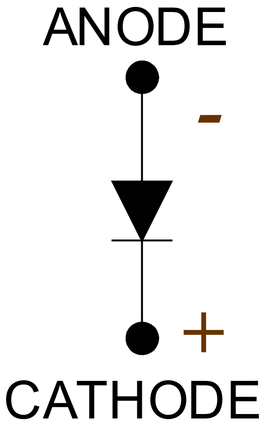
•Typical loads are magnets or capacitor banks

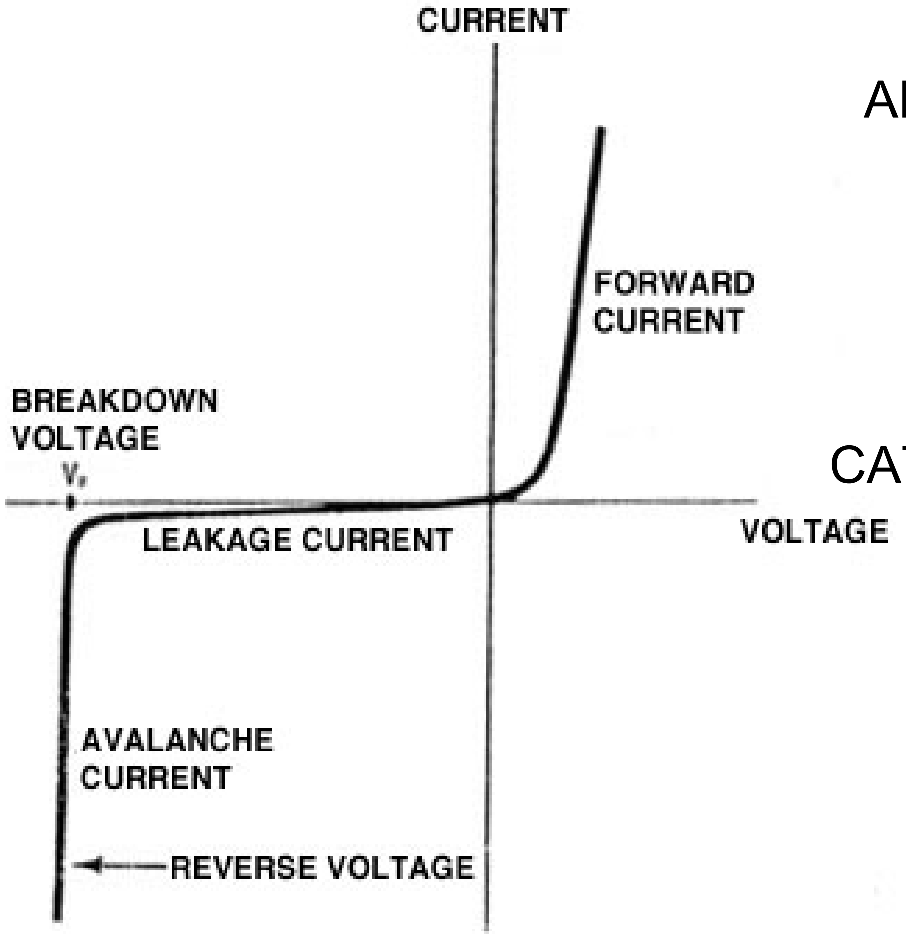
•The bipolar power supplies discussed later are typically used for small corrector magnets are DC-DC converters fed from a common off-line power supply

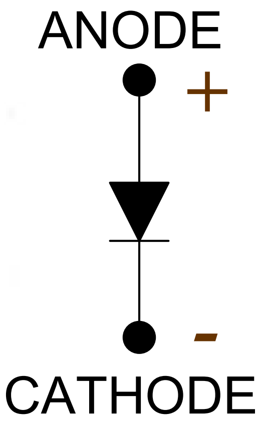
• They can have pulsed outputs as discussed later

Rectifiers - Diode Characteristics



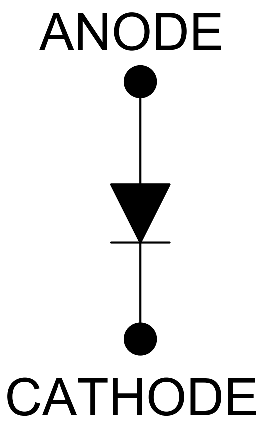






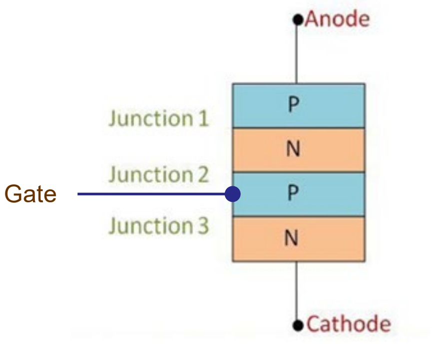
In the reverse direction, there is a small leakage current up until the reverse breakdown voltage is reached

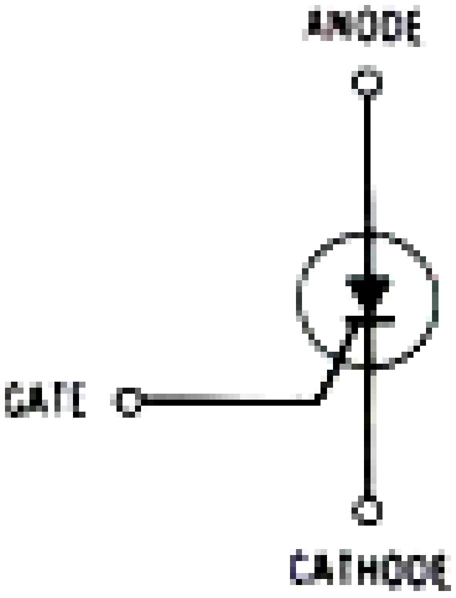
Forward voltage drop, V_f: a small current conducts in forward direction up to a threshold voltage, 0.3V for germanium and 0.7V for silicon



Schematic representation

- *Forward voltage drop,* V_F *or* $V_{F(AV)}$
- *Forward current,* I_F *or* $I_{F(AV)}$
- *Maximum reverse (blocking) voltage,* V_R
- *Average reverse (leakage) current,* $I_{R(AV)}$
- *Forward recovery time,* t_{fr}
- *Reverse recovery time,* t_{rr} *, usually much less than* t_{fr}
- *Peak surge current,* I_{surge}
- *Cooling (air, water, oil, other)*
- *Package style*
- $I = I_S\left(e^{\frac{qV}{nkT}} - 1\right)$ *Shockley equation*

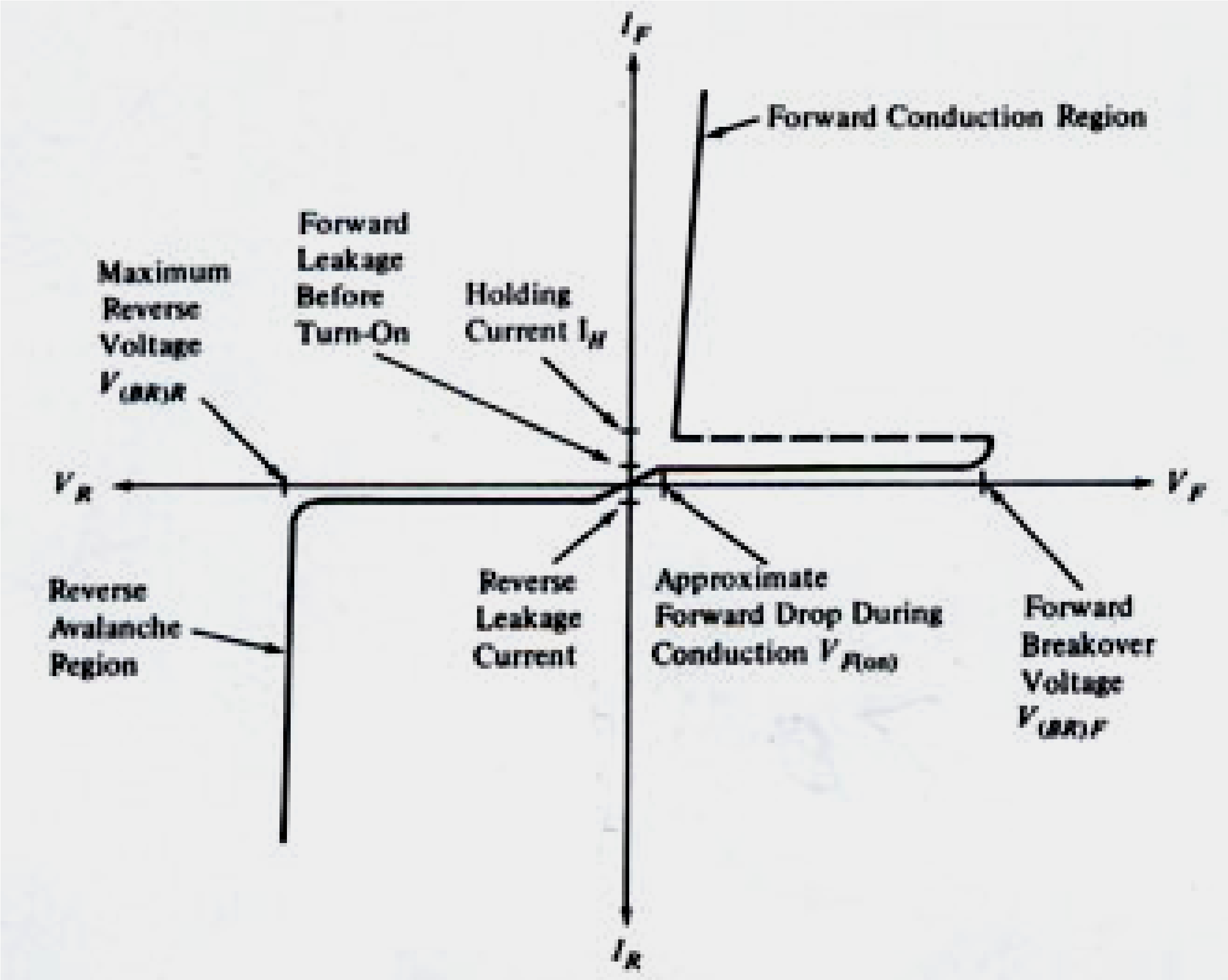




Schematic representation

SCR properties

- *It is simply a conventional rectifier with turn on controlled by a gate signal*
- *It is controlled from the off to on states by a signal applied to the gate-cathode*
- *It has a low forward resistance and a high reverse resistance*
- *It remains on once it is turned on even after removal of the gate signal*
- *The anode-cathode current must drop below the "holding" value in order to turn it off*



V–I characteristics of the SCR with gate open.

Rectifiers - SCR Considerations

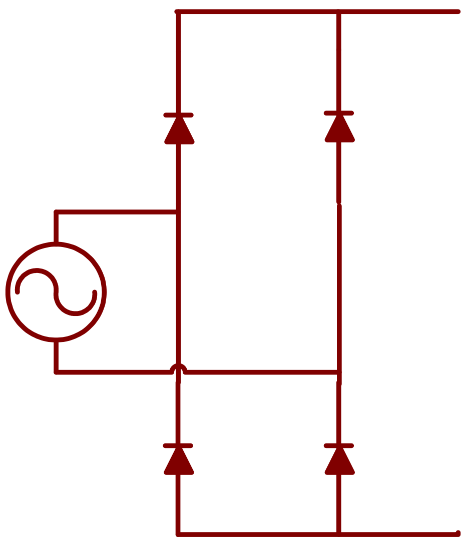
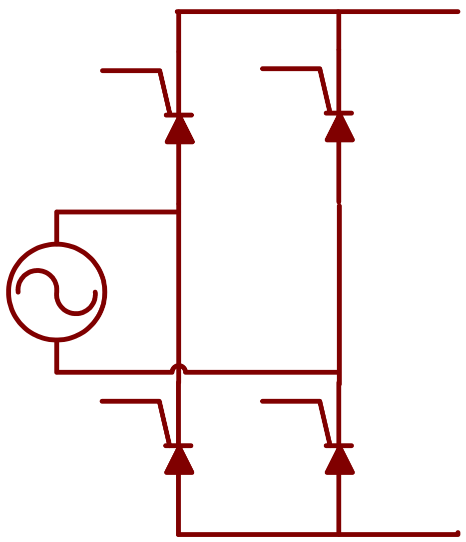
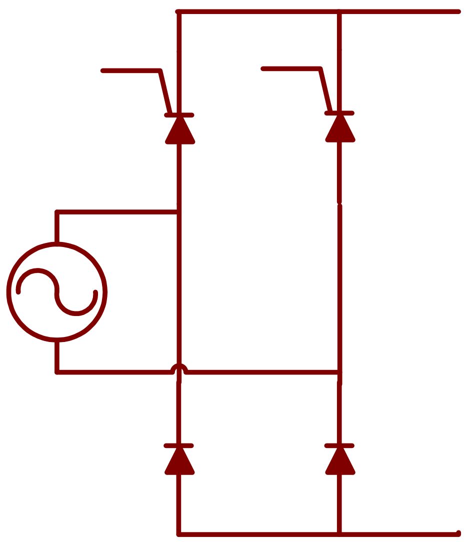
- *Maximum average on-state current,* I_{TAV}
- *RMS on-state current,* I_{TRMS}
- *Gate trigger current minimum,* I_{Gmin}
- *Gate current maximum,* I_{Gmax}
- *Minimum latching current,* I_L
- *Minimum holding current,* I_H
- *Maximum forward di/dt*
- *Peak repetitive reverse voltage,* V_{RRM}
- *Peak forward voltage*
- *Maximum forward dv/dt*
- *Maximum reverse dv/dt*
- *Power dissipation,* P_{AVG}
- *Gate power dissipation,* P_G
- *Maximum junction temperature,* T_{Jmax}

- *A rectifier converts ac voltage to dc voltage*
  - *Classifications*

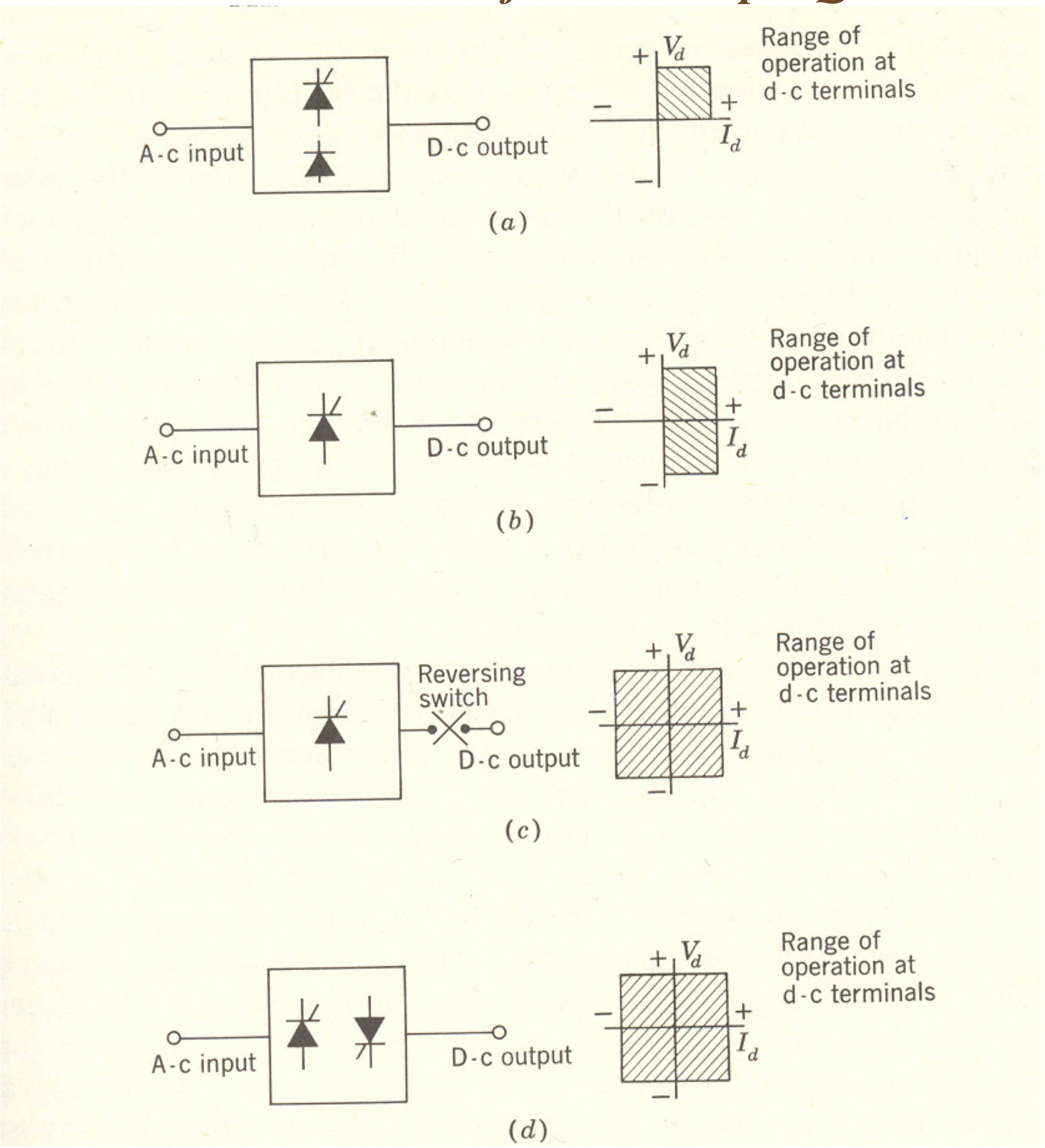
    Uncontrolled rectifiers (diodes)

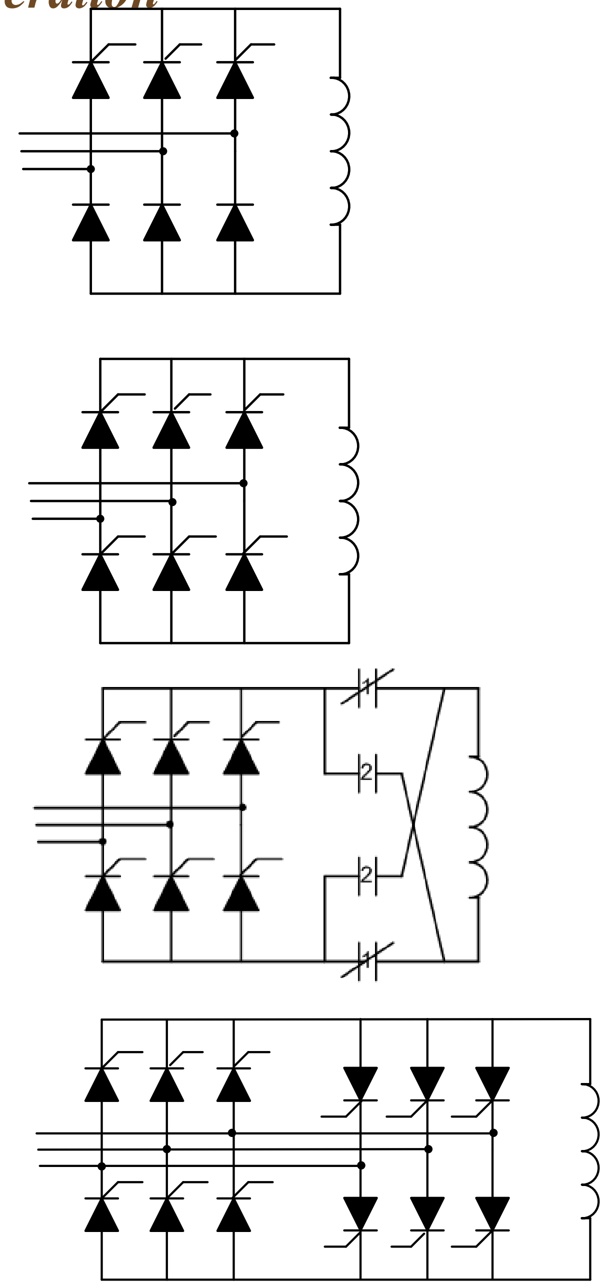
    Controlled rectifiers (all SCRs)

    Semi-controlled rectifiers (SCRs and diodes)

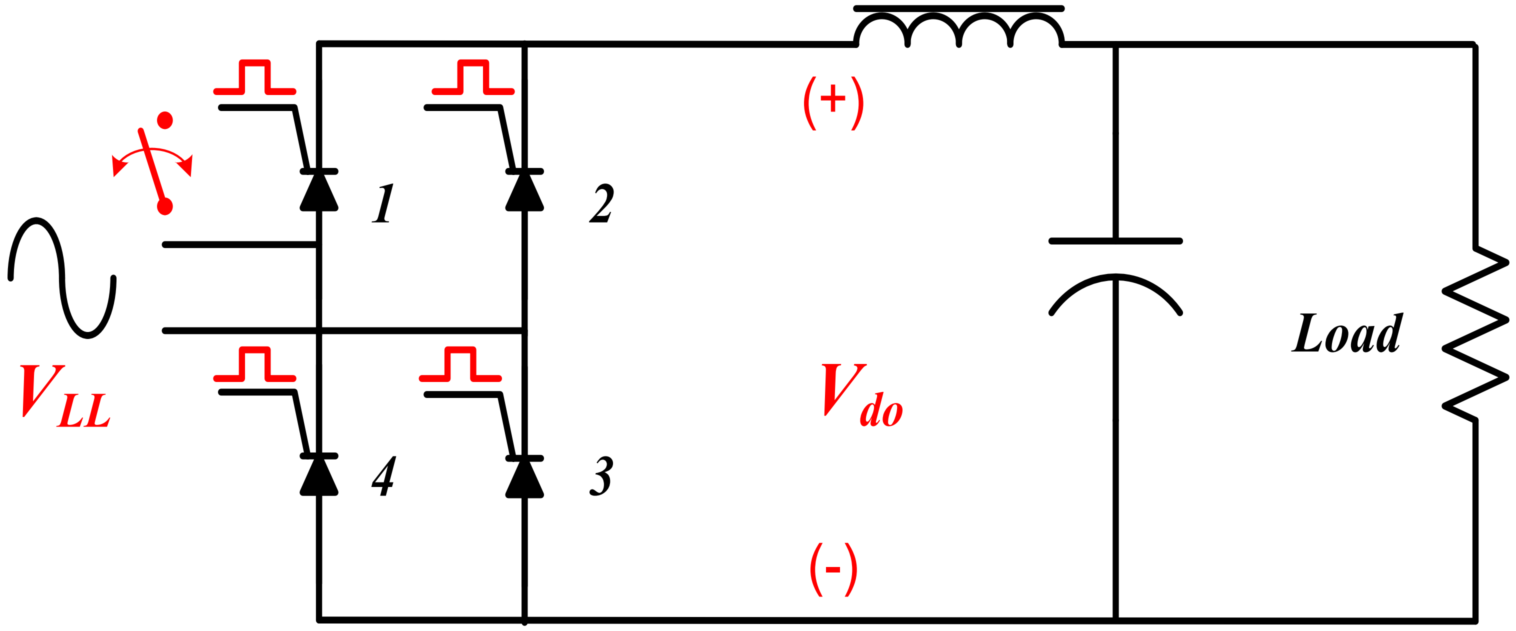




Rectifiers - Multiple Quadrant Operation

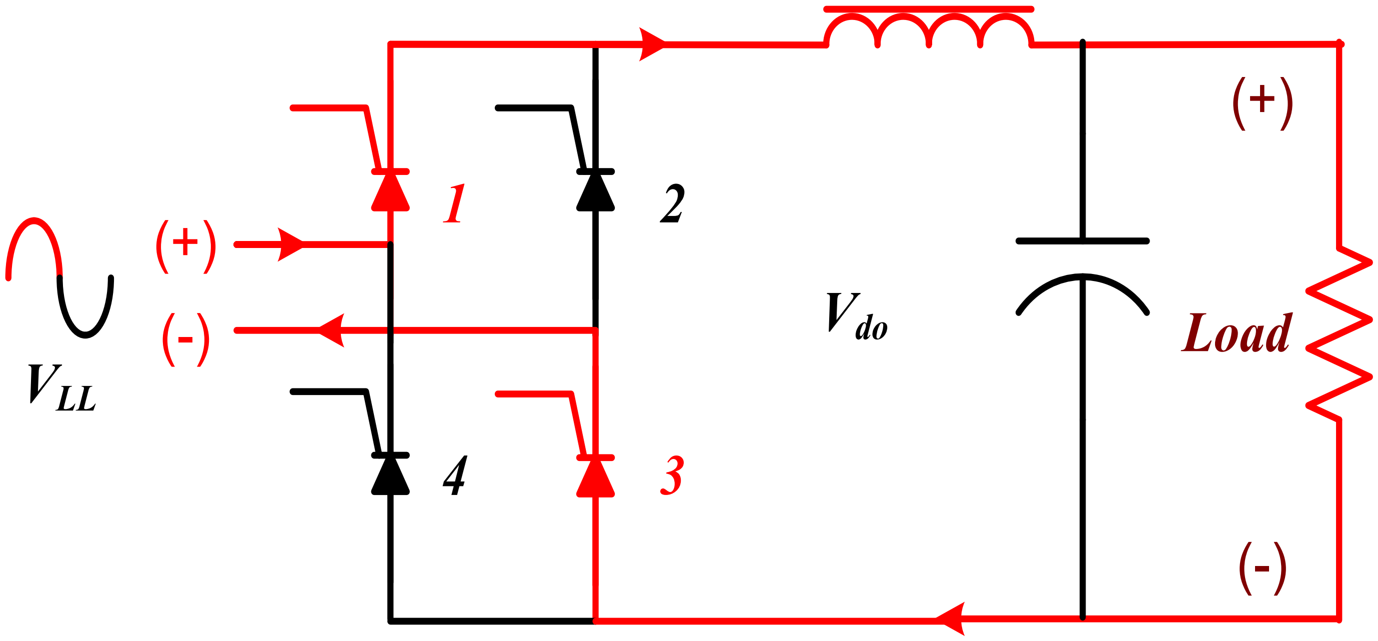




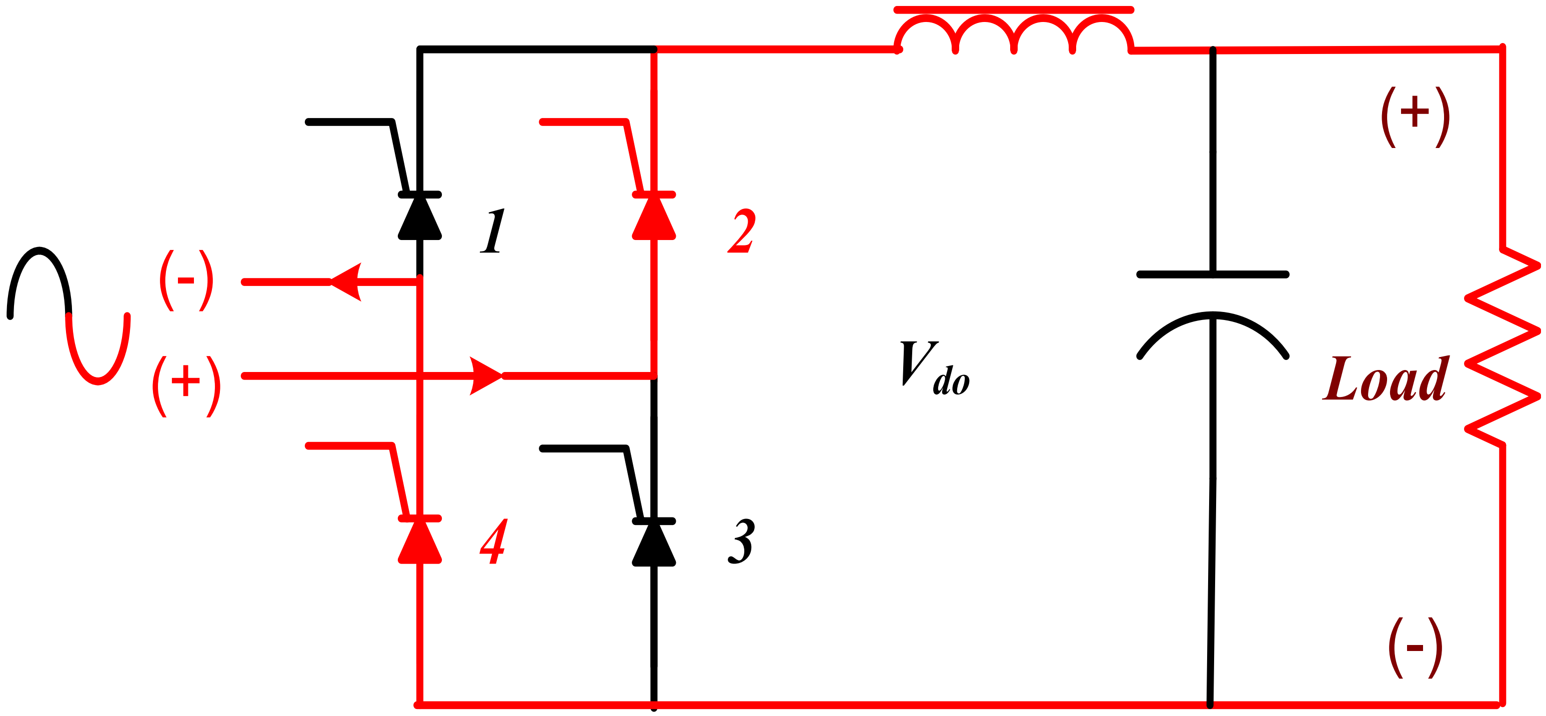
Rectifiers - 1 ϕ Full Wave (q = 2 Pulse)



- *q ≡ the number of possible rectifier states*
- *SCR s are electronic switches*

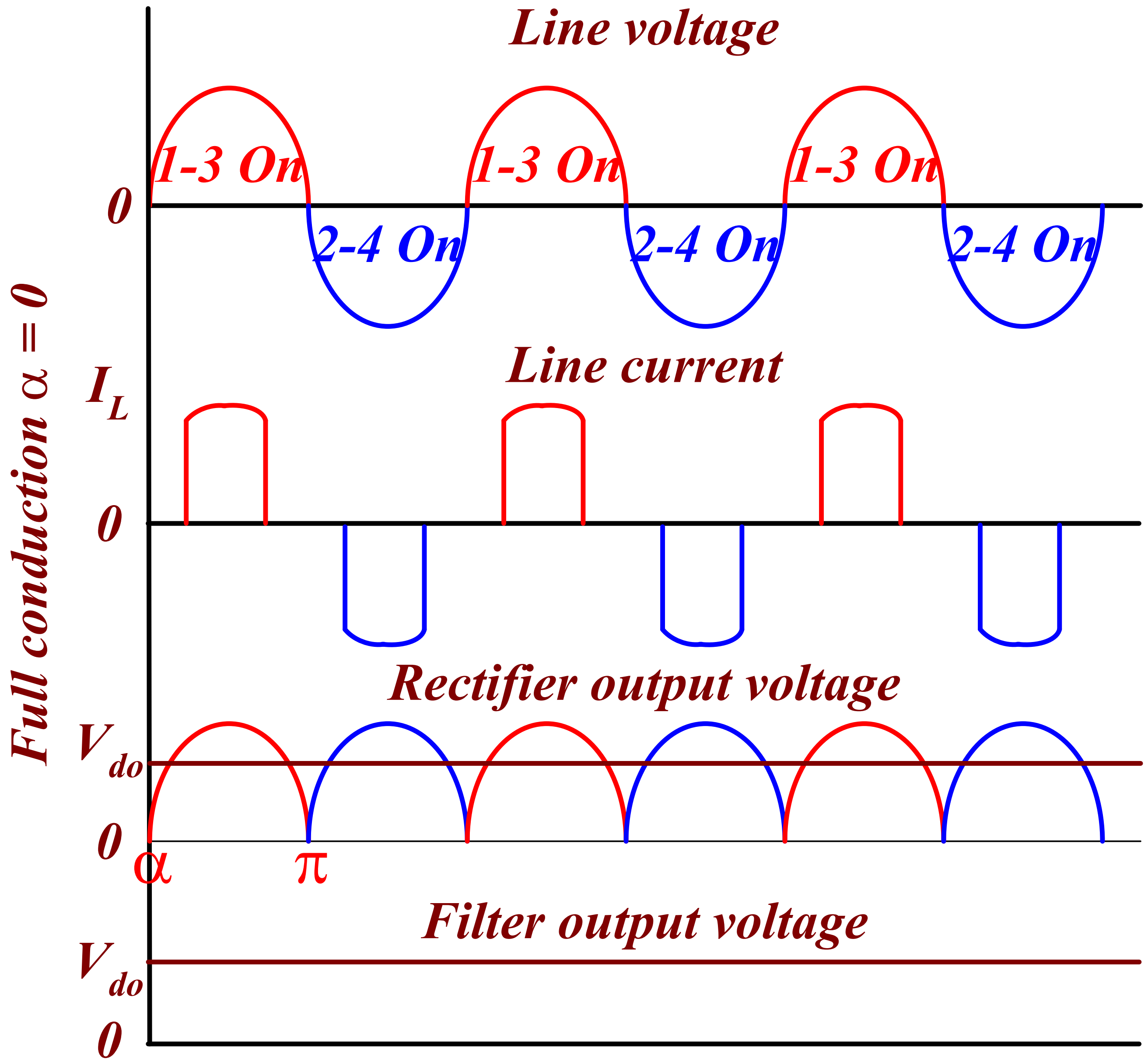


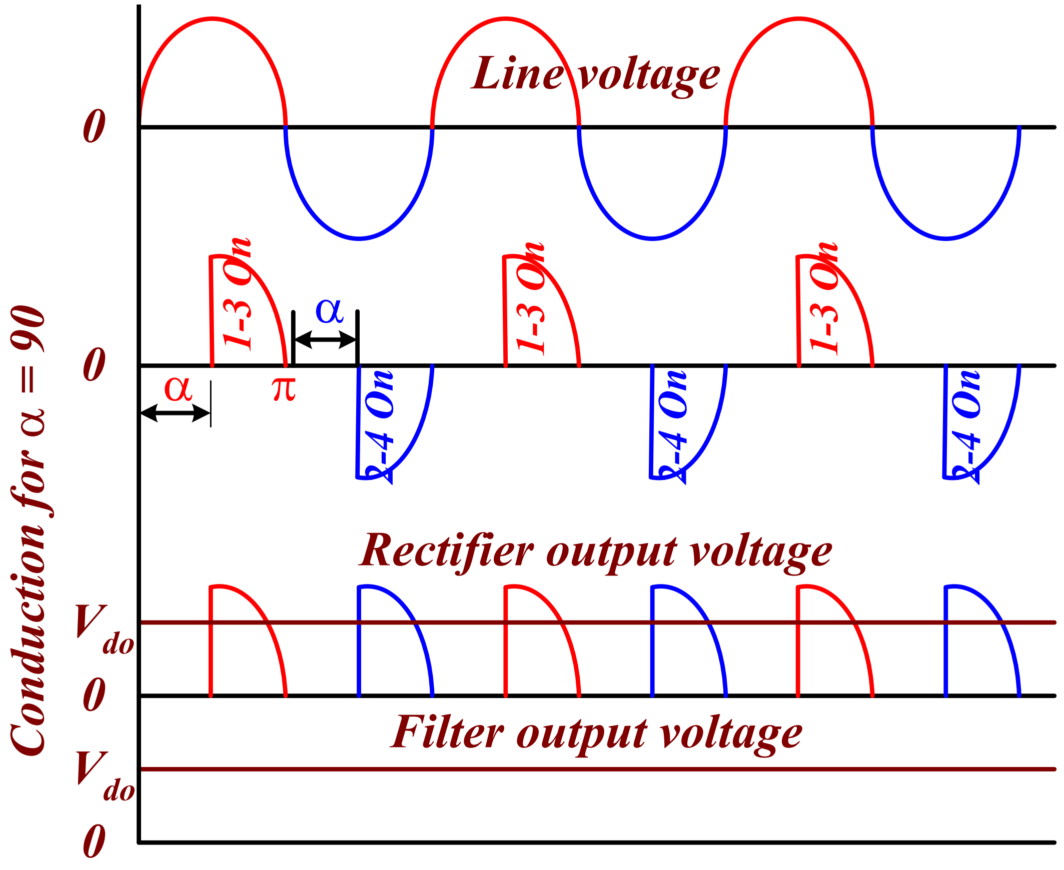
State 1 : SCR s 1 – 3 On



State 2 : SCR s 2 – 4 On

Rectifiers - 1 ϕ Full Wave (q = 2 Pulse)





$$V_{do} = \frac{1}{T}\int_{t}^{T} v_{LL}(t)dt = \frac{1}{T}\int_{t}^{T} \sqrt{2}\, V_{LL} \sin\omega t \; dt = \frac{1}{\omega T}\int_{\alpha}^{\omega T} \sqrt{2}\, V_{LL} \sin\omega t \; d\omega t$$

the SCR gate trigger retard angle range is $0 \le \alpha \le \pi$

$$V_{do} = \frac{\sqrt{2}\, V_{LL}}{\pi}(1+\cos\alpha) \quad \text{for resistive load}$$

Rectifiers - 1 ϕ, Full Wave (q = 2 Pulse) Summary

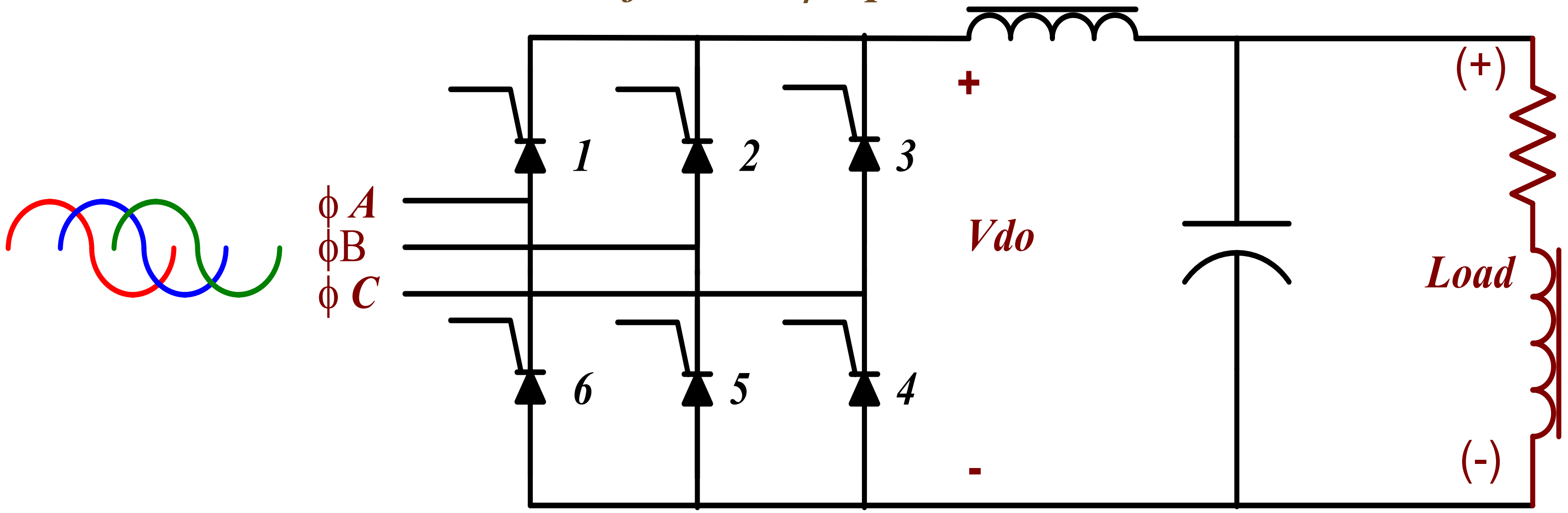
- 2 pulse rectifier – low input power factor, high output ripple
- Ripple frequency is 120 Hz (if input is 60 Hz)
- Large filter needed
- Limited in use to power supplies < 2.5 kW

$$V_{do} = \frac{1}{T}\int_{t}^{T} v_{LL}(t)dt = \frac{1}{T}\int_{t}^{T}\sqrt{2}\,V_{LL}\sin\omega t\;dt = \frac{1}{\omega T}\int_{\alpha}^{\omega T}\sqrt{2}\,V_{LL}\sin\omega t\;d\omega t$$

the SCR gate trigger retard angle range is $0 \le \alpha \le \pi$

$$V_{do} = \frac{\sqrt{2}\,V_{LL}}{\pi}(1+\cos\alpha) \quad \text{for resistive load}$$

Rectifiers - 3 ϕ, q = 6 Pulse

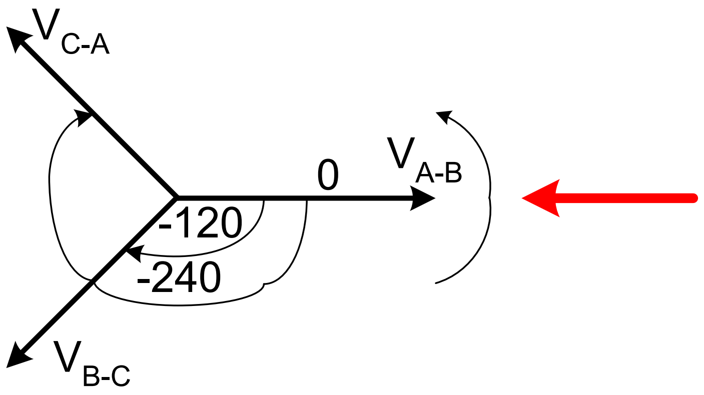


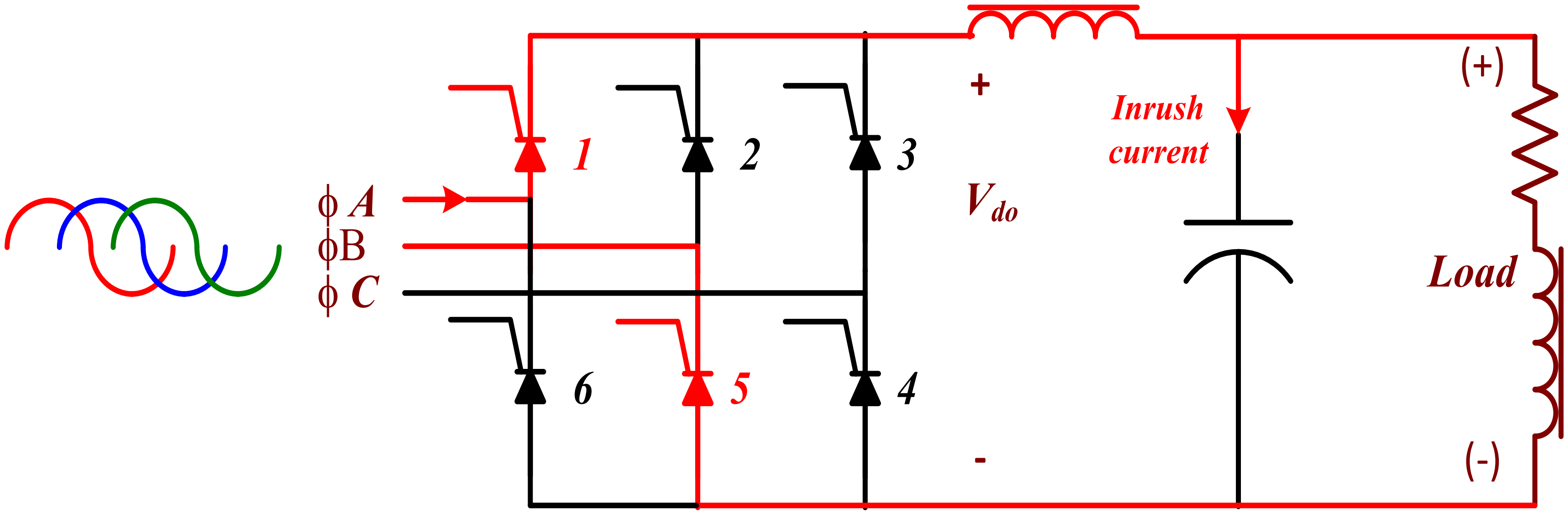
Assuming the American standard phase rotation of

$V_{A-B} = |V| e^{j0}$ $V_{B-C} = |V| e^{-j120}$ $V_{C-A} = |V| e^{-j240}$

The thyristor firing sequence is:

1-5, 1-4, 2-4, 2-6, 3-6, 3-5

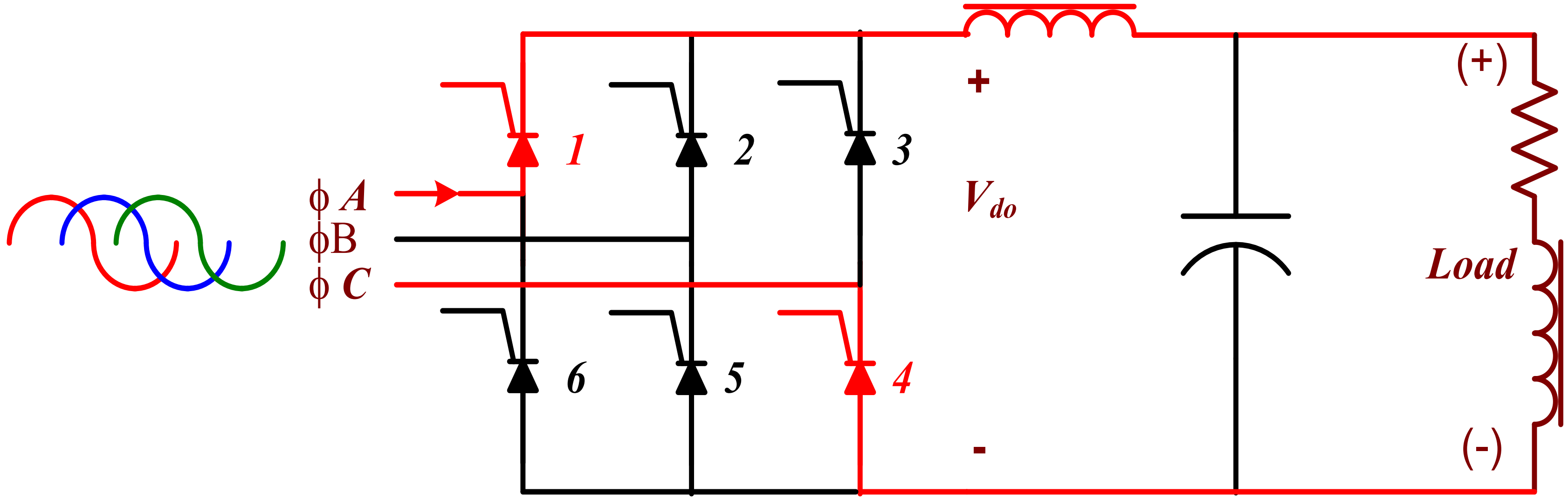




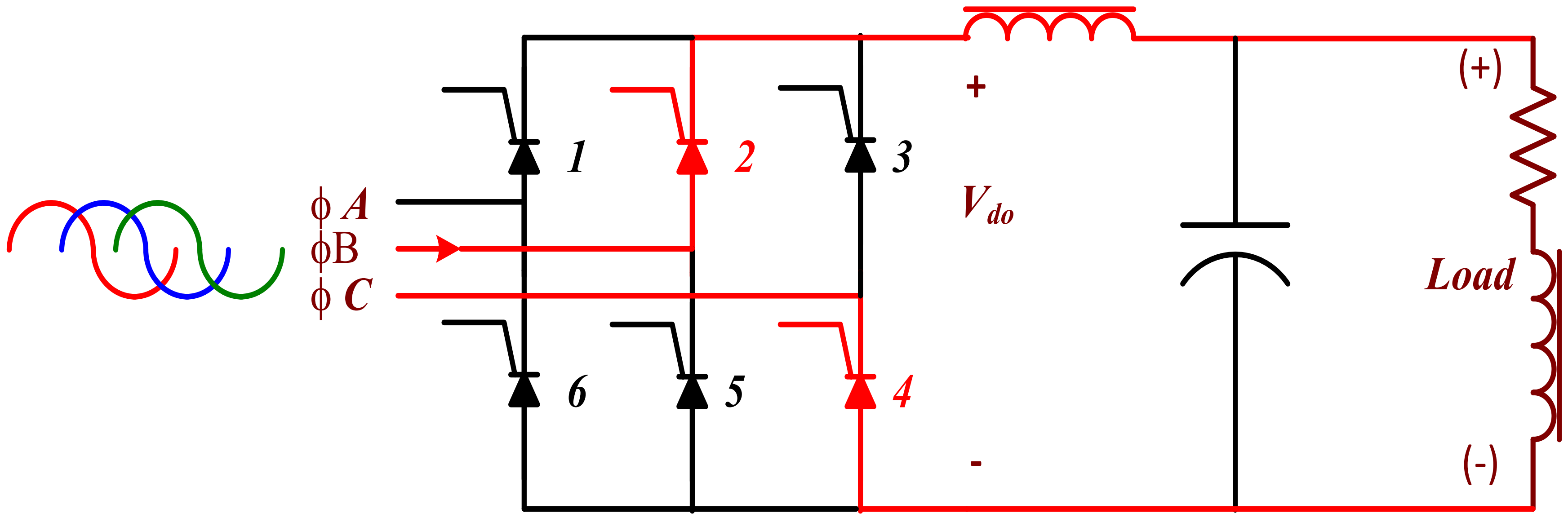
State 1: A-B (+) SCR s 1 – 5 On

Note: Phase SCRs from full retard to full forward slowly to bring the rectifier output voltage up slowly and reduce the capacitor inrush current

Rectifiers - 3 ϕ, q = 6 Pulse

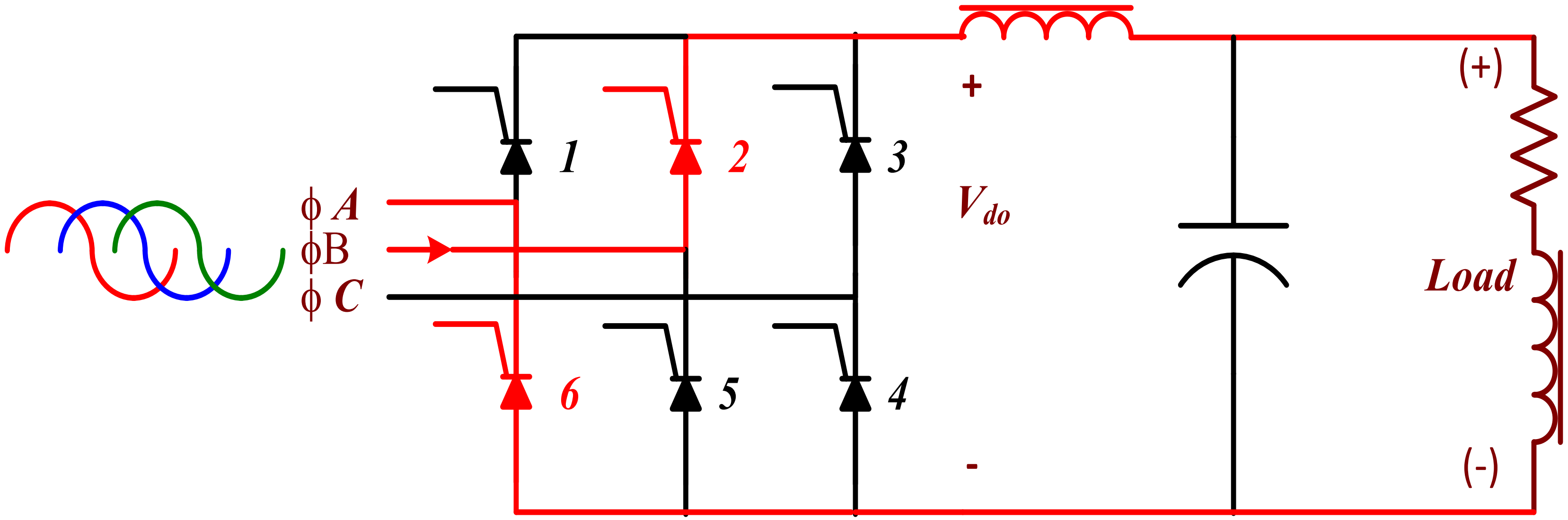


State 2 : A-C (+), 5 off, SCR s 1 – 4 On



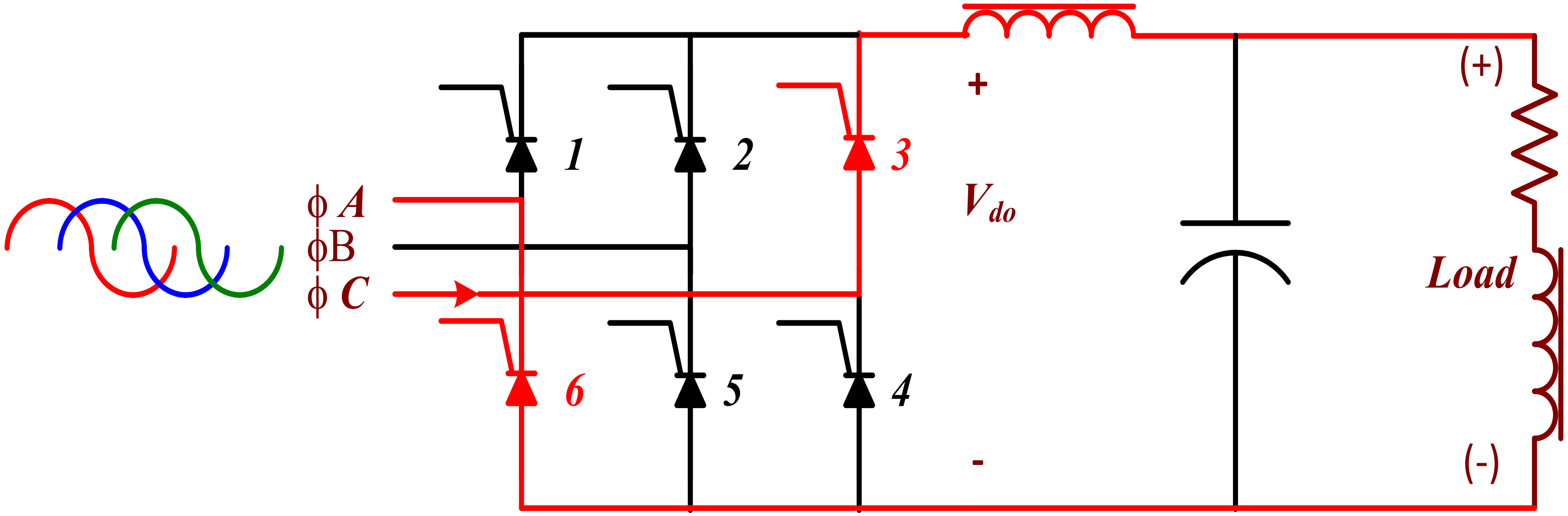
State 3 : B-C (+), 1 off, SCR s 2 – 4 On

3 ϕ, $q = 6$ Pulse Rectifier



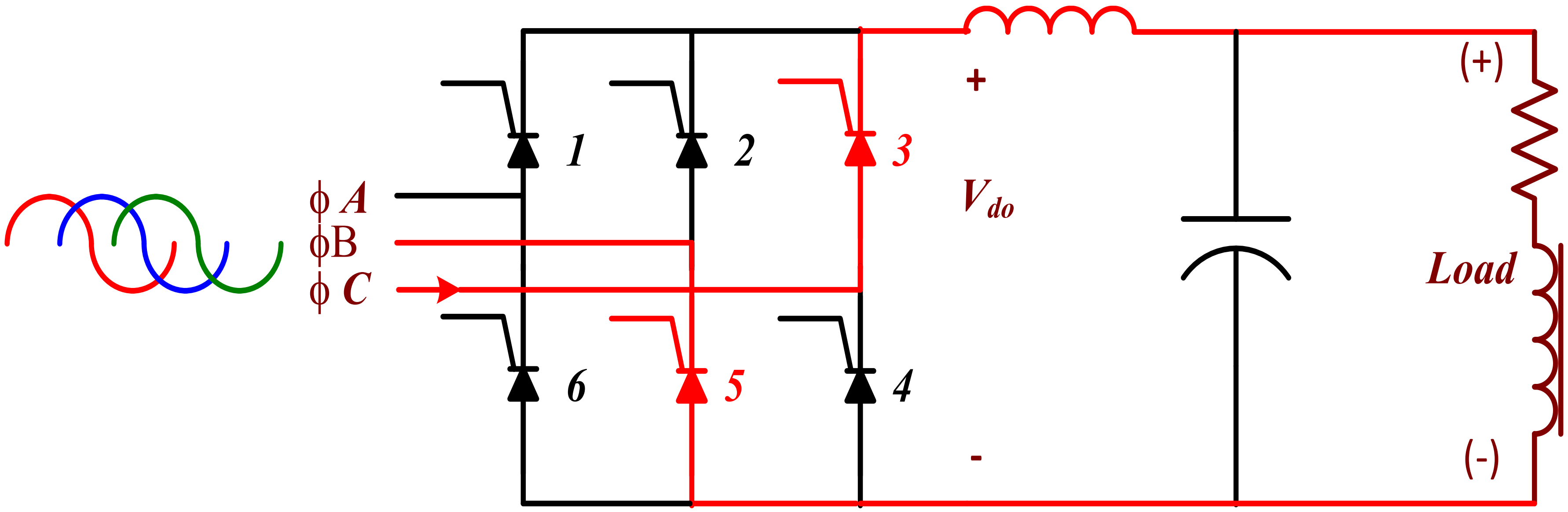
State 4 : B-A (+), 4 off, SCR s 2 – 6 On

$3\ \phi,\ q = 6$ Pulse Rectifier



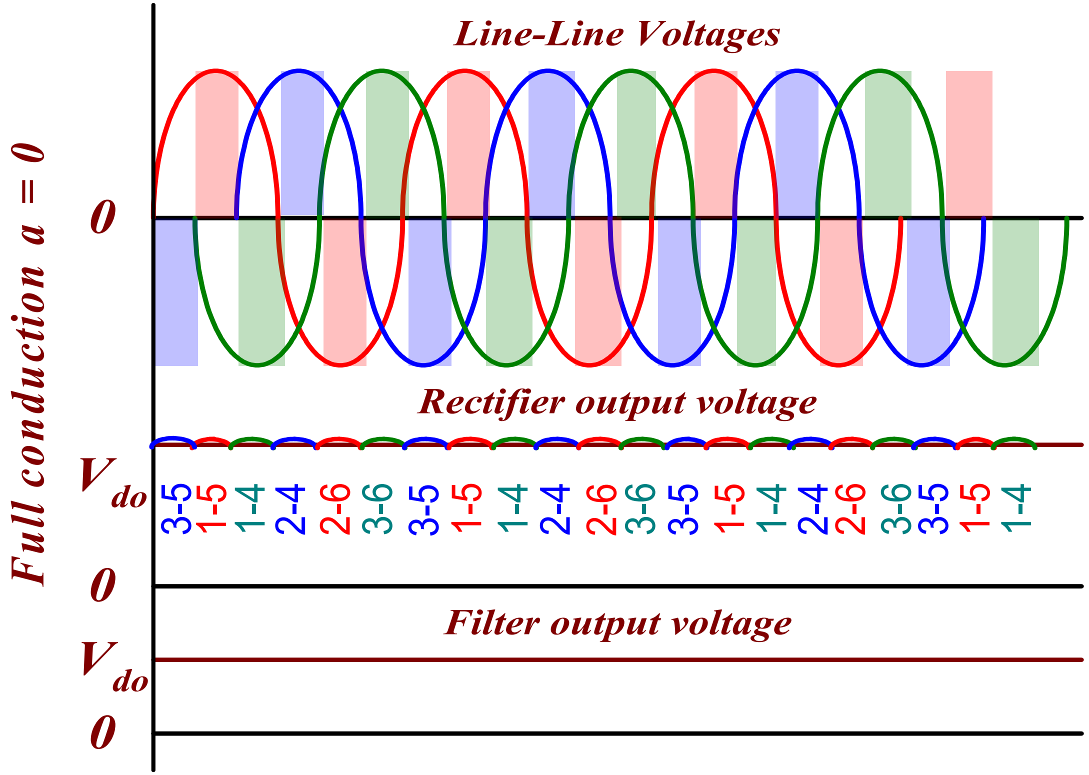
State 5 : C-A (+), 2 off, SCR s 3 – 6 On

3 ϕ, q = 6 Pulse Rectifier



State 6 : C-B (+), 6 off, SCR s 3 – 5 On

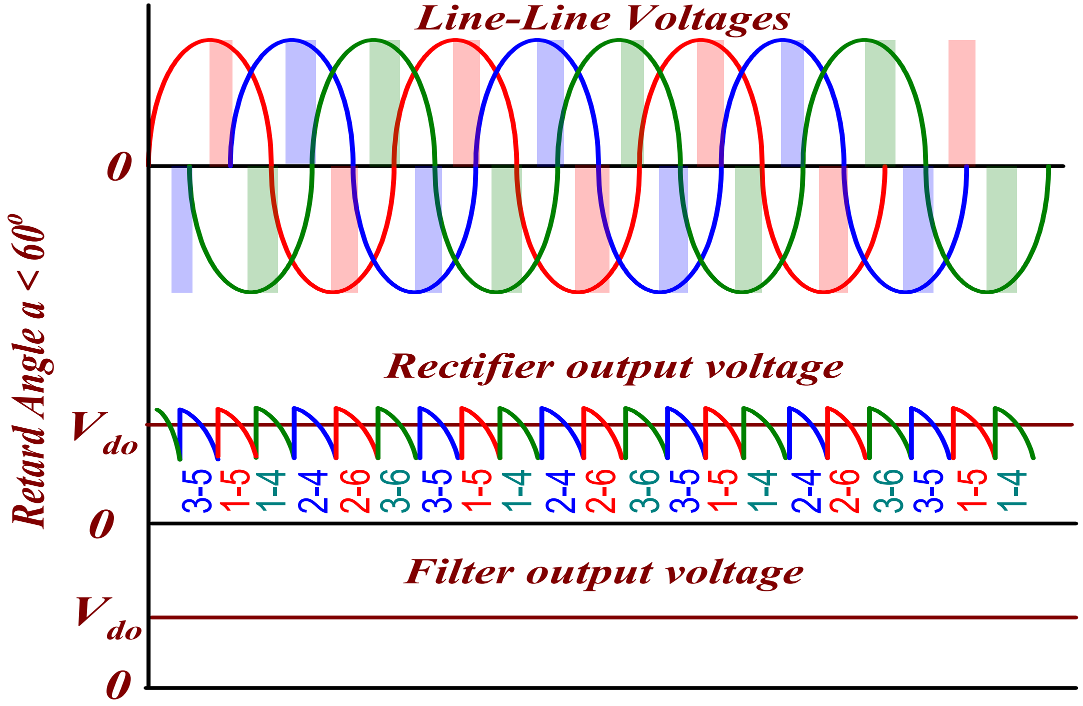
3 ϕ, q = 6 Pulse Rectifier Waveforms



$$V_{do} = \frac{3\sqrt{2}}{\pi} V_{LL} \cos\alpha$$

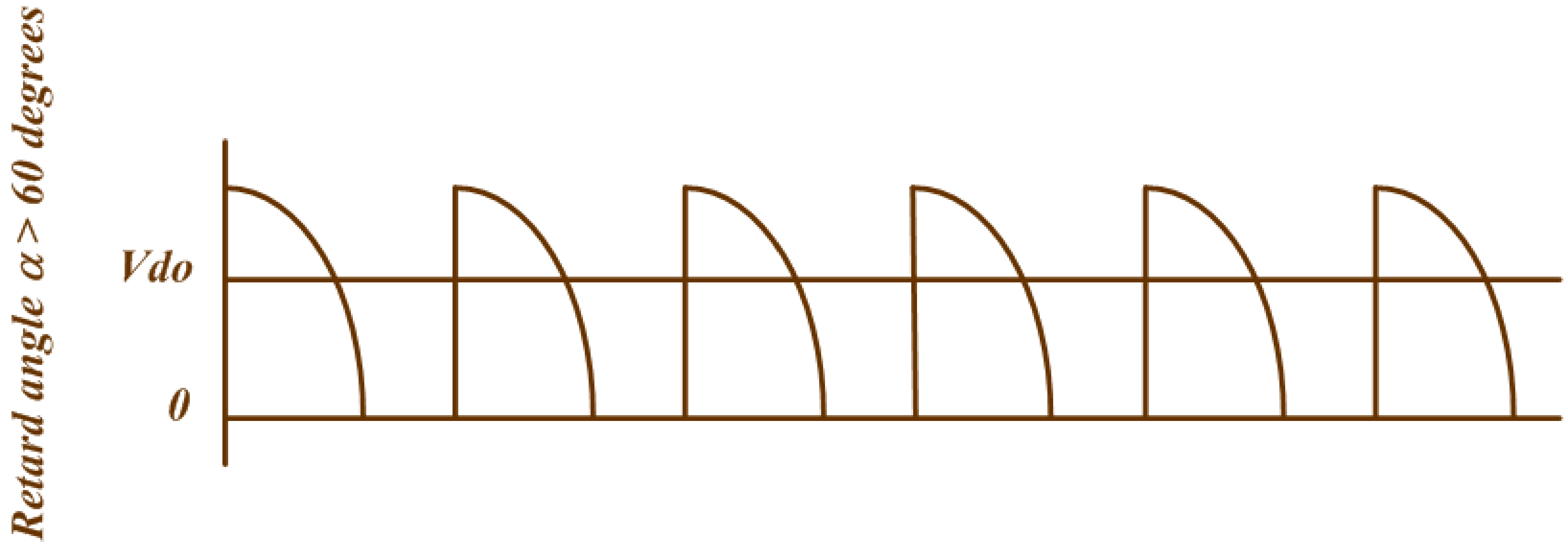
where α is the gate trigger retard angle and conduction is continuous

3 ϕ, q = 6 Pulse Rectifier Waveforms



For $0 \le \alpha \le \dfrac{\pi}{3}$ *where* α *is the gate trigger retard angle and conduction is continuous*

$$V_{do} = \frac{3\sqrt{2}}{\pi} V_{LL} \cos\alpha$$



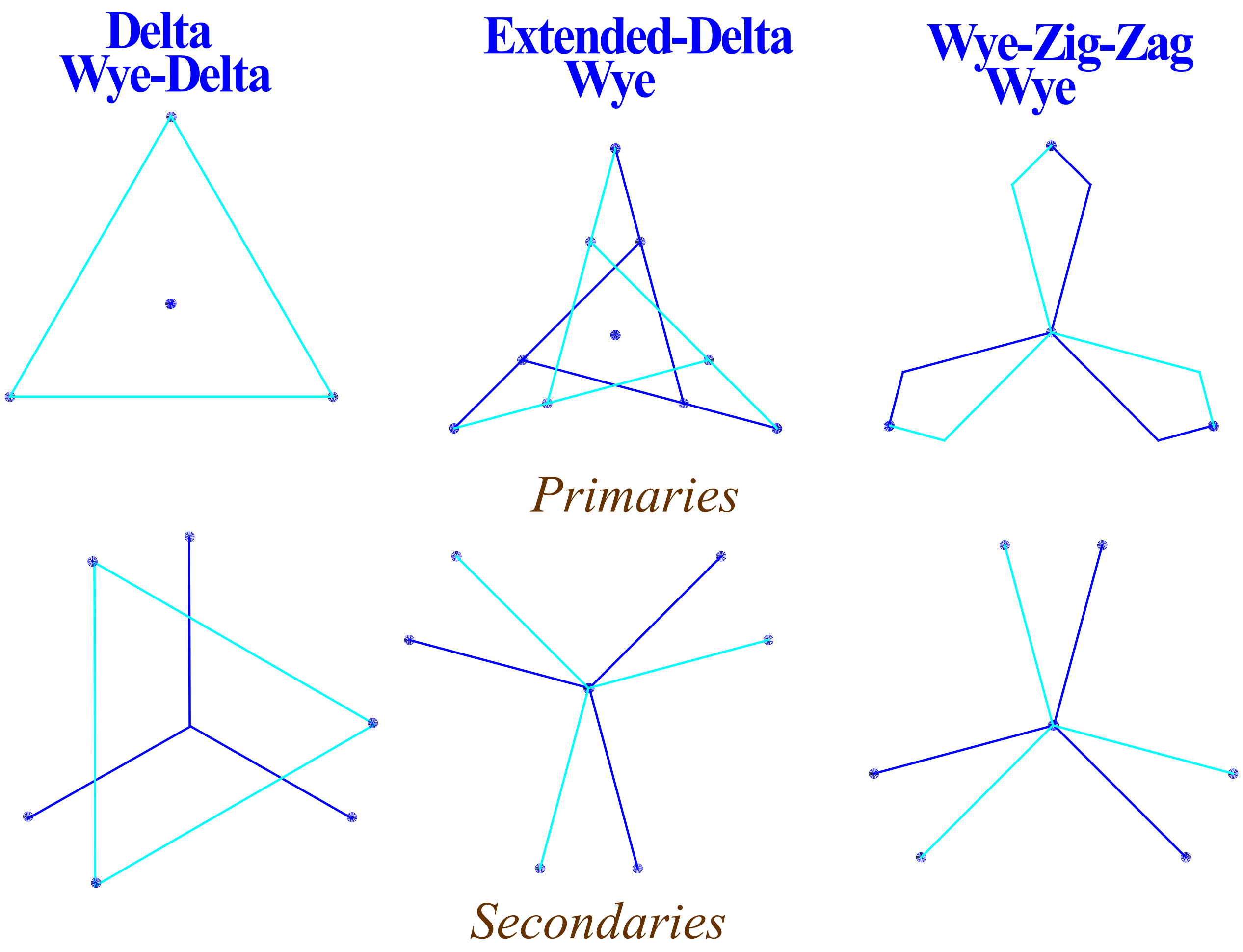
$$For\ \frac{\pi}{3} < \alpha \le \frac{2\pi}{3}\ \ where\ conduction\ can\ be\ discontinuous$$

$$V_{do} = \frac{3\sqrt{2}}{\pi} V_{LL}\left(1+\cos\left(\alpha+\frac{\pi}{3}\right)\right)\ \ for\ resistive\ load$$

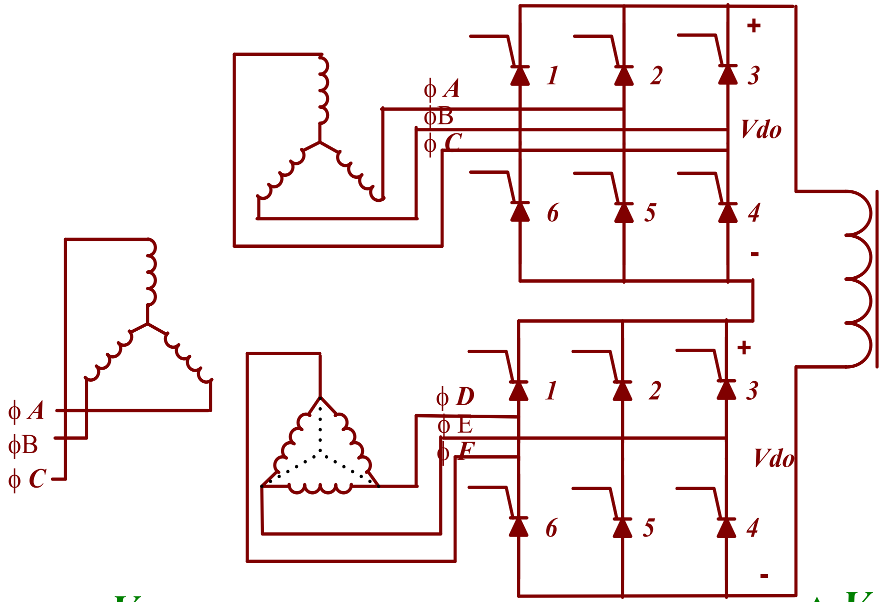
$3\ \phi,\ q = 6$ Pulse Rectifier Summary

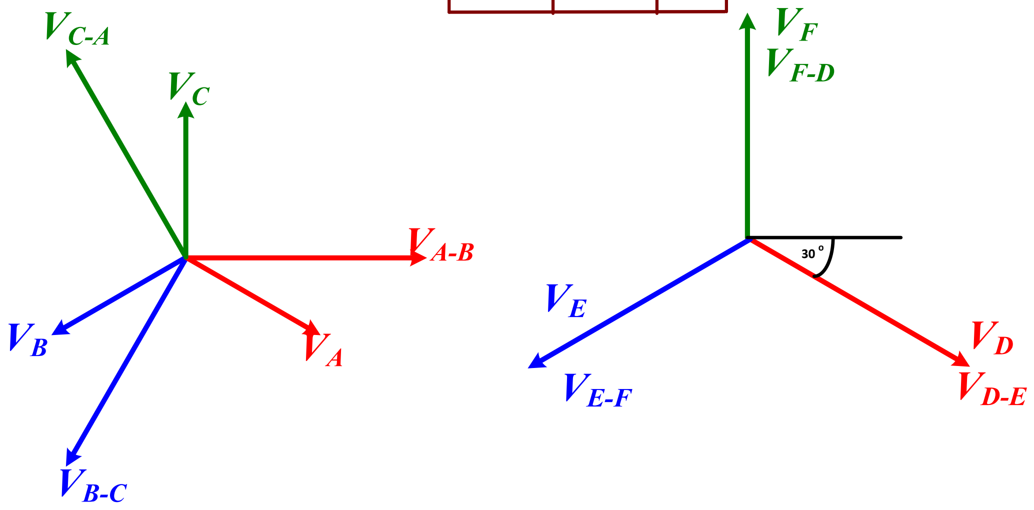
- *6 pulse – high input PF $\rightarrow$ 0.95*
- *Use soft-start to limit filter capacitor inrush current.*
- *Output ripple frequency is 360 Hz for 60 Hz input*
- *Relatively low output ripple and easy to filter with small LC*
- *Limited to loads < 350 kW*
- *Diodes or SCRs are air or water-cooled depending upon load current*

Phase shifting transformer for 12 Pulse operation



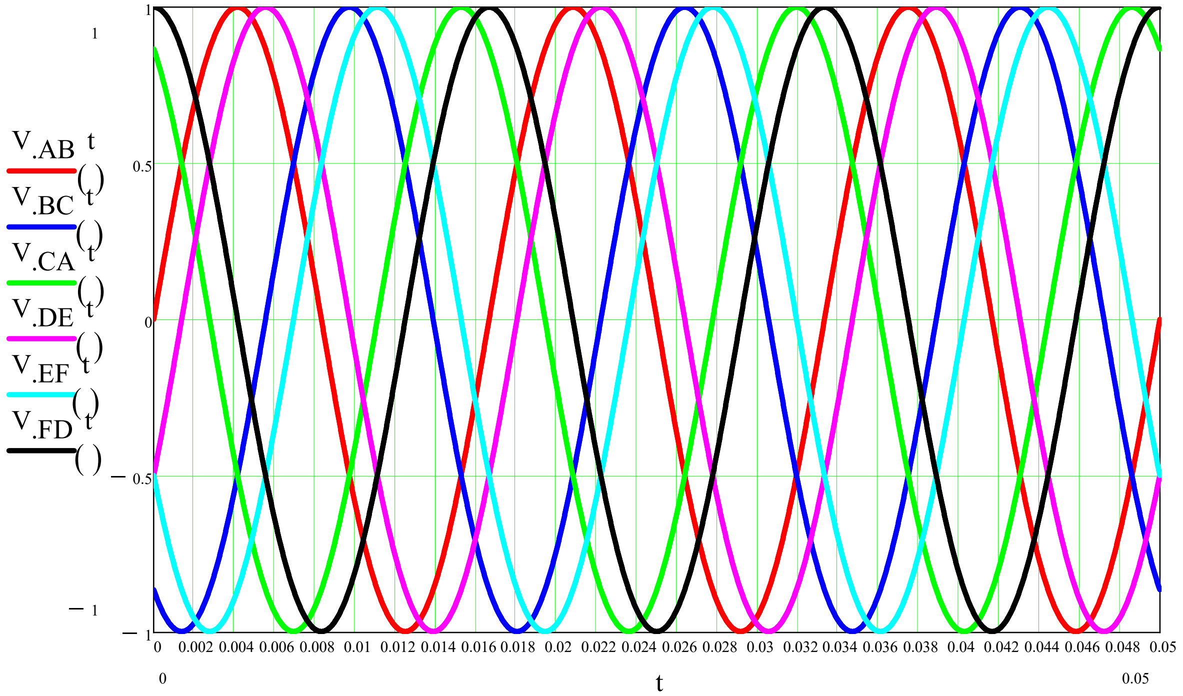
Total Primary Current in Wye-Wye-Delta





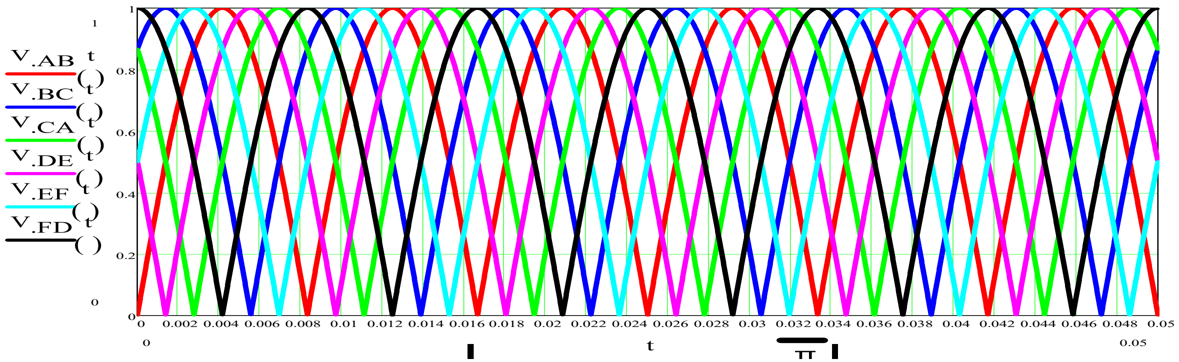
Line	Phase (°)
VA-B	0
VD-E	-30
VB-C	-120
VE-F	-150
VC-A	-240
VF-D	-270

Total Primary Current in Wye-Wye-Delta

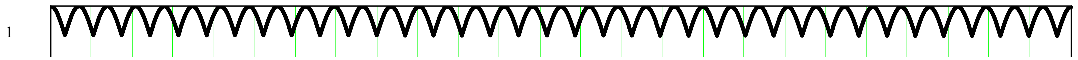


6 cycles (phases) in T=16.67ms

Voltage Output Delta-Wye Rectifier (Line-Line Voltages)

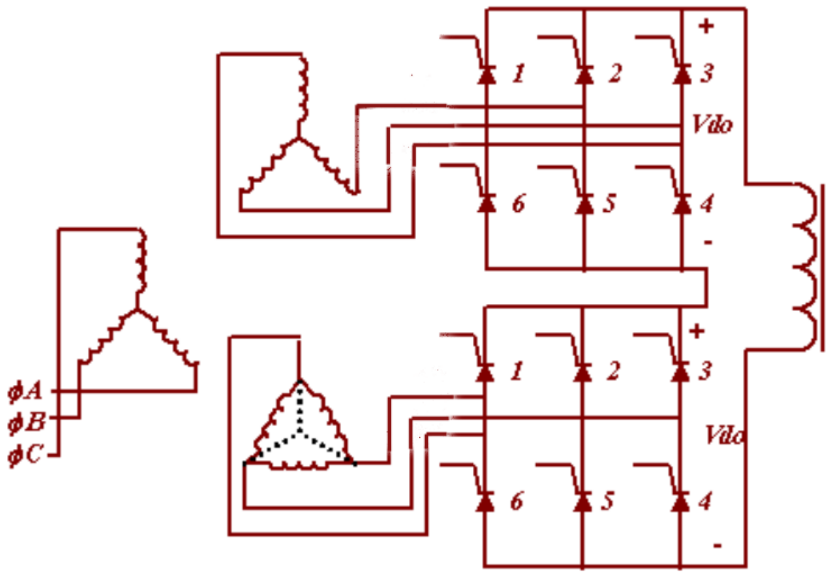


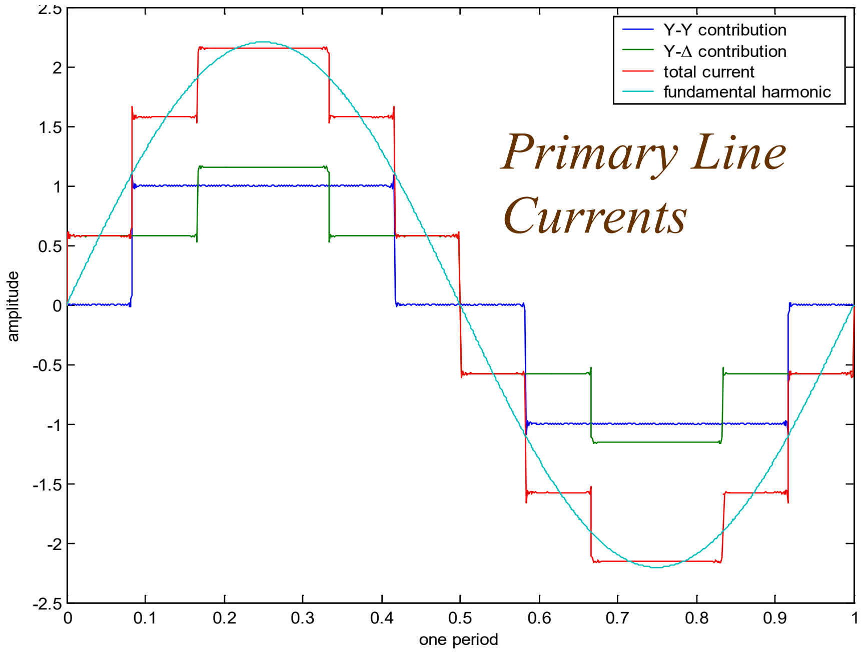
12 cycles in T=16.67ms



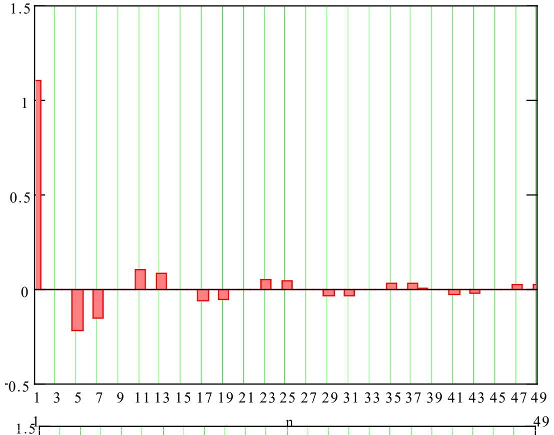
12 pulses in T=16.67ms

Total Primary Current in Wye-Wye-Delta

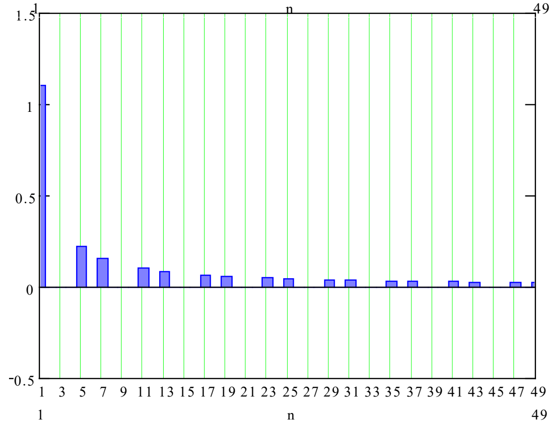




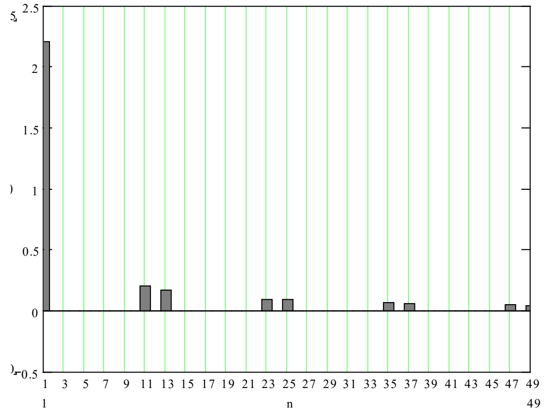
Wye-Wye



Wye-Delta



Wye-Wye-Delta



Addition formulae

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$

$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$

Therefore

$$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$$

$$\sin(A+B) - \sin(A-B) = 2\sin B\cos A$$

and

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\sin\frac{A-B}{2}\cos\frac{A+B}{2}$$

Similarly

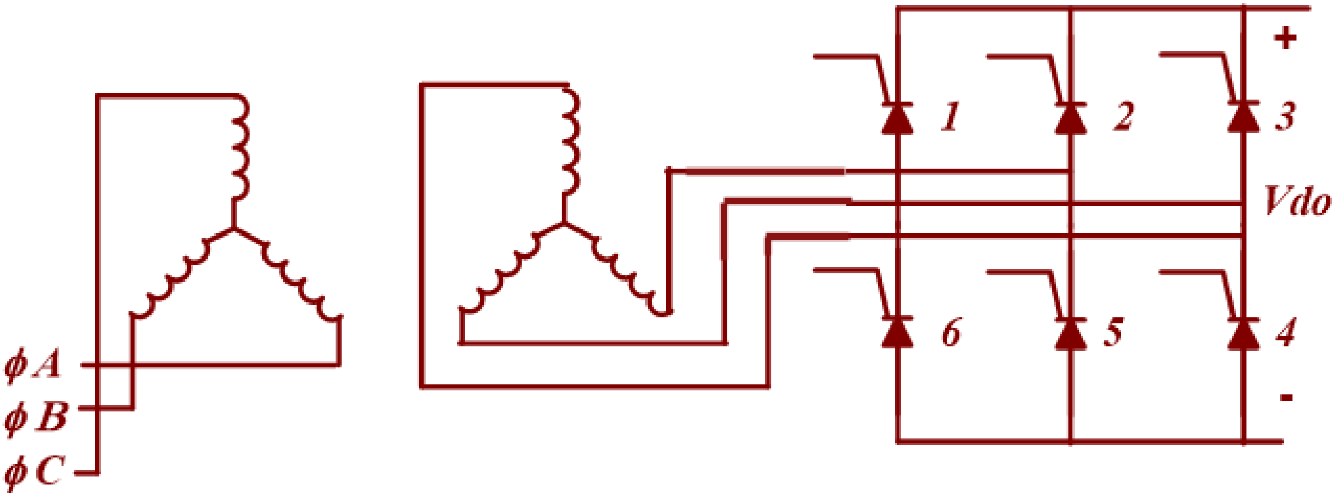
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Three Phase Wye-Wye



$$V_{Ap} = V_{LNp} \sin \omega t$$

$$V_{Bp} = V_{LNp} \sin(\omega t - 2\pi/3)$$

$$V_{Cp} = V_{LNp} \sin(\omega t - 4\pi/3)$$

$$V_{ABp} = V_{Ap} - V_{Bp}$$
$$= V_{LNp}\left[\sin \omega t - \sin(\omega t - 2\pi/3)\right]$$
$$= 2V_{LNp} \sin \pi/3 \cos(\omega t - \pi/3)$$
$$= \sqrt{3}V_{LNp} \sin(\omega t - \pi/3 + \pi/2)$$
$$= \sqrt{3}V_{LNp} \sin(\omega t + \pi/6)$$

$$V_{BCp} = \sqrt{3}V_{LNp} \sin(\omega t - \pi/2)$$

$$V_{CAp} = \sqrt{3}V_{LNp} \sin(\omega t - 7\pi/6)$$

For a transformer ratio, N_{YY}

$$V_s = N_{YY}V_p;\ I_s = I_p/N_{YY}$$

$$V_{ABYs} = \sqrt{3}N_{YY}V_{LNp} \sin(\omega t + \pi/6)$$

$$V_{BCYs} = \sqrt{3}N_{YY}V_{LNp} \sin(\omega t - \pi/2)$$

$$V_{CAYs} = \sqrt{3}N_{YY}V_{LNp} \sin(\omega t - 7\pi/6)$$

$$I_{ABYs} = \left(\sqrt{3}I_{LNp}/N_{YY}\right)\sin(\omega t + \pi/6 + \phi_Z)$$

$$I_{BCYs} = \left(\sqrt{3}I_{LNp}/N_{YY}\right)\sin(\omega t - \pi/2 + \phi_Z)$$

$$I_{CAYs} = \left(\sqrt{3}I_{LNp}/N_{YY}\right)\sin(\omega t - 7\pi/6 + \phi_Z)$$

Spectrum of Wye-Wye

Assume full conduction into a large inductive load

The load current, I_L, is then constant

The current out of the A leg of the transformer is

$$I_{ANYs}(t) = 0 \qquad 0 \le t \le T/12$$
$$= I_L \qquad T/12 \le t \le 5T/12$$
$$= 0 \qquad 5T/12 \le t \le 7T/12$$
$$= -I_L \qquad 7T/12 \le t \le 11T/12$$
$$= 0 \qquad 11T/12 \le t \le T$$

The Fourier series expansion is

$$I_{ANYs}(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi nt}{T} + b_n \sin\frac{2\pi nt}{T}$$

From the symmetry of the waveform,

$$a_0 = a_n = 0$$

$$b_n = \frac{2}{T}\int_0^T I_{ANYs}(t)\sin\frac{2\pi nt}{T}\,dt$$

$$= \frac{2I_L}{T}\left[\int_{T/12}^{5T/12} \sin\frac{2\pi nt}{T}\,dt - \int_{7T/12}^{11T/12} \sin\frac{2\pi nt}{T}\,dt\right]$$

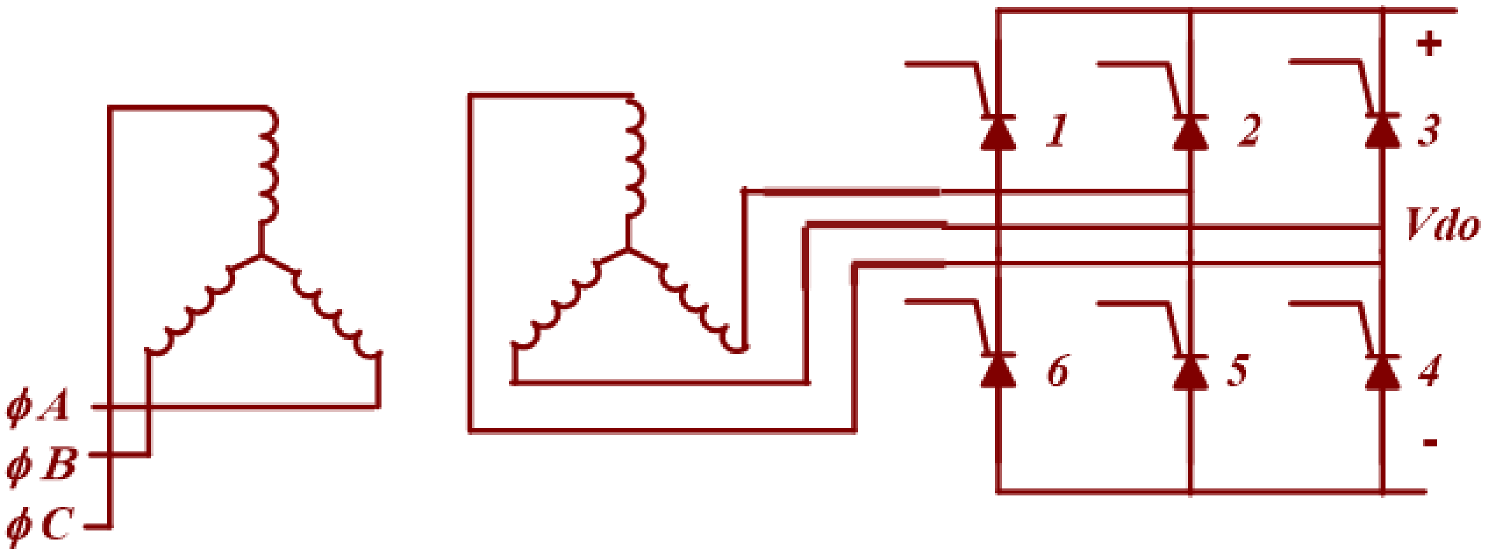
$$= \frac{4I_L}{T}\int_{T/12}^{5T/12} \sin\frac{2\pi nt}{T}\,dt$$

$$= -\frac{2I_L}{n\pi}\cos\frac{2\pi nt}{T}\Bigg|_{T/12}^{5T/12}$$

$$= -\frac{2I_L}{n\pi}\left[\cos(5n\pi/6) - \cos(n\pi/6)\right]$$

$$b_n = \frac{4I_L}{n\pi}\sin\frac{n\pi}{2}\sin\frac{n\pi}{3}$$

Wye-Wye Primary Current

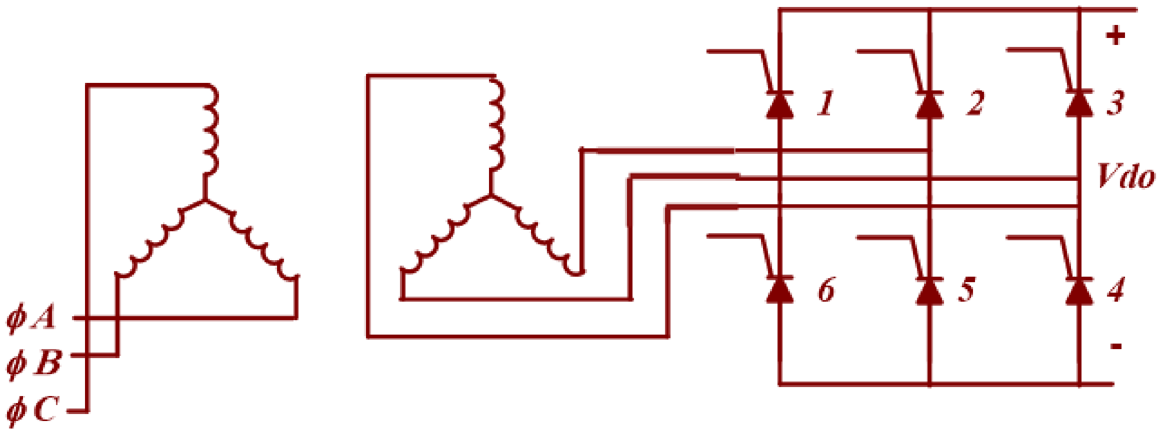


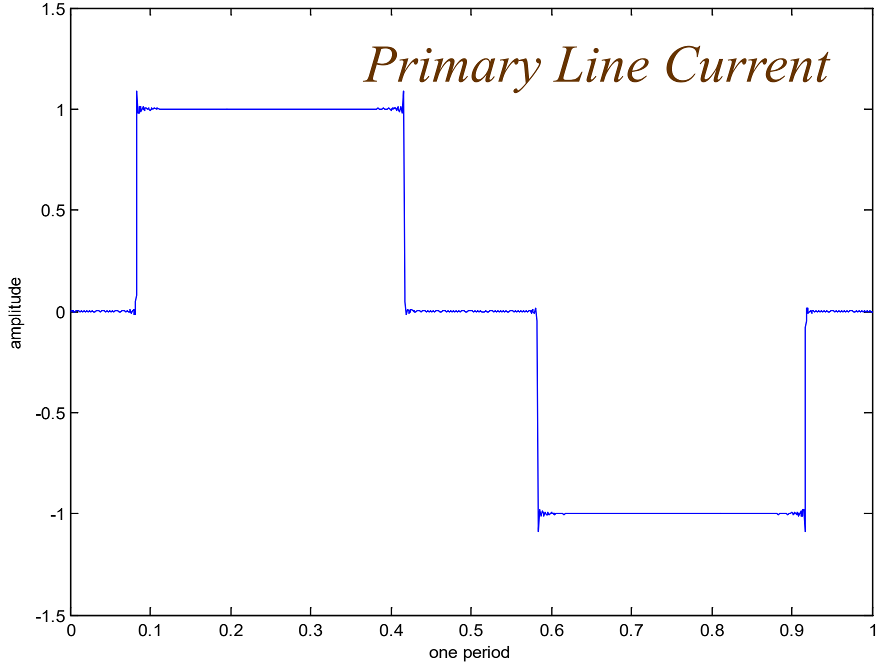
The current on the primary leg of the transformer, due to the YY winding is

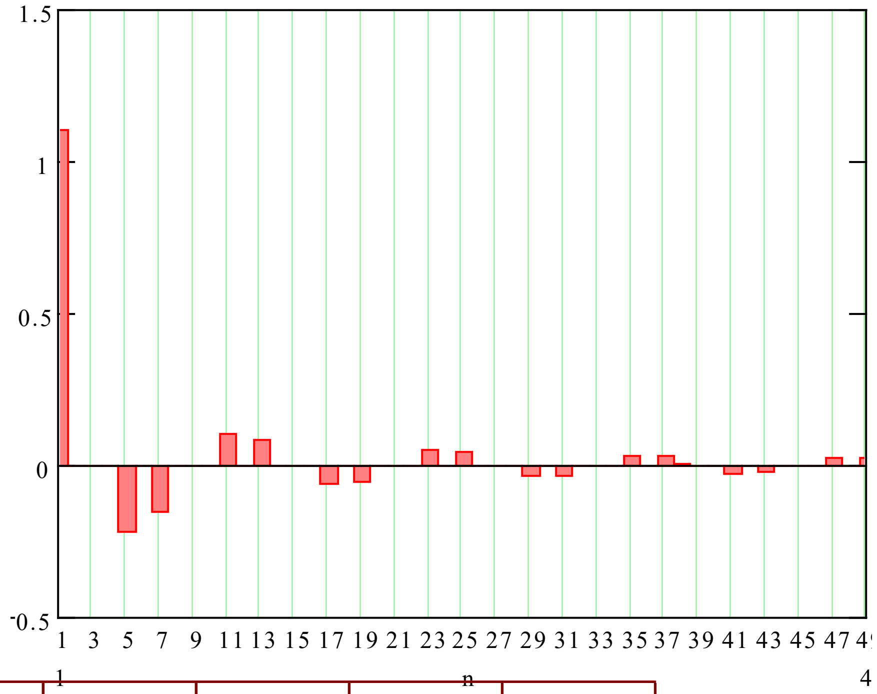
$$I_{ANYp}(t) = N_{YY} \frac{4I_L}{n\pi} \sum_{n=1}^{\infty} \sin\frac{n\pi}{2} \sin\frac{n\pi}{3} \sin\frac{2\pi n t}{T}$$

Note that the first term eliminates all of the even harmonics and the second eliminates all multiples of the third harmonic.

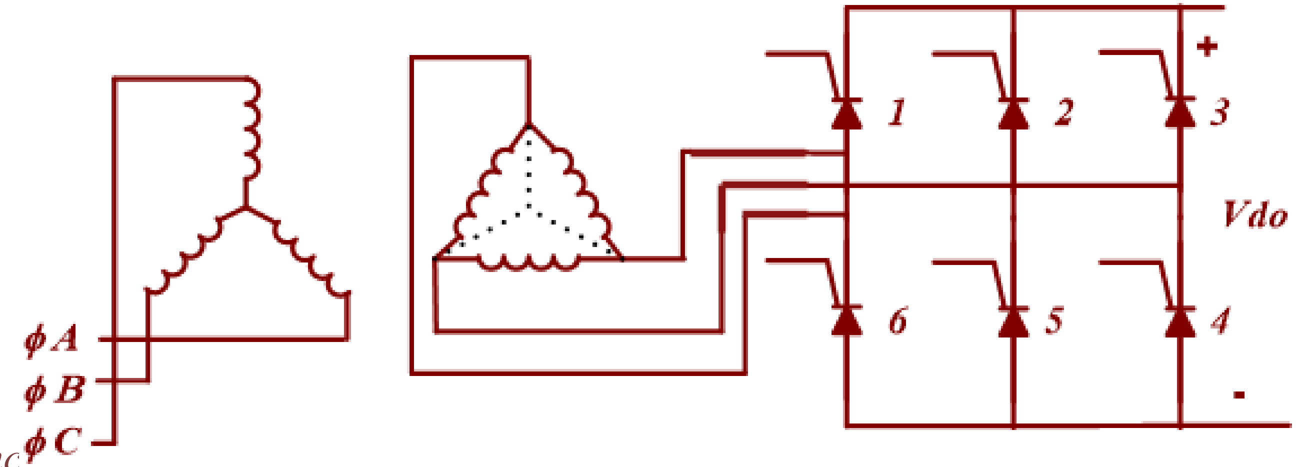
Wye-Wye Primary Current







Harmonic	1	5	7	11	13	15	17	19
Frequency	60	300	420	660	780	900	1020	1140
Amplitude	1.103	-0.221	-0.158	0.100	0.085	0	-0.065	-0.058



In order to have balanc

$$I_{A\Delta s} + I_{B\Delta s} + I_{C\Delta s} = 0$$

When two delta leg A switches conduct

$$I_{B\Delta s} = I_{C\Delta s}$$

so that

$$I_{A\Delta s} + 2I_{B\Delta s} = 0$$

The current through the switch is then

$$I_L = I_{A\Delta s} - I_{B\Delta s}$$

$$I_L = I_{A\Delta s} + \frac{1}{2} I_{A\Delta s}$$

$$I_L = \frac{3}{2} I_{A\Delta s}$$

$$I_{A\Delta s} = \frac{2}{3} I_L$$

For a transformer ratio $N_{Y\Delta}$

$$V_{AB\Delta s} = N_{Y\Delta} V_{LNp} \sin(\omega t)$$

$$V_{BC\Delta s} = N_{Y\Delta} V_{LNp} \sin(\omega t - 2\pi/3)$$

$$V_{CA\Delta s} = N_{Y\Delta} V_{LNp} \sin(\omega t - 4\pi/3)$$

$$I_{AB\Delta s} = (V_{LNp}/N_{Y\Delta}) \sin(\omega t + \phi_Z)$$

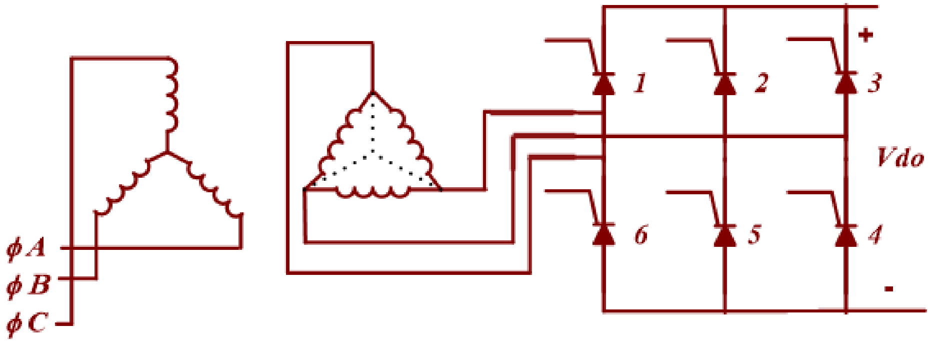
$$I_{BC\Delta s} = (V_{LNp}/N_{Y\Delta}) \sin(\omega t - 2\pi/3 + \phi_Z)$$

$$I_{CA\Delta s} = (V_{LNp}/N_{Y\Delta}) \sin(\omega t - 4\pi/3 + \phi_Z)$$

For equal secondary voltages

$$N_{Y\Delta} = \sqrt{3} N_{YY}$$

Wye-Delta Spectrum



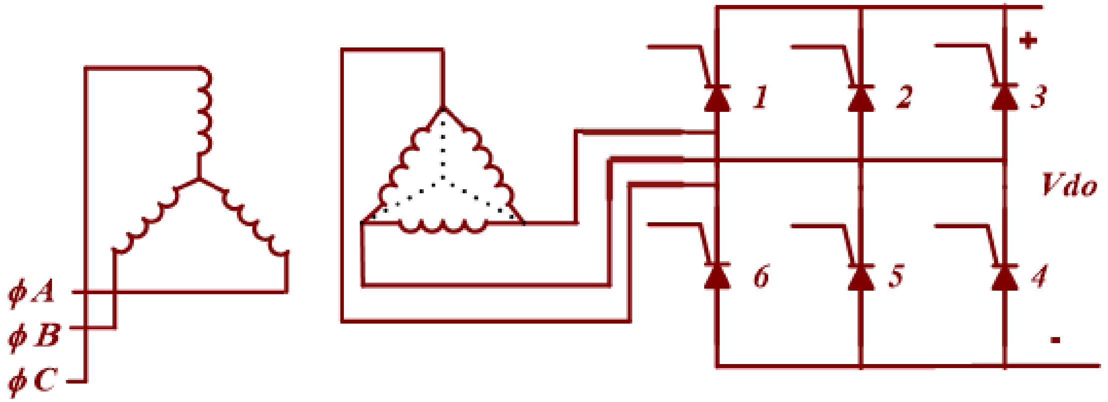
The current through the A winding is

$$
\begin{aligned}
I_{A\Delta s}(t) &= I_L/3 && 0 \le t \le T/6 \\
&= 2I_L/3 && T/6 \le t \le T/3 \\
&= I_L/3 && T/3 \le t \le T/2 \\
&= -I_L/3 && T/2 \le t \le 2T/3 \\
&= -2I_L/3 && 2T/3 \le t \le 5T/6 \\
&= -I_L/3 && 5T/6 \le t \le T
\end{aligned}
$$

$$a_0 = a_n = 0$$

Again, by symmetry, only the b_n terms are non-zero

$$
\begin{aligned}
b_n &= \frac{2}{T}\int_0^{T} I_{A\Delta s}(t)\, sin\frac{2\pi nt}{T}\,dt \\
&= \frac{4I_L}{3T}\left[\int_0^{T/6} sin\frac{2\pi nt}{T}\,dt + 2\int_{T/6}^{T/3} sin\frac{2\pi nt}{T}\,dt + \int_{T/3}^{T/2} sin\frac{2\pi nt}{T}\,dt\right] \\
&= -\frac{2I_L}{3n\pi}\left[cos\frac{2\pi nt}{T}\Big|_0^{T/6} + 2\,cos\frac{2\pi nt}{T}\Big|_{T/6}^{T/3} + cos\frac{2\pi nt}{T}\Big|_{T/3}^{T/2}\right] \\
&= \frac{2I_L}{3n\pi}\left[\left(cos\,0 + cos\frac{\pi n}{3}\right) - \left(cos\frac{2\pi n}{3} + cos\,\pi n\right)\right] \\
&= \frac{4I_L}{3n\pi}\left(cos\frac{n\pi}{6}cos\frac{n\pi}{6} - cos\frac{5n\pi}{6}cos\frac{n\pi}{6}\right) \\
&= \frac{4I_L}{3n\pi}cos\frac{n\pi}{6}\left(cos\frac{n\pi}{6} - cos\frac{5n\pi}{6}\right) \\
&= \frac{8I_L}{3n\pi}cos\frac{n\pi}{6}\,sin\frac{n\pi}{2}\,sin\frac{n\pi}{3}
\end{aligned}
$$



$$I_{A\Delta s}(t) = \frac{8I_L}{3n\pi}\sum_{n=1}^{\infty} \cos\frac{n\pi}{6}\sin\frac{n\pi}{2}\sin\frac{n\pi}{3}\sin\frac{2\pi nt}{T}$$

Note that multiples of the 2^{nd} *and* 3^{rd} *harmonics are also suppressed.*

The $\cos\frac{n\pi}{6}$ *term does not introduce any extra zeros, but it does contribute to the sign of the terms.*

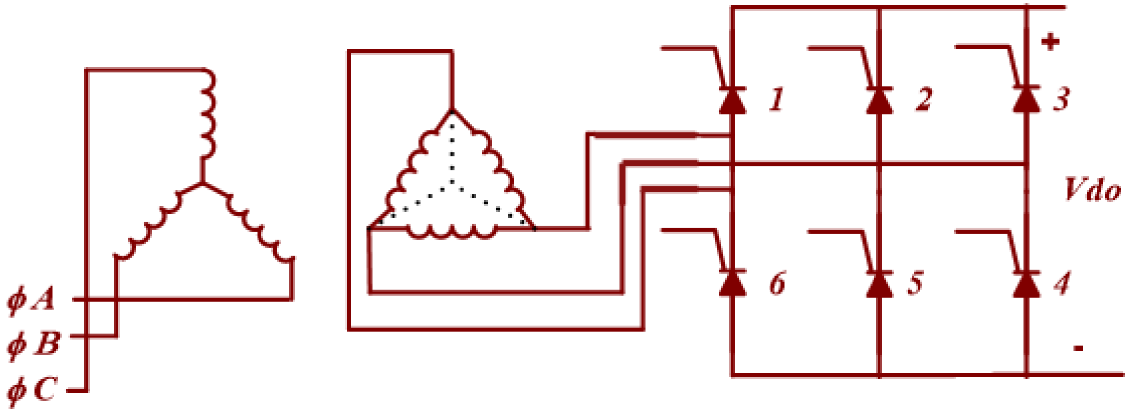
The non-vanishing terms are $n = 1,5,7,11,\cdots$, *for which the magnitude is* $\sqrt{3}/2$.

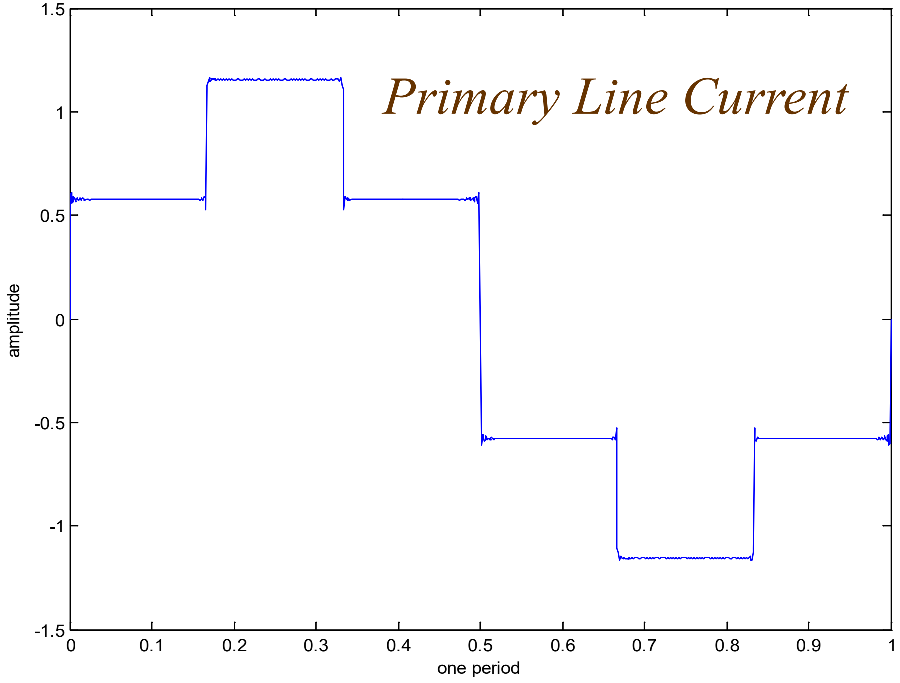
Referred back to the primary, the current is

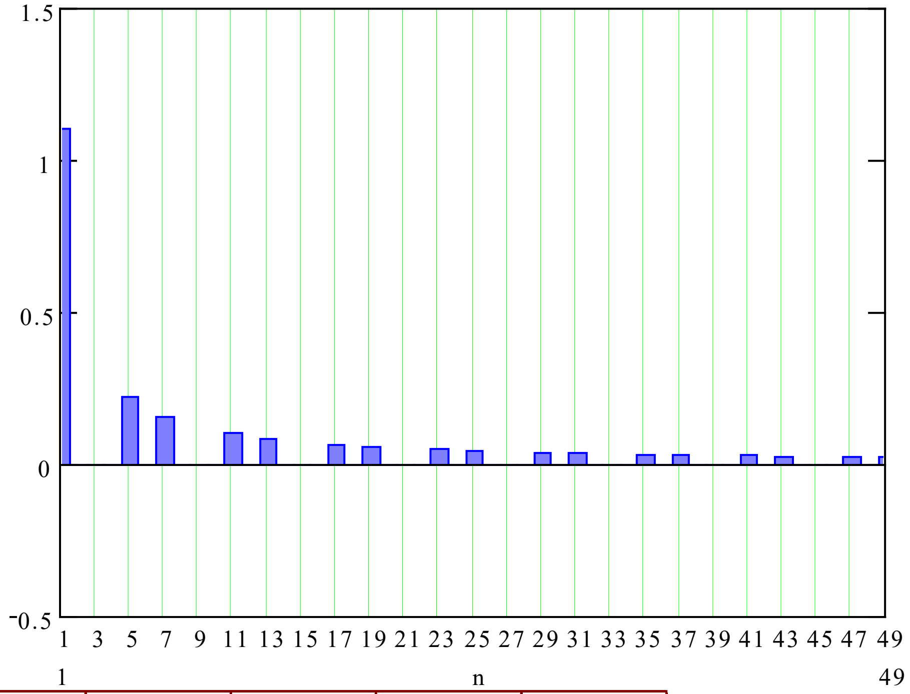
$$I_{A\Delta p}(t) = N_{Y\Delta}\frac{8I_L}{3n\pi}\sum_{n=1}^{\infty} \cos\frac{n\pi}{6}\sin\frac{n\pi}{2}\sin\frac{n\pi}{3}\sin\frac{2\pi nt}{T}$$

$$I_{A\Delta p}(t) = N_{YY}\frac{8\sqrt{3}I_L}{3n\pi}\sum_{n=1}^{\infty} \cos\frac{n\pi}{6}\sin\frac{n\pi}{2}\sin\frac{n\pi}{3}\sin\frac{2\pi nt}{T}$$

Primary Current in the Wye-Delta

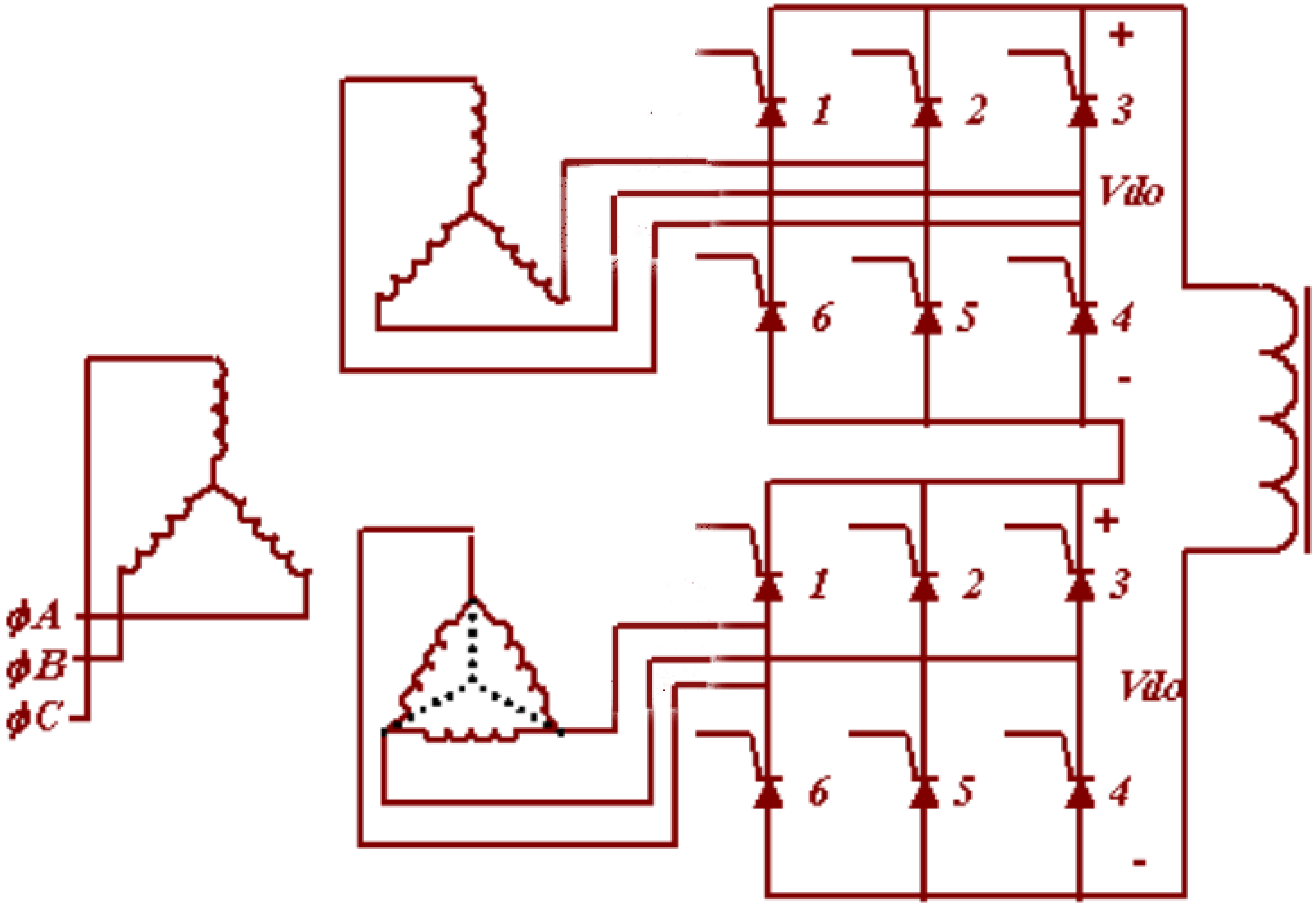






Harmonic	1	5	7	11	13	15	17	19
Frequency	60	300	420	660	780	900	1020	1140
Amplitude	1.103	0.221	0.158	0.100	0.085	0	0.065	0.058

Total Current (Primary Wye Current) in Wye-Wye-Delta



The total current in the A leg of the primary is the sum of these two terms

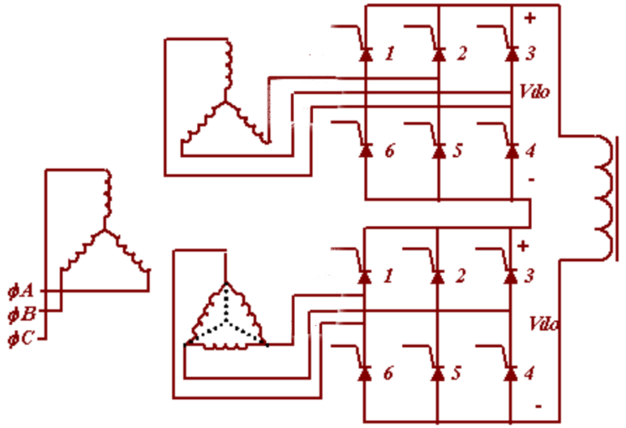
$$I_{Ap}(t) = I_{ANYp}(t) + I_{AN\Delta p}(t)$$

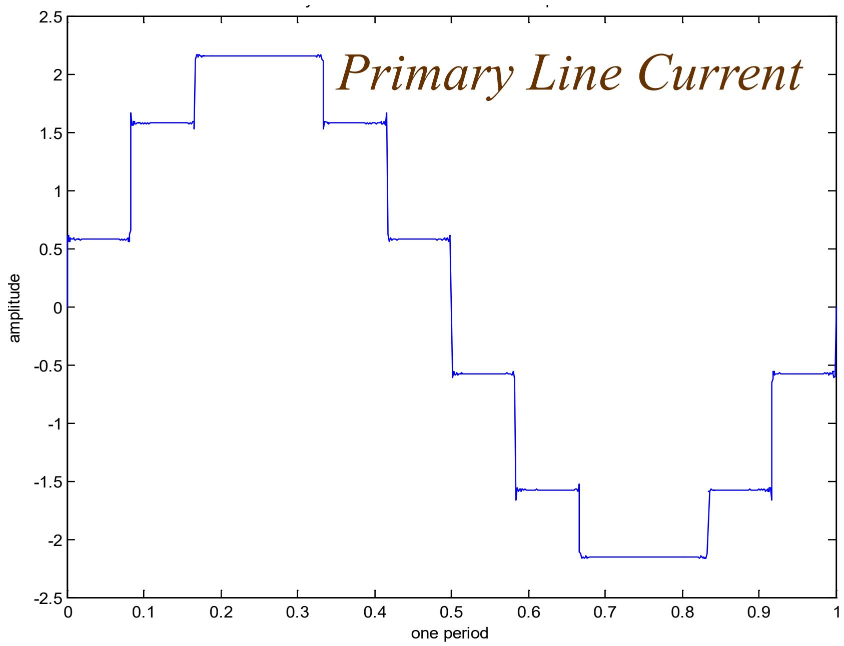
The only non−vanishing terms in both of these series are $n = 1, 5, 7, 11$ *and all other values of* n *which have the same phase*

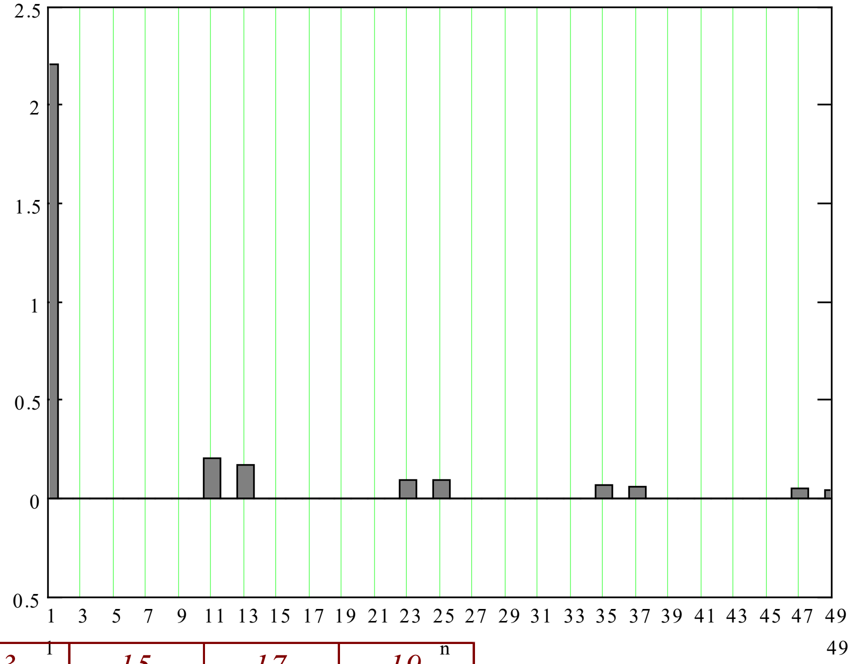
The values of $\cos\frac{n\pi}{6}$ *for these* n *are* $\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$

The surviving terms in each series have the same magnitude, but half have different signs so that the only remaining harmonics in the total balanced 12 pulse bridge are $n = 1, 11, 13, 23, 25, 35, \cdots$ *with coefficient* $N_{YY}\frac{4I_L}{n\pi}$

Total Primary Current in Wye-Wye-Delta

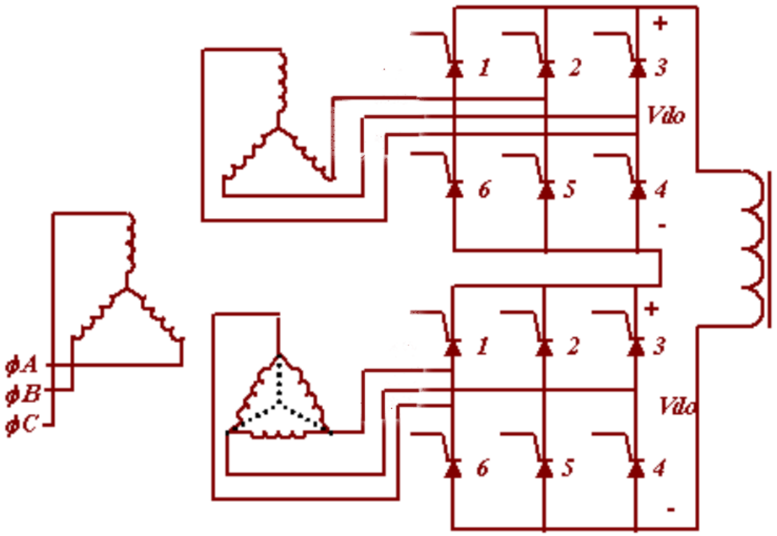


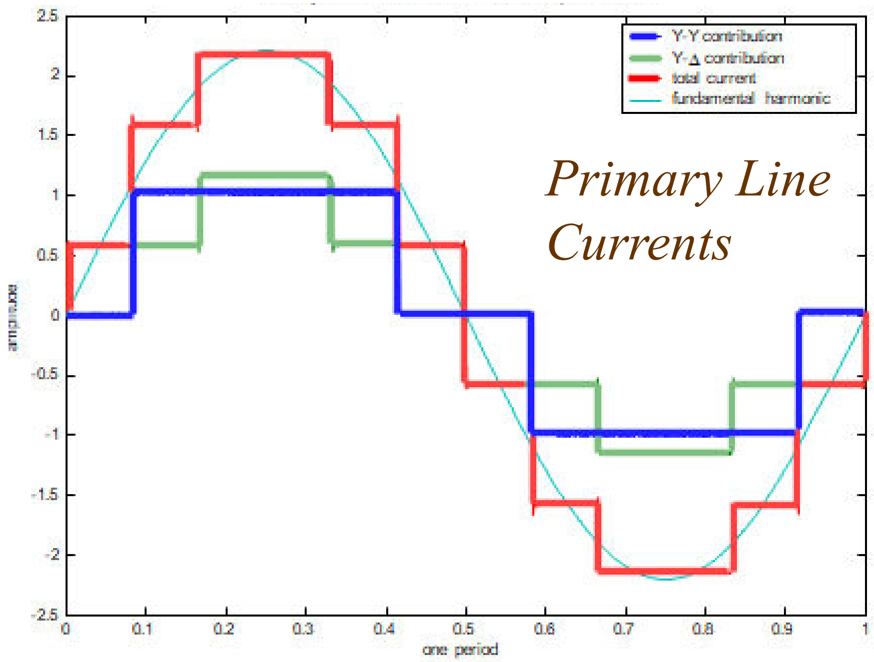




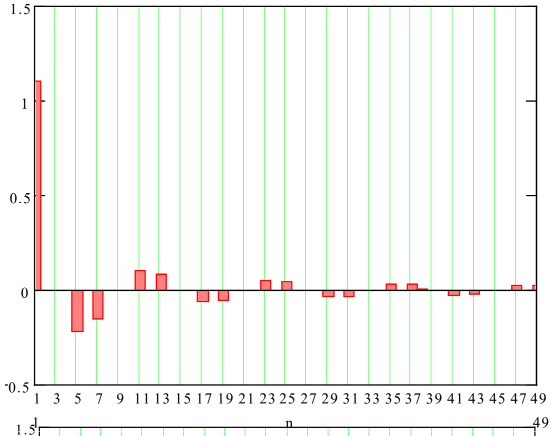
Harmonic	1	5	7	11	13	15	17	19
Frequency	60	300	420	660	780	900	1020	1140
Amplitude	2.205	0	0	0.200	0.085	0.170	0	0

Total Primary Current in Wye-Wye-Delta

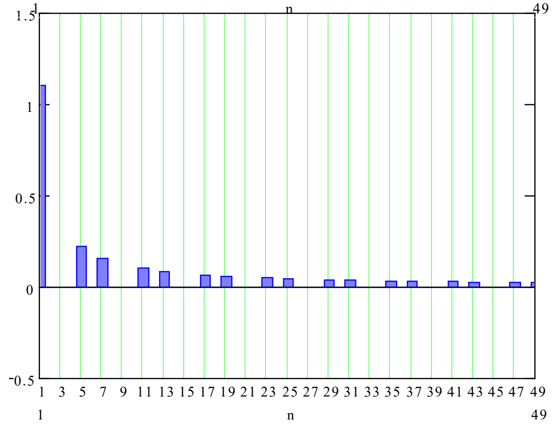




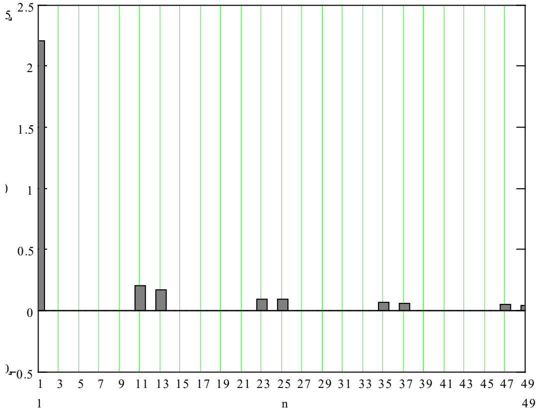
Wye-Wye



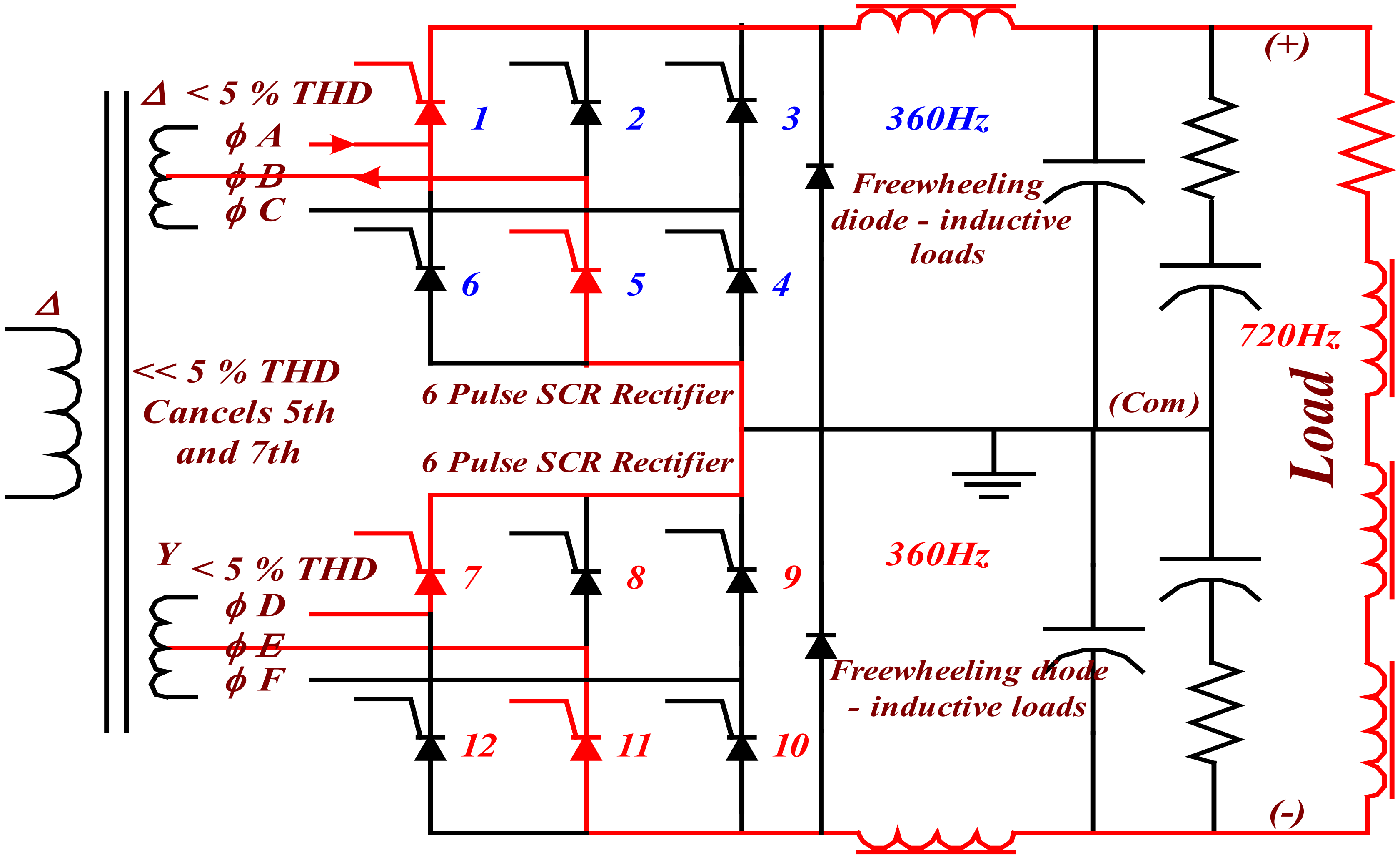
Wye-Delta



Wye-Wye-Delta



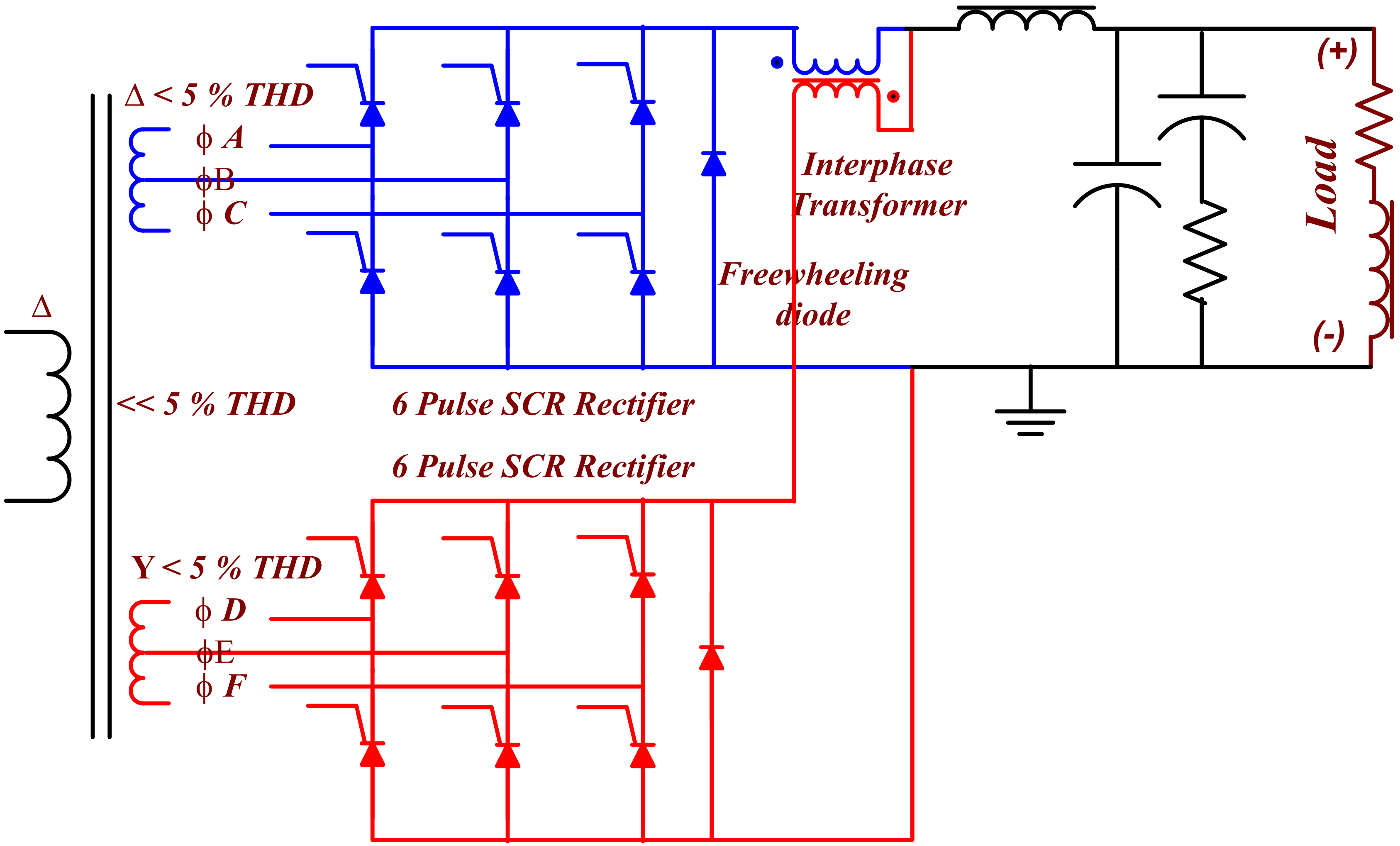
6 ϕ, q = 12 Pulse By Series Bridges



SCR sequence for 30^{O} lagging wye secondary

1-5, 7-11, 1-4, 7-10, 2-4, 8-10, 2-6, 8-12, 3-6, 9-12, 3-5, 9-11

6 ϕ, q = 12 Pulse By Parallel Bridges



Transformer phases and SCR firing sequence are the same as shown for the series-connected bridges

6 ϕ, q = 12 Pulse Rectifiers - Summary

For Both Series And Parallel-Connected Bridges

- *Input transformer Δ primary, Δ – Y secondaries for 6 AC phases*
- *Δ - Y secondaries are phase shifted 30^{O}*
- *5^{th} and 7^{th} harmonics virtually non-existent in input line, << 5 % THD of line voltage < 20 % THD of line current*
- *Very high input PF to 0.97*
- *Output ripple frequency is 720 Hz for 60 Hz input*
- *Use soft-start to limit filter capacitor inrush current*
- *Freewheeling diode for to allow lagging bridge to conduct*
- *For loads $\geq$ 350 kW*

6 ϕ, q = 12 Pulse Rectifiers - Summary

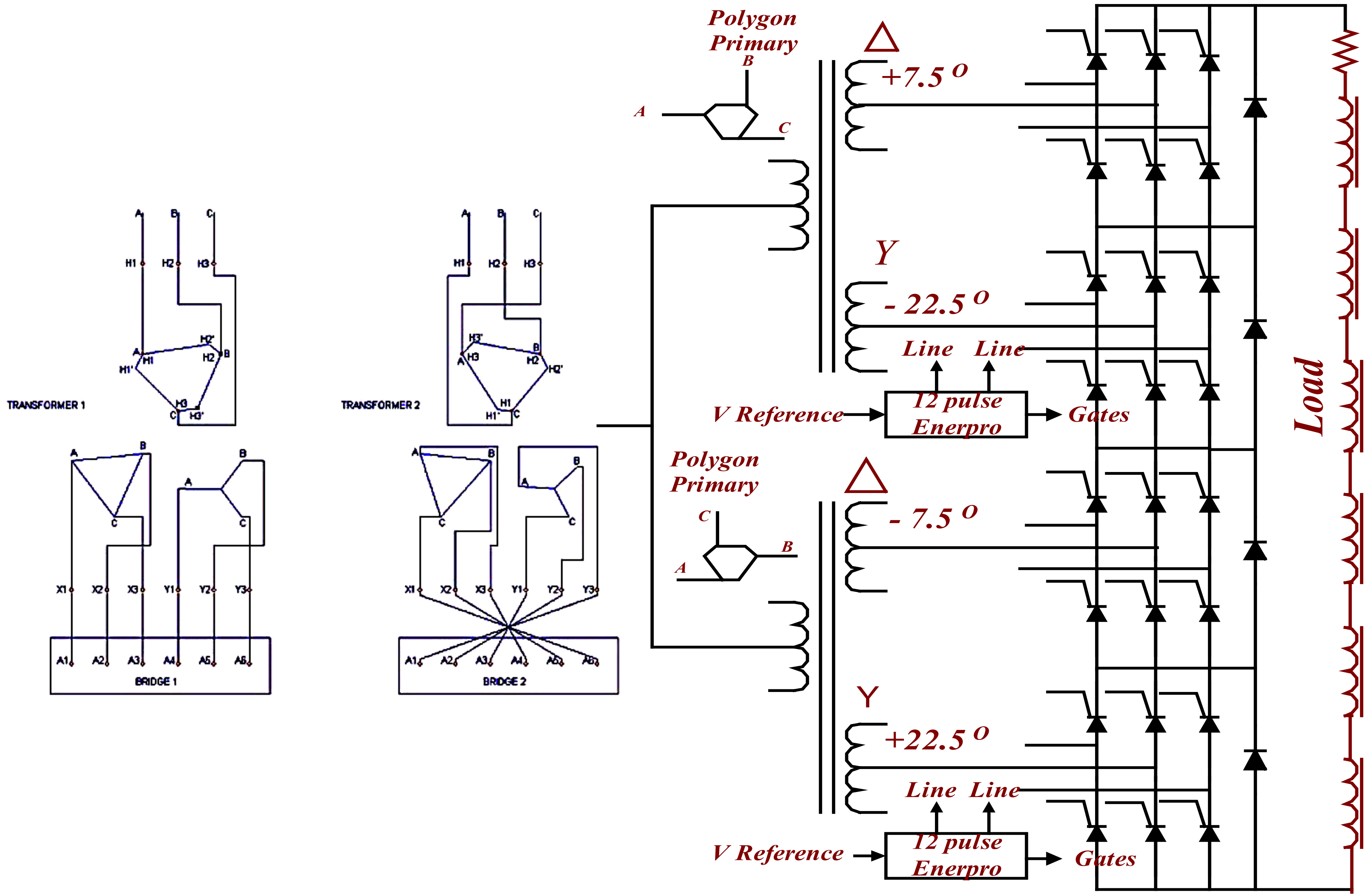
Series-connected bridges

- *For high-voltage, low-current loads*

Parallel-connected bridges

- *For high-current, low-voltage loads ≥ 350 kW.*
- *Inter-phase transformer needed for current sharing*

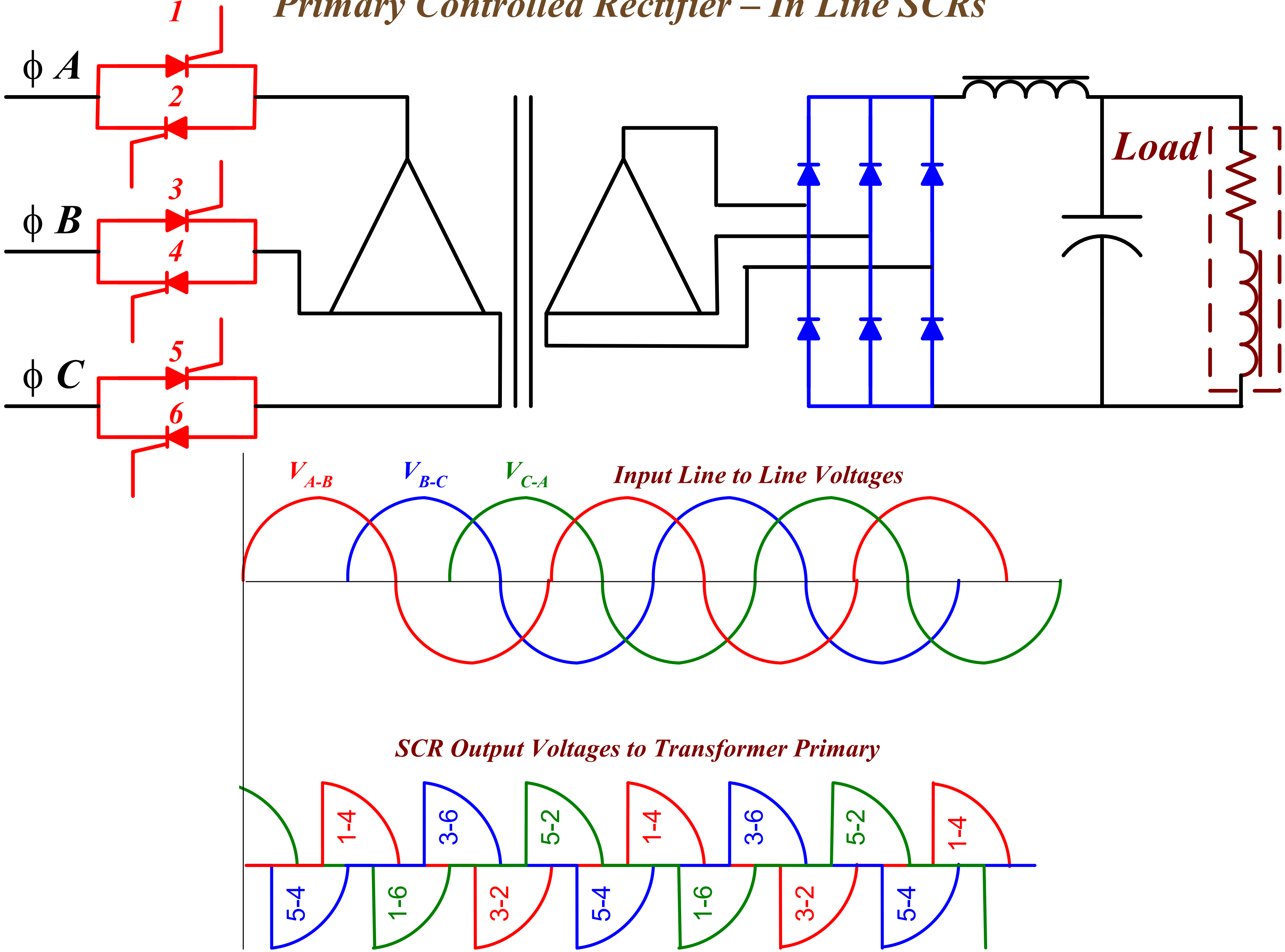
12 ϕ, q = 24 Pulse Rectifier

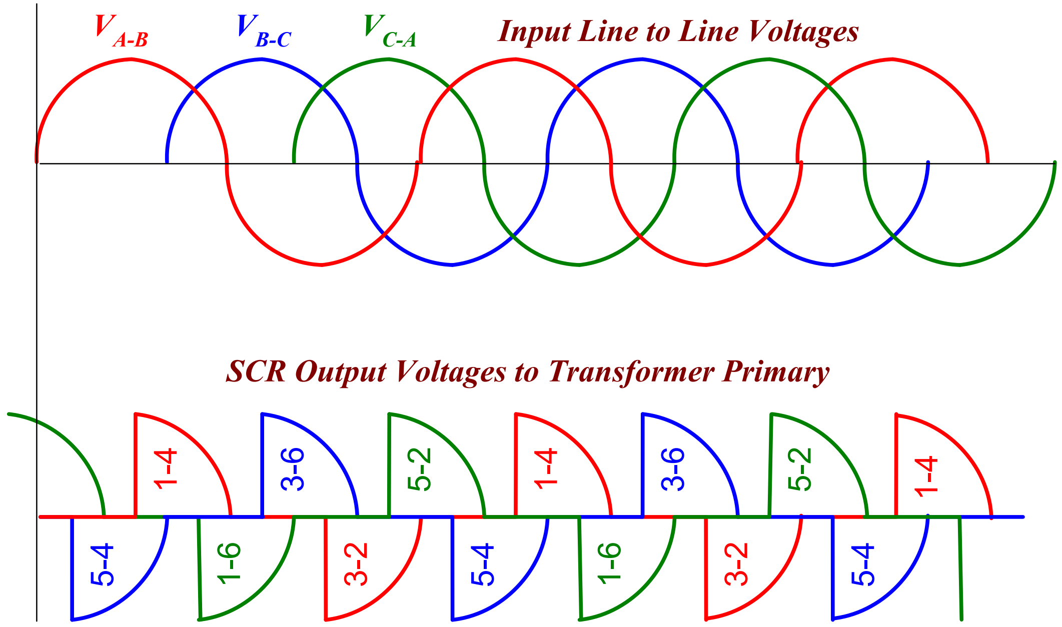


12 ϕ , q = 24 Pulse Rectifier Summary

- *Input transformer polygon primary to + 7.5^O Δ – Y secondaries for -30^O shift*
- *Input transformer polygon primary to - 7.5^O Δ – Y secondaries for +30^O shift*
- *15^O shift between the 4 sets of bridges*
- *For loads ≥ 1 MW DC or Pulsed*

Primary Controlled Rectifier – In Line SCRs

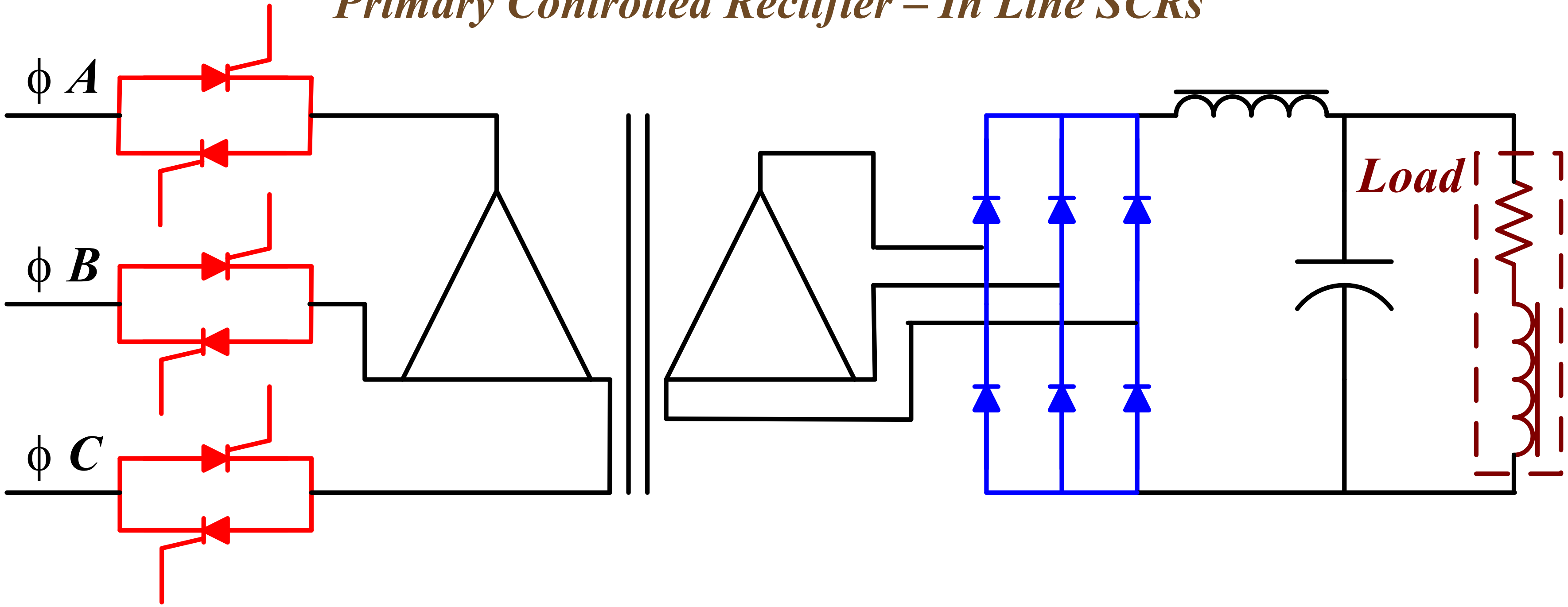




$$V_{RMS} = \sqrt{\frac{1}{\omega T} \int_{\alpha}^{\omega T} \sqrt{2}\, V_{LL} \sin^2 \omega t \; d\omega t}$$

$$V_{do} = \frac{3\sqrt{2}\, V_{RMS}}{\pi} * N$$ *where N is the transformer secondary to primary voltage ratio*

Primary Controlled Rectifier – In Line SCRs



Advantage Compared To Secondary Control

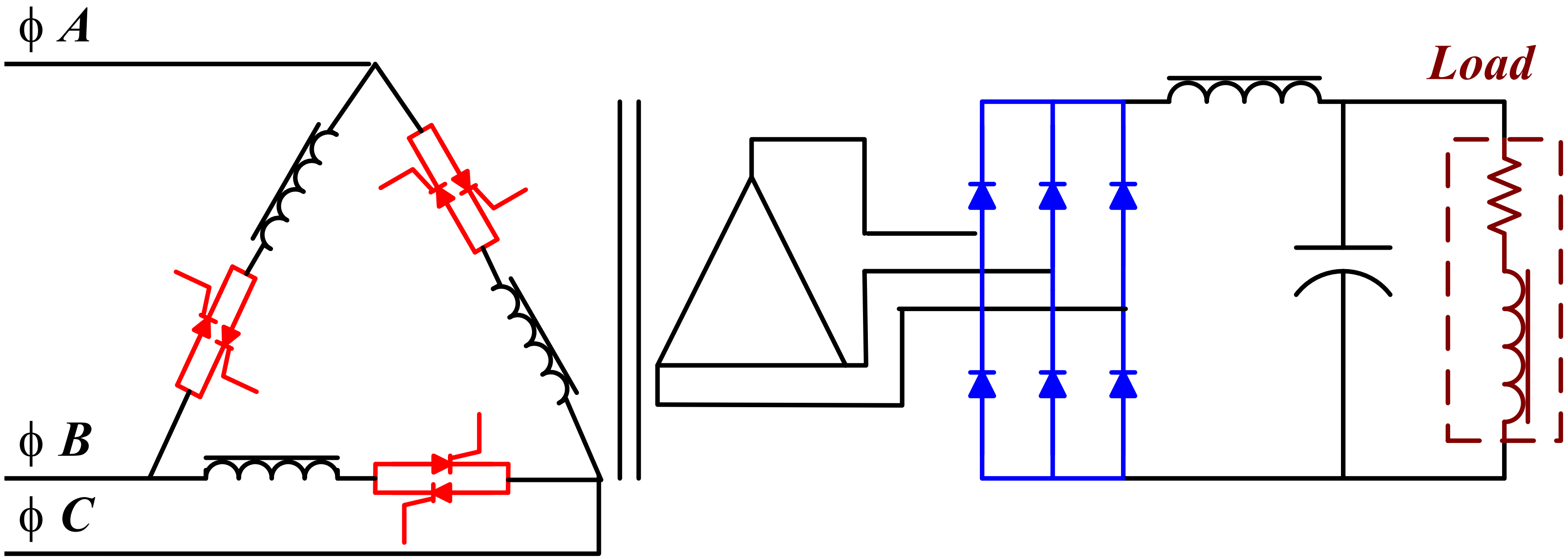
- *Keep SCR controls out of the HV and HV oil*

Disadvantage Compared To secondary Control

- *Twice the semiconductors mean higher losses and lower efficiency*

Similarities

- *PF*
- *Input / output harmonics*
- *Output ripple frequency*



Advantage Compared To In Line SCRs

- $\frac{1}{\sqrt{3}}$ lower SCR current and power (SCR on-voltage is constant)

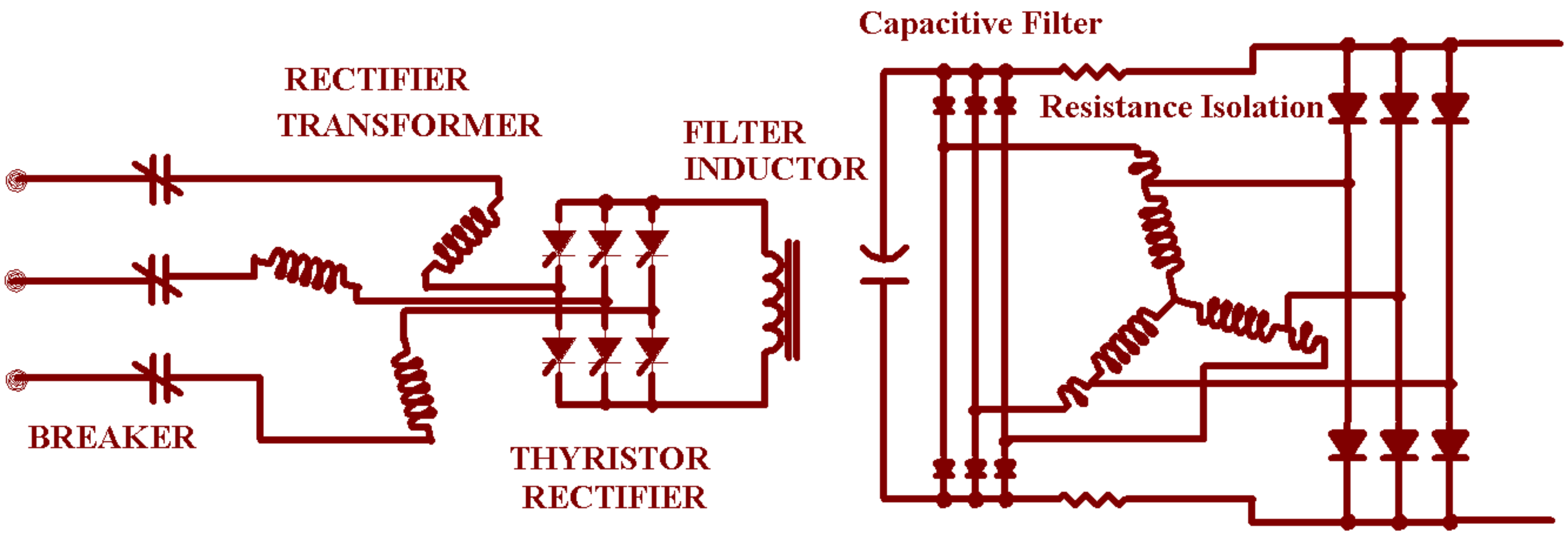
Disadvantage

- Transformer wiring more complex

Similarities

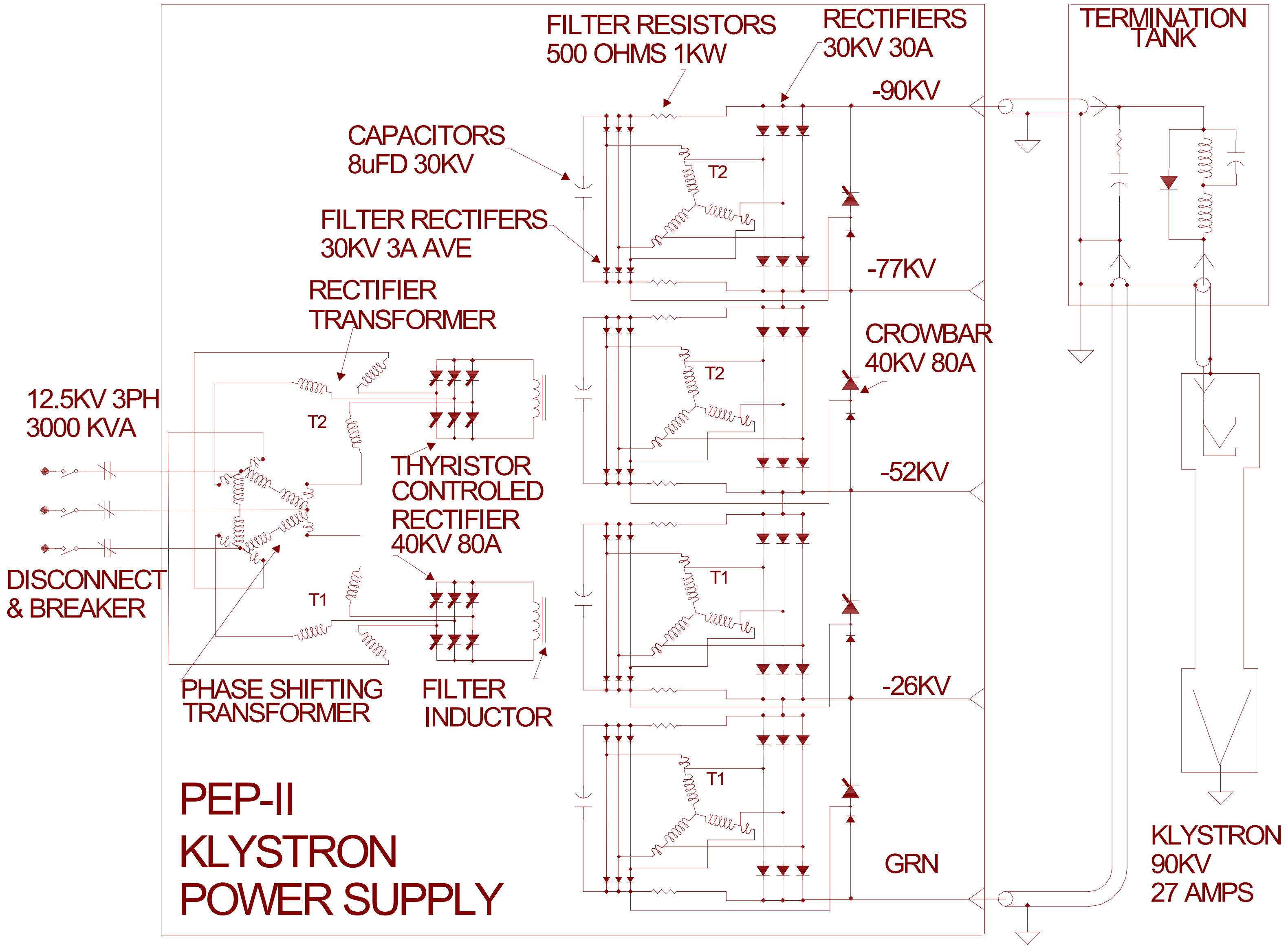
- Other characteristics similar to In Line SCR controller

6 Pulse SCR Star Point Rectifier



- *Primary SCR in open wye with filter inductor in lower voltage primary*
- *High voltage secondary with diodes and filter capacitor isolated from main load*
- *Protected against secondary faults. High output impedance, capacitor bank isolated from load*
- *Secondary uses diodes only.*

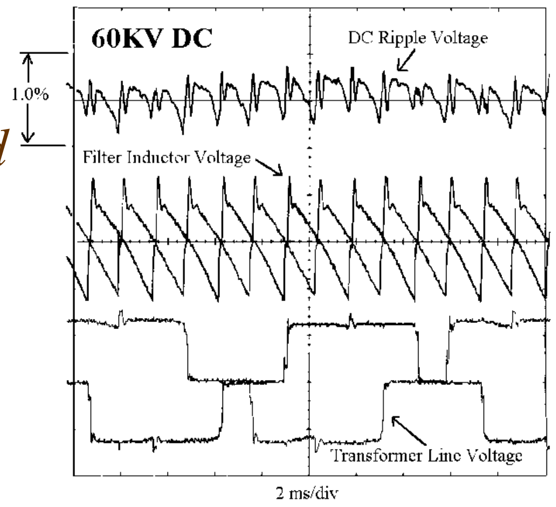
Multi-Phase SCR Star Point Rectifier with Isolated filter

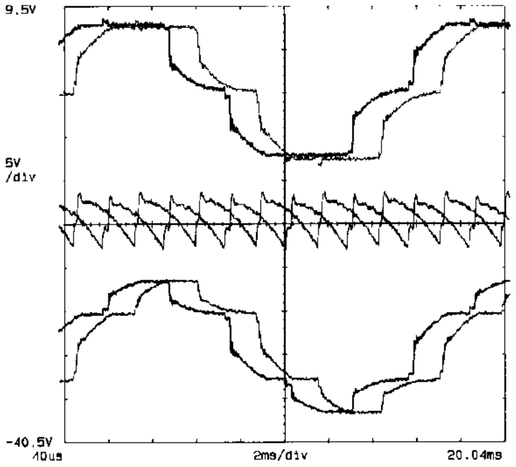


6 Phase SCR Star Point Rectifier

- *Large Joules under Load with Fault from filter Capacitor and Inductor*

KLYSTRON POWER SUPPLY

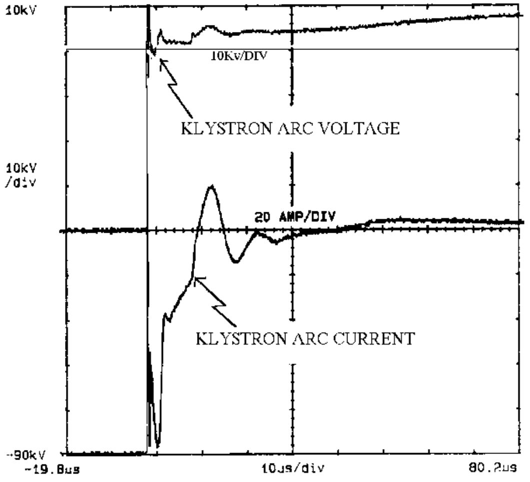




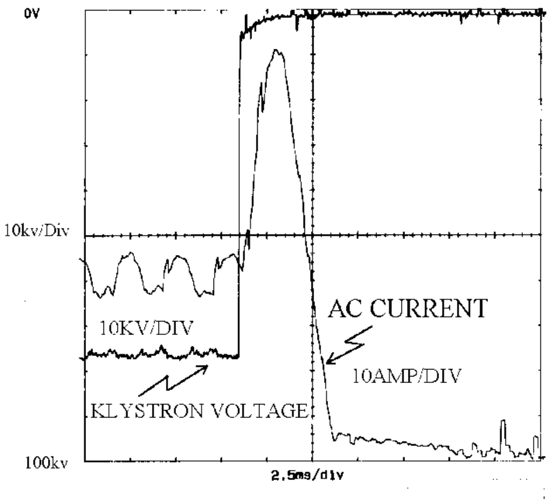
- *Large Joules under Load Fault from filter Capacitor*

- *Low Joules under Fault*
- *Filter Loss $V_{max} * I_{ripple}$ or ~ 5% of Load*

KLYSTRON ARC VOLTAGE/CURRENT



AC CURRENT WITH KLYSTRON ARC



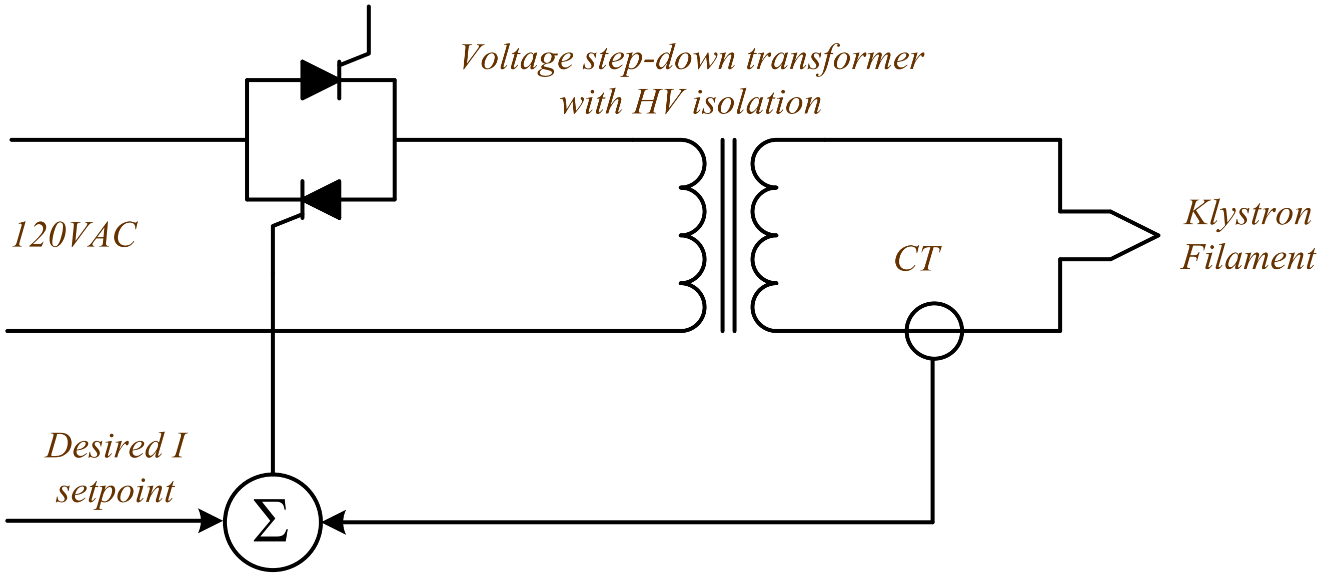
- *Low Joules under Fault*
- *Filter Loss 5% $V_{max} * I_{ripple}$ or ~0.03% of Load*

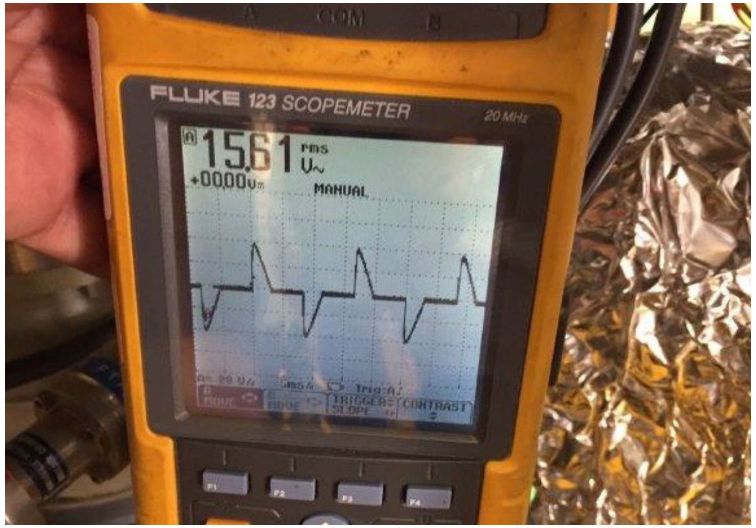
Enerpro FCOG-1200

- *12 pulse operation*
- *600 VAC L-L*
- *Soft start and stop*
- *Phase loss detection*
- *Instant gate inhibit*
- *Phase reference sense*

http://www.enerpro-inc.com

- *Klystron filaments need power. In some situations, DC power is undesirable. SLAC experience is that DC power can cause certain electrolysis effects that erode the filaments. Hence we sometimes avoid DC and use AC controllers*
- *So we also briefly discuss AC controllers (Variacs and electronic types), their waveforms, and their suitability to power klystron filaments*
- *We must also be aware that in certain situations AC powered filaments surrounded by a DC magnetic field (such as in an electron beam gun) can cause filament flexing and early filament failure from mechanical stress. We need to use DC power for these filaments.*





$$I(\omega t) = I_{pk} \sin \omega t$$

$$I_{RMS} = \sqrt{\frac{1}{\omega T} \int_{\alpha}^{\pi} \left(I_{pk} \sin \omega t\right)^2 \, d\omega t}$$

$$I_{RMS} = \frac{I_{pk}}{\sqrt{\pi}} \sqrt{\frac{\pi}{2} - \frac{\alpha}{2} + \frac{1}{4} \sin 2\alpha}$$

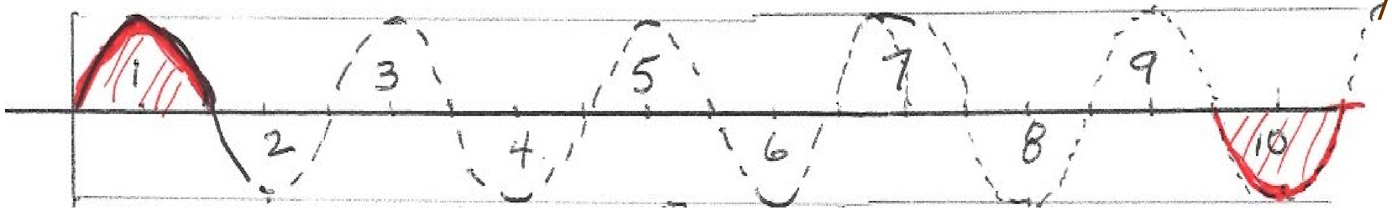
•For duty cycles < 50% firing time is two half cycles

• 50% firing and non-firing time are equal at 2 halves on, 2 halves off

• > 50% non-firing time is one-half cycle

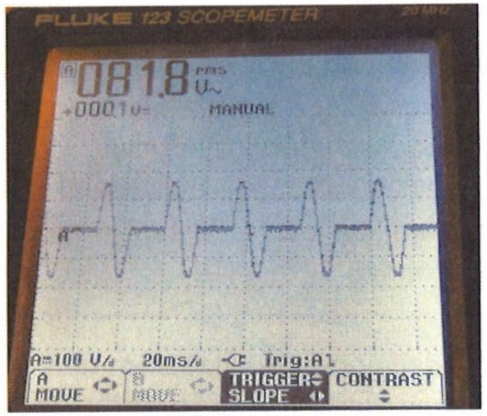
$$I_{RMS} = \frac{\sqrt{\frac{1}{\omega T}\int_0^{\omega T}\left(I_{pk}\sin\omega t\right)^2 d\omega t}}{T_{on}/T_{cycle}}$$

I (amps)

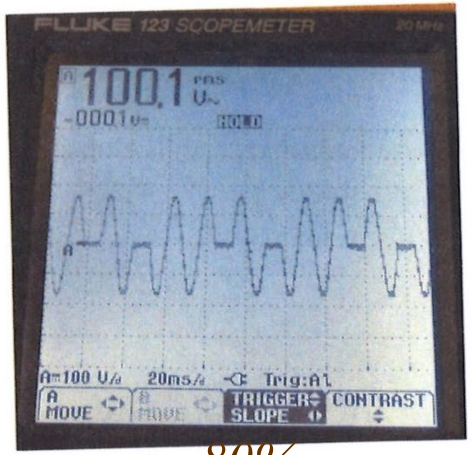


time

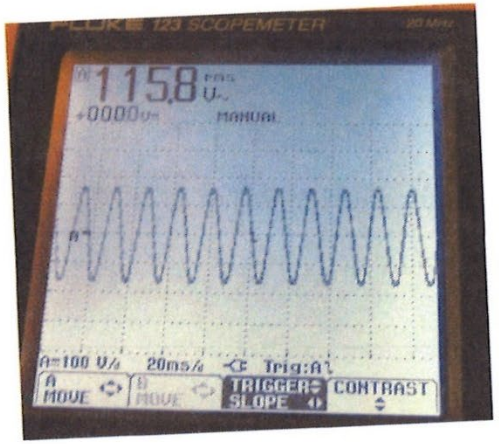
*2 on, 8 off, =20% duty cycle   8*8.3ms = 66.4ms off*



50%



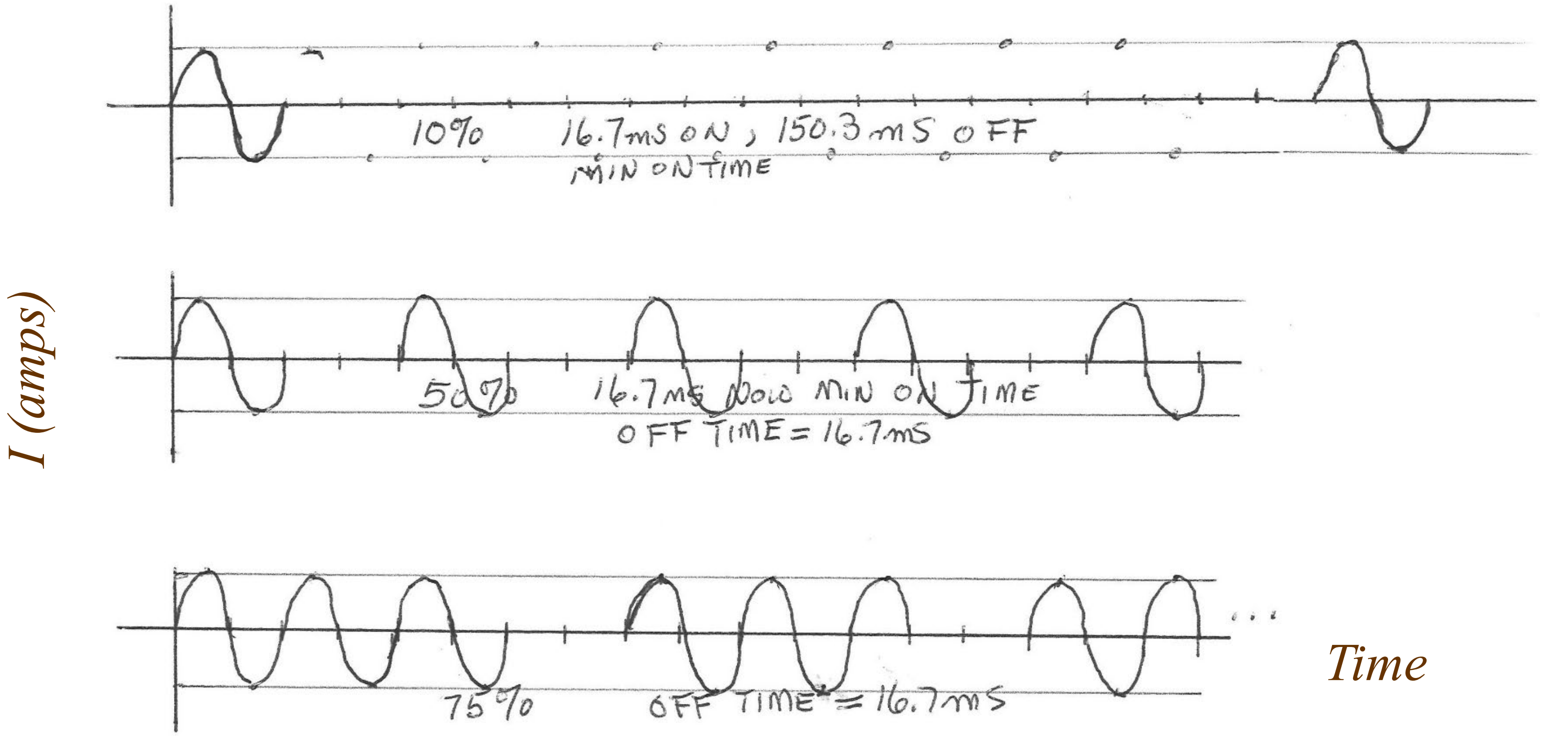
80%



100%

Fixed Amplitude AC Controllers – Variable Burst Firing

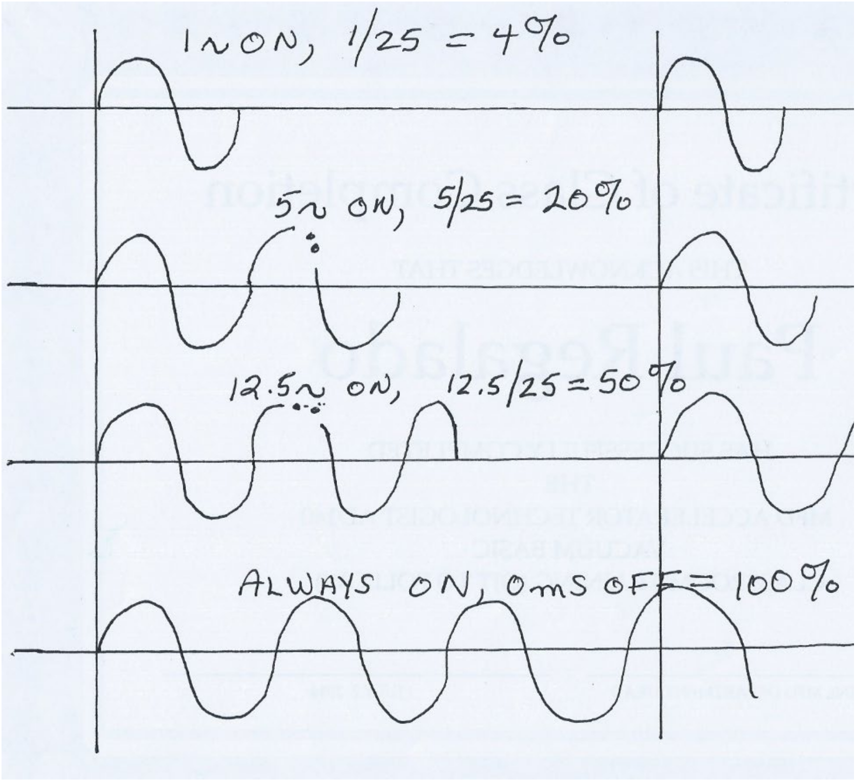
- *0 to 50% of set-point, on time is 16.7ms.* ***Off time*** *is varied to achieve control*
- *51% to 100%,* ***off time*** *is 16.7ms. Power is controlled by varying the on cycles*



$$I_{RMS} = \frac{\sqrt{\frac{1}{\omega T}\int_0^{\omega T}\left(I_{pk}\sin\omega t\right)^2 d\omega t}}{T_{on}/T_{cycle}}$$

Fixed Cycle Time 25 periods to 1000 periods – Use 25 periods here

I (amps)

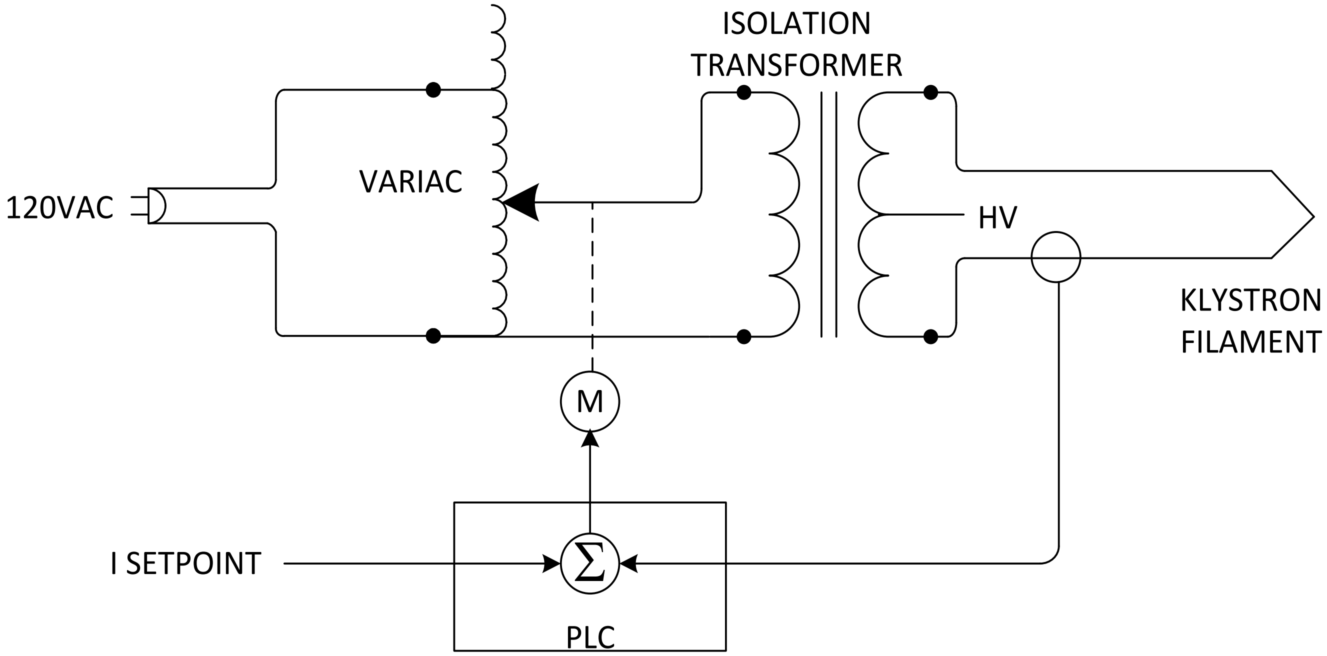


time

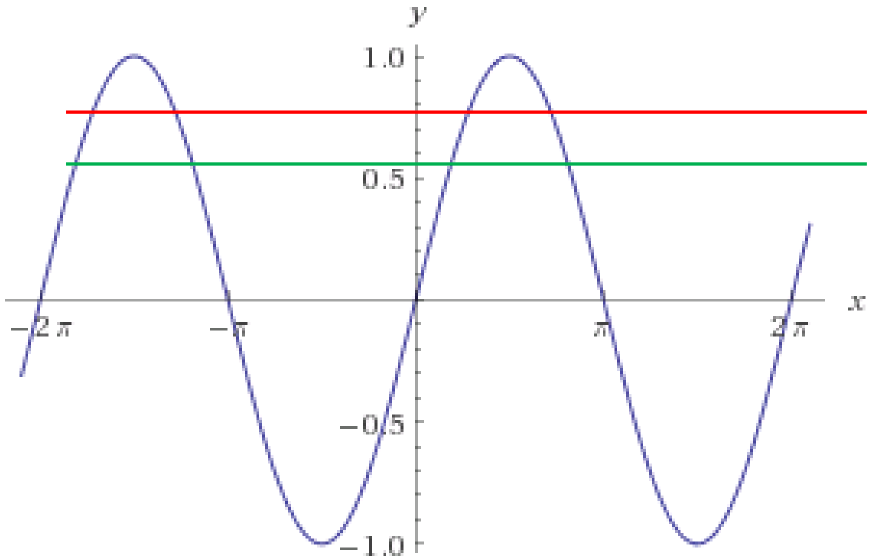
$$I_{RMS} = \frac{\sqrt{\frac{1}{\omega T}\int_0^{\omega T}\left(I_{pk}\sin\omega t\right)^2 d\omega t}}{T_{on}/T_{cycle}}$$

$$T_{cycle} = 25\,periods = 417.5ms$$

Variable Amplitude AC – Variac Controller



Requires motor driven Variac. More maintenance than solid-state, few manufacturers, difficult to obtain spare parts in future



$$I_{AVG}(DC) = \frac{1}{\omega T}\int_0^{\omega T} I_{pk} \sin \omega t \, d\omega t$$

$$I_{AVG} = 0.636 * I_{pk}$$

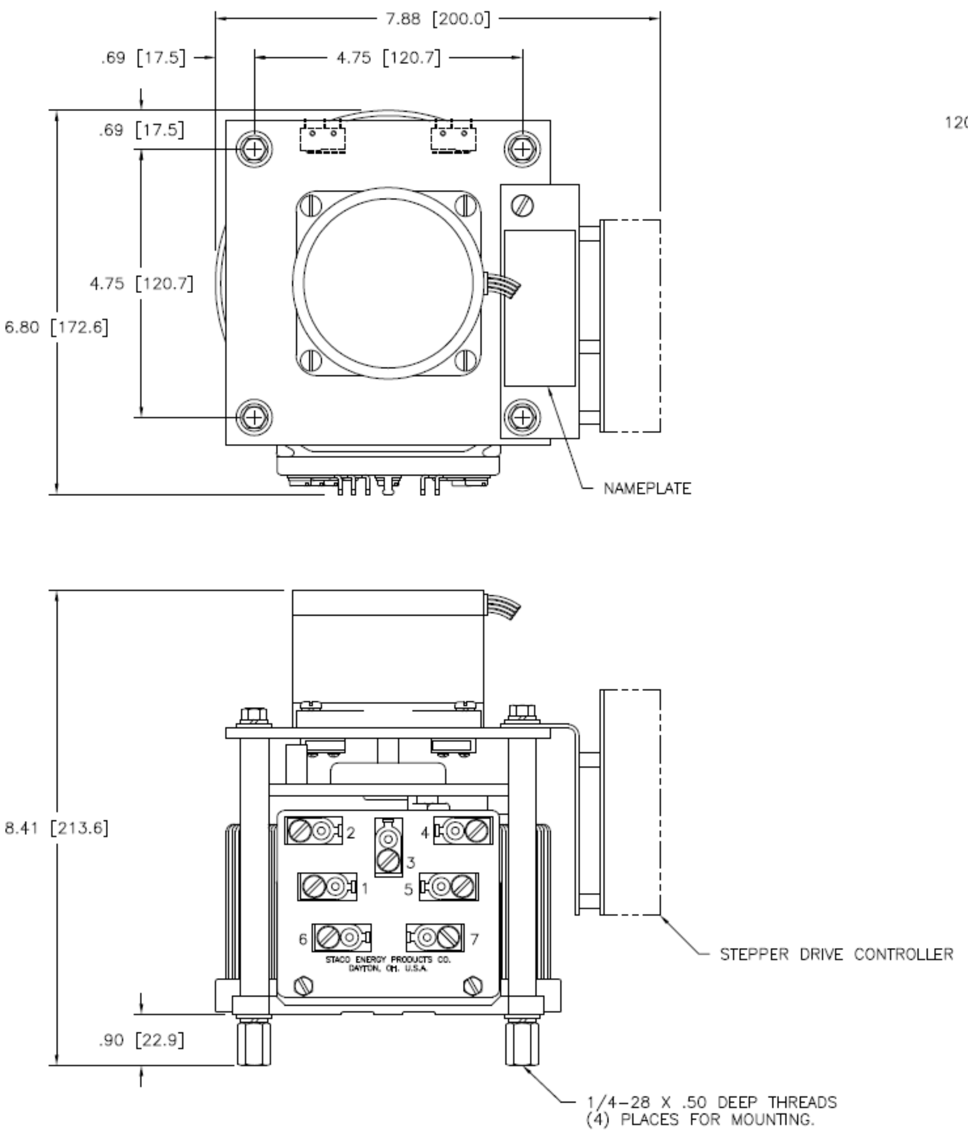
$$I_{pk} = 1.57 * I_{AVG}$$

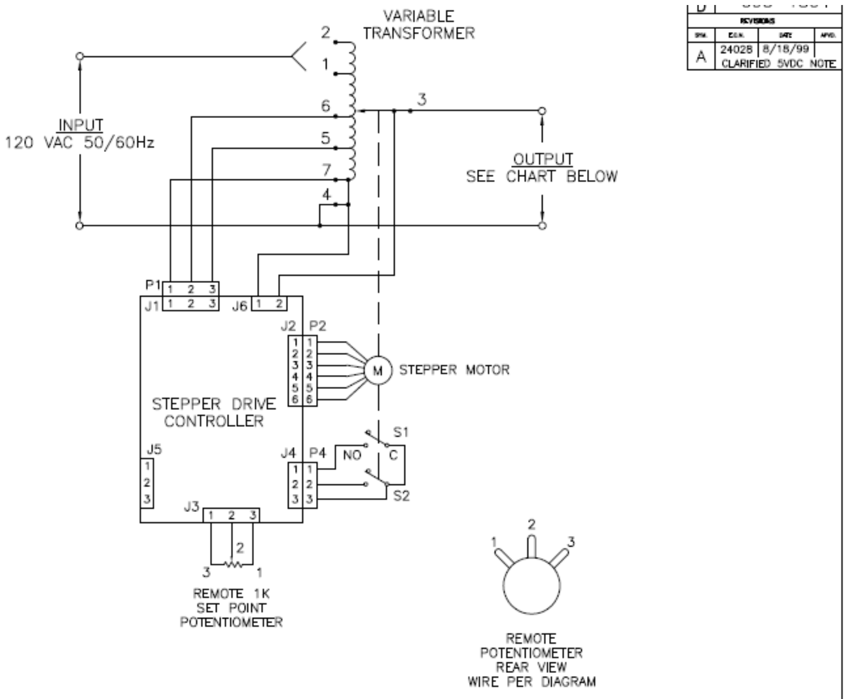
$$I_{RMS} = \sqrt{\frac{1}{\omega T}\int_0^{\omega T} (I_{pk} \sin \omega t)^2 \, d\omega t}$$

$$I_{RMS} = \frac{I_{pk}}{\sqrt{2}}$$

$$I_{pk} = 1.41 * I_{RMS}$$

- *Sinusoidal varying current –mechanical and thermal stress on filament*
- *I and V peaks only as large as needed*





SET POINT ADJUSTMENT:
THE OUTPUT SET POINT MAY BE ADJUSTED WITH THE REMOTE 1K POTENTIOMETER PROVIDED OR WITH A 0–5 VDC SIGNAL TO THE J3 CONNECTOR ON THE STEPPER DRIVE CONTROL. MAKE CONNECTIONS TO J3–2(+) & J3–3(–). A NOMINAL +5 VDC SETPOINT SUPPLY IS AVAILABLE AT J3–1 (ADJUSTABLE AT TRIMPOT R22).

SPECIFICATIONS

WIRING	INPUT VOLTS	INPUT HERTZ	OUTPUT VOLTS	CONSTANT CURRENT LOAD MAX. AMPS	CONSTANT CURRENT LOAD MAX. KVA	CONSTANT IMPEDANCE LOAD MAX. AMPS	CONSTANT IMPEDANCE LOAD MAX. KVA	TERMINAL CONNECTIONS INPUT	TERMINAL CONNECTIONS OUTPUT
SINGLE PHASE	120	50/60	0–120	15	1.80	20	2.40	2–4	4–3
			0–140	15	2.10	–	–	1–4	4–3

TITLE: SPEC. CONTROL DRAWING STEPPER DRIVEN VARIABLE TRANSFORMER MODEL SD1510

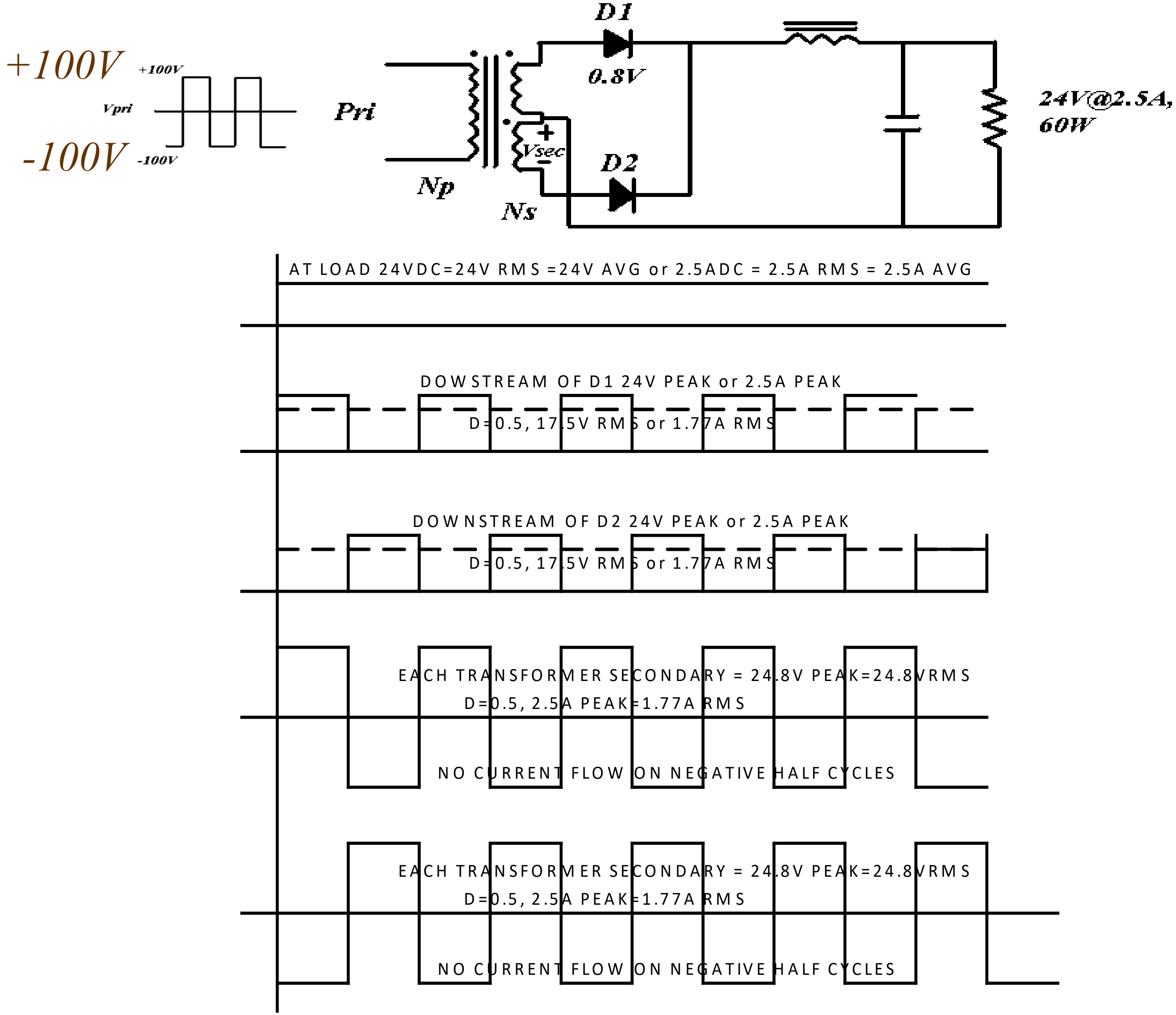
STACO ENERGY PRODUCTS CO. DAYTON, OHIO U.S.A.

DRAWN BY: S.A. SMITH | DATE: 12/7/95 | SCALE: 1=1 | SHEET 1 OF 1 | DWG. SIZE: D | DWG. NO. 095–1804

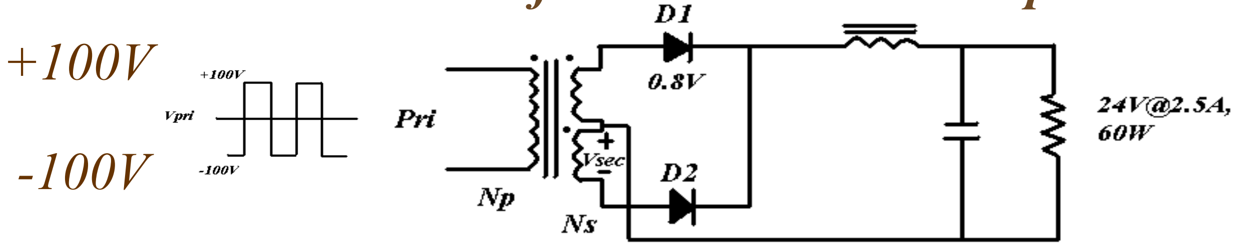
AC Controllers for Filaments

Controller Type	Type	Stress Types
Variac	*Variable Amplitude AC*	*Least thermal stress from AC current – no off time*
Intelligent half-cycle	*Fixed Amplitude AC*	*Thermal stress from AC current – short off time*
Burst Variable	*Fixed Amplitude AC*	*Thermal stress from AC current – long off time*
Burst Fixed	*Fixed Amplitude AC*	*Thermal stress from AC current – longest off time*
Phase Angle Triggered	*Fixed Amplitude AC*	*Thermal and mechanical stresses from chopped AC current*

Transformer Primer - Example



Transformer Primer - Example



$V_{load} = 24VDC = V_{peak} = V_{rms} \quad I_{load} = 2.5A = I_{peak} = I_{rms}$

$P_{load} = 24V * 2.5A = 60W$

$V_{secrms} = V_{secpeak} * \sqrt{D} = 24.8V * \sqrt{0.5} = 17.5V \; each \; winding$

$V_{secrms} = \sqrt{17.5V^2 + 17.5V^2} = 24.8V \; both \; windings$

$I_{secrms} = \sqrt{1.77A^2 + 1.77A^2} = 2.5A \; total \; from \; both \; windings$

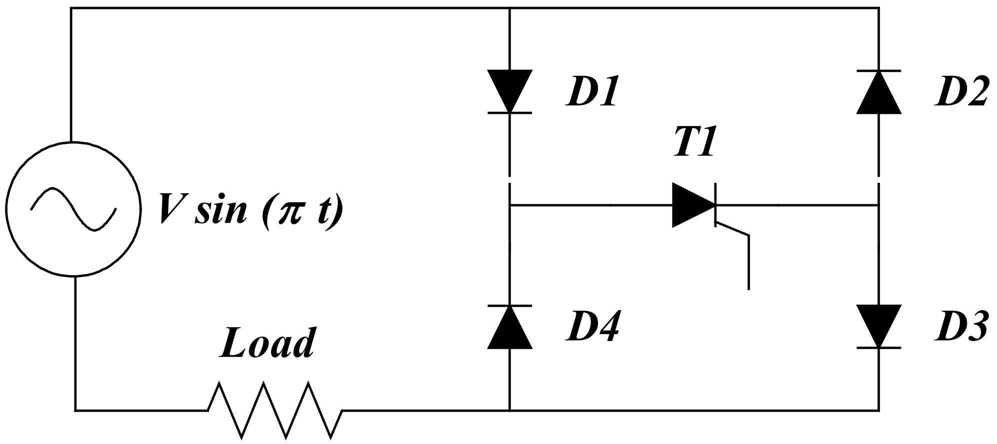
$P_{sec} = V_{secrms} * I_{secrms}$=24.8V*2.5A=62W both secondaries

$V_{prirms} = V_{pripeak} * \sqrt{D} = 100V * \sqrt{1} = 100V$

$I_{prirms} = \frac{V_{rmssec}}{V_{rmspri}} * I_{secrms} = \frac{24.8V}{100V} * 2.5A$=0.62A

P_{pri} =100V*0.62A=62W

$Eff = \frac{P_{load}}{Psec} * 100\% = \frac{60W}{62W} * 100\%$ =96.8%



Assume ideal components in the phase-controlled circuit above. For a purely resistive load:

A. Explain how the circuit operates

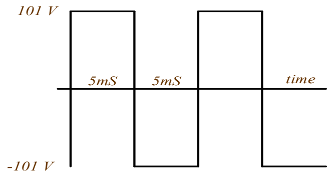
B. Draw the load voltage waveform and determine the boundary conditions of the delay angle α

C. Calculate the average load voltage and average load current as a function of α

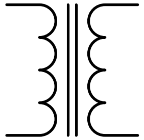
D. Find the RMS value of the load current. Help: $\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$

Given the following:

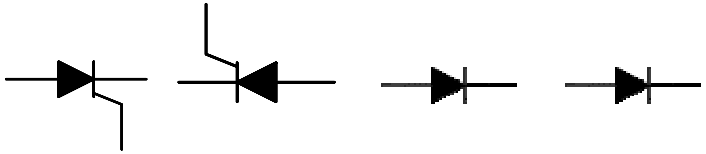
- *Input voltage waveform*



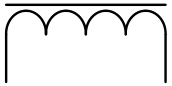
- *Lossless transformer*

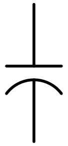


- *Two SCRs and two diodes each with conducting voltage drop of 1V.*



- *Inductor, lossless, with very large inductance. Capacitor, large and lossless*





- *Resistor, 10 ohms, capable of very large power dissipation*

- *Circuit operating under steady-state conditions (i.e. all transients have subsided)*

Problem

A. With the SCRs triggering retard angle at zero degrees, arrange the circuit to provide a full-wave, rectified, and properly low-pass filtered DC output of 200V into the 10ohm load resistor.

B. Calculate the load current and power

C. Determine the needed transformer turns ratio.

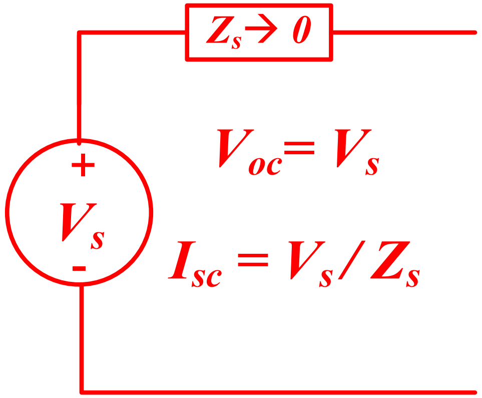
D. Calculate the circuit efficiency

Increase the SCRs trigger retard angle to 90 degrees and
E. Calculate the new output voltage, current, and power

F. Determine the new circuit efficiency

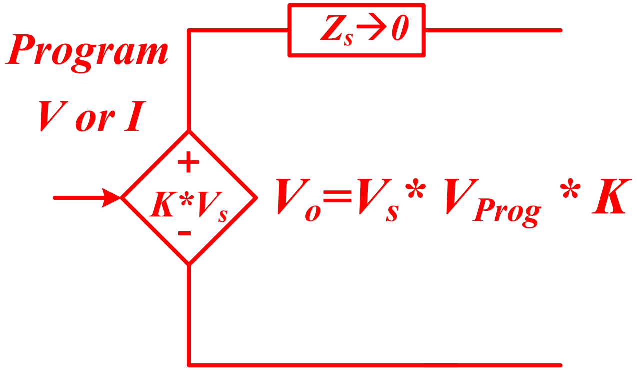
Thevenin Voltage Sources

Independent



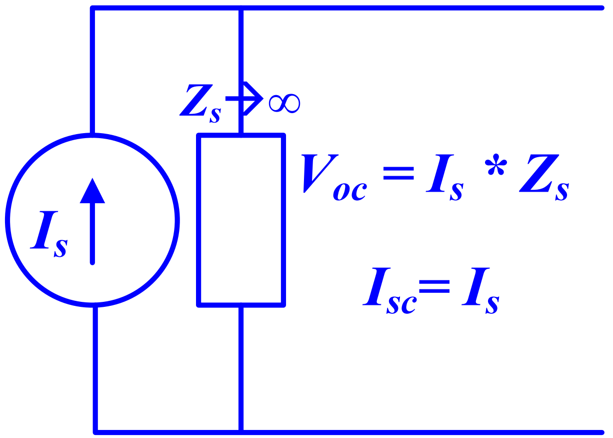
- *A way to analyze any complex source and load network*
- *Provides a constant output voltage regardless of the output current*
- *Fixed DC output voltage*

Dependent



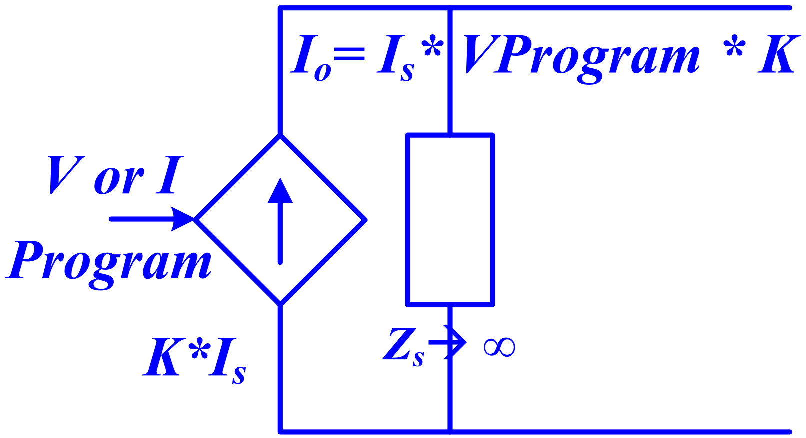
- *Provides a constant output voltage regardless of the output current*
- *Continuously adjustable*
- *V_o dependent on V_{Prog} (V_{Ref})*

Independent



- *Provides a constant output current regardless of the output voltage*
- *Fixed DC output current*

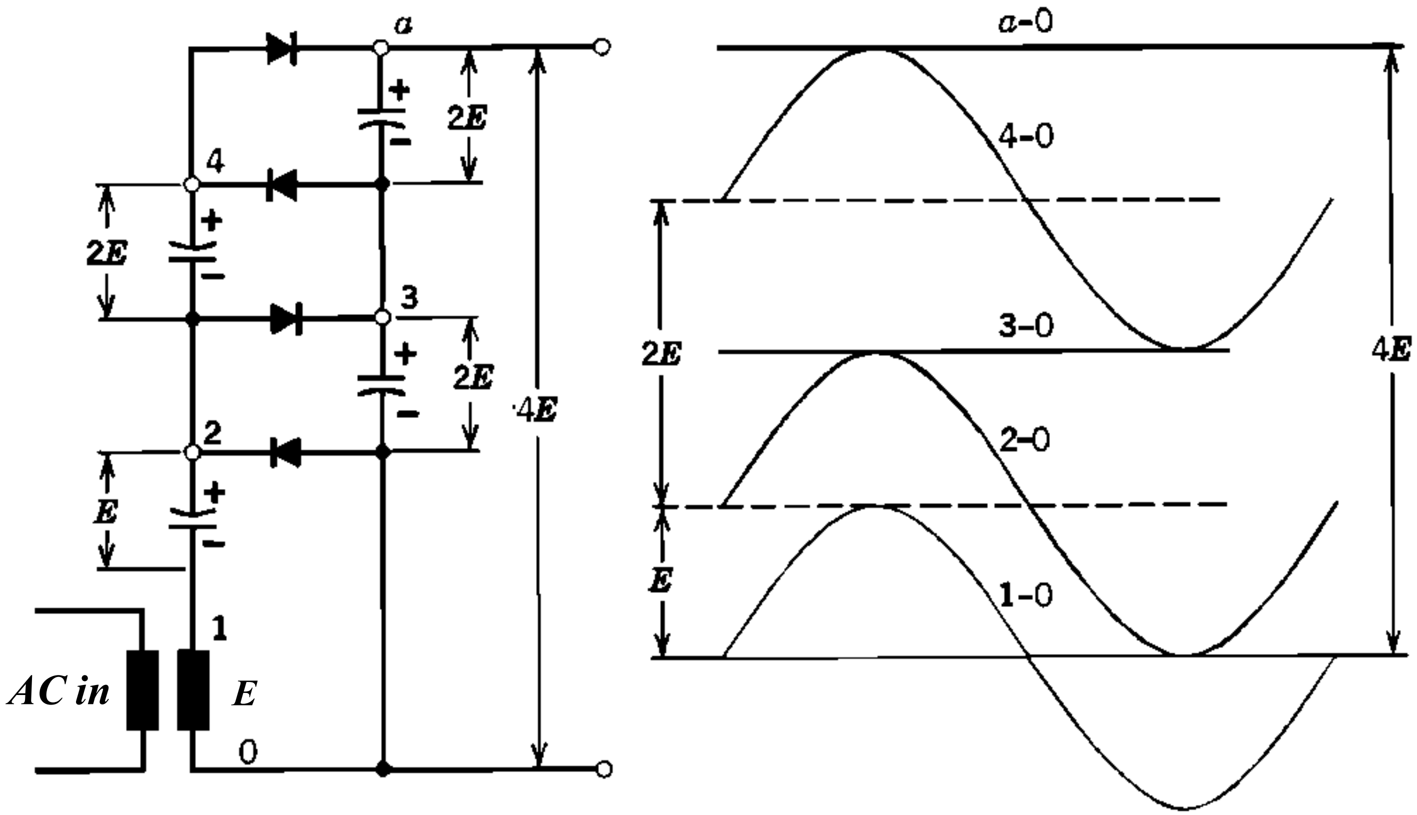
Dependent

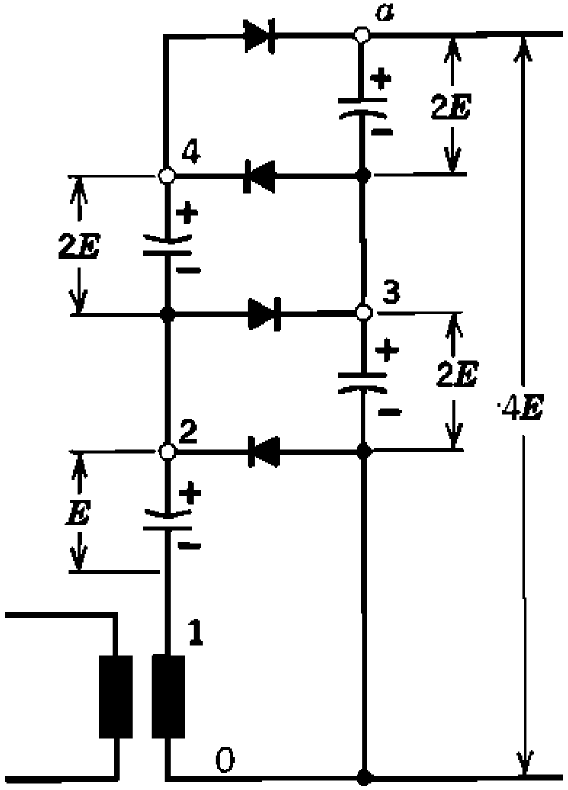


- *Provides a constant output current regardless of the output voltage*
- *Continuously adjustable*
- I_o *dependent on* V_{Prog} *(* V_{Ref}*)*

High Voltage Low Current DC supplies

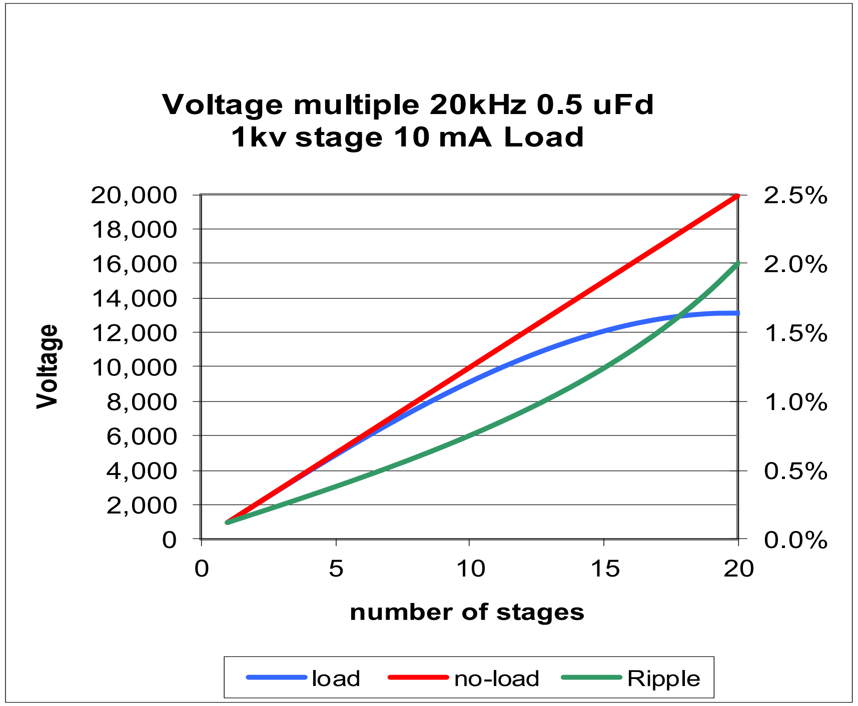
Voltage Multipliers, Cockroft Walton or Cascade Supplies





- Voltage multipliers or cascaded supplies
- Electron beam gun supplies and deflector supplies
- Half-wave, full-wave, three-phase, or six phase
- 20kV to 1,000 kV, 0 to 10 mA DC
- Requires high frequency input drive ~ 5 kHz to 50 kHz, but at low instantaneous power
- Provides low frequency, but high instantaneous power output
- Advantages – simple, reliable, inexpensive
- Disadvantages- low output power, poor regulation high output ripple, high output Z, 1st stage draws high current

High Voltage Multiplier DC supplies

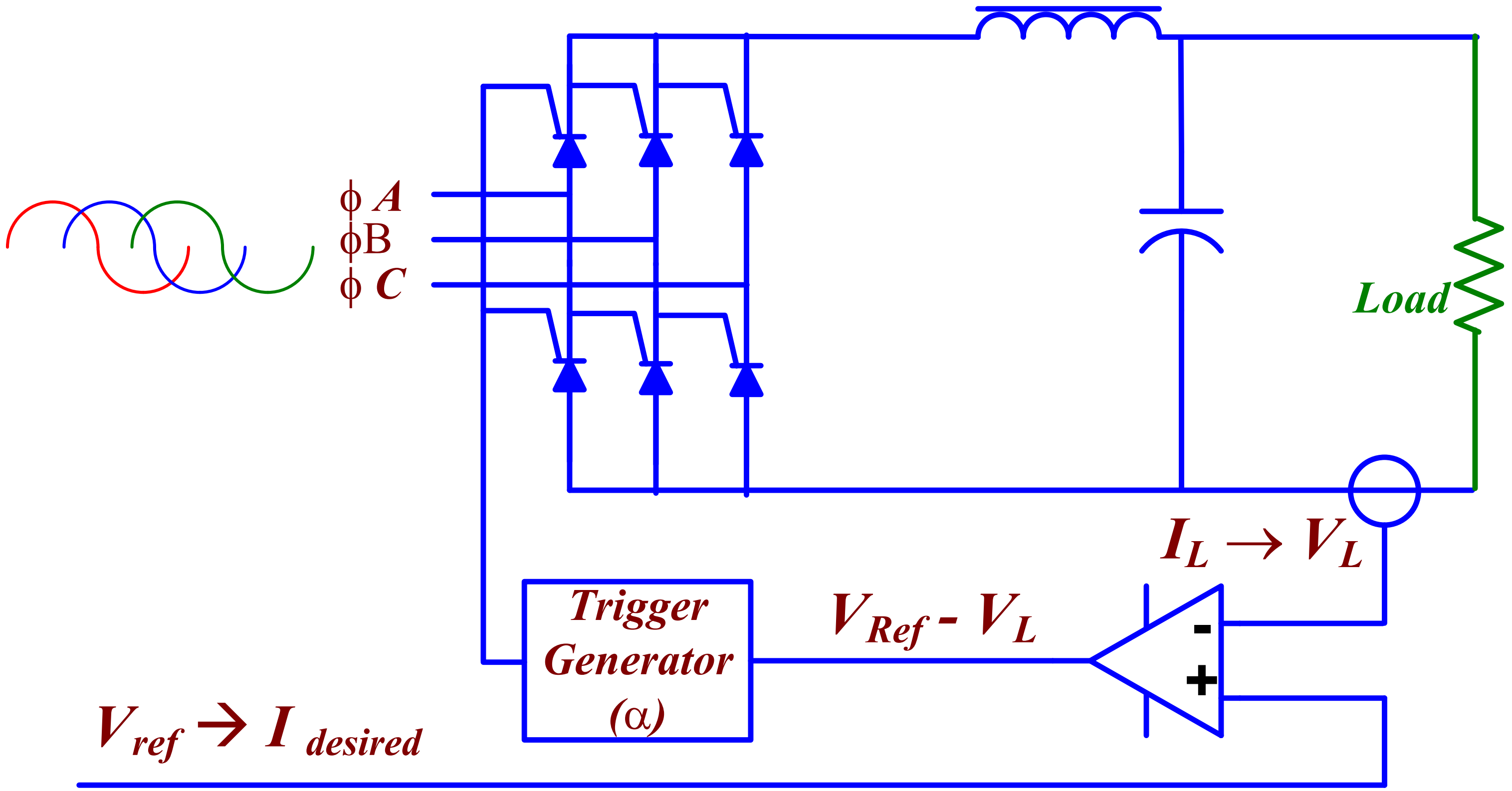


•*Disadvantages:*

Poor regulation $E_{drop} = (I_{load} / (f*C)) * (2/3*n^3 + n^2/2 - n/6)$

Large ripple $E_{ripple} = (I_{load}/(f * C))*n*(n+1)/2$

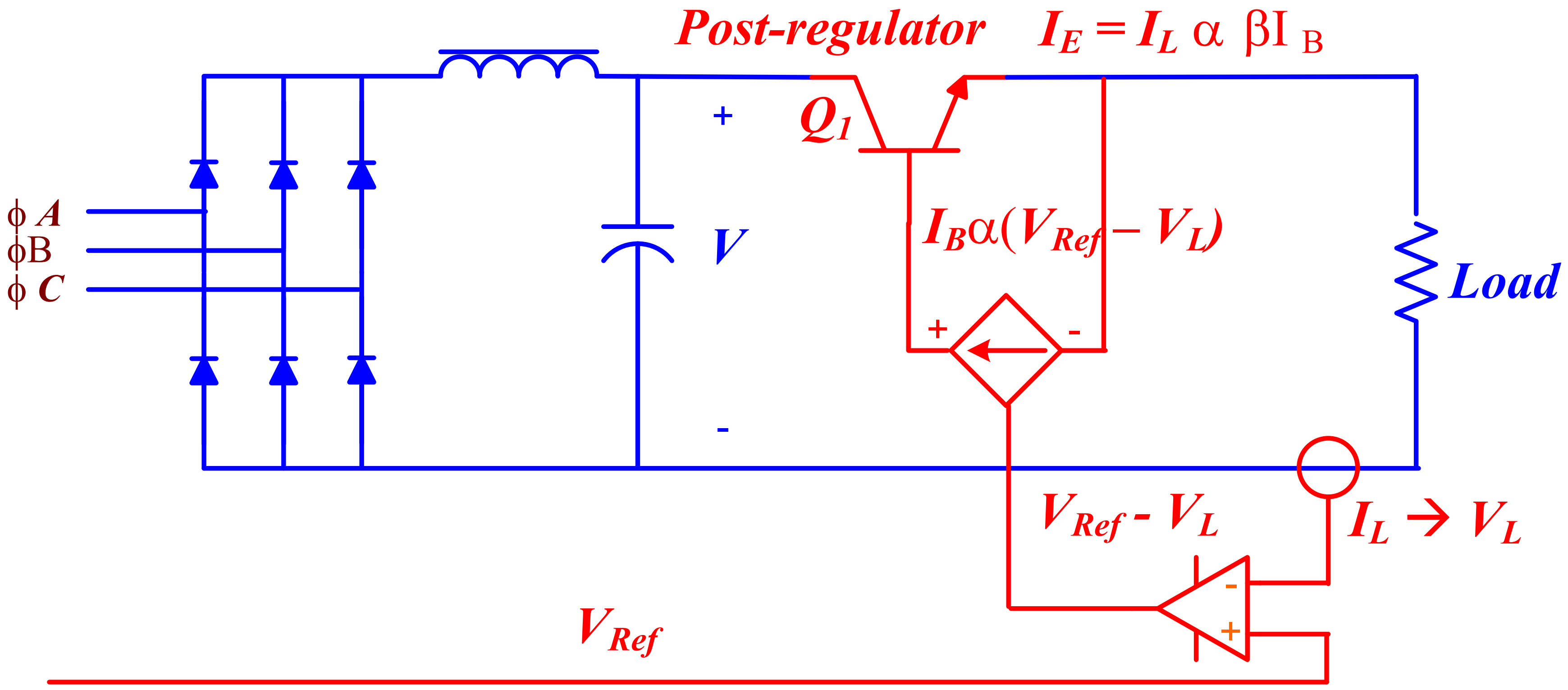
SCR Rectifier / Regulator Current Source



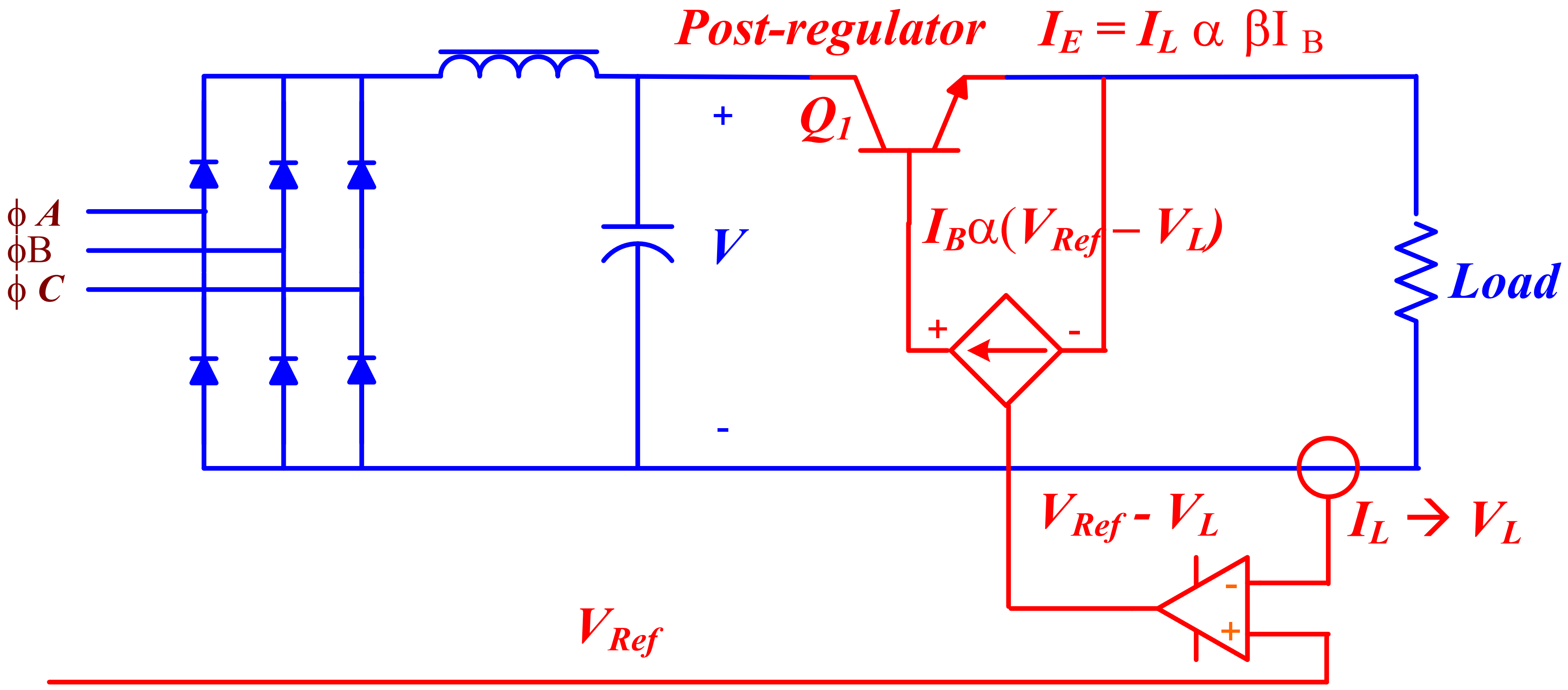
Reference Change	$V_{Ref}\uparrow$	$V_{Ref} - V_L \uparrow$	$\alpha\downarrow,\ I_L\uparrow$
Reference Change	$V_{Ref}\downarrow$	$V_{Ref} - V_L\downarrow$	$\alpha\uparrow,\ I_L\downarrow$
Load I Correction	$I_L\uparrow$	$V_{Ref} - V_L\downarrow$	$\alpha\uparrow,\ I_L\downarrow$
Load I Correction	$I_L\downarrow$	$V_{Ref} - V_L\uparrow$	$\alpha\downarrow,\ I_L\uparrow$

Disadvantage: *Line commutated, low bandwidth, some fast changes not regulated*

Diode Rectifier With Linear Post-Regulator To Improve Response



Reference Change	V_{Ref} ↑	$I_B \alpha V_{Ref} - V_L$ ↑	$I_E = I_L$ ↑
Reference Change	V_{Ref} ↓	$I_B \alpha V_{Ref} - V_L$ ↓	$I_E = I_L$ ↓
Load I Correction	I_L ↑	$I_B \alpha V_{Ref} - V_L$ ↓	$I_E = I_L$ ↓
Load I Correction	I_L ↓	$I_B \alpha V_{Ref} - V_L$ ↑	$I_E = I_L$ ↑



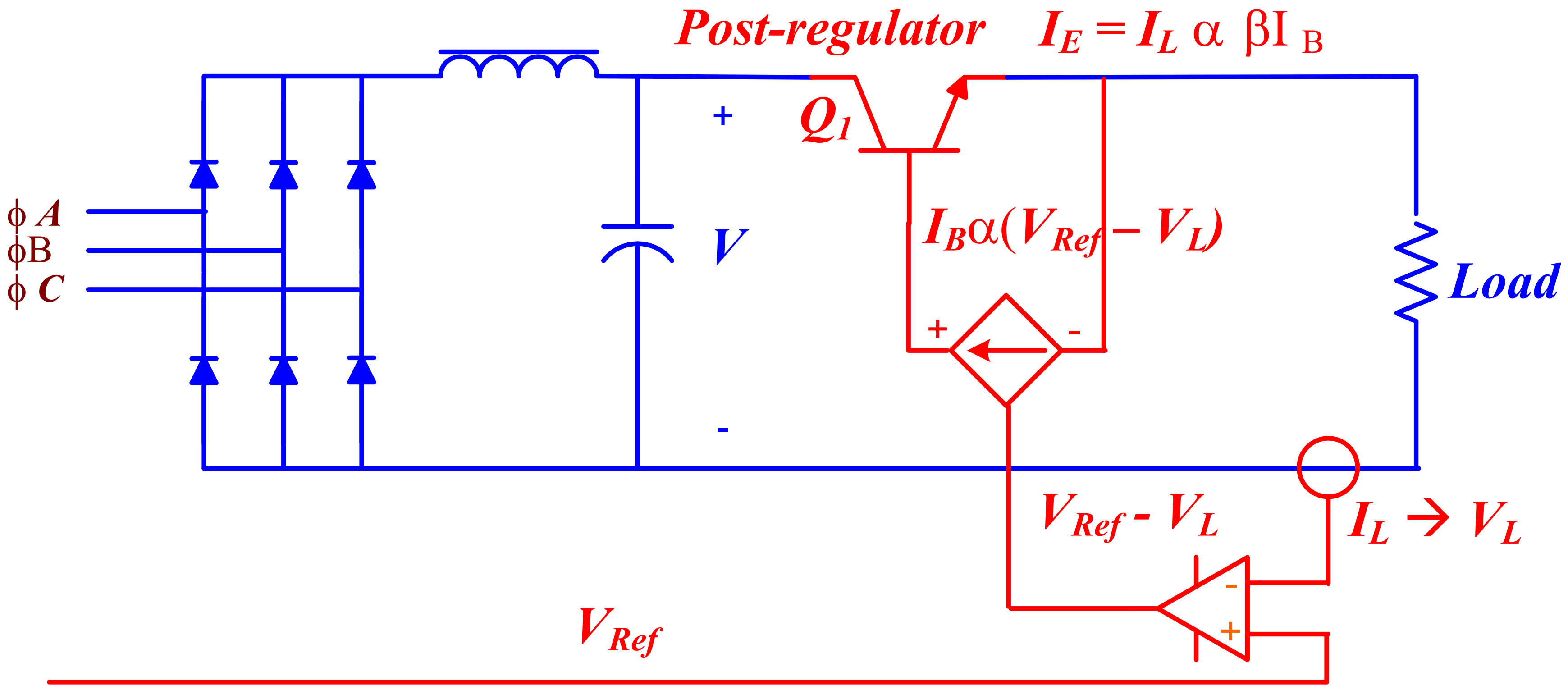
Regulation occurs by changing the transistor Q1 resistance

$$R_{Q1} = \frac{V_{CE}}{I_E} = \frac{V - V_L}{I_L}$$

V is constant, so $\quad$ if $I_L \uparrow, V_L \uparrow, V - V_L \downarrow, R_{Q1} \downarrow$

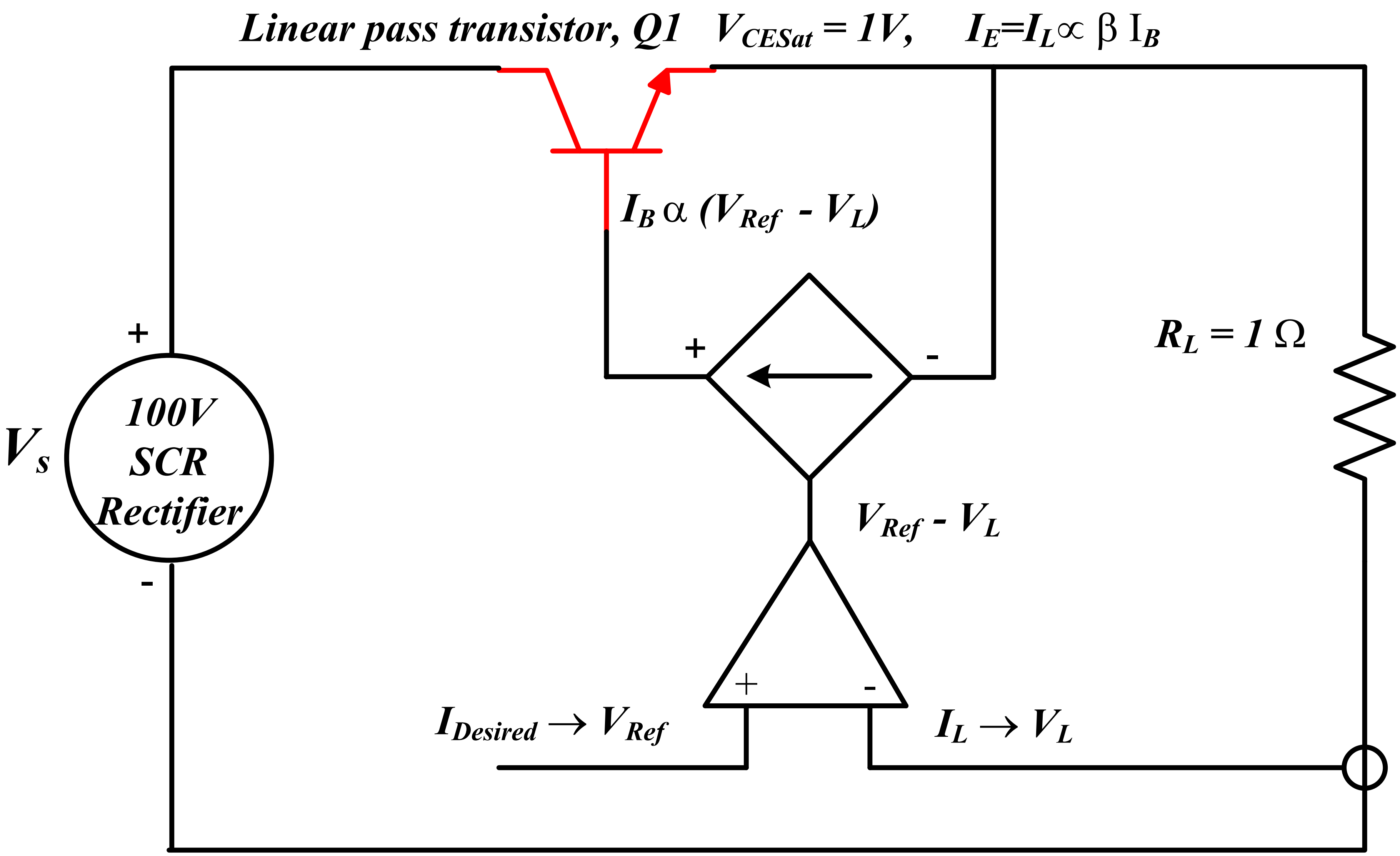
$$\text{if } I_L \downarrow, V_L \downarrow, V - V_L \uparrow, R_{Q1} \uparrow$$

Diode Rectifier With Linear Post-Regulator To Improve Response

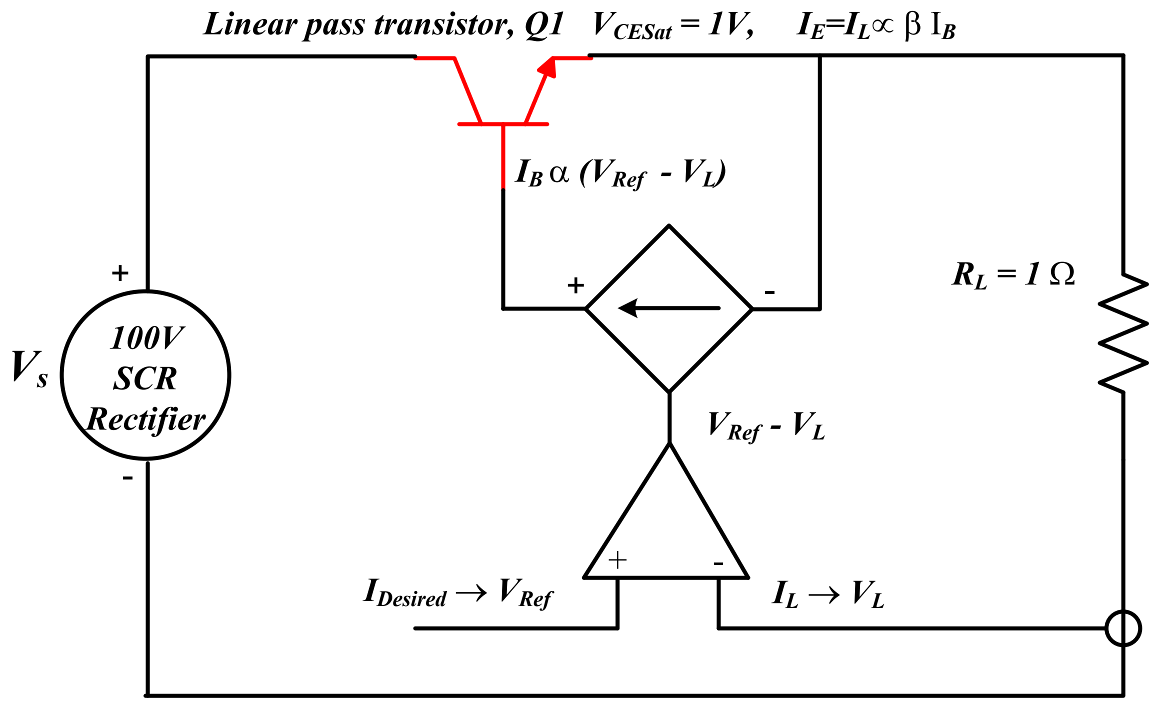


- Output I sensed and deviations due to programming, load or other changes are corrected by changing the resistance of the post-regulator.
- Broader bandwidth than line-commutated type
- Very inefficient topology, except when full output is required

Linear Regulator Disadvantage



Linear Regulator Disadvantage



$$V_S = 100V \qquad I_L = 0 \rightarrow 99\,A \qquad V_{Q1} = V_S - V_L$$

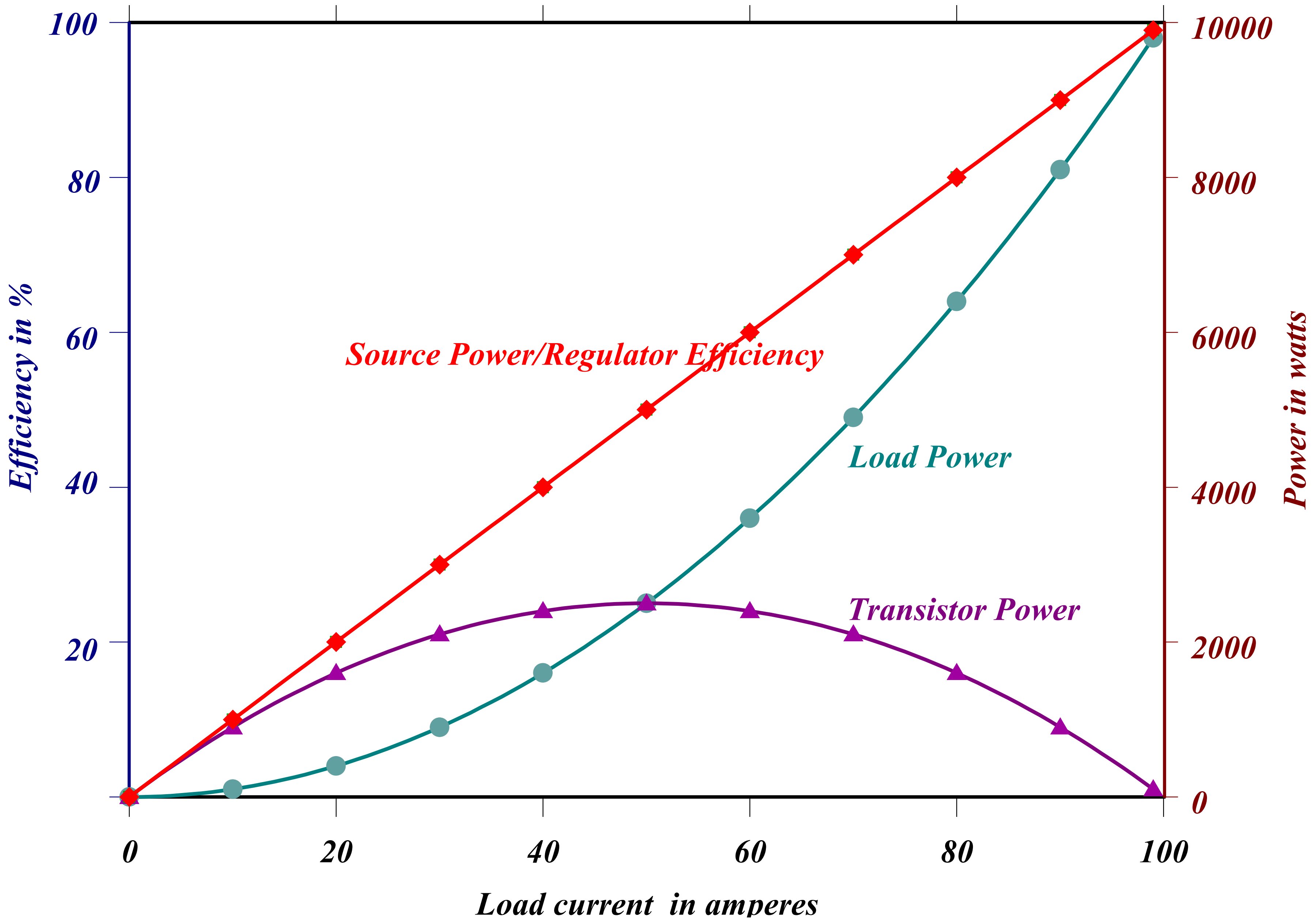
$$I_S = I_L \qquad V_L = I_L * R_L \qquad P_{Q1} = V_{Q1} * I_{Q1}$$

$$P_S = V_S * I_S \qquad P_L = V_L * I_L \qquad Eff = \frac{P_L}{P_S}$$

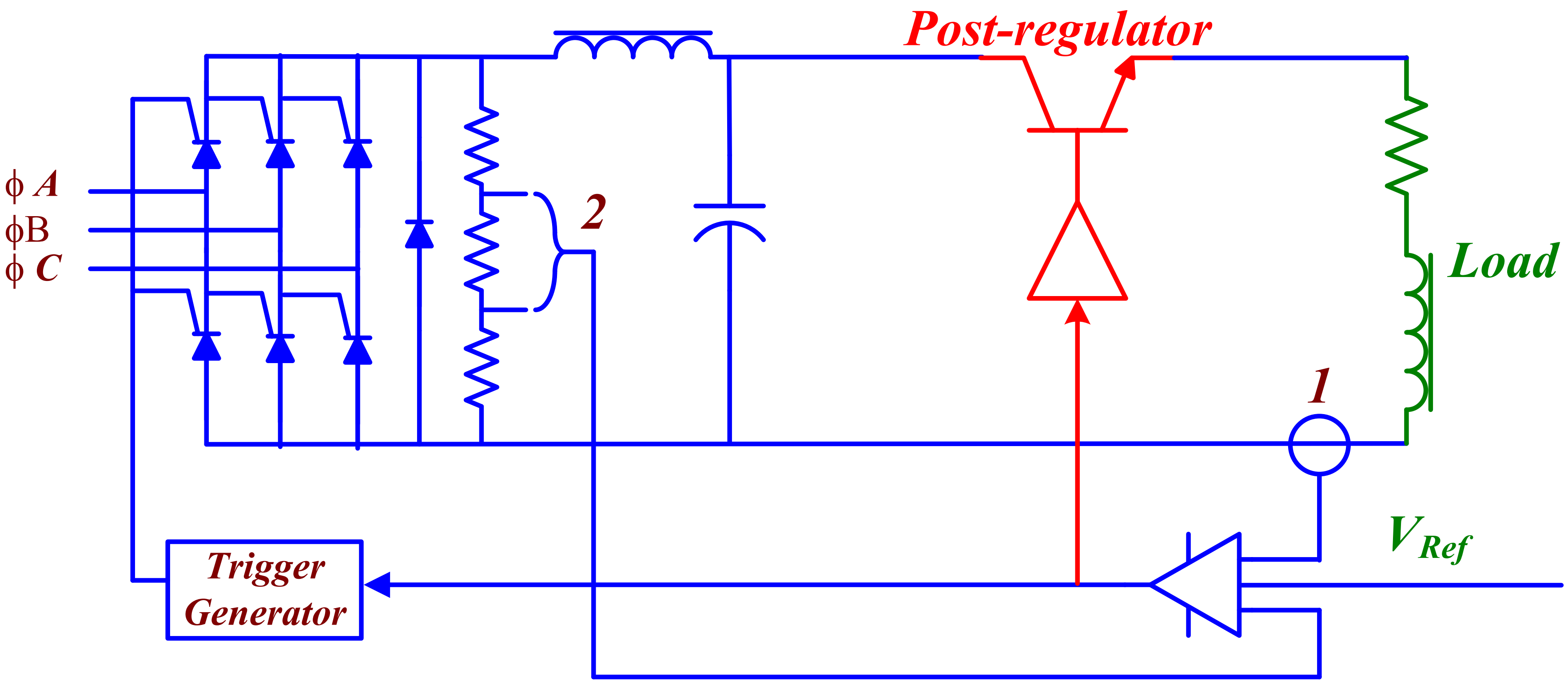
Linear Regulator Disadvantage

Load Amperes I_L = I Desired	Load Volts $V_L = I_L * R_L$	Load Watts $P_L = V_L * I_L$	Q1 Volts $V_{Q1}=V_S - V_L$	Q1 Amperes $I_{Q1}=I_L$	Q1 Watts $P_{Q1}=V_{Q1}*I_{Q1}$	Source Volts $V_S=100$	Source Amperes $I_S=I_l$	Source Watts $P_S=V_S * I_S$	% Efficiency $Eff=P_L / P_S$
0	0	0	100	0	0	100	0	0	0
10	10	100	90	10	900	100	10	1000	10
20	20	400	80	20	1600	100	20	2000	20
30	30	900	70	30	2100	100	30	3000	30
40	40	1600	60	40	2400	100	40	4000	40
50	50	2500	50	50	2500	100	50	5000	50
60	60	3600	40	60	2400	100	60	6000	60
70	70	4900	30	70	2100	100	70	7000	70
80	80	6400	20	80	1600	100	80	8000	80
90	90	8100	10	90	900	100	90	9000	90
99	99	9801	1	99	99	100	99	9900	99

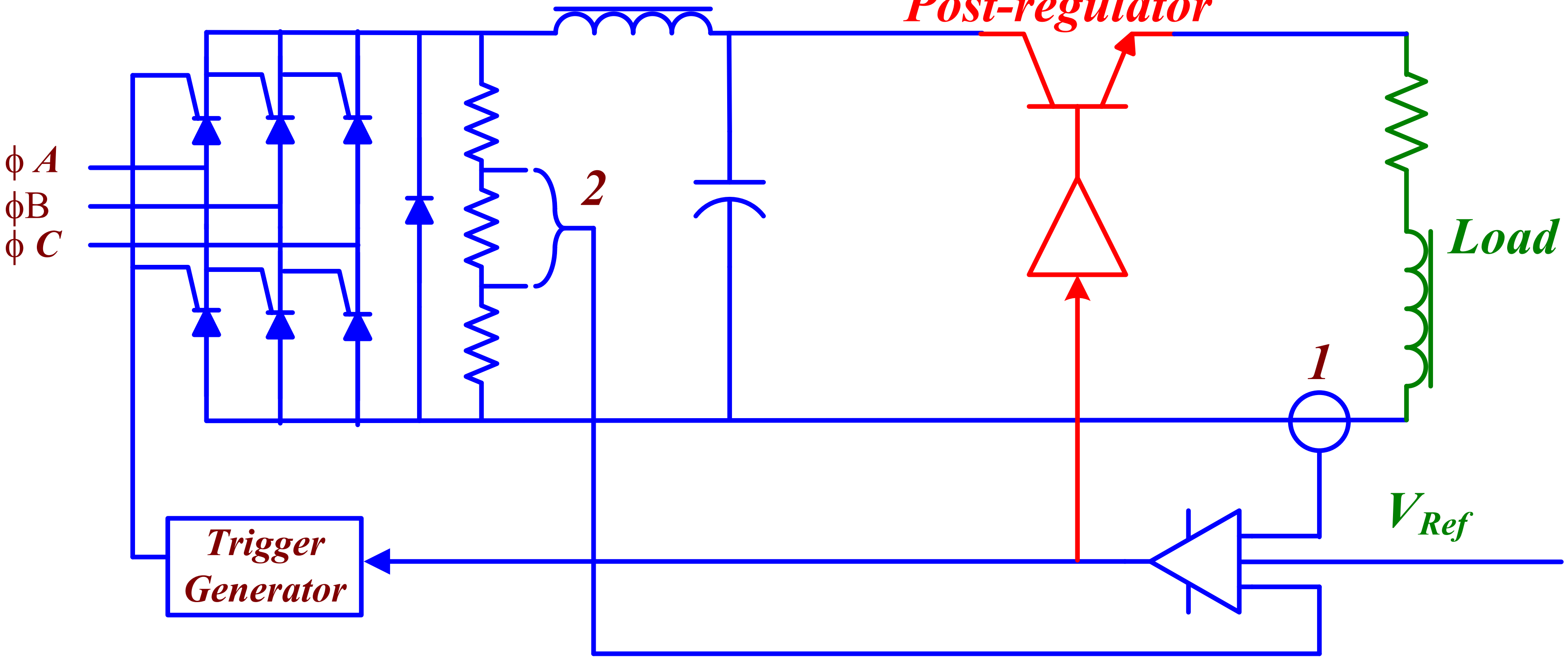
Linear Regulator Disadvantage



SCR Rectifier With Linear Post-Regulator To Improve Efficiency / Response



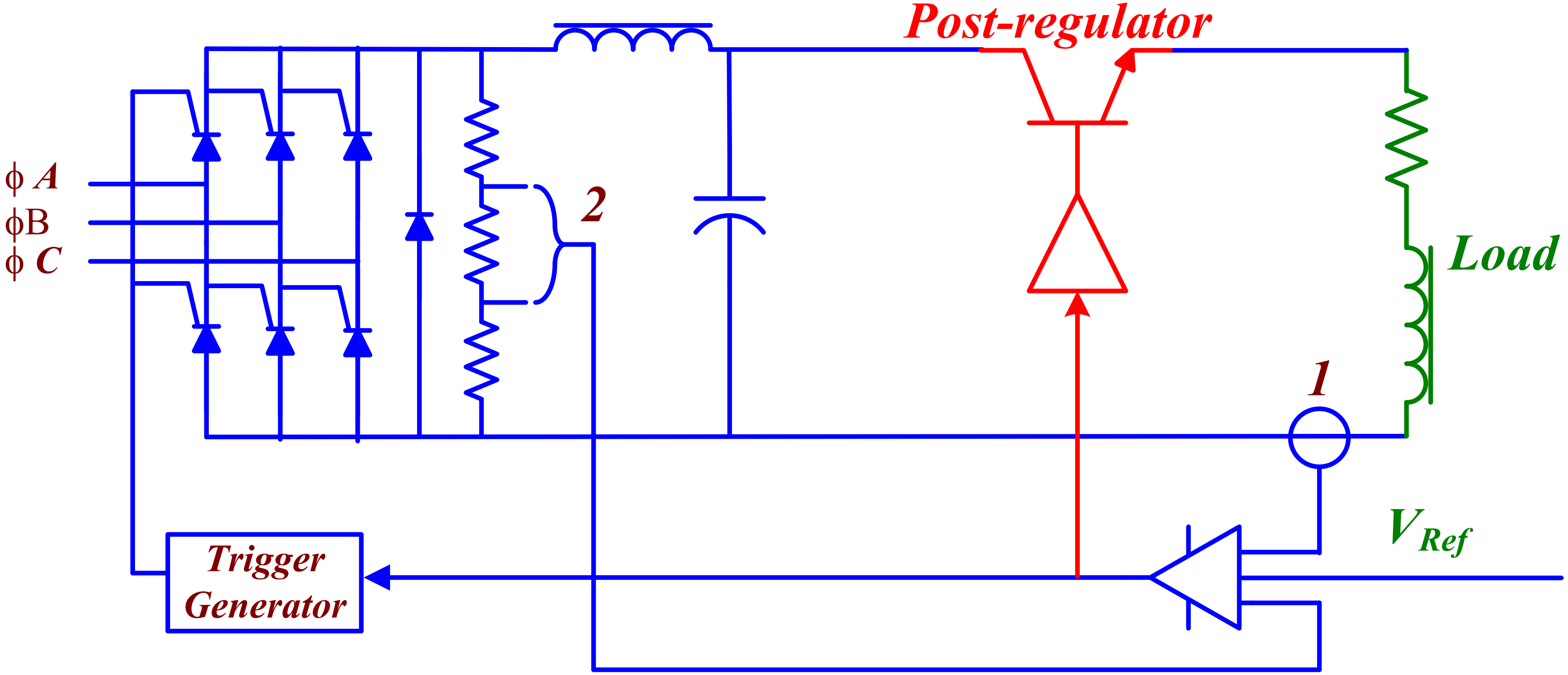
- SCRs full on for full output
- SCRs phased back for lower outputs to improve efficiency.
- Limited range regulation is done by the post-regulator



1. *Output I sensed. Deviations due to load or other changes are corrected by SCR rectifier and post-regulator.*

2. *Rectifier V_O is sensed. Slow line changes corrected by BW-limited SCRs. Fast transients corrected by high BW post-regulator*

3. *Bipolar transistor V_{CE} is monitored. If V_{CE} and/or $V_{CE}*I_E$ exceeds a safe value, SCR firing is advanced and rectifier V_O is increased accordingly*



Disadvantages

- *Large output changes cannot be accommodated by post-regulator. Requires retardation of SCR rectifier pulses to improve efficiency*
- *Low power factor when SCR gate firing is retarded ($V_{load} << V_{line}$)*
- *Implementation of 2 control loops is complex*

The Present – Switchmode Power Supplies
Circa 1990 - Present

Recalling The Recent Past

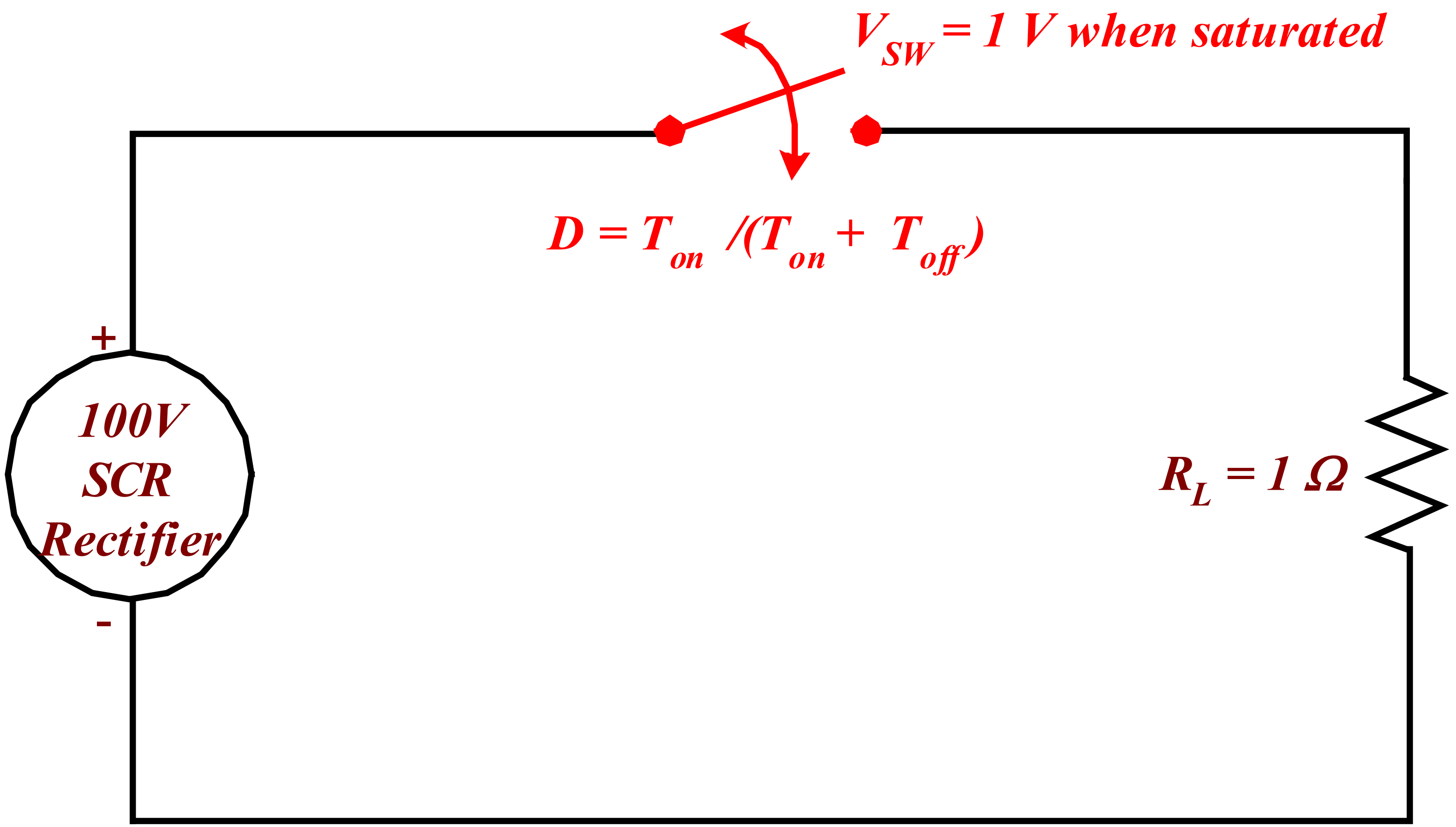
Topology	Disadvantages
• SCRs for rectification and regulation	• Low power factor • High AC line harmonic distortion • Narrow bandwidth • Slow transient response
• SCRs for rectification and gross regulation • Fine regulation by post linear transistors	• Low power factor when line $V \neq$ load V • High AC line harmonic distortion • Complex control loops

The Present Popular Solution

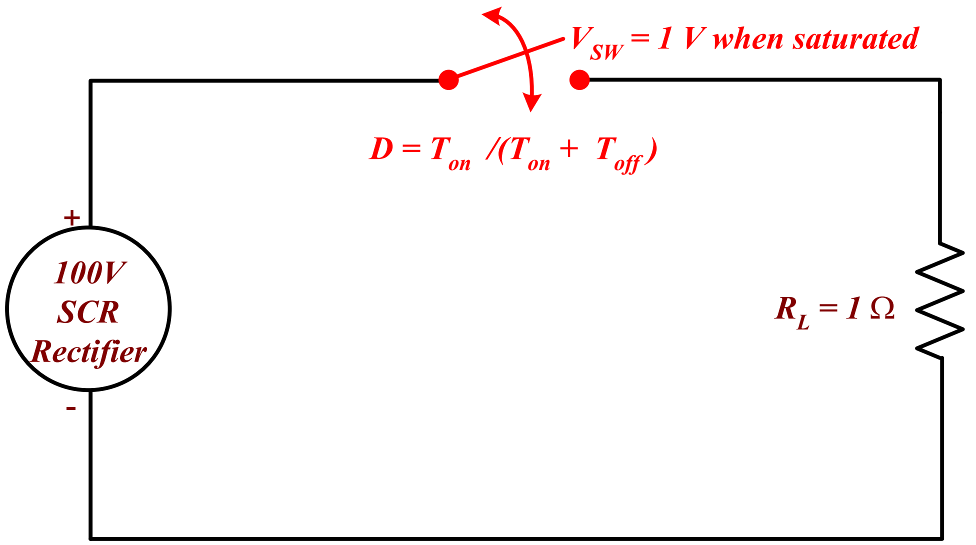
Topology	Advantages
• SCRs (or diodes) for rectification • High speed switches (switch-mode inverters) for regulation	• Rectifier SCRs or diodes are full on – hence high power factor (> 0.9) possible • High PF means low AC line harmonic distortion (< 5% V, < 25 % I) • Fast (10 kHz to 100 kHz) switching means wide bandwidth (> 100 s of Hz), fast transient response (microseconds) • Fast switching means more corrections per unit time – better output stability • Simple control loops compared to SCR rectifier/post-regulator combination • Fast switching, high frequency operation for electrically and physically smaller transformers and filter components

The Present Popular Solution (Continued)

Topology	*Disadvantages*
• *SCRs (or diodes) for rectification* • *High speed switches (switch-mode inverters) for regulation*	• *High speed, fast-edge switching can generate conducted and radiated electromagnetic interference (EMI)*


V_{SW} = 1 V when saturated
D = T_{on} /(T_{on} + T_{off})
+
100V
SCR
Rectifier
-
R_L = 1 Ω

The Switchmode Advantage



$$D = \frac{I_{Lavg}}{I_{peak}} = \frac{I_{Lavg}}{99\,A}$$

$$V_S = 100V$$

$$V_{SWRMS} = 1V * D^{1/2}$$

$$V_{Lavg} = I_{Lavg} * R_L$$

$$I_S = I_{SW} = I_L$$

$$I_{SWRMS} = 99\,A * D^{1/2}$$

$$V_{LRMS} = 99\,V * D^{1/2}$$

$$Eff = \frac{P_L}{P_S} * 100\%$$

$$P_S = P_{SW} + P_L$$

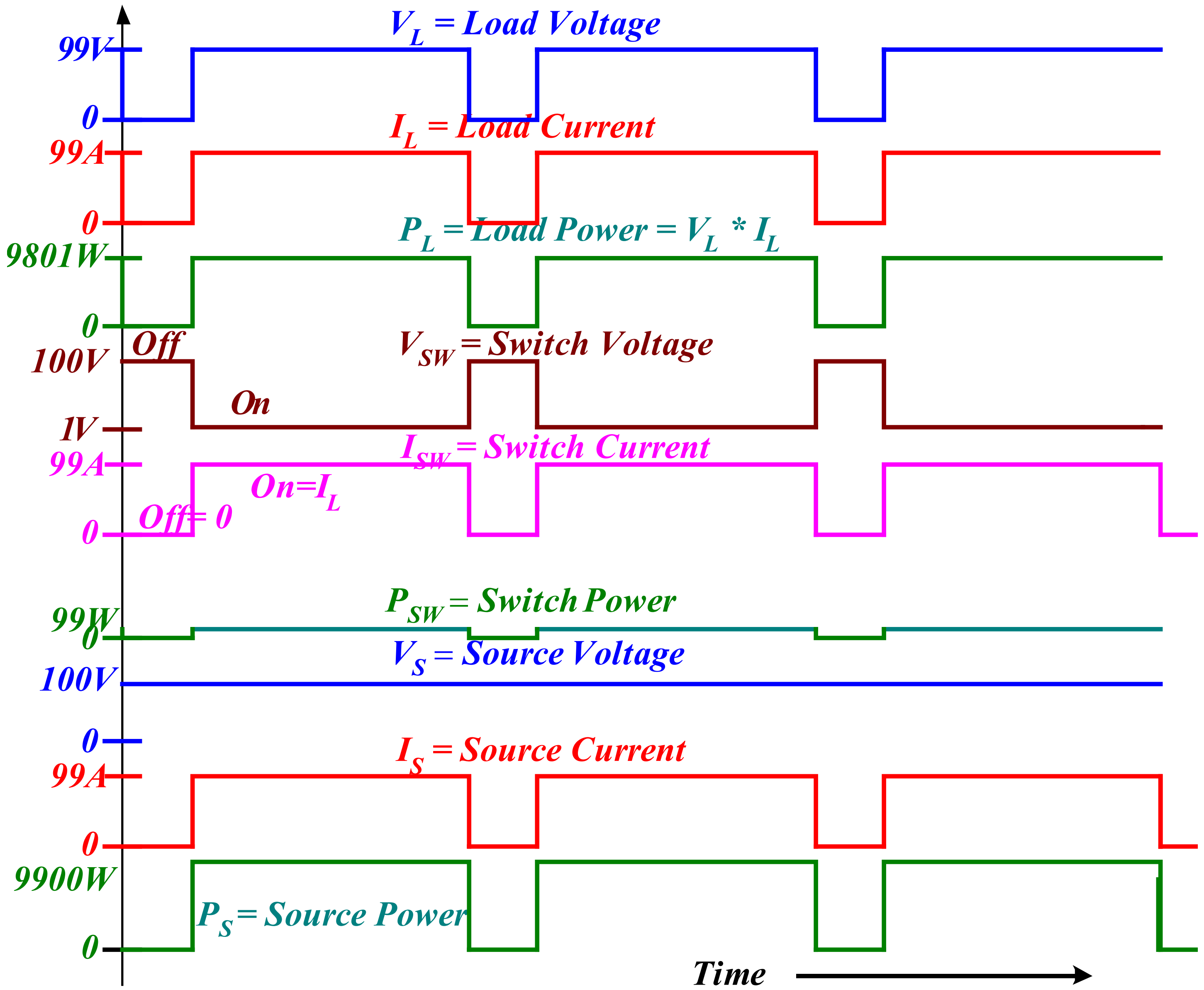
$$P_{SW} = V_{SWRMS} * I_{SWRMS}$$

$$I_{Lavg} = 0 \rightarrow 99\,A$$

$$I_{LRMS} = 99\;\;A * D^{1/2}$$

$$P_L = V_{LRMS} * I_{LRMS}$$

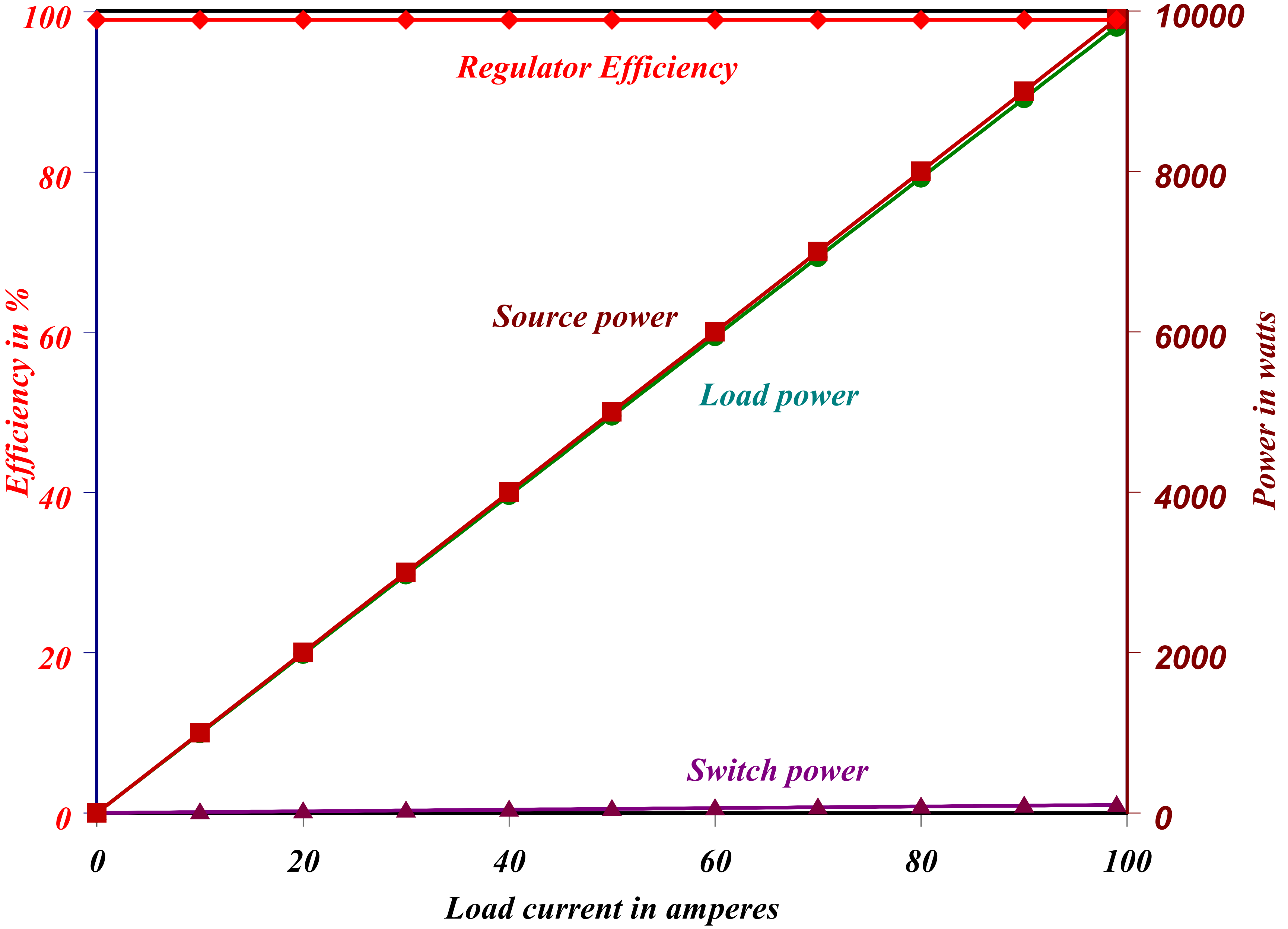
The Switchmode Advantage - Waveshapes



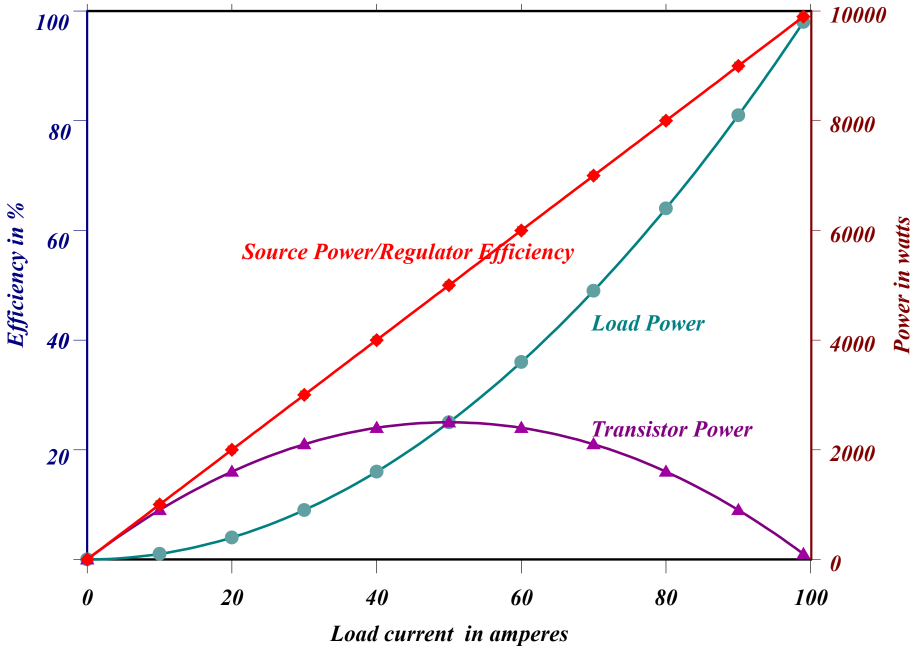
The Switchmode Advantage - Calculations

Avg Load Amps I_{Lavg} = $I_{Desired}$	Duty Factor=I_{Lavg} / Ipeak	Average Load V $V_{Lavg} = I_{Lavg} * R_L$	Load Volts RMS V_{Lrms}=99V*D^0.5	Load Amps RMS I_{Lrms}=99A*D^0.5	Load Power $P_{Lavg} = V_{Lrms} * I_{lrms}$	Switch Volts RMS V_{SWrms} = 1V * D^0.5	Switch Amps RMS I_{SWrms}=99 * D^0.5	Switch Power $P_{SWavg} = V_{SWrms} * I_{SWrms}$	Source Power $P_S = P_{Lrms} + P_{SWrms}$	% Efficiency Eff= $P_L / P_S * 100\%$
0	0	0	0	0	0	0.00	0.0	0	0	NA
10	0.101	10	31	31	990	0.32	31.5	10	1000	99
20	0.202	20	44	44	1980	0.45	44.5	20	2000	99
30	0.303	30	54	54	2970	0.55	54.5	30	3000	99
40	0.404	40	63	63	3960	0.64	62.9	40	4000	99
50	0.505	50	70	70	4950	0.71	70.4	50	5000	99
60	0.606	60	77	77	5940	0.78	77.1	60	6000	99
70	0.707	70	83	83	6930	0.84	83.2	70	7000	99
80	0.808	80	89	89	7920	0.90	89.0	80	8000	99
90	0.909	90	94	94	8910	0.95	94.4	90	9000	99
99	1	99	99	99	9801	1.00	99.0	99	9900	99

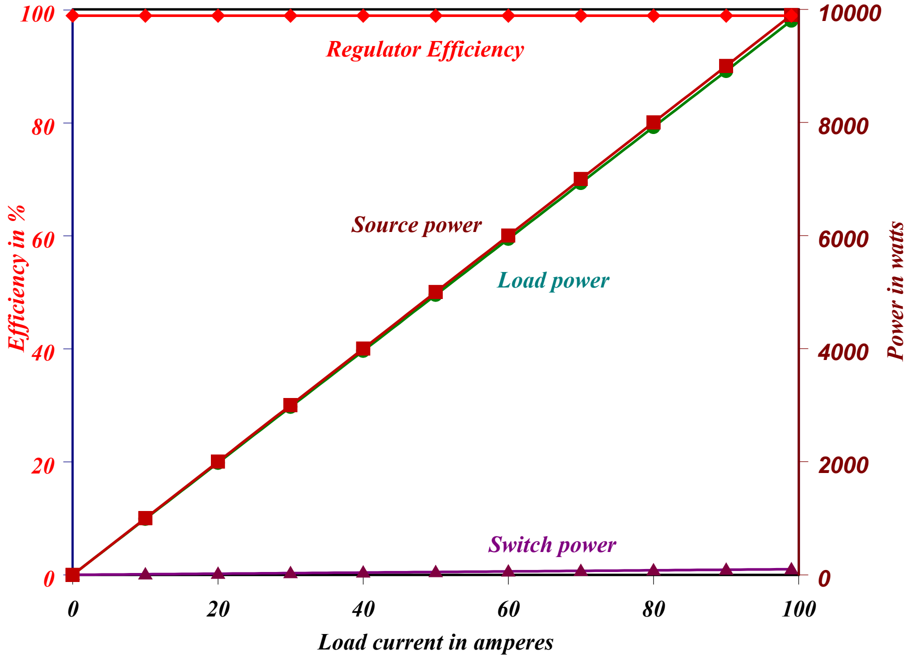
The Switchmode Advantage - Plots



Linear regulator



Switchmode regulator



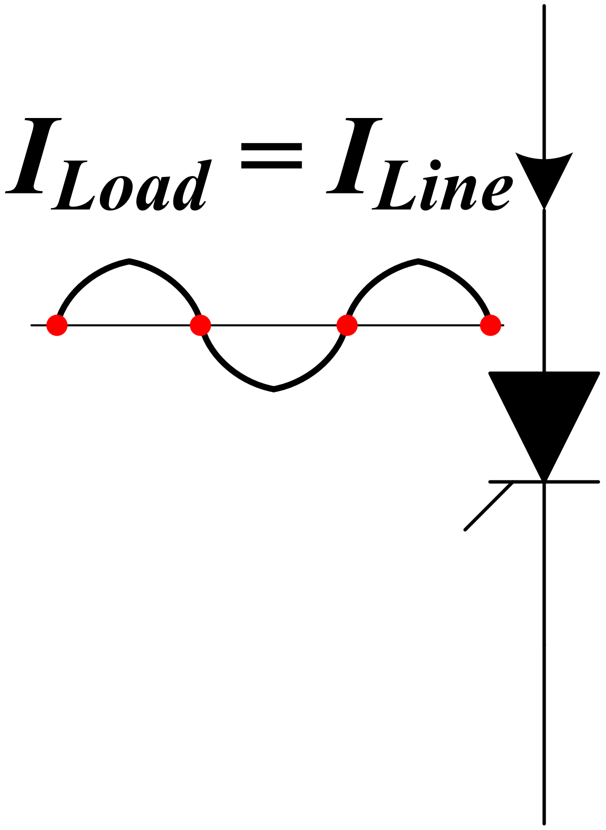
SCR Regulation Vs Switchmode Regulation

	SCR	Switchmode
Efficiency	Low at low load, high at full load	High, whether low or high load
Operating frequency	60 Hz	10 kHz to 1,000 kHz
Transient Response	Tens of milliseconds	Tens of microseconds
Short-term-stability	100s of ppm	10s of ppm
Input filter	Large	Smaller, HF regulator provides supplemental filtering
Isolation/Line-matching transformer	Large and upstream of the rectifiers	Smaller because of high frequency. Downstream of the regulator
Output filter	None	High frequency ripple = smaller size
Power factor	Low when output is low	Always high
Line distortion	High when output is low	Always low
EMI	High when output is low	High, but higher frequency, easier to filter

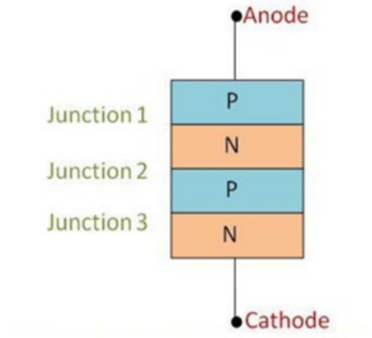
Linear Vs. Switchmode– Advantage Summary

Linear	***Switchmode***
Output current/voltage is adjusted by varying pass transistor resistance	*Output current/voltage is adjusted by varying switch duty factor*
Transistor voltage and current are in phase so transistor power loss is high	*Switch voltage and current are out of phase so switch power is low*
Efficiency is dependent upon the output operating point and is maximum at 100 % load	*Efficiency is high and relatively constant*

Line Commutated Switches



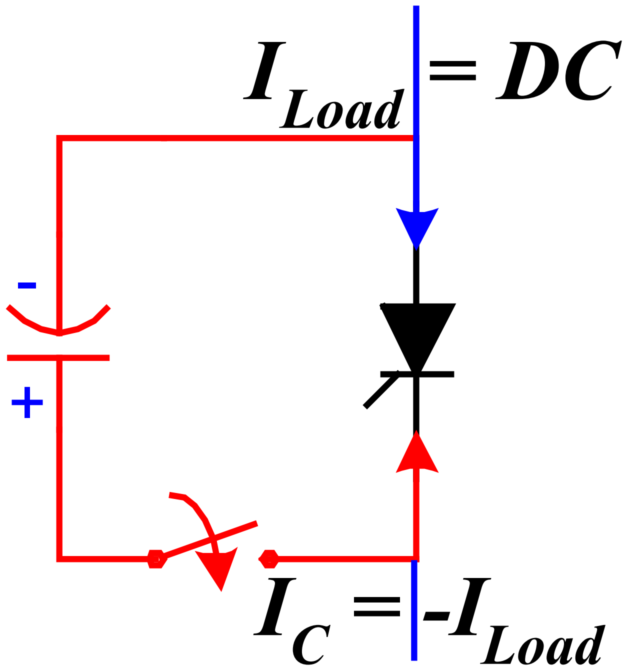
- Typically thyristor (4 - element) family devices SCRs, Triacs



- Employ natural current zero occurs each 1/2 cycle for turnoff
- Slow, tied to 60Hz line and no turnoff control
- Not suitable as fast switch

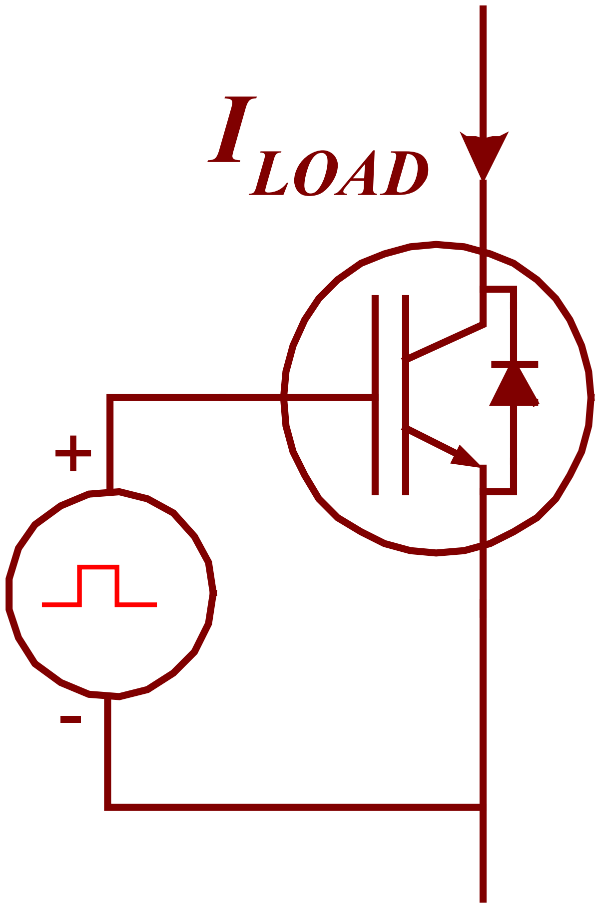
Force Commutated

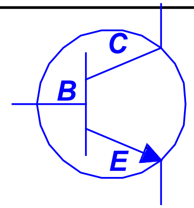
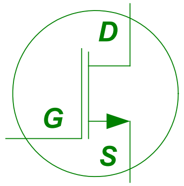
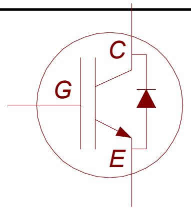
- *Typically SCRs, Triacs*
- *Artificial current zero is manufactured by pre-charged capacitor $I_c = - I_{Load}$*
- *Complex and power-consuming charging and discharging circuits for capacitor*
- *Not suitable approach for fast switches*



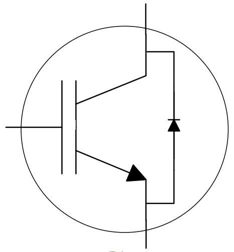
Self Commutated

- Devices have the ability to turn on or turn off by the application of a forward or reverse bias to the control elements (gate – emitter)
- Typically Bipolar Junction Transistors (BJTs), Metal Oxide Semiconductor Field Effect Transistors (MOSFETs) or Insulated Gate Bipolar Junction Transistors (IGBTs)
- Only self-commutated switches used in modern switchmode power supplies



Self-Commutated Device	*Bipolar Junction Transistor (BJT)*	*Metal Oxide Field Effect Transistor (MOSFET)*	*Insulated Gate Bipolar Transistor (IGBT)*
Symbol			
Available Ratings	600 V, $10 \rightarrow 100$ A 1000V, $10 \rightarrow 100$A	150 V, $10 \rightarrow 600$ A 600 V, $10 \rightarrow 100$ A 1200V, $10 \rightarrow 100$A	600 V, $10 \rightarrow 800$ A 1200V, $10 \rightarrow 2400$A 1700V, $50 \rightarrow 2400$A 3300V, $200 \rightarrow 1500$A 6500V, $200 \rightarrow 800$A
Switching Speed	$DC \le fs \le 2$ kHz	$DC \le fs \le 1{,}000$ kHz	$DC \le fs \le 20$ kHz
Vce or Vds f(Vge/Vgs, Ic/Id)	0.5 V $\rightarrow$ 1.5 V	1.5 V $\rightarrow$ 6 V	1.0 $\rightarrow$ 3.0V
*Conduction Loss (Vce*Ic) or (Vds*Id)*	*Lowest*	*Highest*	*Reasonable*
Control Mode	*Current*	*Voltage*	*Voltage*

- *Used in vast majority of switchmode power supplies, except MOSFETs for corrector / trim bipolars*
- *Voltage controlled device - faster than BJT*
- *MOSFET faster, but V_{DS} too large*
- *20 kHz for PWM*
- *Robust, failure rate < 50 FITs*
- *Commercially available since 1990*



IGBT

IGBT Availability	
600V	10 → 800A
1200V	10 → 2400A
1600 / 1700V	50 → 2400A
2500 / 3300V	200 → 1500A
4500 / 6500V	200 → 800A
Available as 6-pack, half-bridge, single switch	

Manufacturers of IGBTs and IGBT Gate Drivers

ABB	*https://new.abb.com/semiconductors*
Fuji Electric (Collmer)	*https://www.fujielectric.com/products/semiconductor/*
Infineon (Eupec, IRF)	*https://www.infineon.com/cms/en/product/power*
Hitachi	*http://www.hitachi-power-semiconductor-device.co.jp/en/*
Littlefuse (IXYS, Westcode)	*https://www.littelfuse.com/products/power-semiconductors/discrete-igbts.aspx*
Mitsubishi	*http://www.mitsubishielectric.com/semiconductors/*
Power Integrations	*https://gate-driver.power.com*
On Semiconductor(Fairchild)	*https://www.onsemi.com/home.do*
Powerex	*https://www.pwrx.com/Home.aspx*
Renesas (Intersil)	*https://www.renesas.com/us/en/products/power.html*
Semikron	*https://www.semikron.com/*
Toshiba	*https://toshiba.semicon-storage.com/us/product.html*

- *There are many topologies, but most are combinations of the types that will be discussed here.*
- *Each topology contains a unique set of design trade-offs*

  Voltage stresses on the switches

  Chopped versus smooth input and output currents

  Utilization of the transformer windings
- *Choosing the best topology requires a study of*

  Input and output voltage ranges

  Current ranges

  Cost versus performance, size and weight

Two Broad Categories

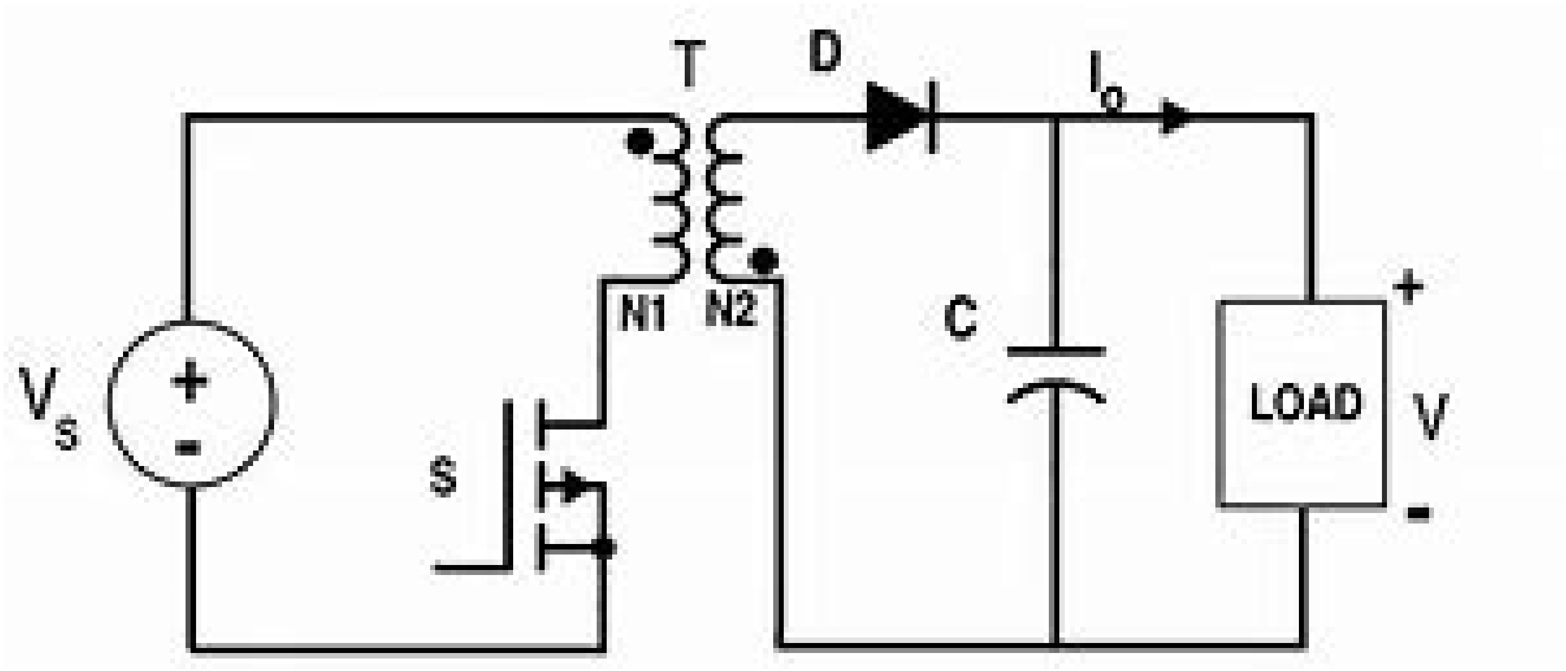
Flyback Converters

- Buck-Boost converter where the line-to-load matching/isolation transformer doubles as the output filter choke
- Advantage – reduction of one major component
- Disadvantage – constrained to low power applications. Not employed in accelerator power supplies

Forward Converters

- The line-to-load matching/isolation transformer is separate from the output filter choke
- May be used in low and high power systems. Used in the vast majority of accelerator power supplies
- Disadvantage – the increased cost and space associated with a separate transformer and choke

Flyback Converter



$$V_O = \frac{D}{1-D} * V_S * \frac{N_2}{N_1}$$

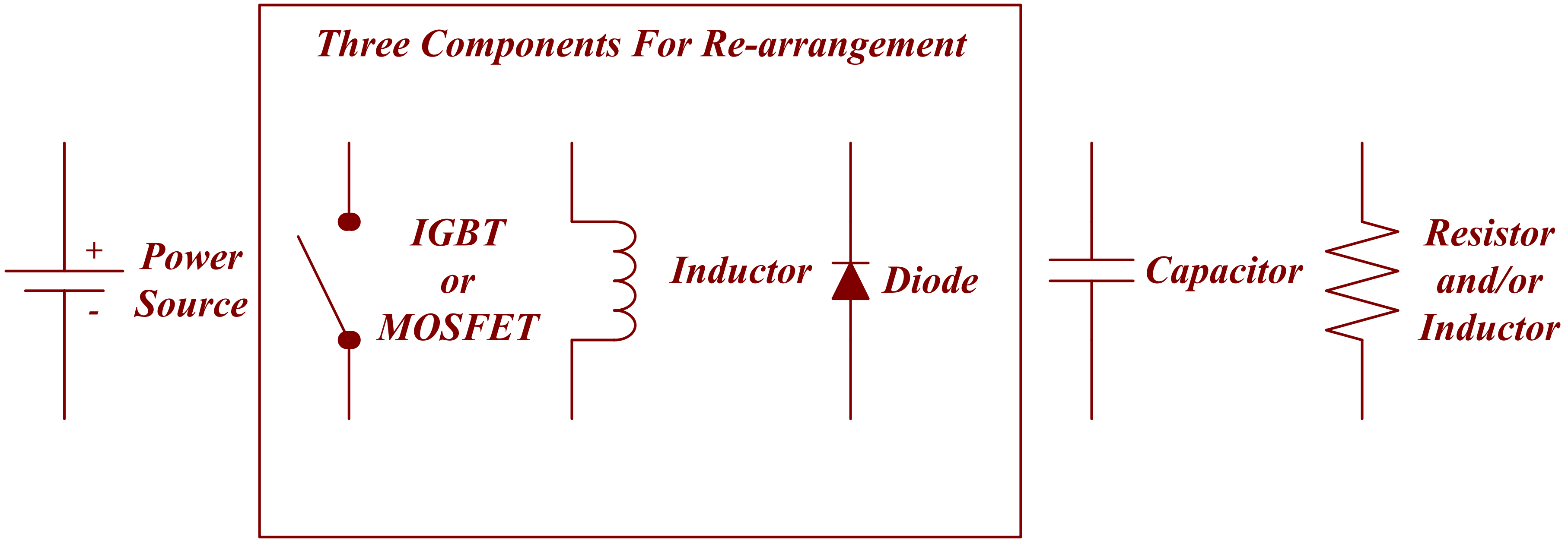
Typical Forward Converters Listed In Order Of Increasing Use

- *Half-bridge Converter*
- *Boost Regulator*
- *Buck Regulator*
- *Full-bridge Converter*

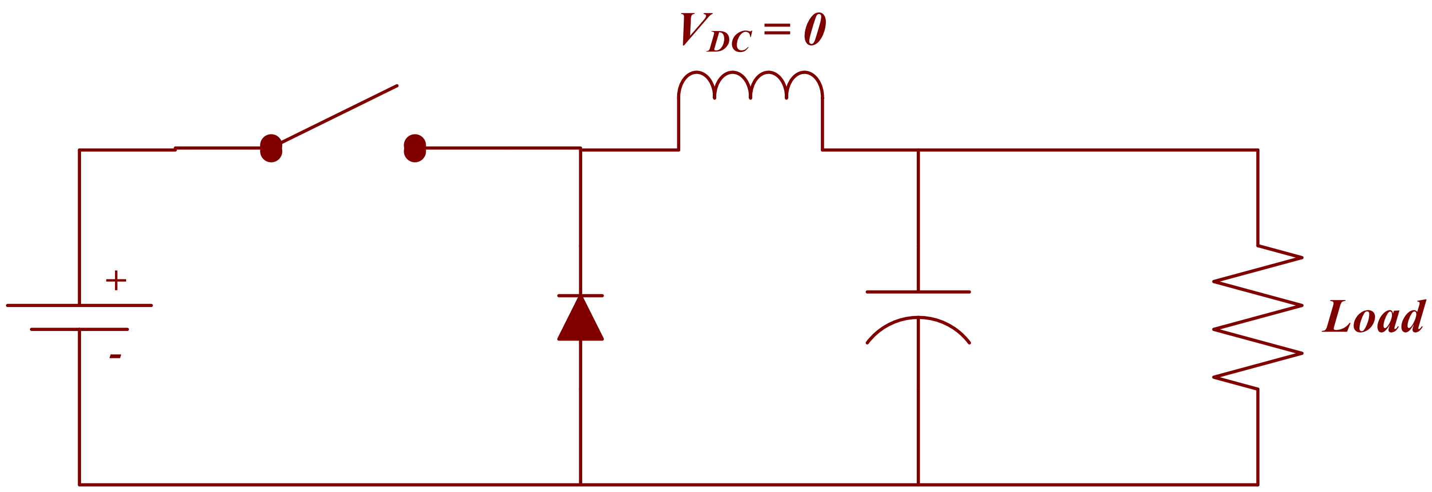
Typical Forward Converters Listed in Order of Increasing Complexity

- *Buck Regulator*
- *Boost Regulator*
- *Half-bridge Converter*
- *Full-bridge Converter*

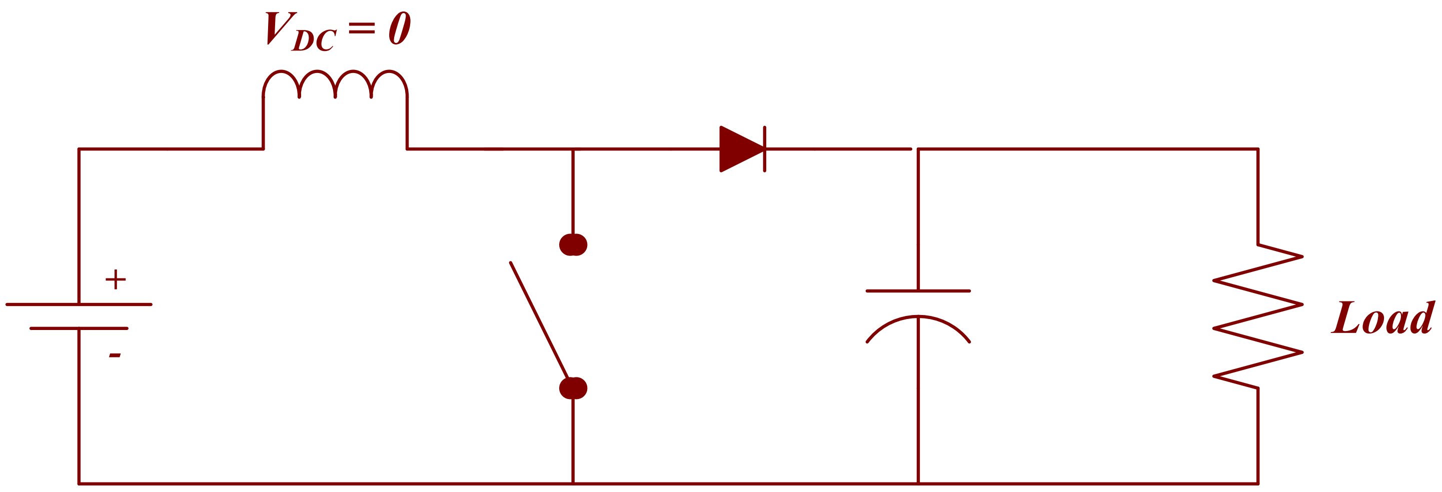
Basic switchmode tool kit



Most fundamental switchmode converter topologies are constructed by rearranging the three components

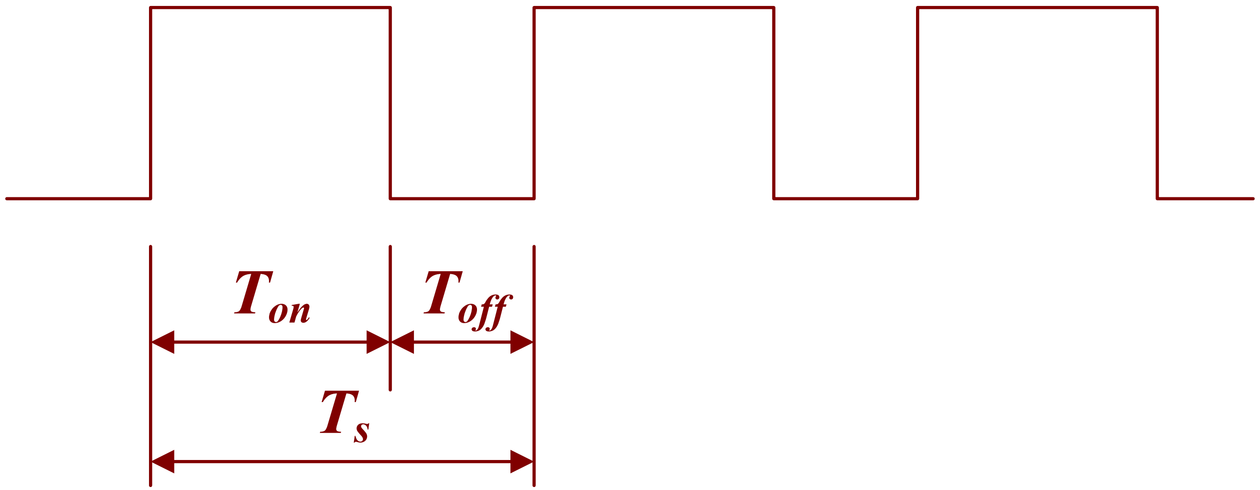


Buck



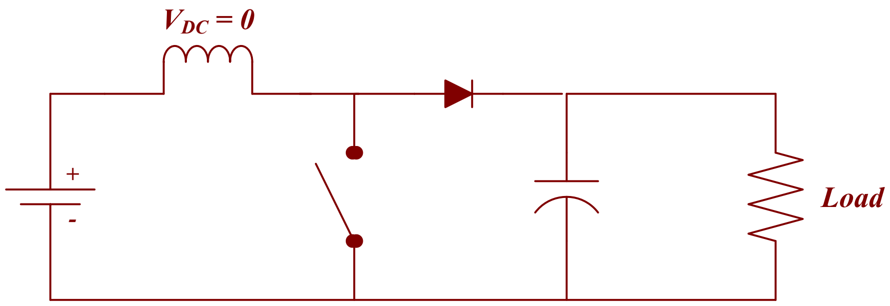
Boost

Definition of the Pulse Width Modulated (PWM) Waveform

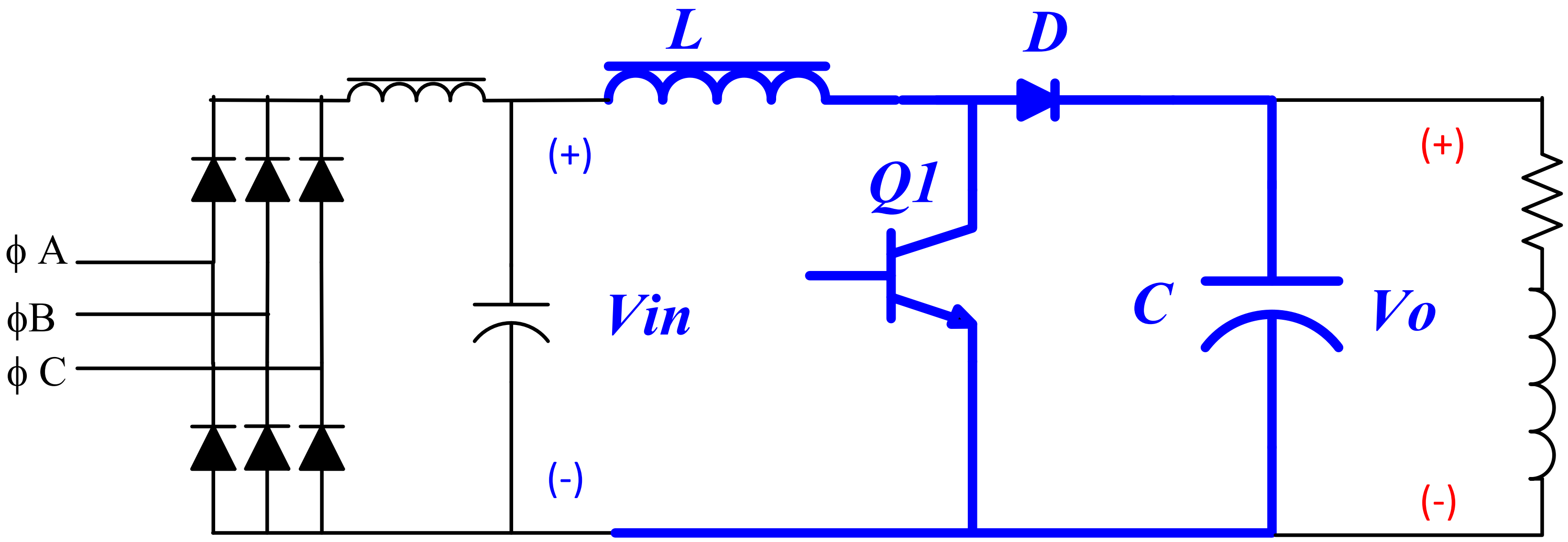


$$Duty\ Cycle = Duty\ Ratio = D = \frac{T_{on}}{T_{on} + T_{off}} = \frac{T_{on}}{T_s}$$

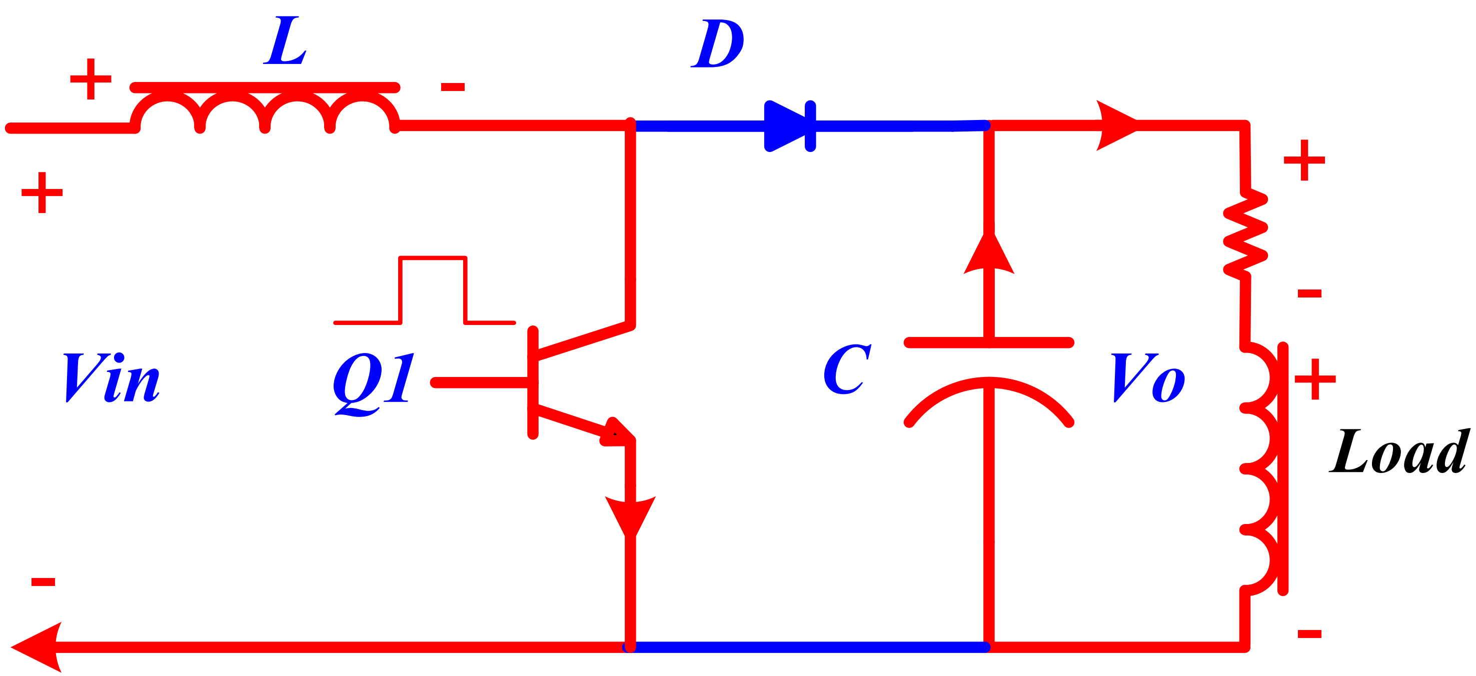
$$D' = 1 - D = \frac{T_{off}}{T_{on} + T_{off}} = \frac{T_{off}}{T_s}$$



Boost

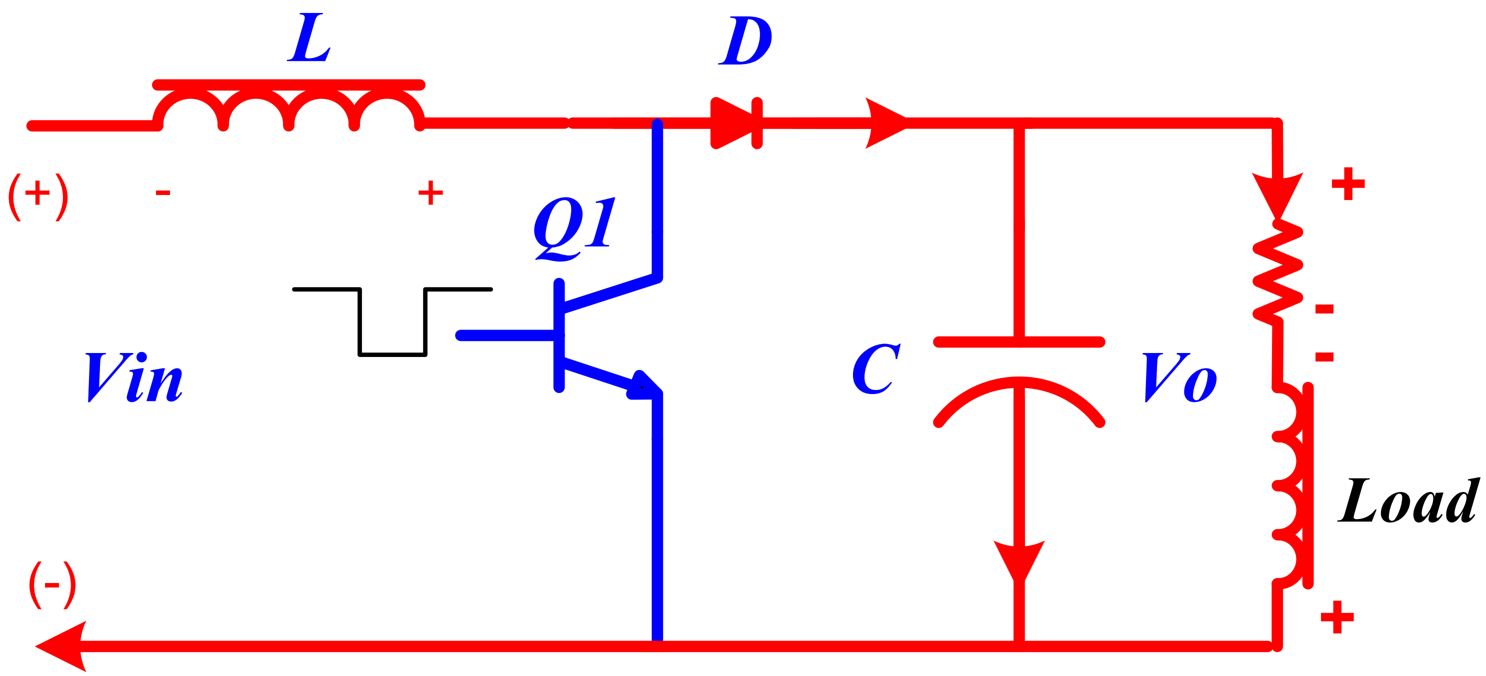


- *Boosts the input voltage to a higher output voltage* $V_o = V_{in}/(1 - D)$
- *Input current is smooth (continuous)*



State 1 – Power Transfer

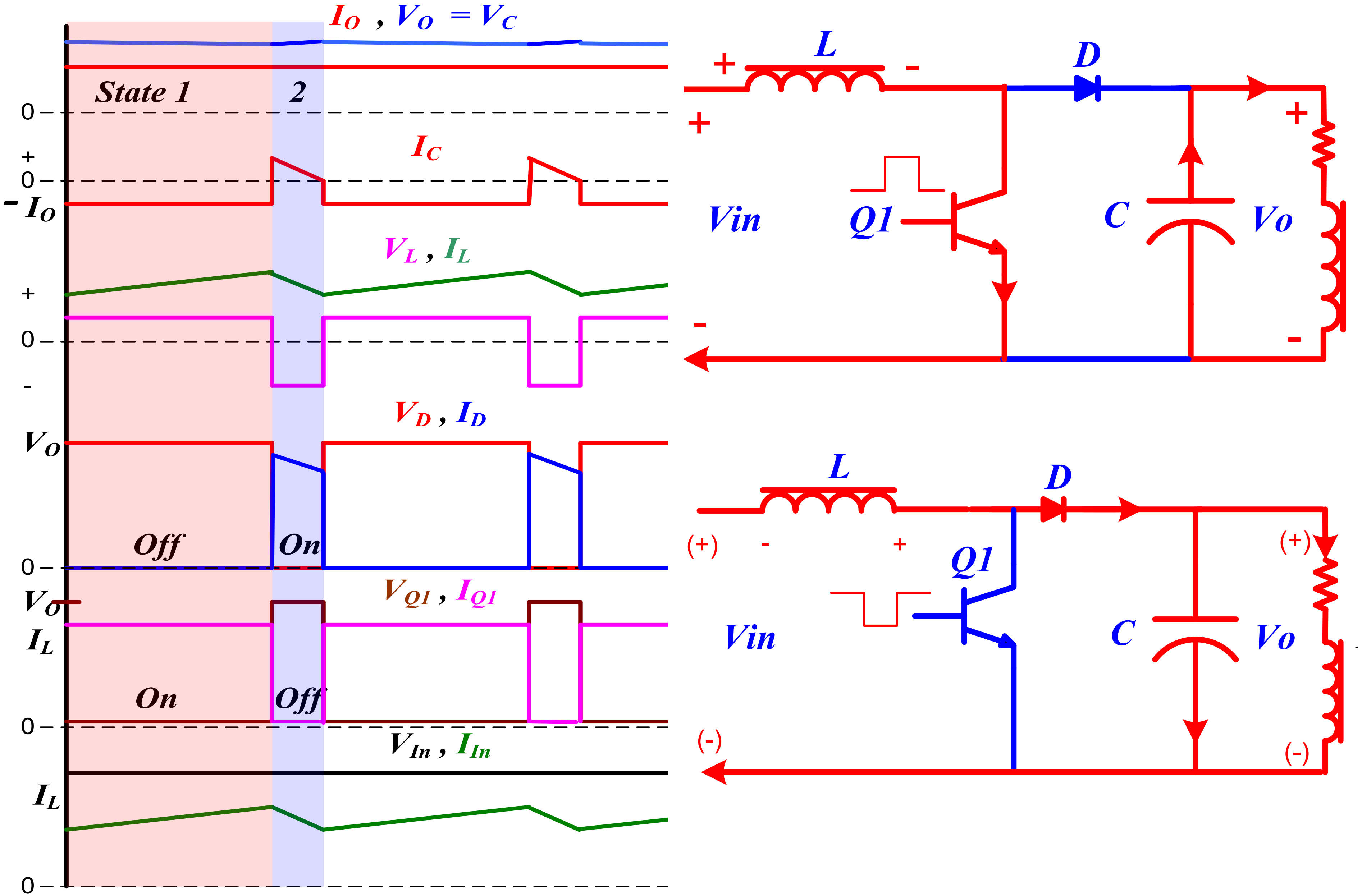
- *Switching device Q1 turned on by square wave drive circuit with controlled on-to-off ratio (duty factor, D)*
- *V_{in} impressed across L*
- *Current in L increases linearly in forward direction*
- *Diode D is reversed biased (open)*
- *Capacitor C discharges into the load*



State 2 - Regulation

- Q1 turned off. L polarity reverses.
- $V_O = V_{In} + V_L$, $V_L = V_O - V_{In}$
- $V_O > V_{In}$, L current decreases linearly
- Diode D is forward biased (closed)
- Capacitor C is recharged

Topologies - Boost Converter Waveforms



Summary

- *Output polarity is the same as the input polarity*

- *In steady-state, L volt-seconds with Q1 on = volt-seconds with Q1 off*

$$V_{In} * t_{on} = (V_O - V_{In}) * t_{off}$$

$$V_O = V_{In} *(t_{on} + t_{off}) / t_{off}$$

$$V_O = V_{In} / (1 - D)$$

- *Output voltage is always greater than the input voltage because $D \leq 1$*

- *IGBT duty factor (D) range 0 to 0.95*

- *Limitation of D yielding greater output voltage is the limitation on the input current through the inductor and diode*

- *Output voltage is not related to load current so output impedance is very low (approximates a true voltage source).*

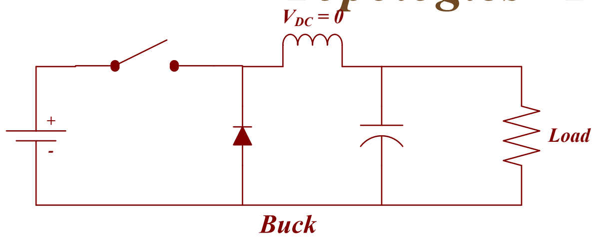
Some Advantages

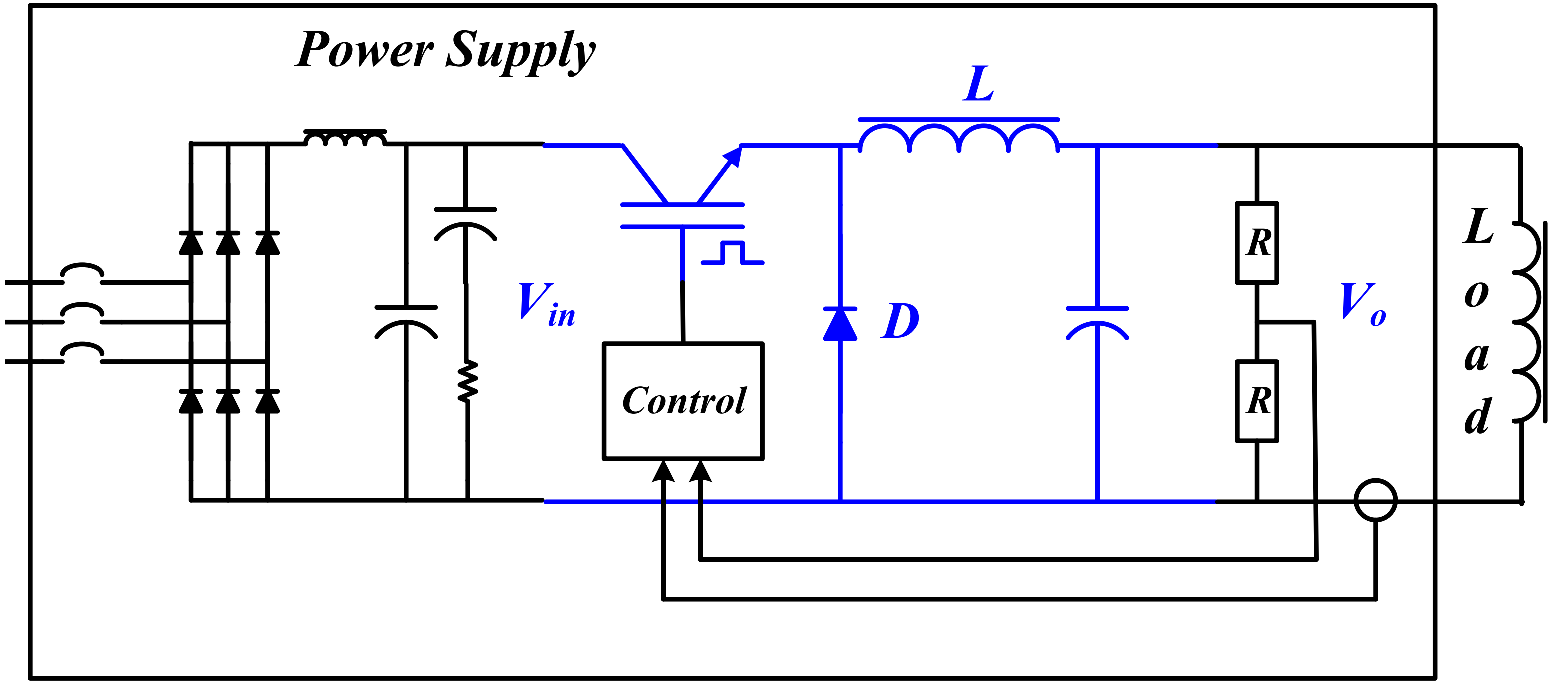
- *Few components, 1 switch – simple circuit, high reliability if not overstressed*
- *Input current is always continuous, so smaller input filter capacitor needed*

Some Disadvantages

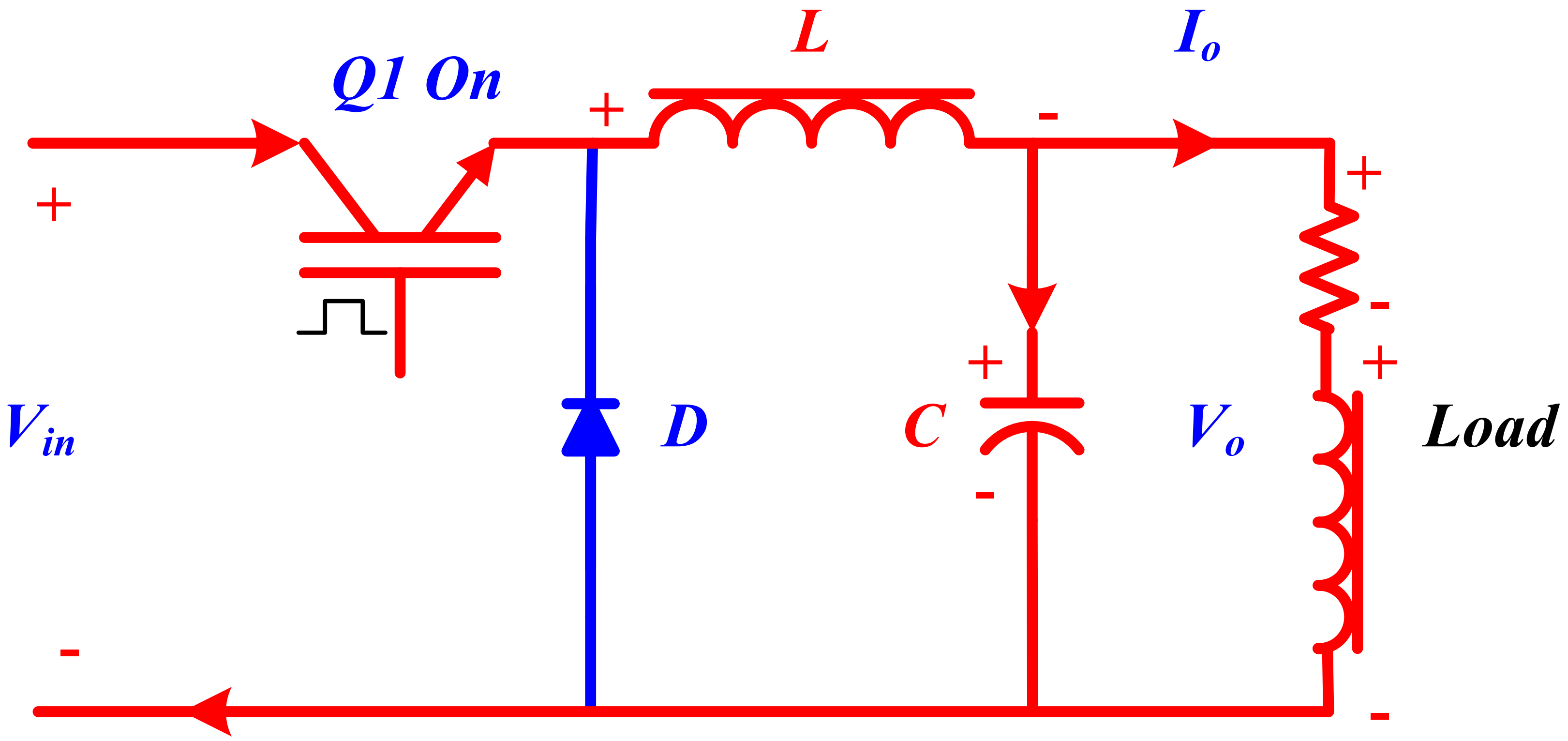
- *Capacitor C current is always discontinuous so a much larger output capacitor is needed for same output ripple voltage*
- *Output is DC and unipolar so no chance of high-frequency transformer or bipolar output*
- *Low frequency transformer must be used in front of the Boost for isolation and to match the line voltage to the load voltage*
- *Minimum output voltage equal input voltage*

Topologies - Buck Converter (Regulator)



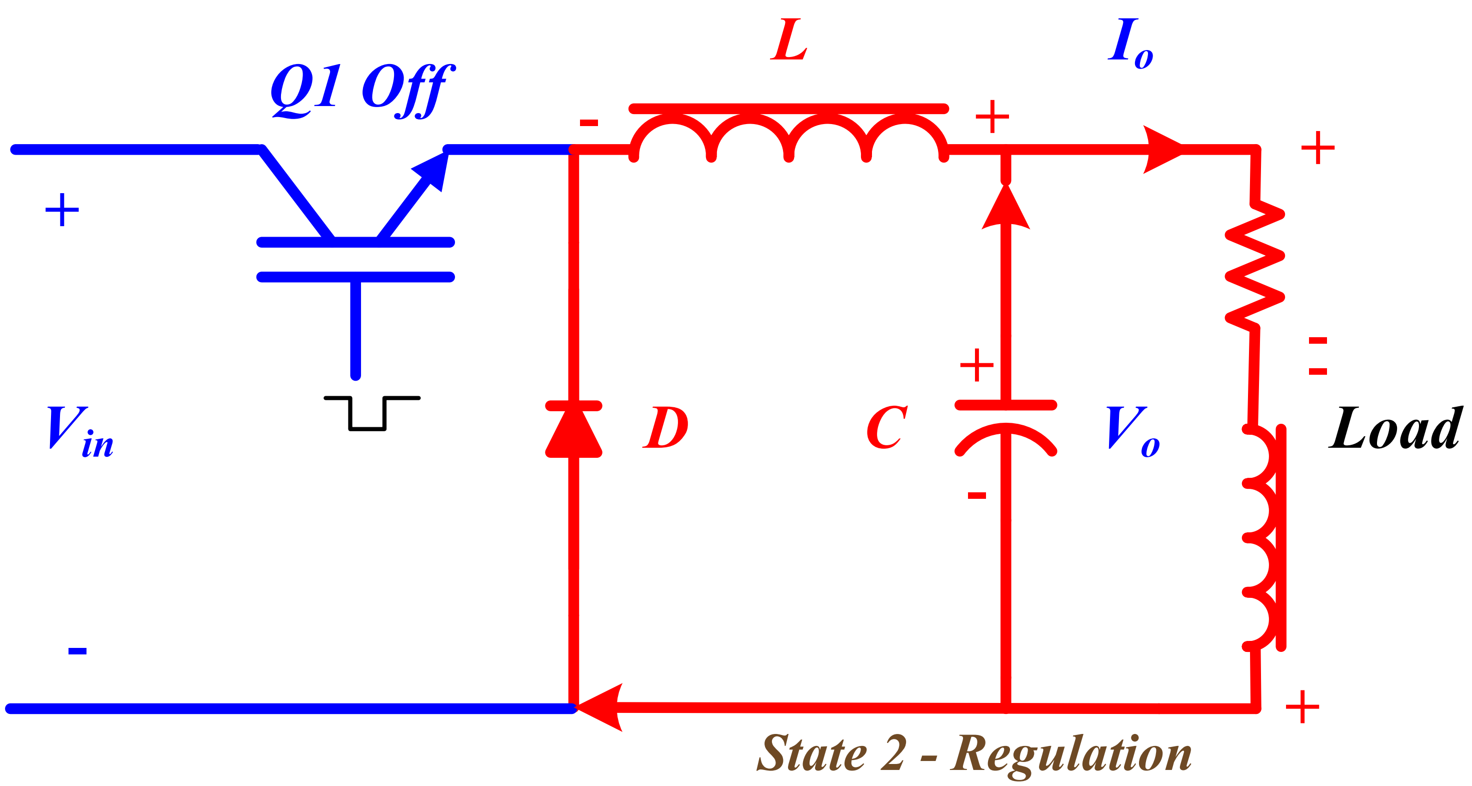


- *Used in the majority of switchmode power supplies*
- *Bucks the input voltage down to a lower voltage*
- *Perhaps the simplest of all*
- *Input current discontinuous (chopped) – output current smooth*



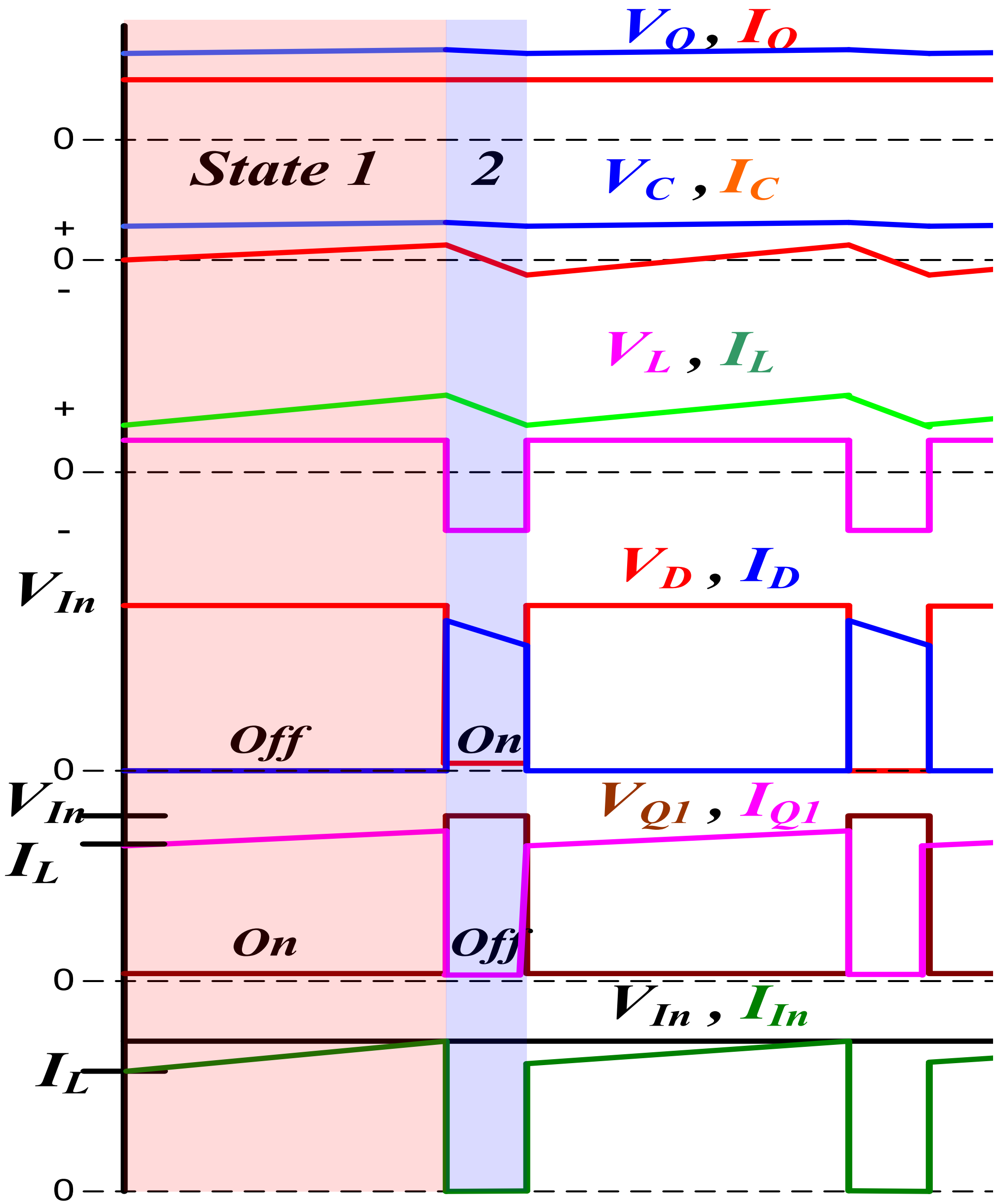
State 1 – Power Transfer

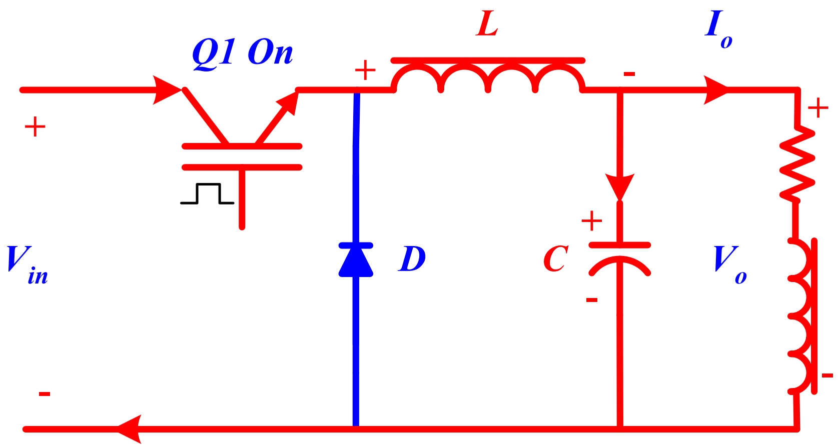
- *Switching device Q1 turned on by square wave drive circuit with controlled on-to-off ratio (duty factor, D)*
- *$V_{in} - V_o$ impressed across L*
- *Current in L increases linearly*
- *Capacitor C charges to Vo*

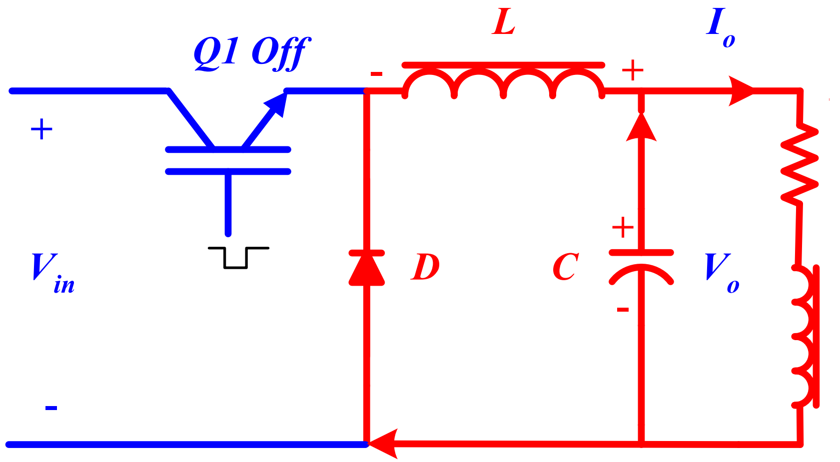


- *Switching device Q1 turns off*
- *Voltage across L reverses: – Vo impressed across L*
- *Diode D turns on*
- *Current in L decreases linearly*
- *C discharges into the Load*

Topologies - Buck Converter Waveforms

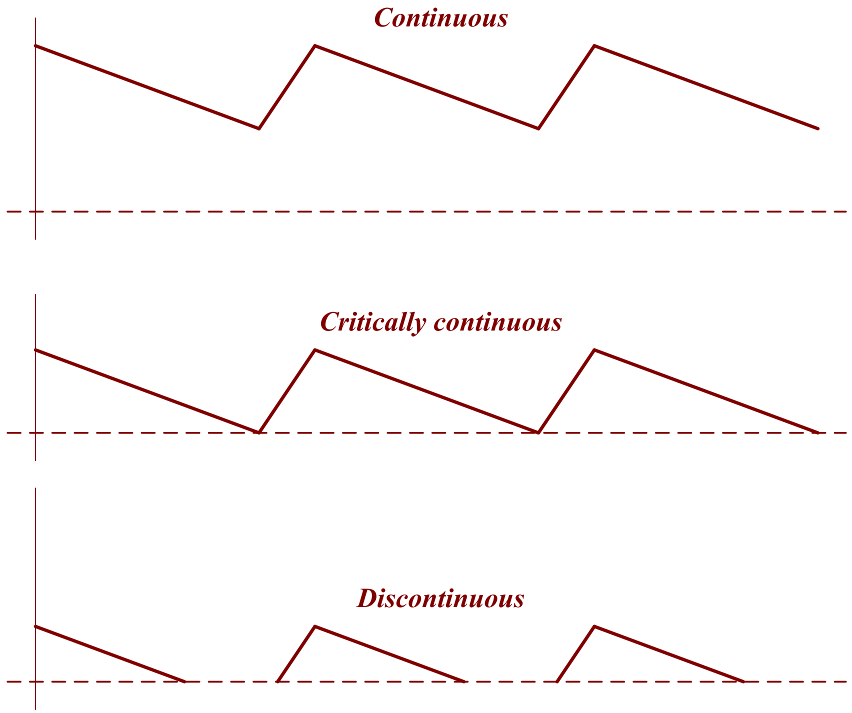






Topologies - Buck Converter Conduction

Buck converter inductor current can be continuous, critically continuous or discontinuous



Discontinuous current is caused by:
- *Too light a load*
- *Too small an inductor*
- *Too small filter capacitor*
- *Discontinuous difficult to control output and output ≠ D* Vin*

Summary

- *Output polarity is the same as the input polarity*
- *In steady-state L volt-seconds with Q1 on = volt-seconds with Q1 off*

$$(V_{In} - V_O) * t_{on} = (V_O * t_{off})$$

$$V_O = V_{In} * t_{on} / (t_{on} + t_{off}) = V_{In} * D$$

- *Output voltage is always less than the input voltage because* $D \leq 1$
- *Switch duty factor (D) range 0 to 0.95*
- *Output voltage is not related to load current so output impedance is very low (approximates a true voltage source)*

An Advantage

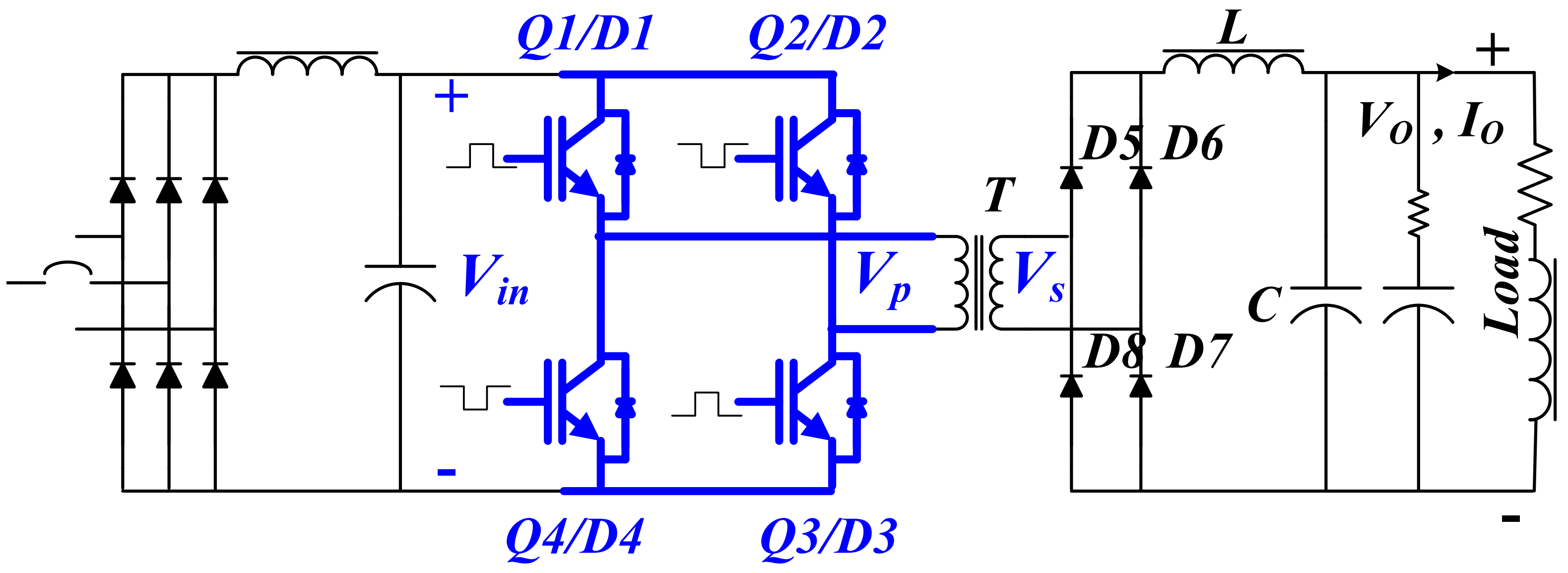
- Few components, 1 switch – simple circuit, high reliability if not overstressed

Disadvantages

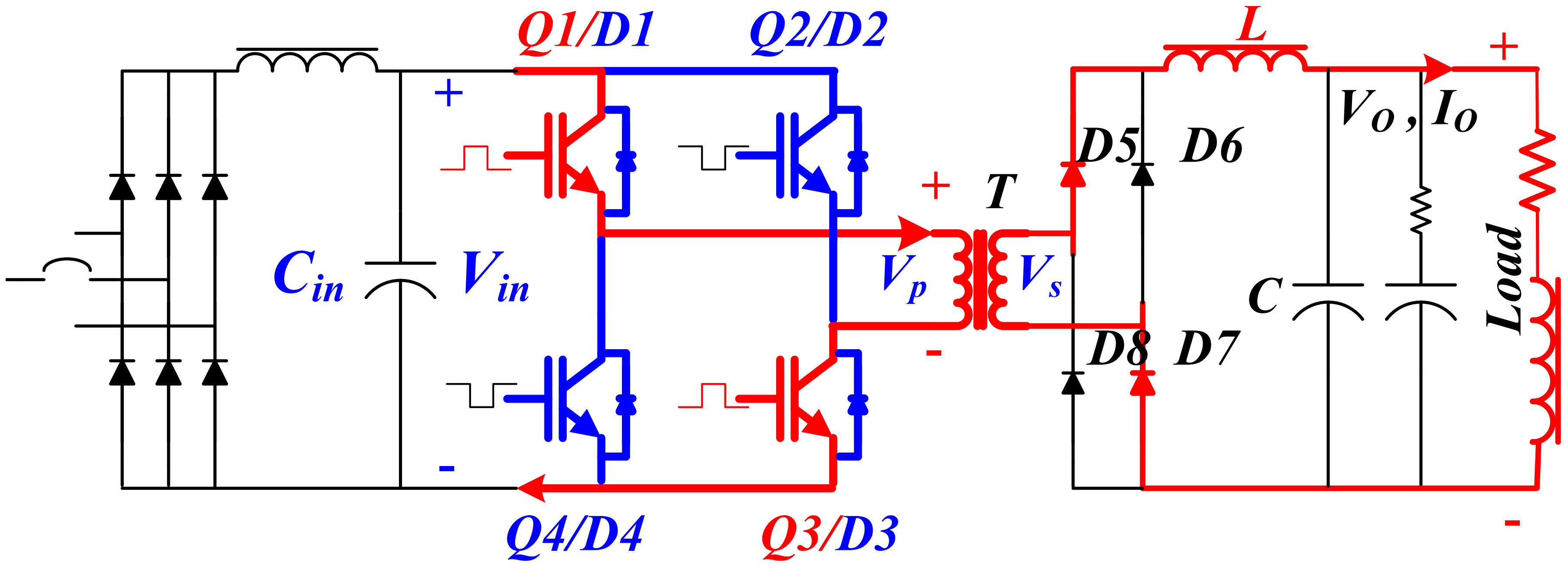
- Output is DC and unipolar so no chance of high-frequency transformer or bipolar output
- Low frequency transformer must be used in front of the Buck for isolation and to match the line voltage to the load voltage

Application

- Used very widely in accelerator power systems, typically for large power supplies (perhaps ≥ 350 kW and used in conjunction with a 12-pulse rectifier with 6-phase transformer)

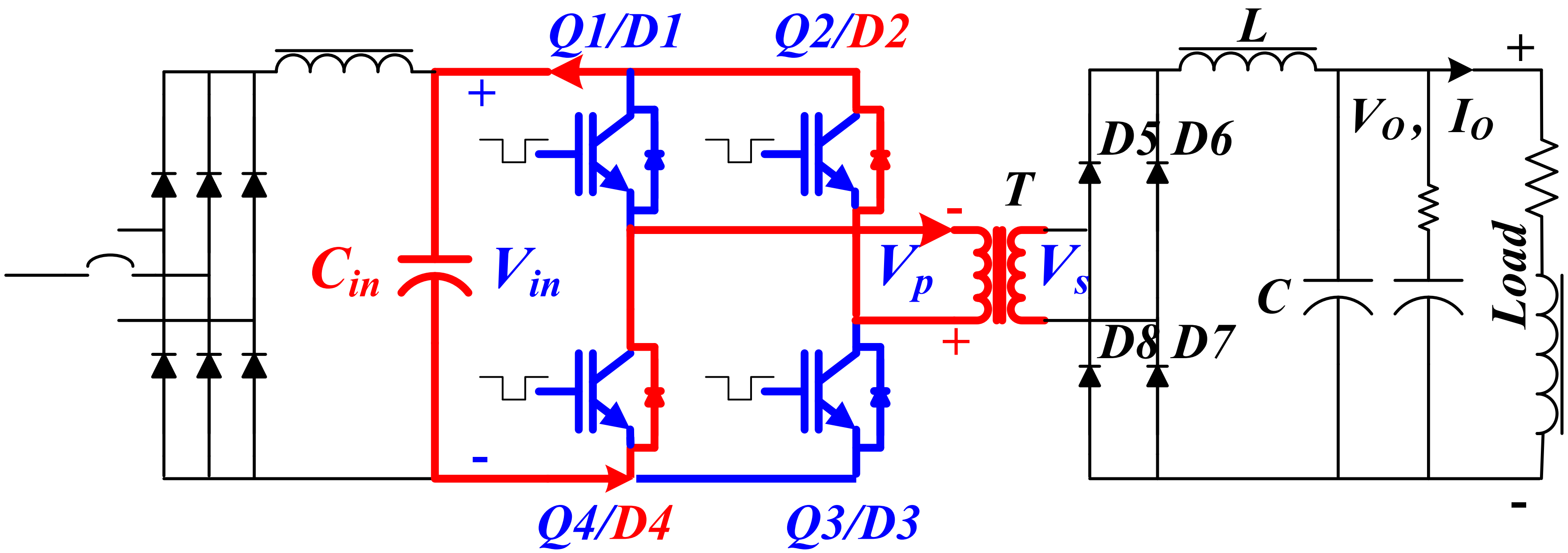


- *Full wave rectifier, output ripple is multiples of the input frequency*
- *Equal in popularity to buck topology for high-power converters*
- *Used when line and load voltages are not matched*
- *Voltage stress on switches = input voltage*
- *Good transformer utilization, power is transmitted on both half-cycles*



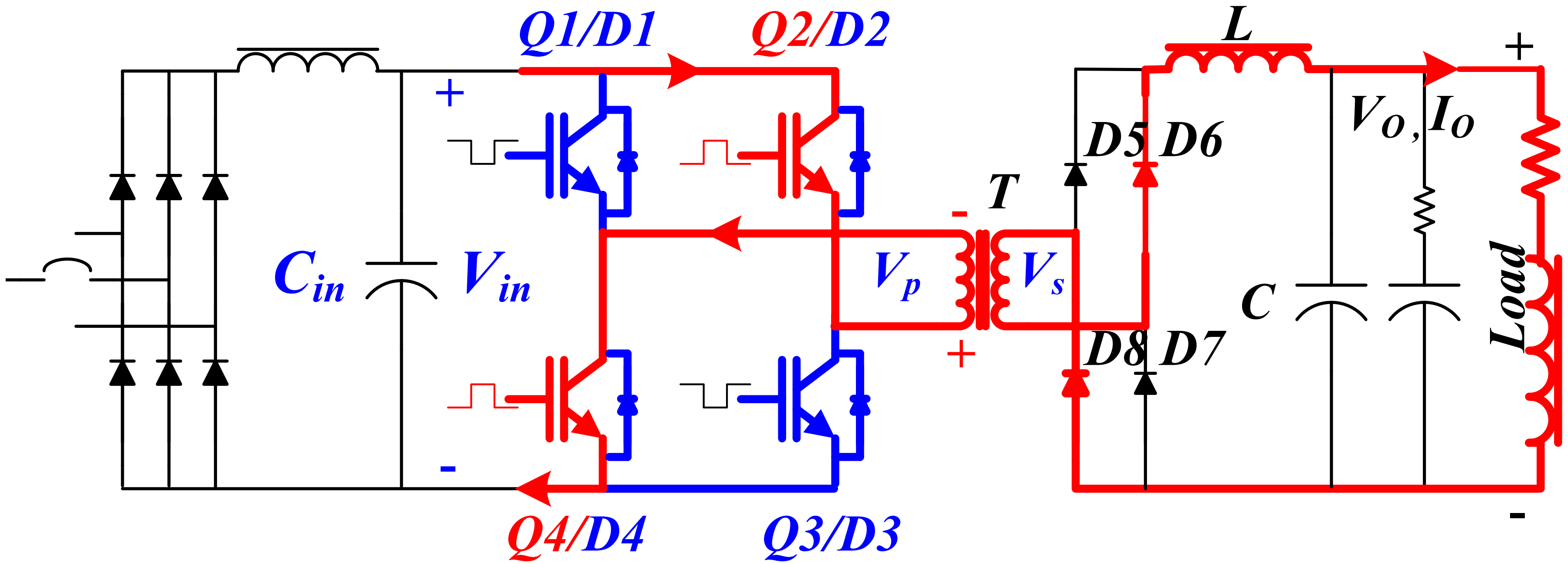
State 1 - Power

- Power is derived from the input rectifier and slugs of energy from C_{in}
- Q1 and Q3 are closed. Current flows through Q1 and the primary winding of T and Q3
- A voltage (V_{in}) is developed across the primary winding of T. A similar voltage is (V_{in} *N) is developed across the secondary winding of T
- The secondary voltage causes rectifiers D5 and D7 to conduct current



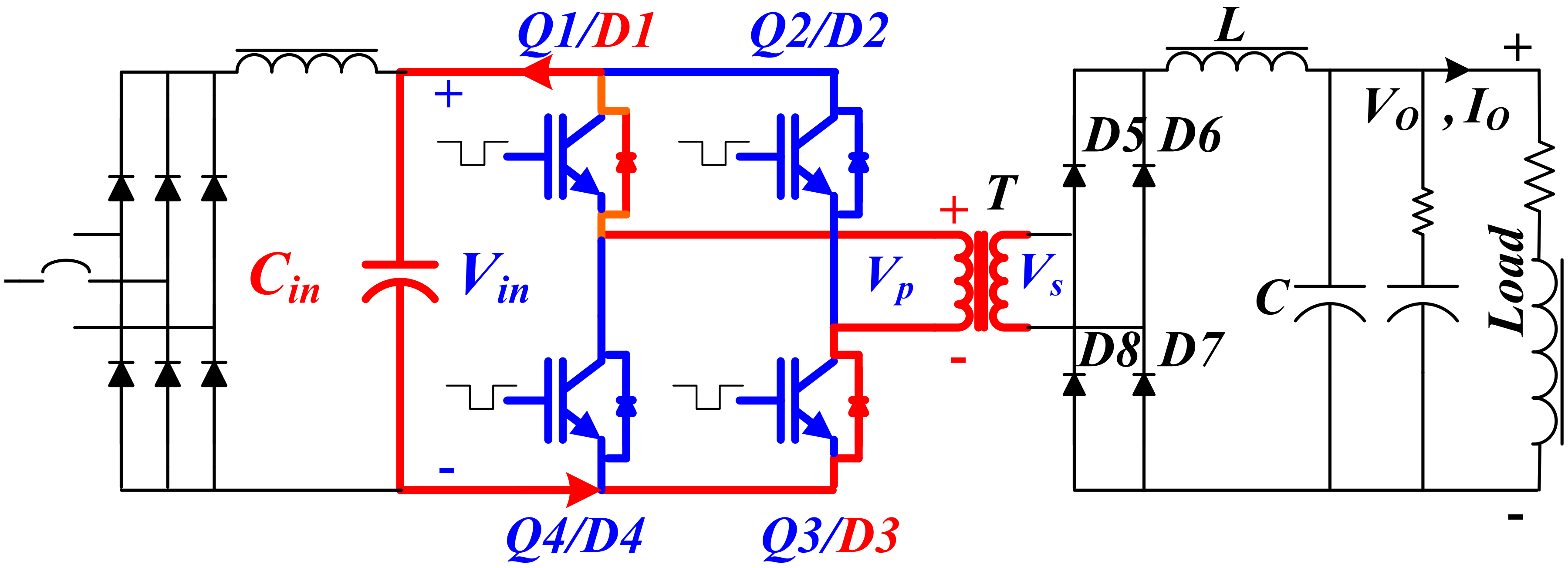
State 2 - Power Off

- Q1 and Q3 are turned off. All switches are off
- C_{in} recharges
- The transformer primary current flows in the same direction but the voltage reverses polarity. This causes D2 and D4 to conduct. Stored leakage inductance energy is returned to the input filter capacitor. The transformer current decays to zero.
- The secondary rectifiers D5, D6, D7 and D8 are all off



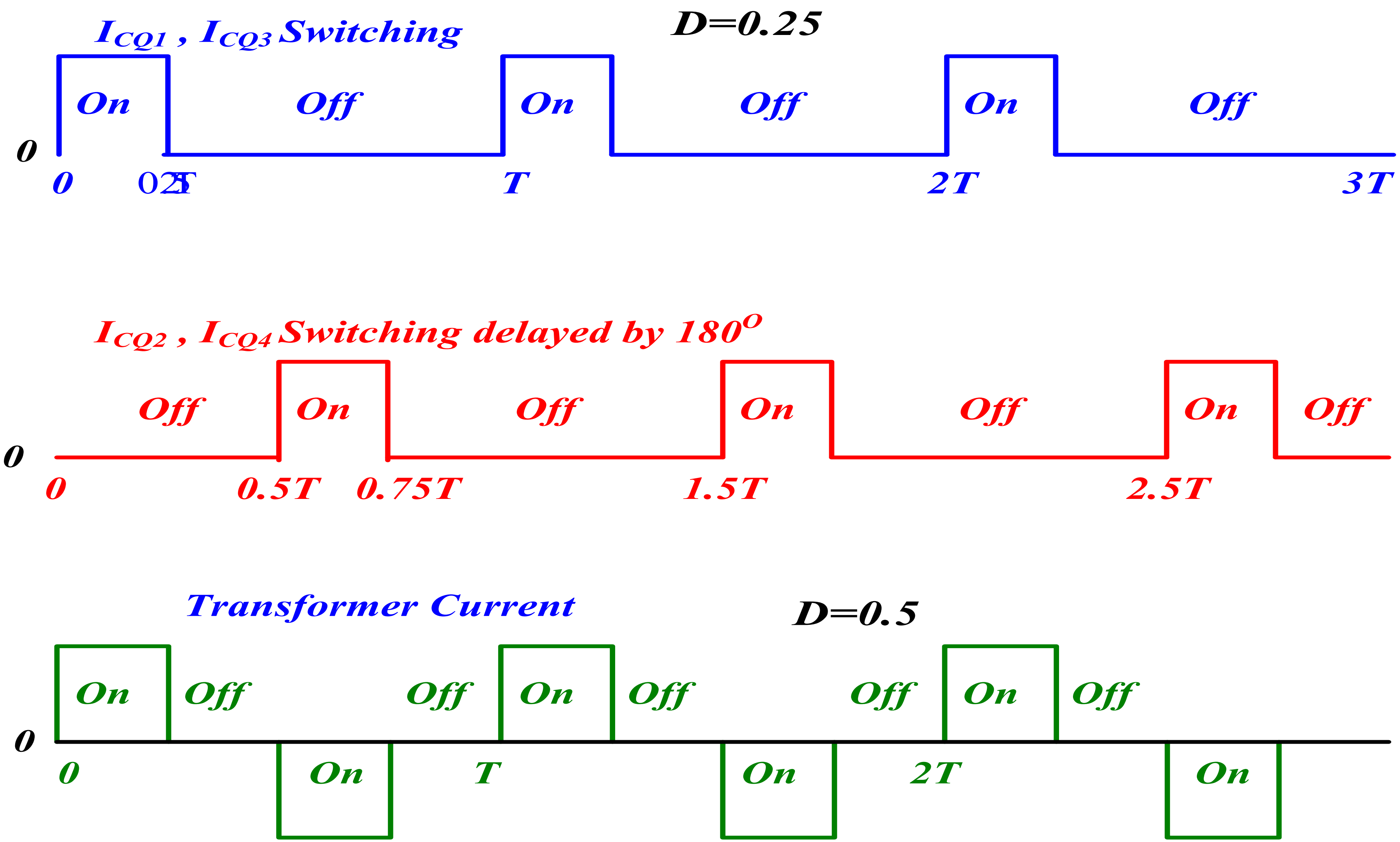
State 3 - Power

- *Power is derived from the input rectifier and slugs of energy from C_{in}*
- *Q2 and Q4 are closed and current flows through Q2, the primary winding of T and Q4*
- *A voltage (V_{in}) is developed across the primary winding of T. A similar voltage (Vin*N) is developed across the secondary winding of T*
- *The secondary voltage causes rectifiers D6 and D8 to conduct current*

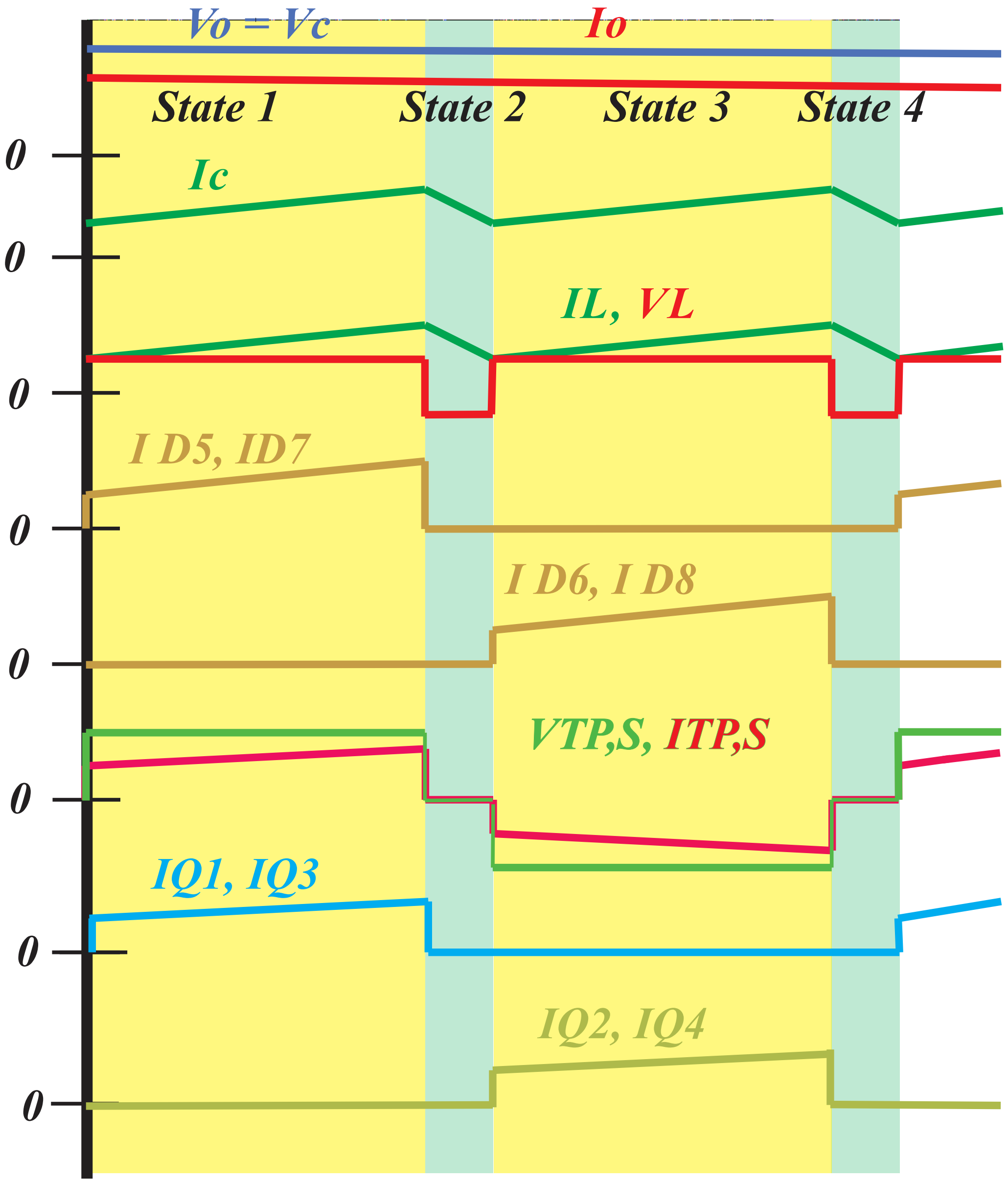


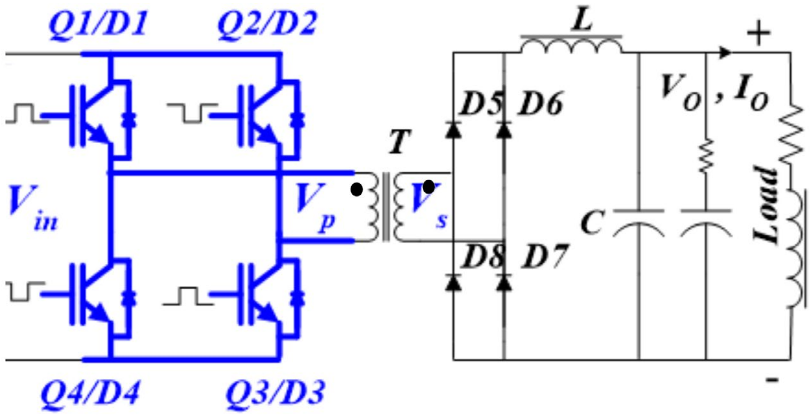
State 4 – Power Off

- Q2 and Q4 are turned off. All switches are off
- C_{in} recharges
- The current in the transformer primary flows in the same direction but the voltage reverses polarity. This causes D1 and D3 to conduct. Stored leakage inductance energy is returned to the input filter capacitor. The transformer current goes to zero.
- The secondary rectifiers D5, D6, D7 and D8 all turn off



I_{CQ1} , I_{CQ3} Switching
D=0.25
On
Off
On
Off
On
Off
0
0
025T
T
2T
3T
I_{CQ2} , I_{CQ4} Switching delayed by 180^{O}
Off
On
Off
On
Off
On
Off
0
0
0.5T
0.75T
1.5T
2.5T
Transformer Current
D=0.5
On
Off
Off
On
Off
Off
On
Off
0
0
On
T
On
2T
On


Vo = Vc
Io
State 1
State 2
State 3
State 4
0
Ic
0
IL, VL
0
I D5, ID7
0
I D6, I D8
0
VTP,S, ITP,S
0
IQ1, IQ3
0
IQ2, IQ4
0


Q1/D1
Q2/D2
Vin
Vp
Vs
T
Q4/D4
Q3/D3
D5
D6
D8
D7
L
C
Vo , Io
Load
+
-

Topologies - Full Bridge Waveforms

- *Some inductive energy can be recovered to recharge input filter C_{in}*
- *Same pulses applied to Q1 & Q3 and the same, but 180^O delayed, pulses are applied to Q2 & Q4*
- *Switching sequence is Q1 & Q3 are turned on, then turned off after providing the required ON time*
- *After delay (to account for finite switch turn off and turn on), Q2 & Q4 are turned on. After providing the required ON time, Q2 & Q4 are turned off.*
- *Sequence repeats*
- *Q1 and Q4 or Q2 and Q3 are never turned on together*
- *Only the leading edge (or trailing) edge of the gating and current pulse move*
- *Symmetrical +/- pulse obtained. Must be rectified to provide a DC output*
- *The output ripple is twice the switching frequency*

Advantages

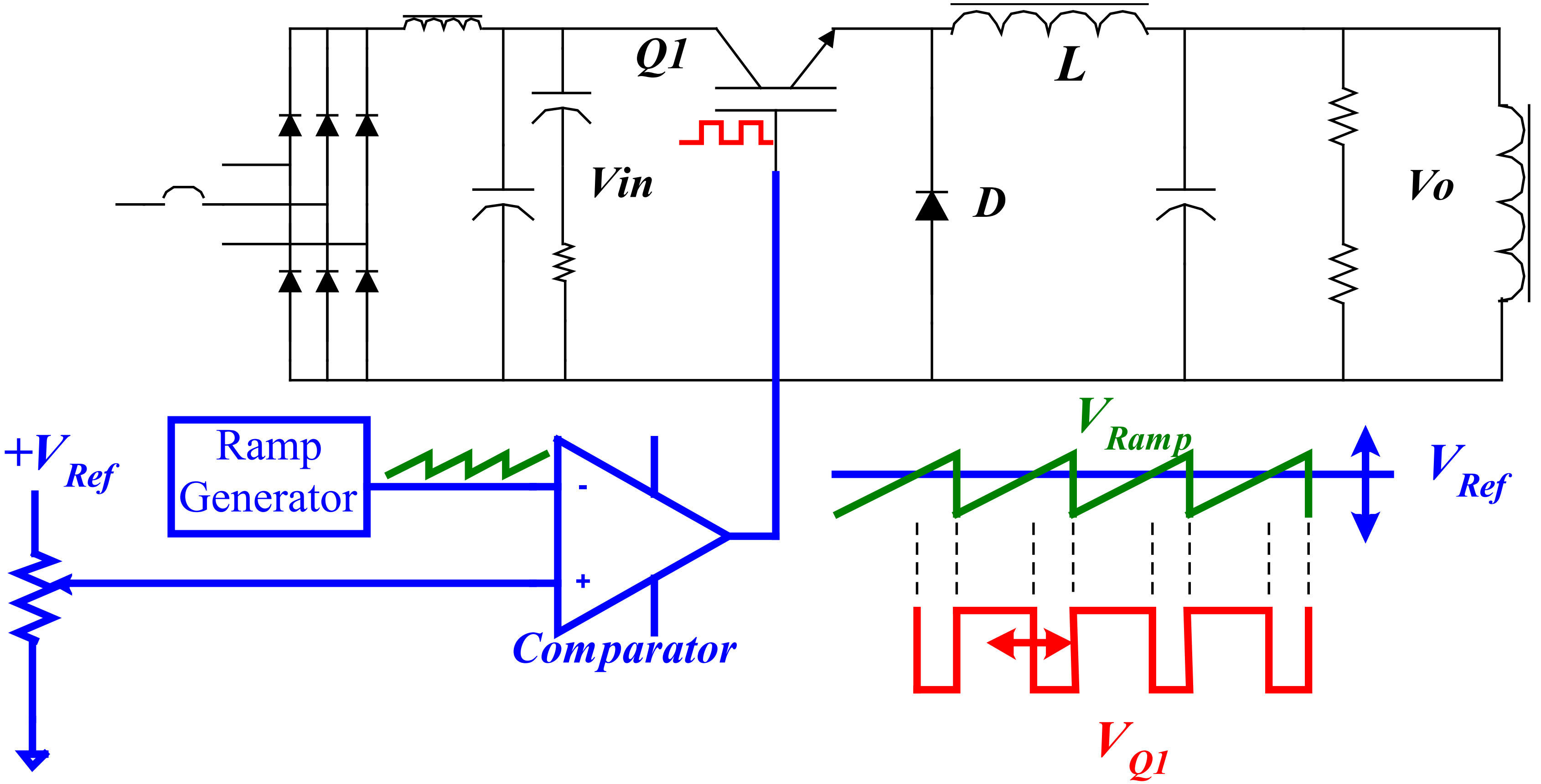
- *Simple primary winding needed for the main transformer, driven to the full supply voltage in both directions*
- *Power switches operate under extremely well-defined conditions. The maximum stress voltage will not exceed the supply line voltage under any conditions.*
- *Positive clamping by 4 energy recovery diodes suppresses voltage transients that normally would have been generated by the leakage inductances.*
- *The input filter capacitor C_{in} is relatively small*
- *Modest part count for high reliability.*
- *Can be used with or without line-to-load matching transformer*
- *Transformer matches the load to the input line.*
- *With transformer unipolar output, without transformer, used for bipolar operation*
- *Capable of high power output (500 kW)*

Disadvantage

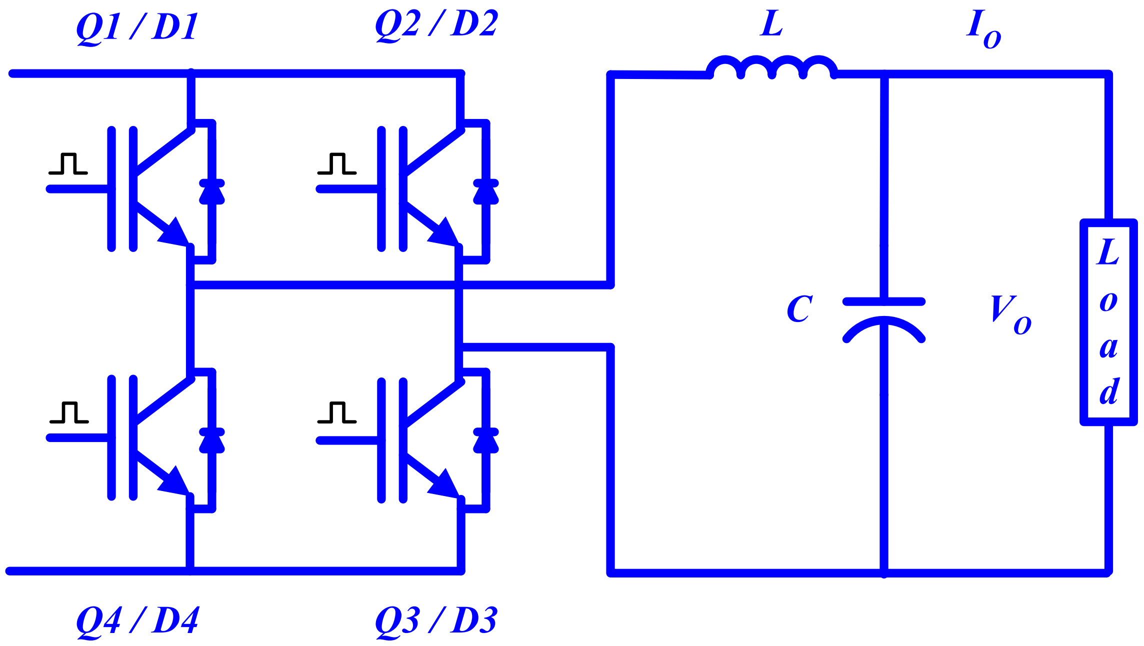
- *Four (4) switches are required, and since 2 switches operate in series, the effective saturated on-state power loss is somewhat greater than in the 2 switch, half-bridge case. In high voltage, off-line switching systems, these losses are acceptably small.*

Converter Type	Topology	V_o	P_o	Transformer	Output Type
Buck	1 switch	$V_o = V_{in} * D$	Any	Not possible	Unipolar
Boost	1 switch	$V_o = V_{in} / (1-D)$	I_{in} limits Po	Not possible	Unipolar
Full Bridge	4 switches Minor switch losses	$V_o = V_{in} * D * n$	Any	Possible	Unipolar/ bipolar

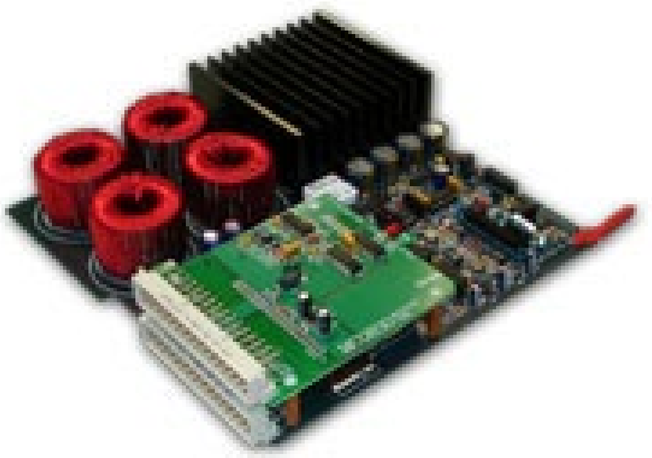
Pulse Width Modulation (PWM) Techniques



V_{Ref} ↑	$V_{Ref} - V_{Ramp} = V_{Q1}$ pulse width ↑	V_O ↑
V_{Ref} ↓	$V_{Ref} - V_{Ramp} = V_{Q1}$ pulse width ↓	V_O ↓


Q1 / D1
Q2 / D2
L
I_o
C
V_o
Load
Q4 / D4
Q3 / D3

PWM - Bipolar Bridge



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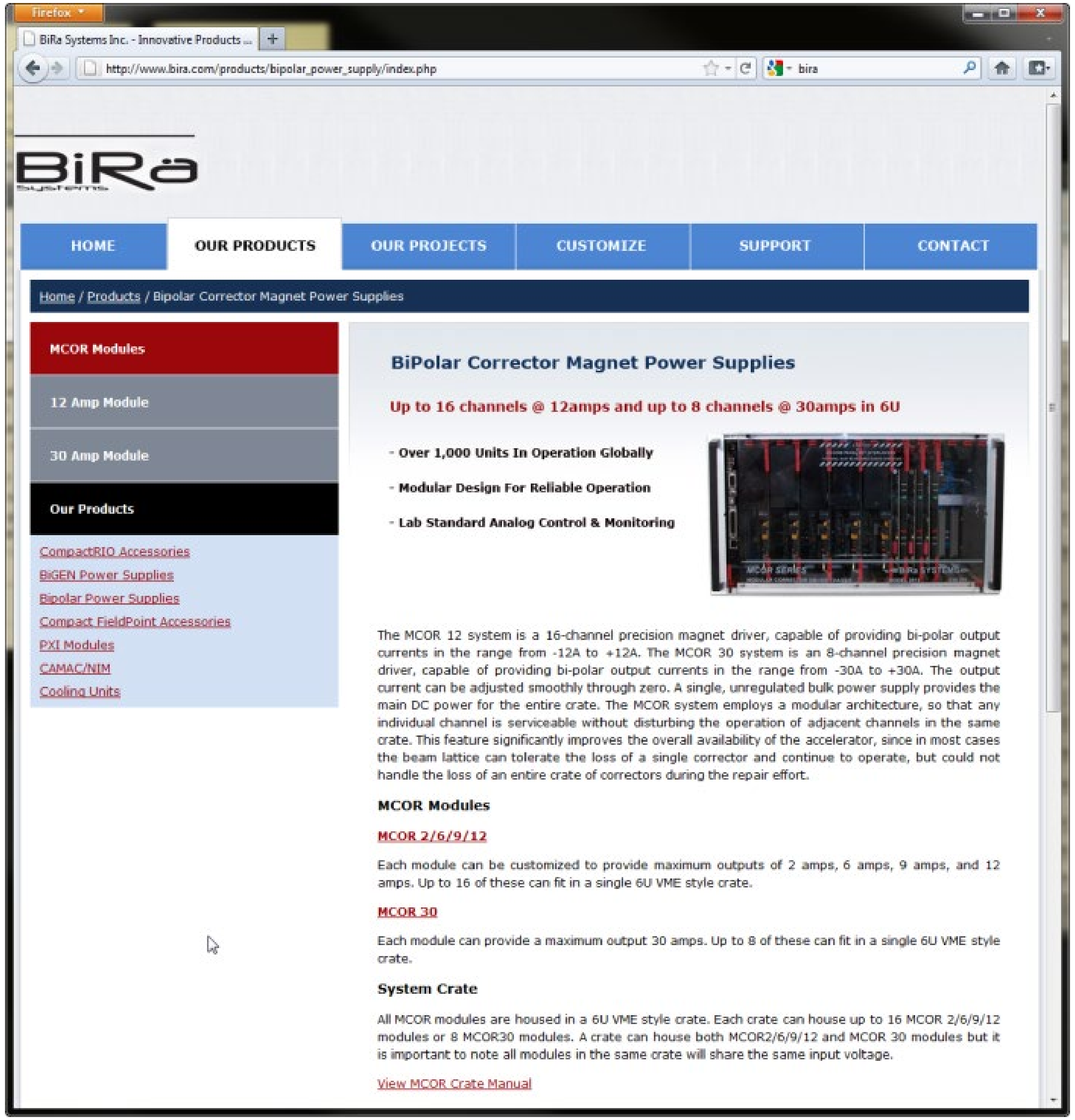
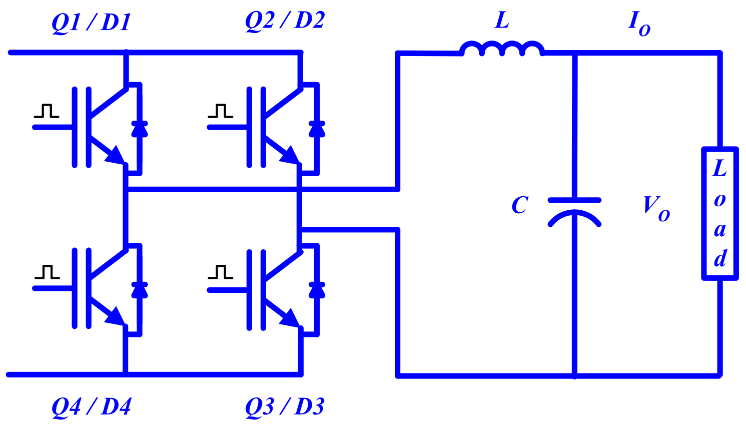


Generalities



- *Diagonal switching*
- *Two PWMs are usually employed*
- *Switches Q1 and Q3 are the + output leg*
- *Switches Q2 and Q4 are the – output leg*
- *An output rectifier is not required*
- *Since the output desired DC, but contains + and – components, a non-polarized output filter capacitor must be used*
- *2 and 4 quadrant operation is possible*

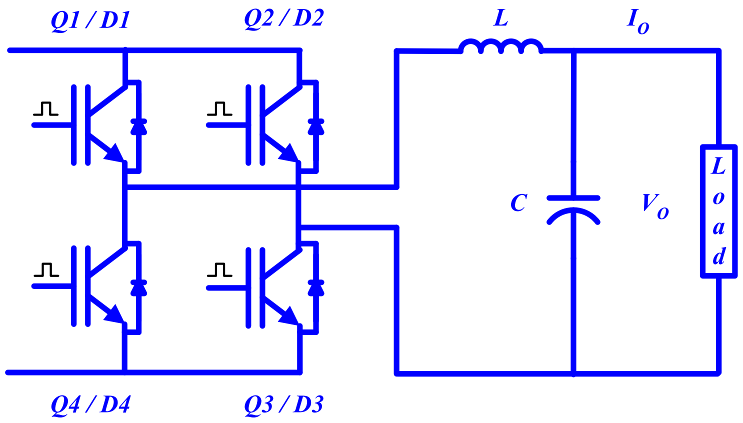
Two types of PWM

- *Sign/magnitude in which the sign of the reference signal determines which pair of switches to turn on and the magnitude determines the pulse duration/duty factor*

- *“50/50” scheme in which there are 2 separate, complimentary PWM signals*

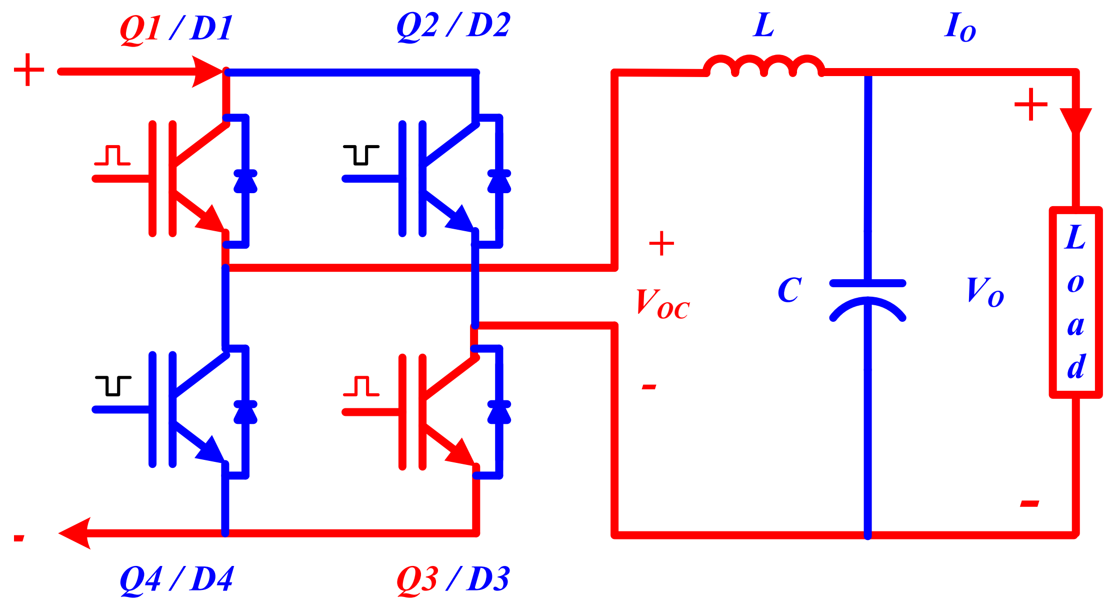
PWM - Bipolar Bridge – Sign / Magnitude PWM

Reference Signal	Q1/Q3 D	Q2/Q4 D
0	Off	Off
+25%	0.25	Off
+50%	0.50	Off
+75%	0.75	Off
+100%	1.00	Off
-25%	Off	0.25
-50%	Off	0.50
-75%	Off	0.75
-100%	Off	1.00

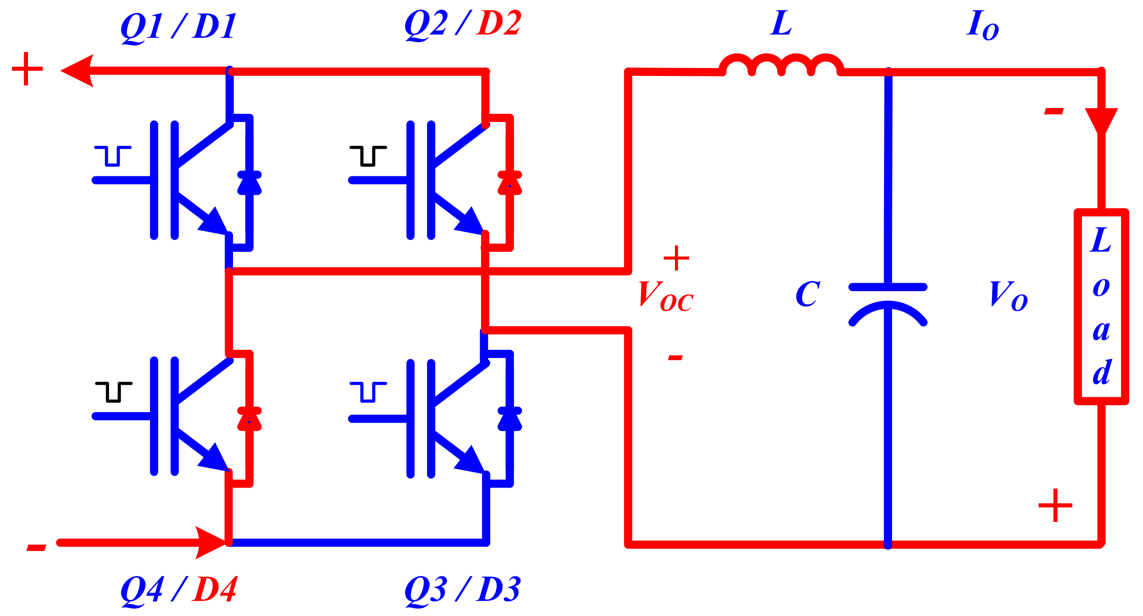


- *Switch only one leg at a time*
- *The 2 switches in the active leg switch on and off together*

Bipolar Bridge – Sign / Magnitude PWM – (+) Output

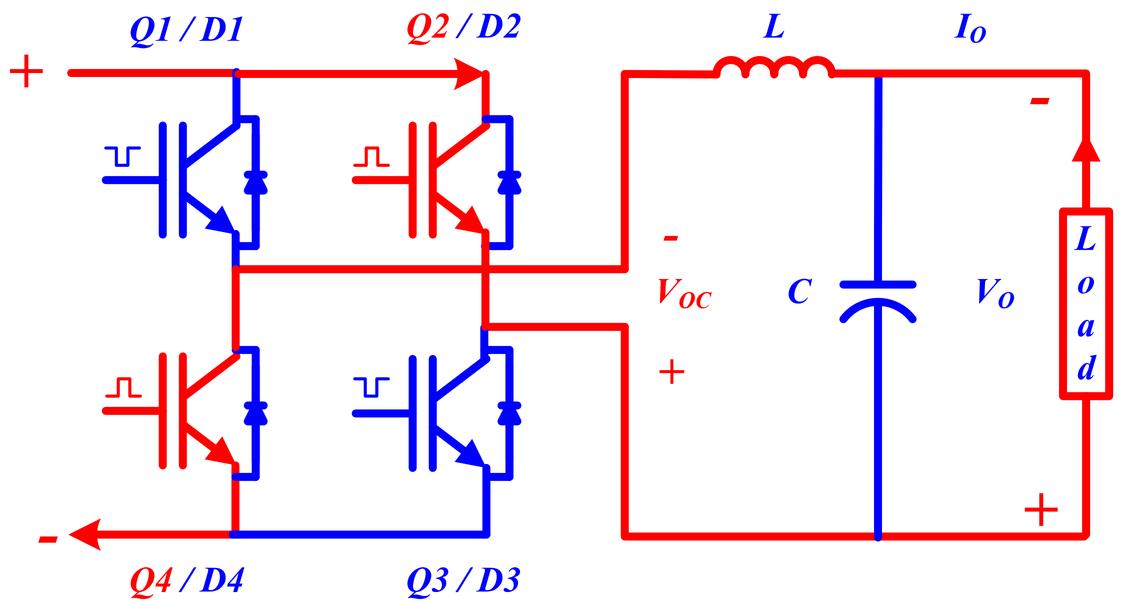


Q1/Q3 closed

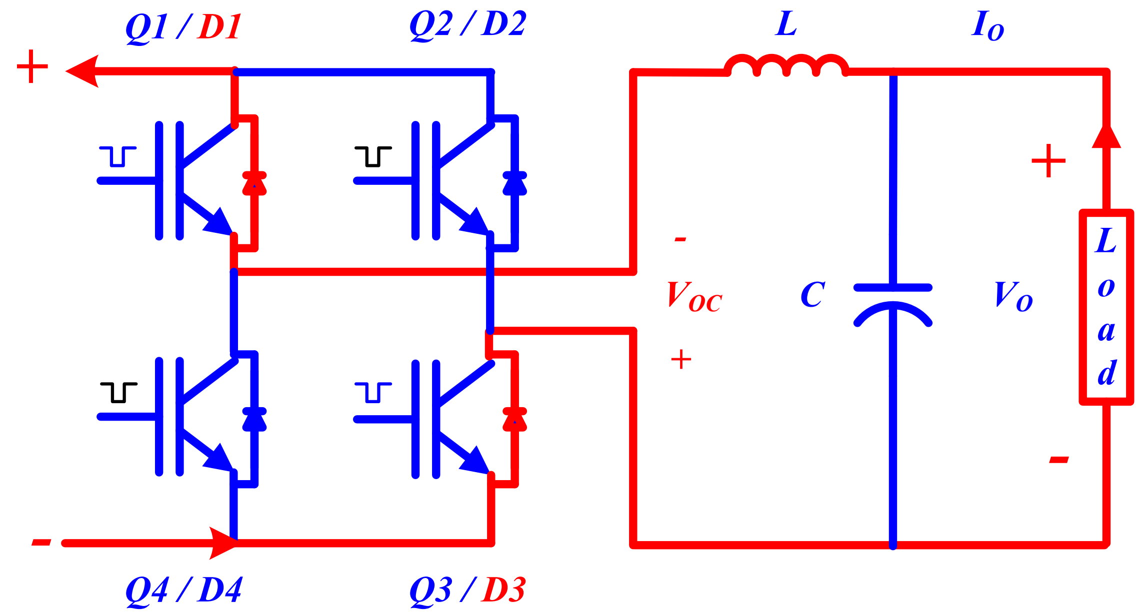


Q1/Q3 opened

Bipolar Bridge – Sign / Magnitude PWM – (-) Output

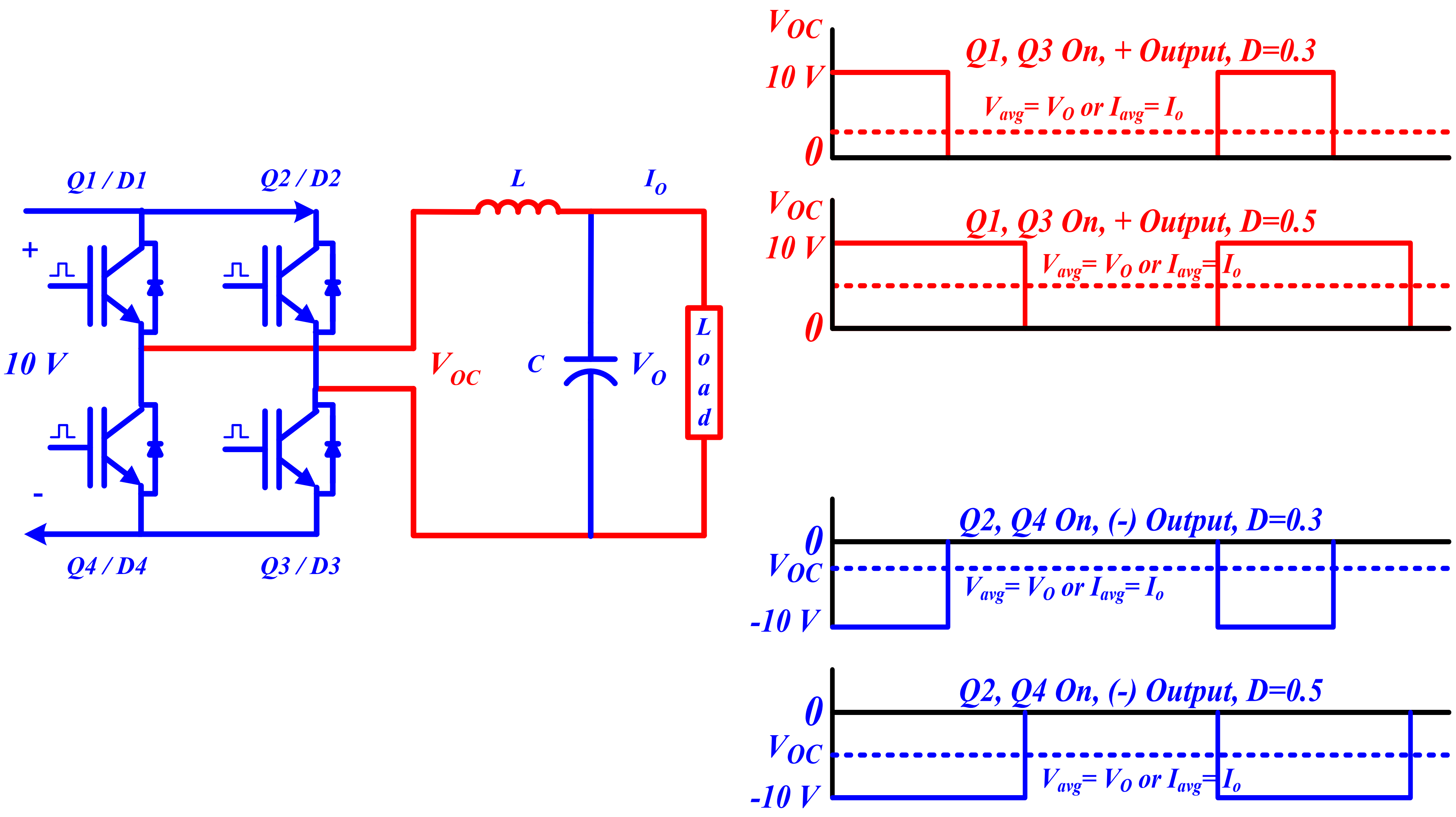


Q2/Q4 closed



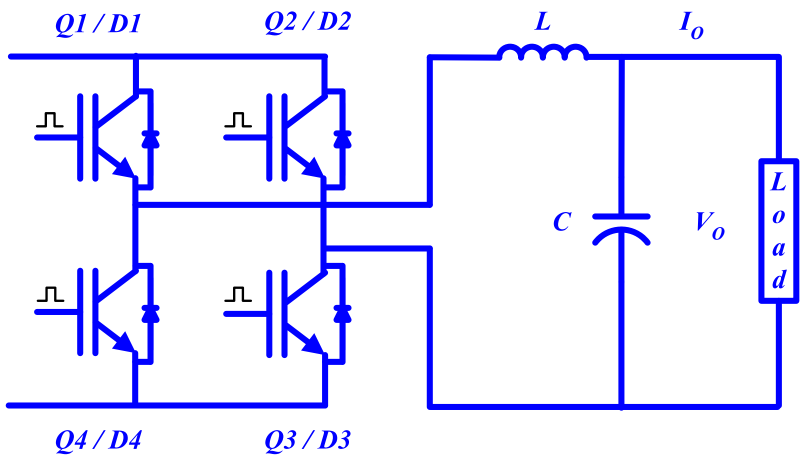
Q2/Q4 opened

Bipolar Bridge – Sign / Magnitude PWM - Waveforms



"50/50" Bipolar PWM

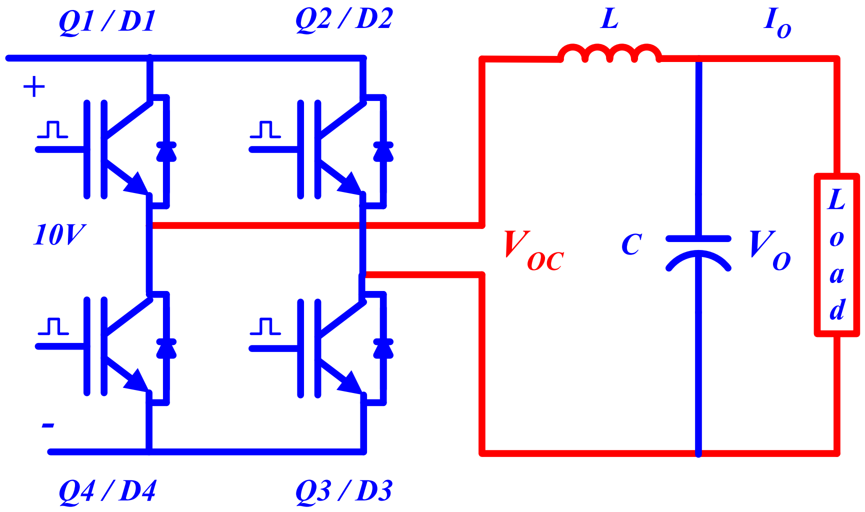
Desired Output Reference Signal	Q1/Q3D	Q2/Q4 D
-100%	0.0%	100.0%
-75%	12.5%	87.5%
-50%	25.0%	75.0%
-25%	37.5%	62.5%
0%	50.0%	50.0%
25%	62.5%	37.5%
50%	75.0%	25.0%
75%	87.5%	12.5%
100%	100.0%	0.0%



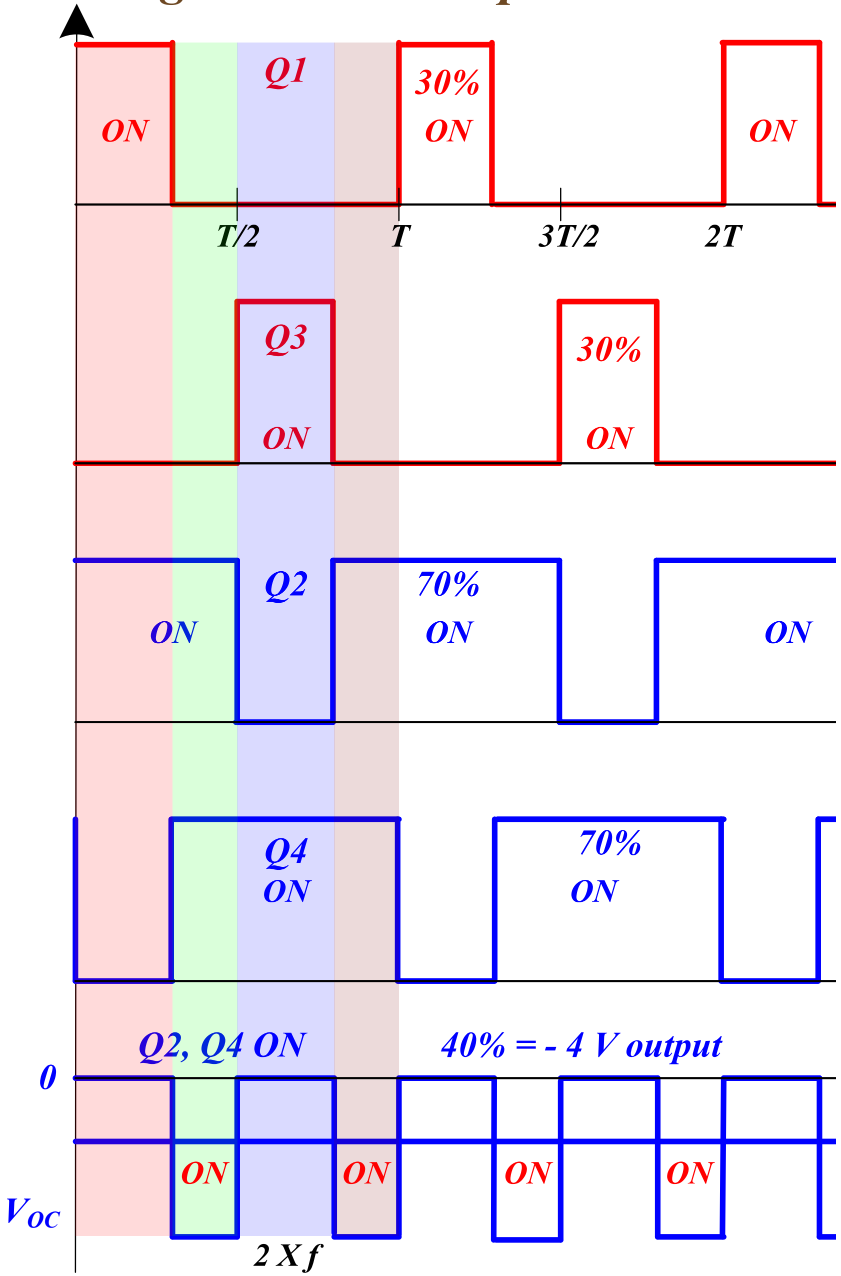
- Both bridge legs are always active
- Q1/Q3 (+) bridge
- Q2/Q4 (-) bridge
- Q1/Q3 180 ^{O} phase shifted
- Q2/Q4 180 ^{O} phase shifted
- Q1 is complement of Q4
- Q2 is complement of Q3

PWM - "50/50" Bipolar Switching For - 4 V Output

Q1/Q3 on 30%   Q2/Q4 on 70%

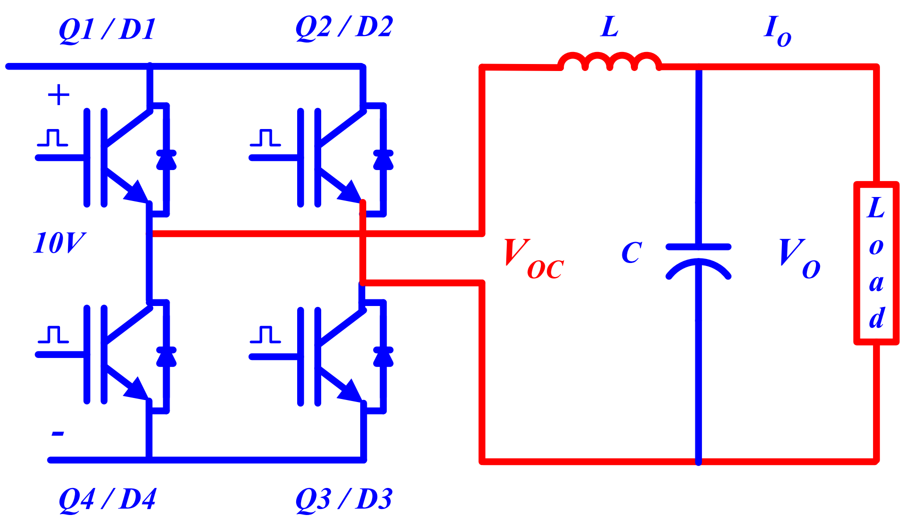


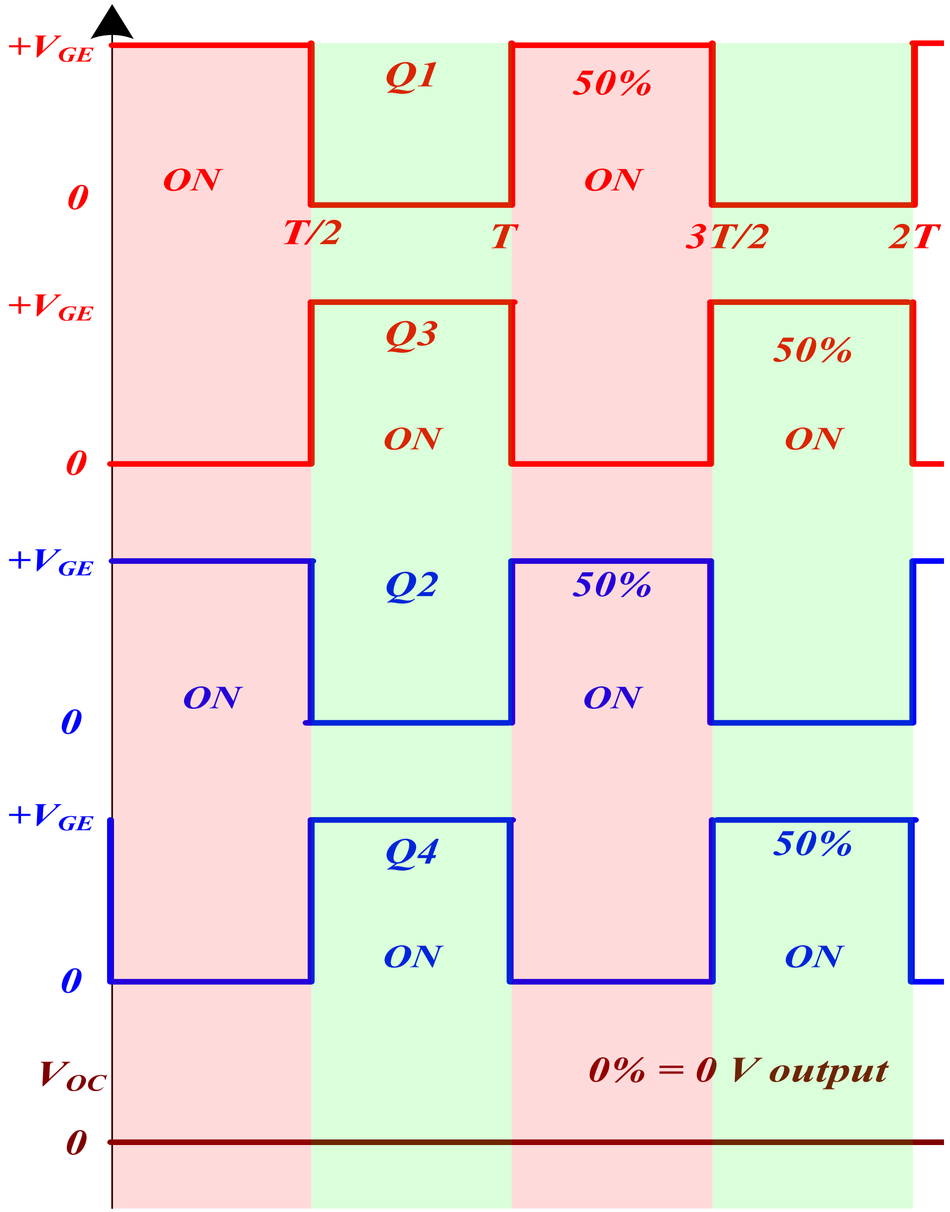
	Q1	Q3	Q2	Q4	V_{OC}
1	On	Off	On	Off	0 V
2	Off	Off	On	On	-10 V
3	Off	On	Off	On	0 V
4	Off	Off	On	On	-10 V



PWM - "50/50" Bipolar Switching For 0 V Output

Q1/Q3 on 50% Q2/Q4 on 50%

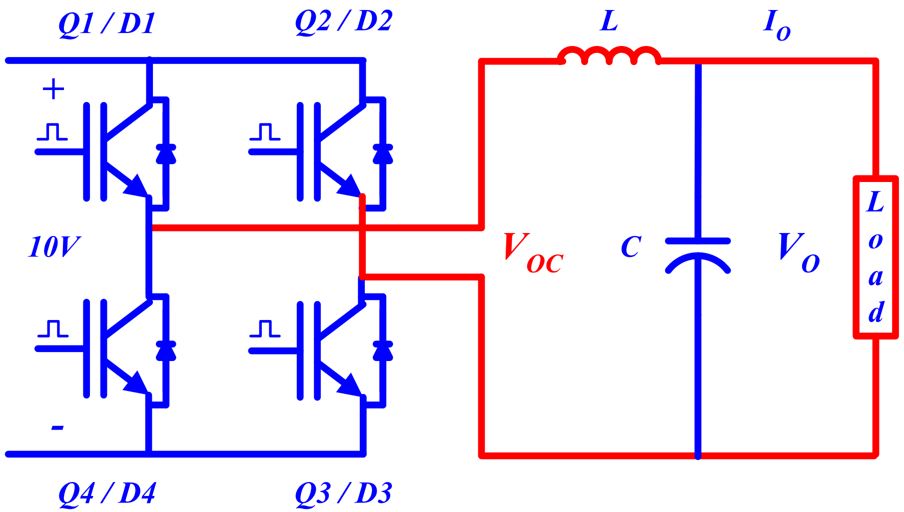




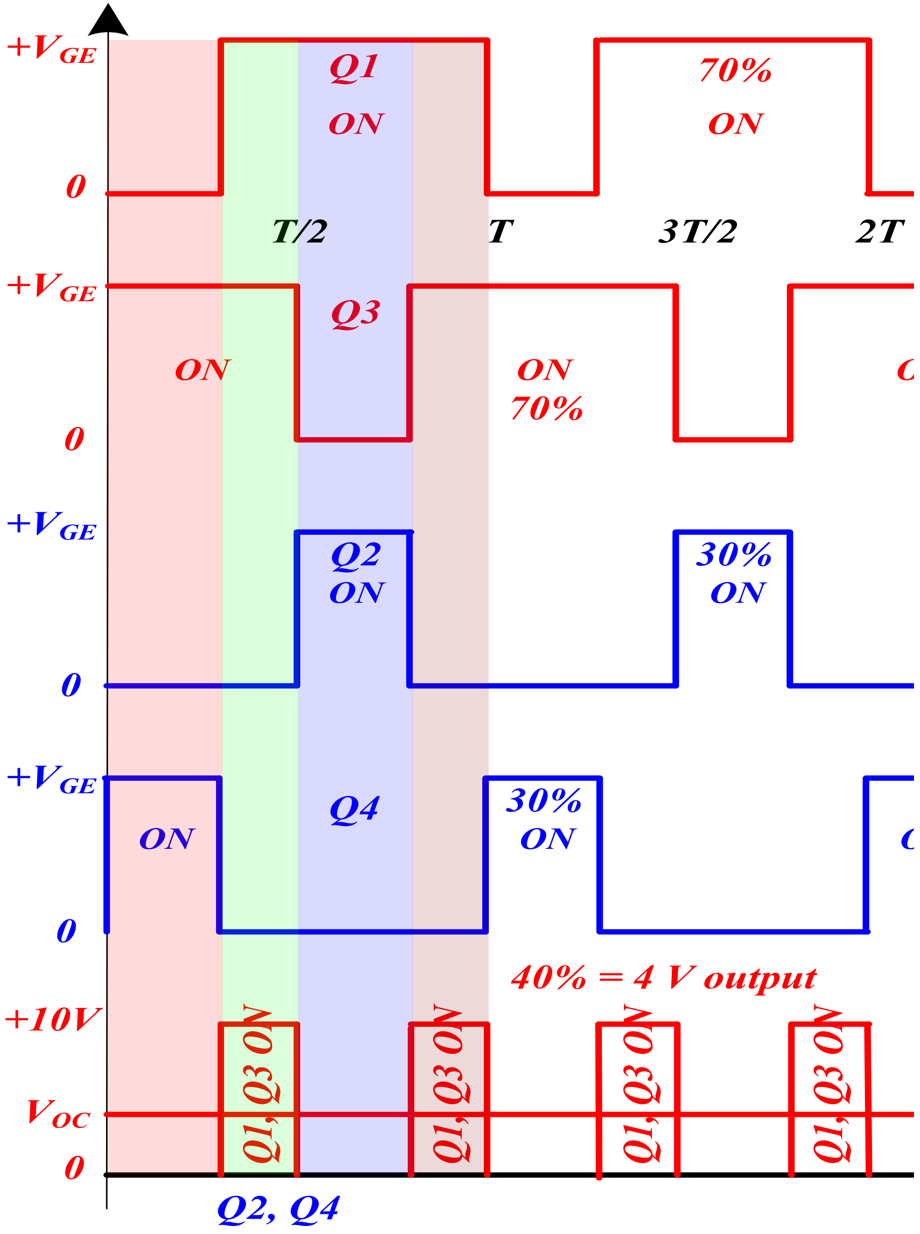
	Q1	Q3	Q2	Q4	VOC
1	On	Off	On	Off	0 V
2	Off	On	Off	On	0 V
3	On	Off	On	Off	0 V
4	Off	On	Off	On	0 V

PWM - “50/50” Bipolar Switching For + 4 V Output

Q1/Q3 on 70%   Q2/Q4 on 30%



	Q1	Q3	Q2	Q4	V_{OC}
1	Off	On	Off	On	0 V
2	On	On	Off	Off	+10 V
3	On	Off	On	Off	0 V
4	On	On	Off	Off	+10 V

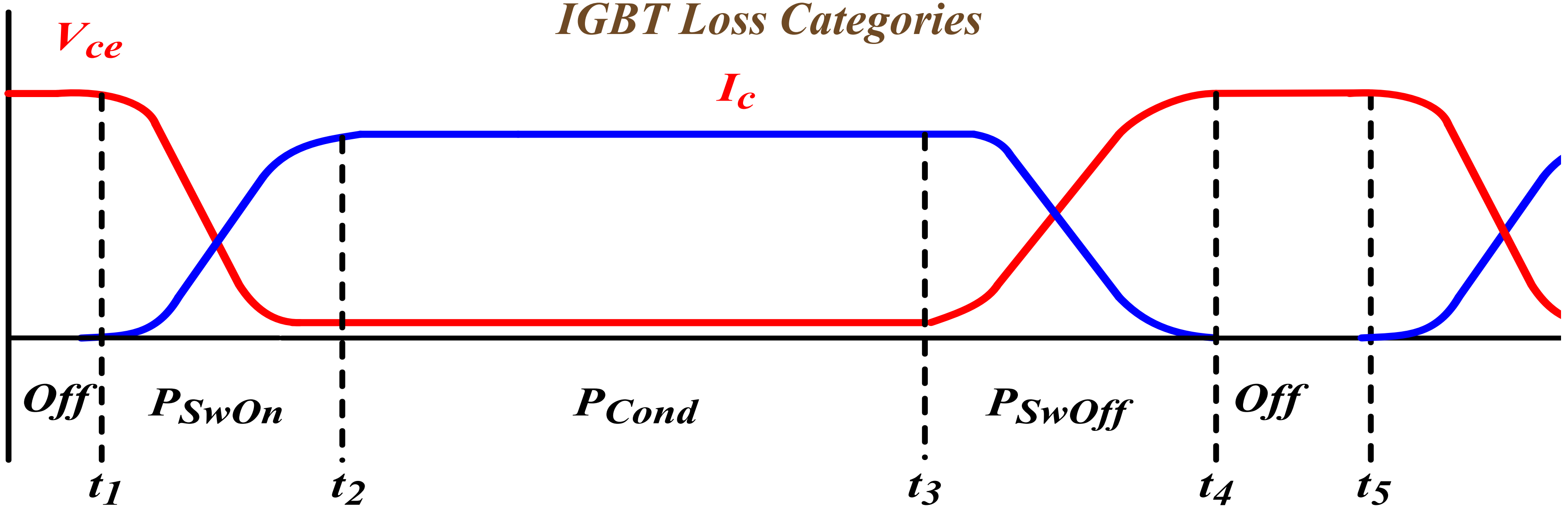


PWM - Bipolar PS PWM Strategies Compared

PWM Type	*Advantages*	*Disadvantages*
Sign/Magnitude		*Output voltage is 1X the switching frequency – difficult to filter* *Zero crossing transitions are discontinuous*
"50/50"	*Output voltage pulse 2X the switching frequency. Easier to filter* *Smoothest transitions through zero.*	

Conducting and Switching Losses

IGBT Loss Categories



Turn-on losses

$$P_{SwOn} = \frac{1}{t_5 - t_1} * \int_{t_1}^{t_2} v_{CE}(t) * i_C(t) * dt$$

Conduction losses

$$P_{Cond} = \frac{1}{t_5 - t_1} * \int_{t_2}^{t_3} V_{CE} * I_C * dt$$

Turnoff losses

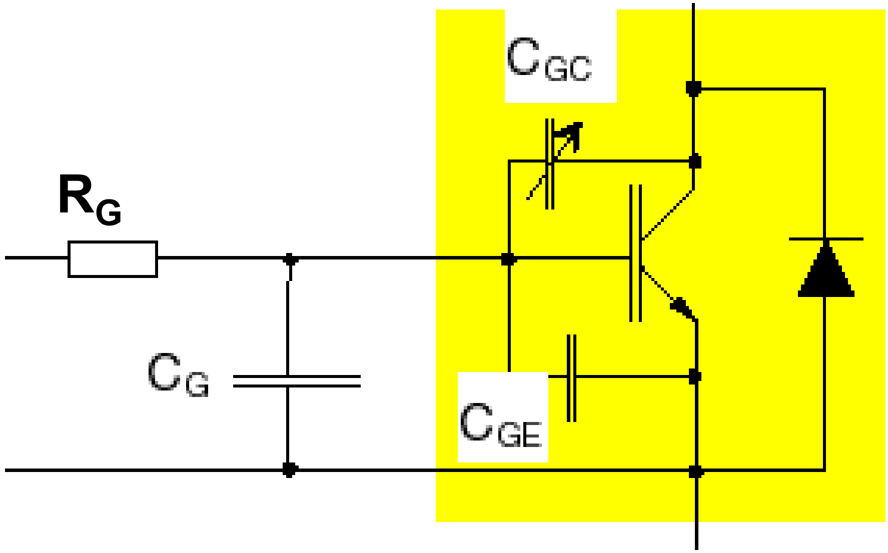
$$P_{SwOff} = \frac{1}{t_5 - t_1} * \int_{t_3}^{t_4} v_{CE}(t) * i_C(t) * dt$$

Reducing Conducting and Switching Losses

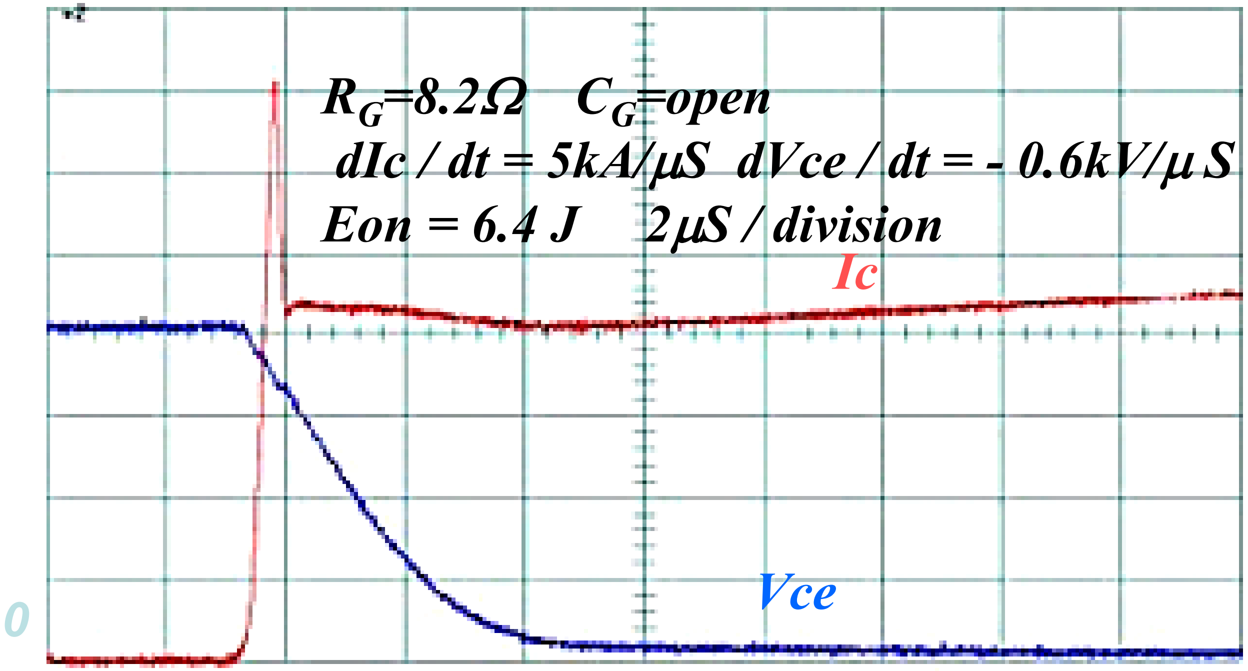
Reduce losses for greater efficiency and:

- *Smaller AC distribution system*
- *Less heat load into cooling water system*
- *Less heat into buildings and building HVAC*
- *Reduce IGBT dissipation*

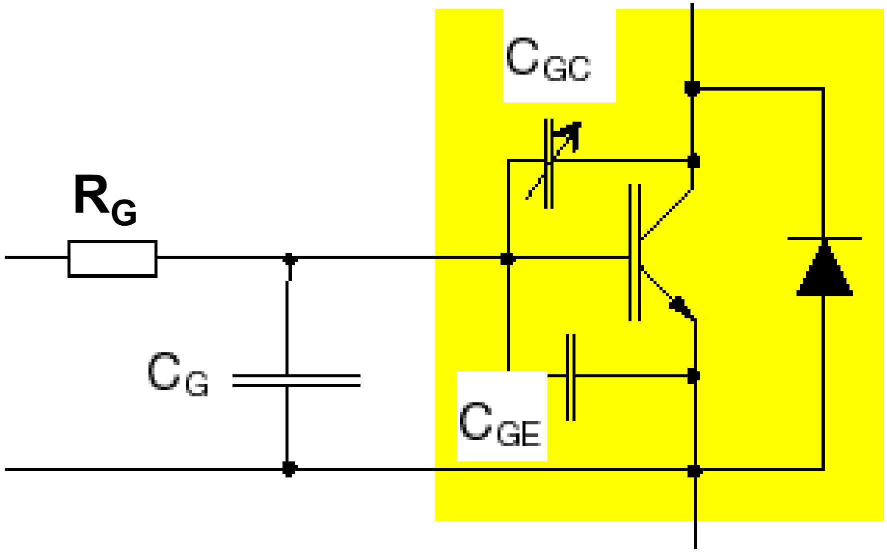
Reducing Turn On Losses By Varying RG

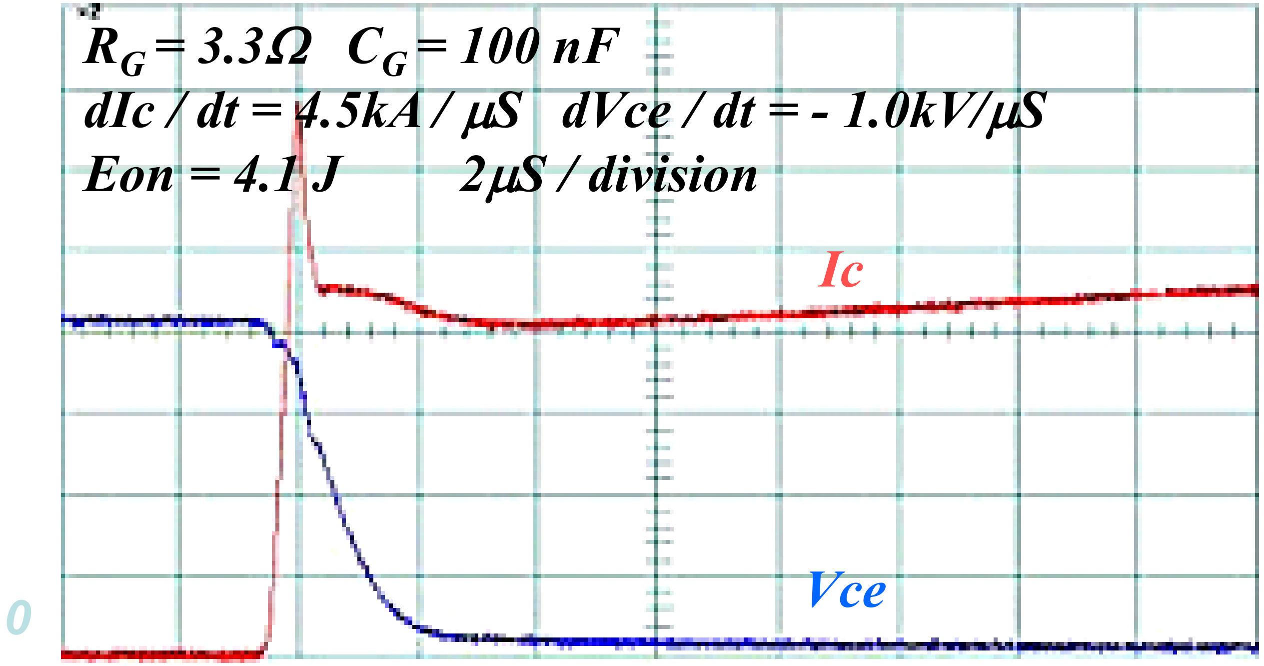


https://www.infineon.com/dgdl/Infineon-PCIM_2015_Slew_rate_control_of_discrete_IGBT_and_CoolMOS_reaches_targets_far_beyond_the_gate_resistor_regime-Article-v01_00-EN.pdf?fileId=5546d46250cc1fdf0150d23a77d35e54

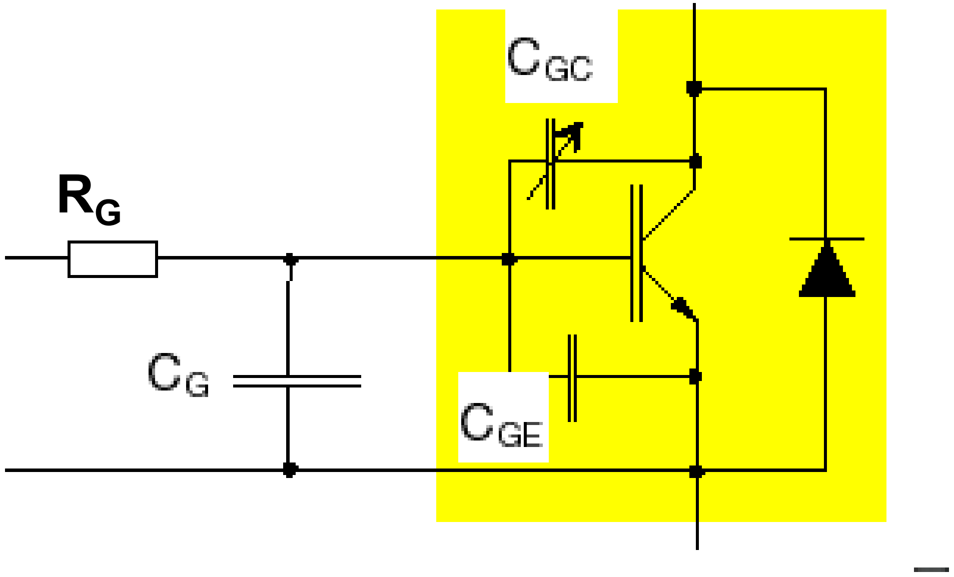


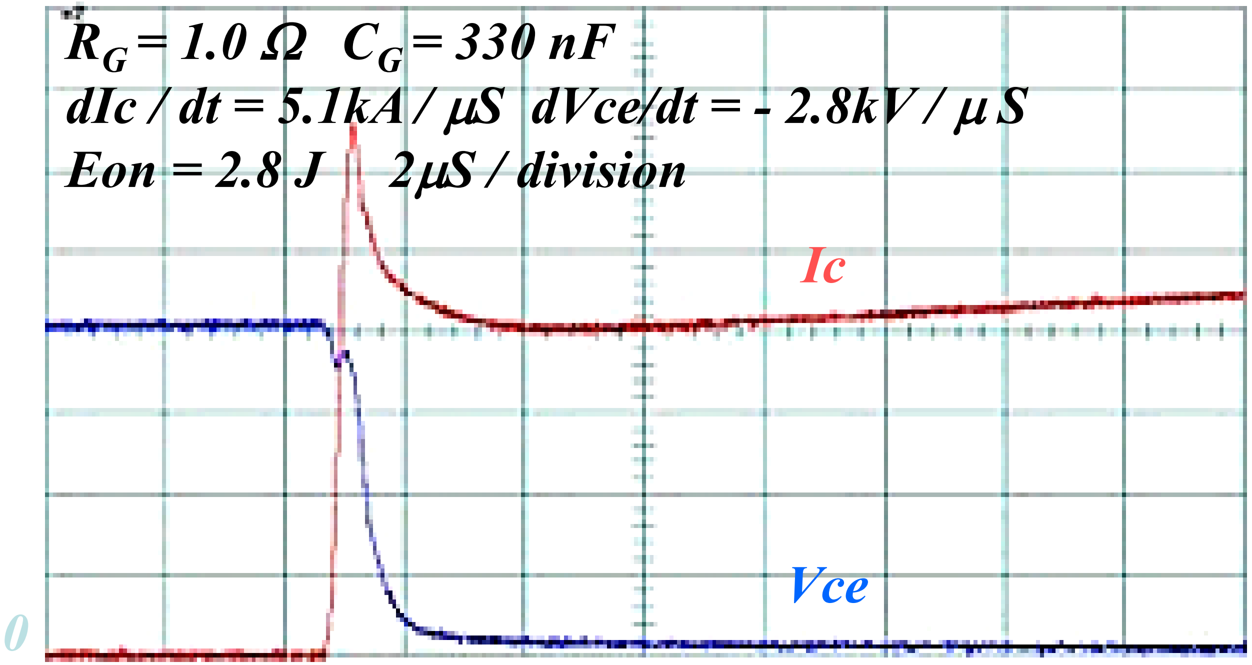
Reducing Turn On Losses By Varying RG





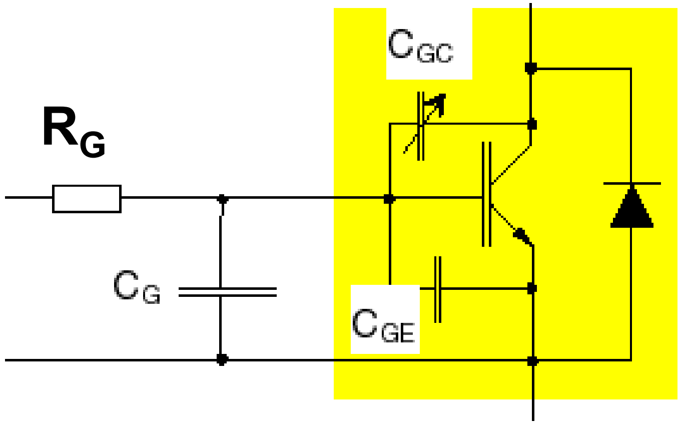
Reducing Turn On Losses By Varying RG



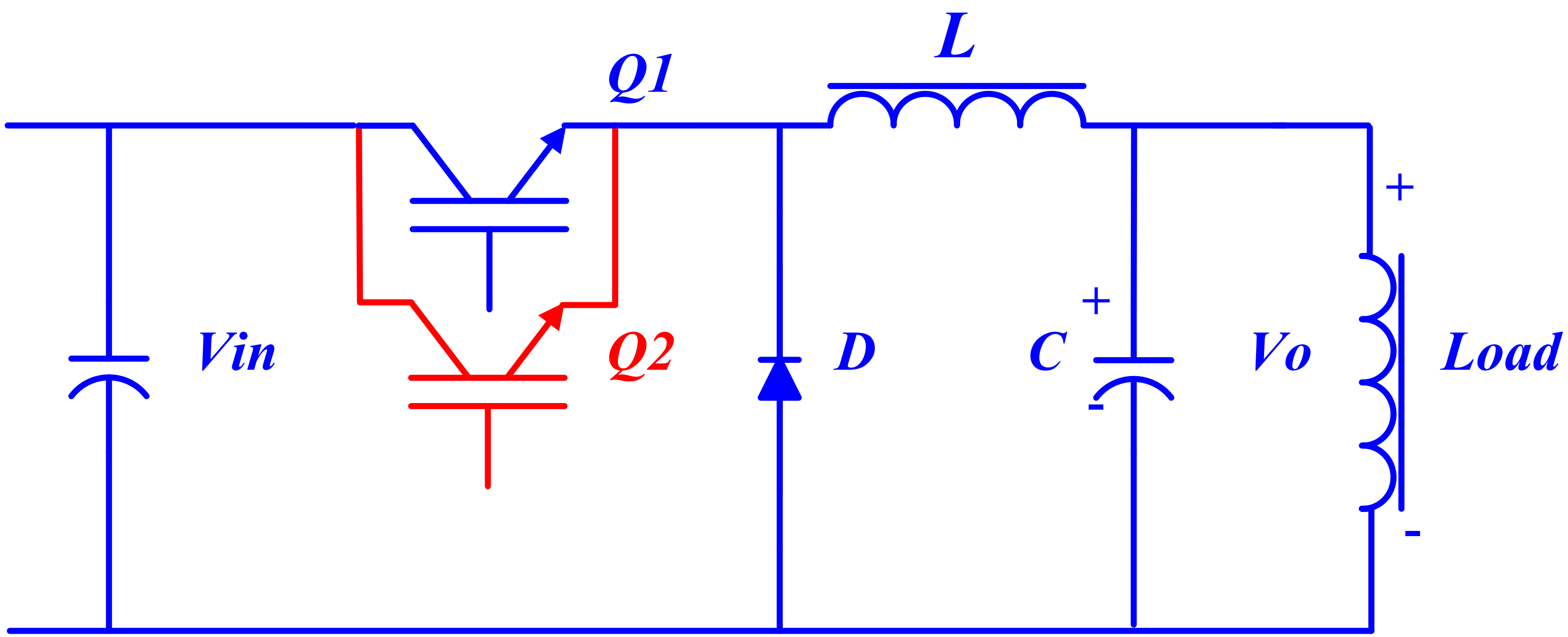


Reducing Turn On Losses By Varying RG

Case	R_G	C_G	dV_{CE}/dt	E_{On}
1	8.2 Ω	open	-0.6 kV/μs	6.4 J
2	3.3 Ω	100 nF	-1.0 kV/μs	4.1 J
3	1.0 Ω	330 nF	-2.8 kV/μs	2.8 J

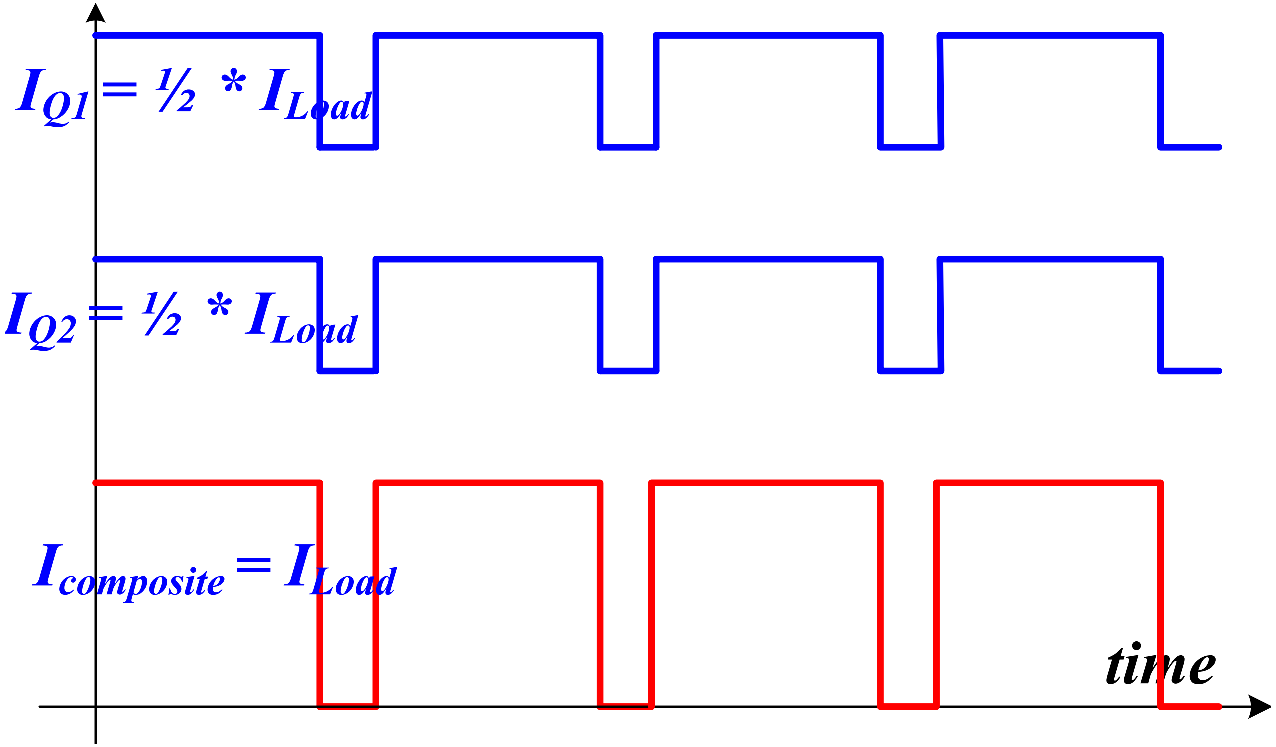


- $P_{Dis} \propto -\frac{dV_{CE}}{dt}^{-1}$
- $\frac{dV_{CE}}{dt}$ is controlled via R_G
- A faster $\frac{dV_{CE}}{dt}$ reduces losses but likely increases EMI



- *If the current rating of a single switch is insufficient (conduction loss is too great), add another switch in parallel.*
- *There are then 2 ways to switch Q1 and Q2, switch them ON and OFF together or stagger their On and OFF times*

Conduction Loss Reduction By Simultaneous Switching of Q1 and Q2



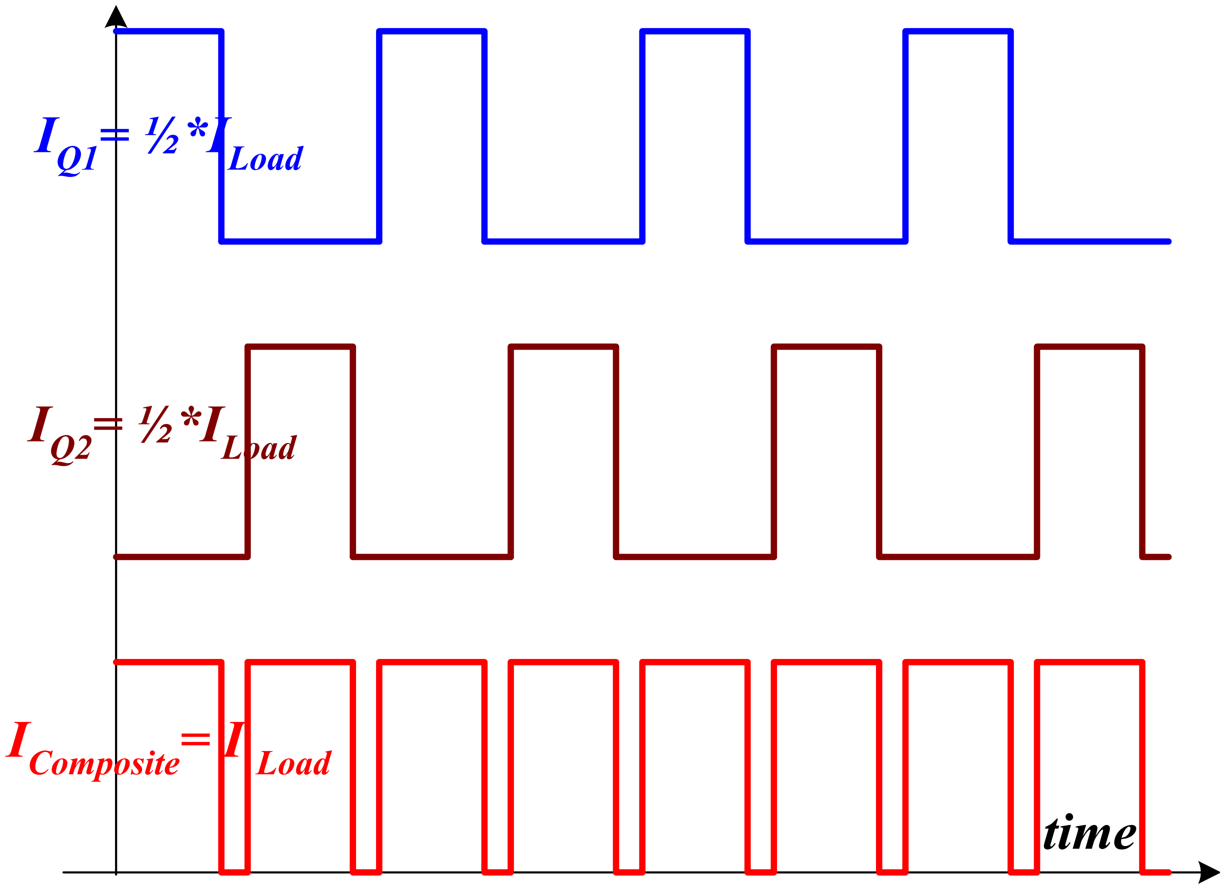
$$V_{RMS\,2Sw-EaSw} = V_{RMS-1Sw} \qquad I_{RMS\,2Sw-EaSw} = \frac{1}{2} * I_{RMS-1Sw}$$

$$P_{Ave1Sw} = V_{RMS1Sw} * I_{RMS1Sw}$$

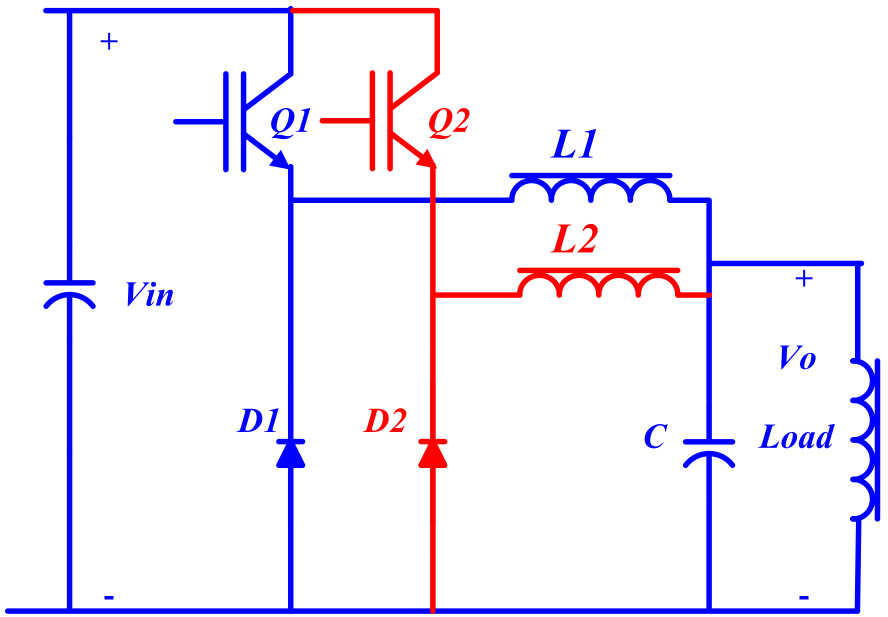
$$P_{Ave2Sw-EaSw} = V_{RMS1Sw} * \frac{1}{2} I_{RMS1Sw} = \frac{1}{2} * P_{Ave1Sw}$$

The composite frequency is the same as in Q1 and Q2

Conducted Loss Reduction By Staggered Switching of Q1 and Q2

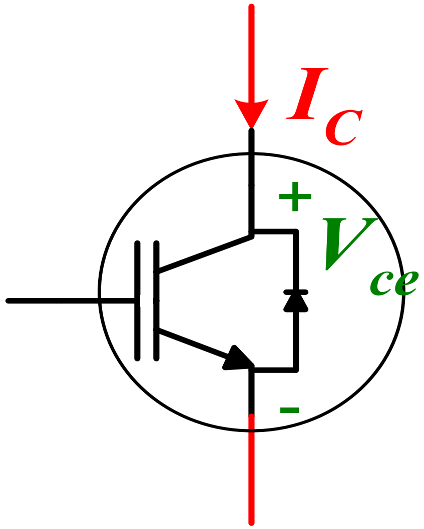


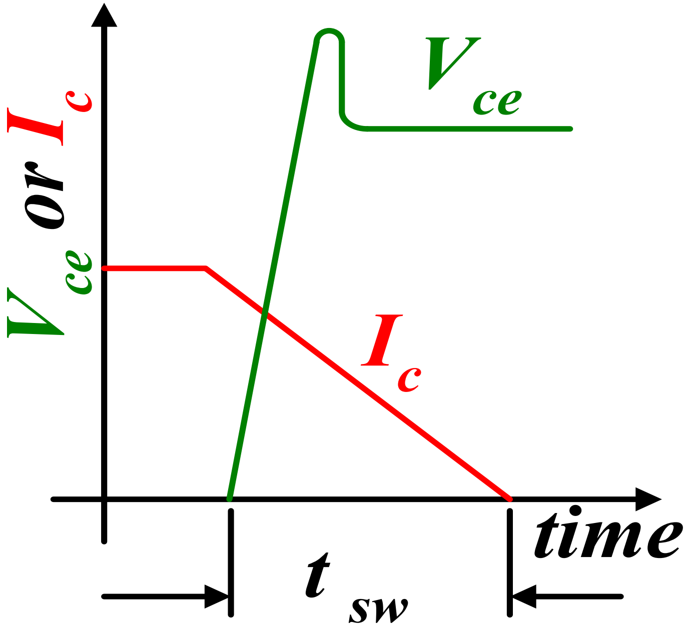
- *Duty factor is each switch is halved*
- *P_{ave} in each switch is 1/2 that of the single switch case*
- *The composite frequency is twice that of Q1 and Q2*



Features:

- *A second switch Q1 is added.*
- *Q1 and Q2 are staggered switched*
- *D2 is added, L2 is added*
- *Current in D1, D2 is 1/2 the load current*
- *Current in L1, L2 is 1/2 the load current*
- *L1, L2 energy 1/4 that of single inductor since* $E = \frac{1}{2} L I^2$





$$P_{SwOff} = \frac{1}{T} \int_0^{t_{Sw}} v_{CE}(t)\, i_C(t)\, dt$$

Snubber Analysis

Semiconductor switches undergo stresses during switching

- *Voltage spikes can exceed maximum voltage rating*
- *Current spikes can exceed maximum current rating*
- *Power dissipation at maximum voltage and current may be excessive*

Snubbers are used to address these issues

Elementary calculations can give insight into snubber operation and design

- *Techniques we will use are similar to those needed in other power supply circuits*

Switch Turn-off Loss

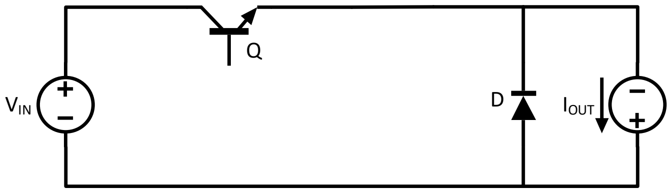
Ideal switch:

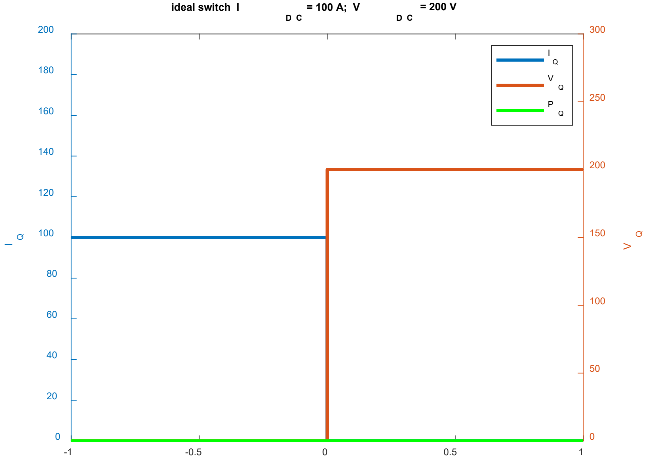
- *Opens instantaneously* V_{SW}: $0 \rightarrow V_{IN}$
- *Current transfers to diode* I_{SW}: $I_{OUT} \rightarrow 0$
- *Switching power* $= 0$

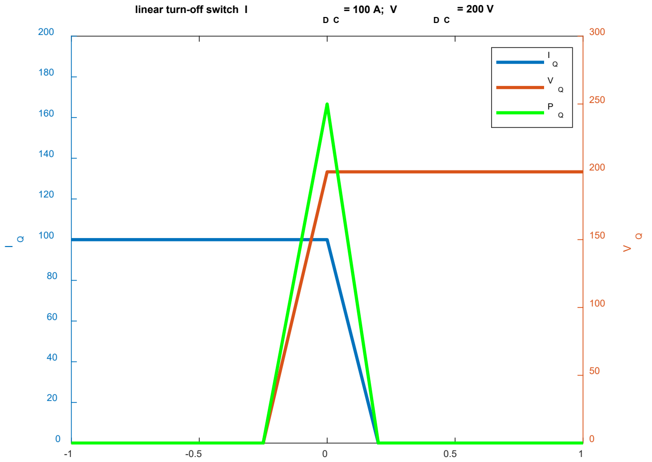
Real switch:

- *Assume that* I, V *change linearly with time*
  - *Reasonable approximation to understand concepts*
- Q *starts to open with* $V_Q = 0$
- V_Q *must reach* V_{IN} *before* $V_D = 0$ *and diode conducts*
- $P_Q = f_{SW} \int_{t_{ON}}^{t_{OFF}} v_Q(t) i_Q(t) dt$

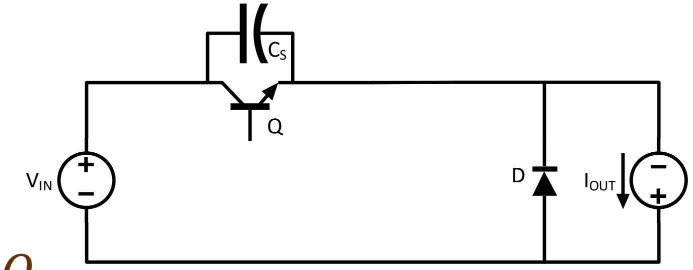
$$= \frac{1}{2} V_{IN} I_{OUT} (t_{OFF} - t_{ON}) \cdot f_{SW}$$







Switch Turn-off Loss Reduction: Shunt Q with Capacitor



Intuition:

- I_{OUT} *flows through* C *and* Q: $I_{OUT} = i_Q + i_C$
  - *Lower* i_Q *means less power dissipated in* Q
- *When* $i_C = I_{OUT}$, $i_Q = 0$ *and* $P_Q = 0$
- I_{OUT} *flowing in* C *linearly increases* v_C *until* $v_C = V_{IN} \Rightarrow$ *Diode turns on*

Calcs: By assumption current in Q decreases linearly, dropping to 0 *at* $t = t_1$:

$$i_Q = [1 - (t/t_1)]I_{OUT} \Rightarrow i_C = I_{OUT} - i_Q = (t/t_1)I_{OUT}$$

$$v_C(t) = \frac{1}{C}\int_0^t i_C(t')\,dt' = \frac{I_{OUT}}{2Ct_1}t^2 = v_Q(t)$$

At $t = t_1$, $i_C = I_{OUT}$ *and* $v_C = v_Q = \frac{I_{OUT}t_1}{2C} = \alpha V_{IN}$, *where* $\alpha \equiv \frac{I_{OUT}t_1}{2CV_{IN}}$

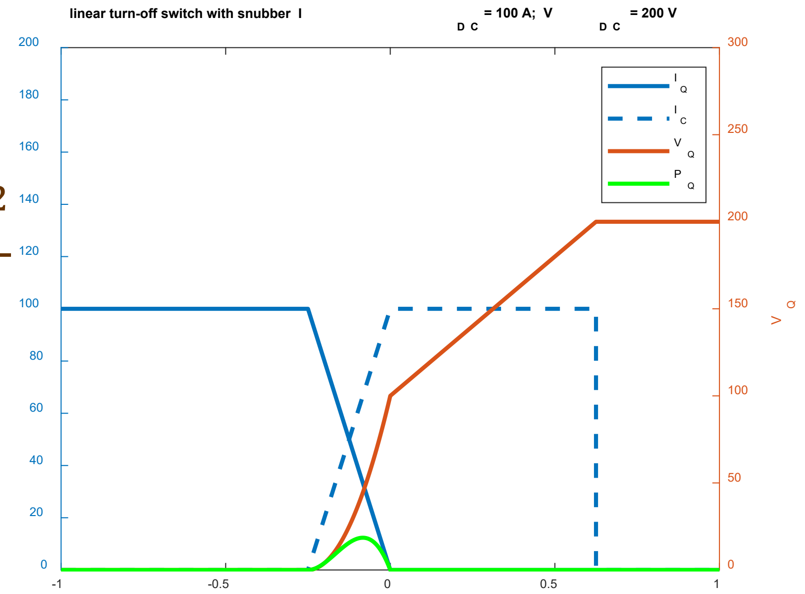
C *continues charging with* I_{OUT}: $v_C(t) = \frac{1}{C}\int_{t_1}^t I_{OUT}\,dt' + \frac{I_{OUT}t_1}{2C} = \frac{I_{OUT}t}{C} - \frac{I_{OUT}t_1}{2C}$

v_C *stops increasing when* $v_C(t_2) = V_{IN} \Rightarrow t_2 = \frac{CV_{IN}}{I_{OUT}} + \frac{t_1}{2} \Rightarrow$ *Diode turns on*

Switch Turn-off Loss Reduction: Shunt Q with Capacitor

$$\mathcal{E}_{QS} = \int_{t_{ON}}^{t_{OFF}} v_Q(t) i_Q(t) dt$$

$$= \int_0^{t_1} \frac{I_{OUT} t^2}{2Ct_1} I_{OUT}[1 - (t/t_1)]\, dt = \frac{(I_{OUT} t_1)^2}{24C}$$



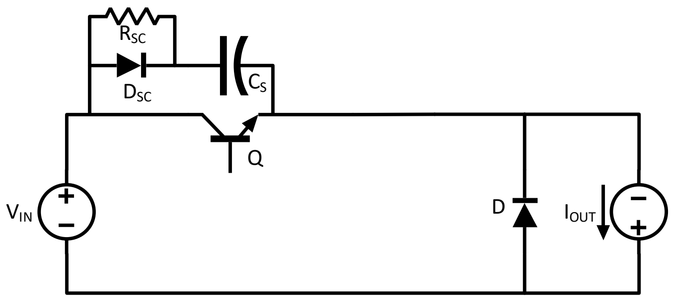
If C is selected such that $v_C(t_1) = \alpha V_{IN}$

$$\mathcal{E}_{QS} = \frac{I_{OUT} t_1}{2CV_{IN}} \frac{V_{IN} I_{OUT} t_1}{12} = \frac{\alpha V_{IN} I_{OUT} t_1}{12}$$

We can make this $\ll \frac{V_{IN} I_{OUT}(t_1 + t_2)}{2} = \mathcal{E}_{Q0}$

Other consequences, considerations, and trade-offs:

- *Larger C reduces energy loss in the device*
- *Also increases the charging time and the time at which the diode turns on.*
- *Additional charge in C will create a current spike when Q turns on again*
  - *This current may challenge the instantaneous current and thermal limits of the device.*



We now introduce a damping resistor R_{SC} to:

- *Limit current out of the shunt C_S into Q*
- *Dissipate energy stored in C_S*

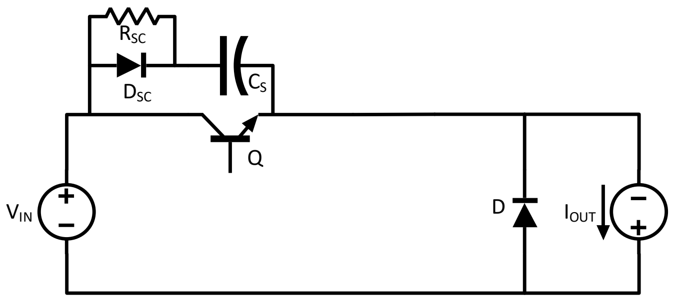
We still want to keep the low impedance charging path to C_S, so we shunt R_{SC} with a diode D_{SC}. The value of R_{SC} is chosen

- *Large enough to limit current from C_S through Q: $V_{IN}/R_{SC} < I_{Q_{MAX}} - I_{OUT}$*
- *Small enough to discharge C_S during t_{ON} of Q: $R_{SC}C_S \ll t_{ON}$*

We now have two sources of energy that need to be dissipated

- $\mathcal{E}_{QS} = \frac{I_{OUT}t_1}{2CV_{IN}} \frac{V_{IN}I_{OUT}t_1}{12}$ *proportional to* $\frac{1}{C_S}$ *from the turn-off*
- $\mathcal{E}_C = \frac{1}{2}C_S V_{IN}^2$ *proportional to C_S from the snubber*

C_S can be chosen such that $\mathcal{E}_{QS} + \mathcal{E}_C < \mathcal{E}_{Q0}$



Trade-offs in component choices:

- *Small value of C_S*
  - *Increased energy dissipated in Q*
  - *Higher impedance for high frequency noise; less filtering*
  - *More high frequency ringing with parasitic inductances*
  - *Increased value of $dv_Q/dt = i_c/C_S$*
  - *Less stored energy in C_S*
- *Large value of C_S*
  - *Larger stored energy in capacitor*
  - *More energy dissipation in circuit*
  - *More current flows through Q at switch turn-on*
  - *Larger energy dissipation time constant for same R_{SC}*

Switch Turn-on Loss – Dual Case $(V \leftrightarrow I)$ $(C \leftrightarrow L)$

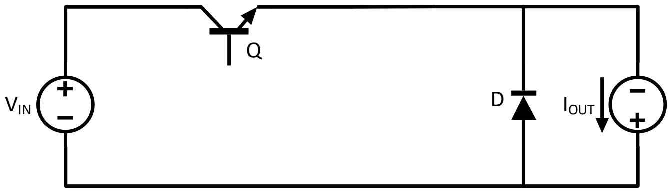
Ideal switch:

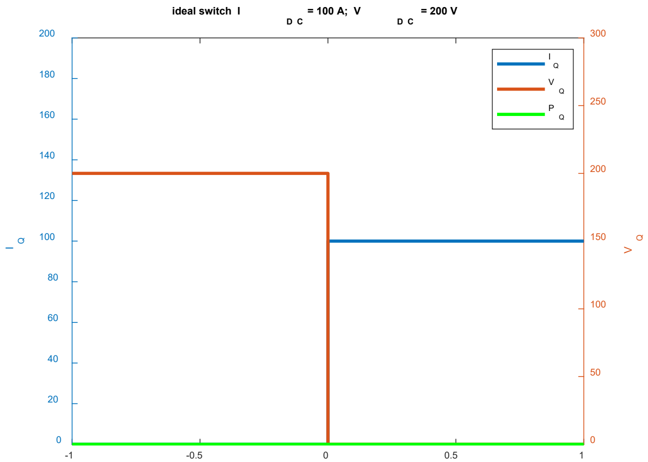
- *Closes instantaneously* V_{SW}: $V_{IN} \rightarrow 0$
- *Current transfers to switch* I_{SW}: $0 \rightarrow I_{OUT}$
- *Switching power* $= 0$

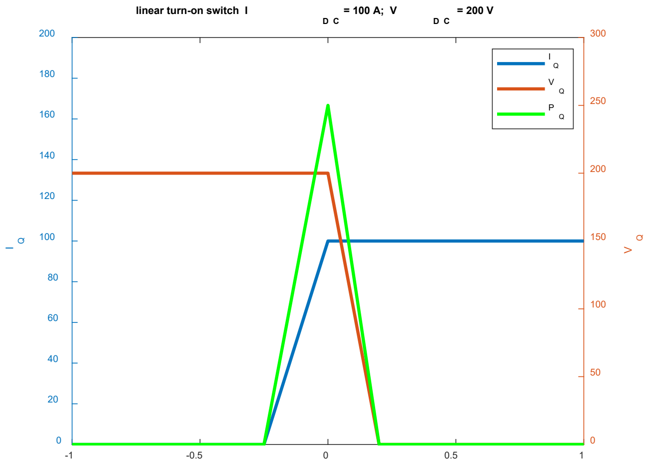
Real switch:

- *Assume that* I, V *change linearly with time*
    - *Reasonable approximation to understand concepts*
- *Q starts to close*
- *Once* $V_Q \neq V_{IN}$ $V_D \neq 0$ *and diode stops conducting*
- $P_Q = f_{SW} \int_{t_{ON}}^{t_{OFF}} v_Q(t) i_Q(t) dt$

$$= \frac{1}{2} V_{IN} I_{OUT} (t_{OFF} - t_{ON}) \cdot f_{SW}$$



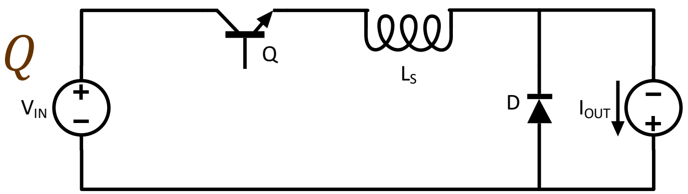




Switch Turn-on Loss Reduction: Series Q with Inductor

Intuition:

- *D on* $\Rightarrow V_{IN}$ *drops across Q and L:* $V_{IN} = v_Q + v_L$
  - *Lower* v_Q *means less power dissipated in Q*
- *When* $v_L = V_{IN}$, $v_Q = 0$ *and* $P_Q = 0$
- *Increasing* i_Q *across L creates* v_L *that keeps* $v_Q + v_L = V_{IN}$*; diode on longer*



Calcs: By assumption voltage in Q decreases linearly, dropping to 0 *at* $t = t_1$*:*

$$v_Q = [1 - (t/t_1)]V_{IN} \Rightarrow v_L = V_{IN} - v_Q = (t/t_1)V_{IN}$$

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t')\,dt' = \frac{V_{IN}}{2Lt_1}t^2 = i_Q(t)$$

At $t = t_1, v_Q = 0; v_L = V_{IN}$ *and* $i_L = i_Q = \frac{V_{IN}t_1}{2L} = \alpha I_{OUT}$, *where* $\alpha \equiv \frac{V_{IN}t_1}{2LI_{OUT}}$

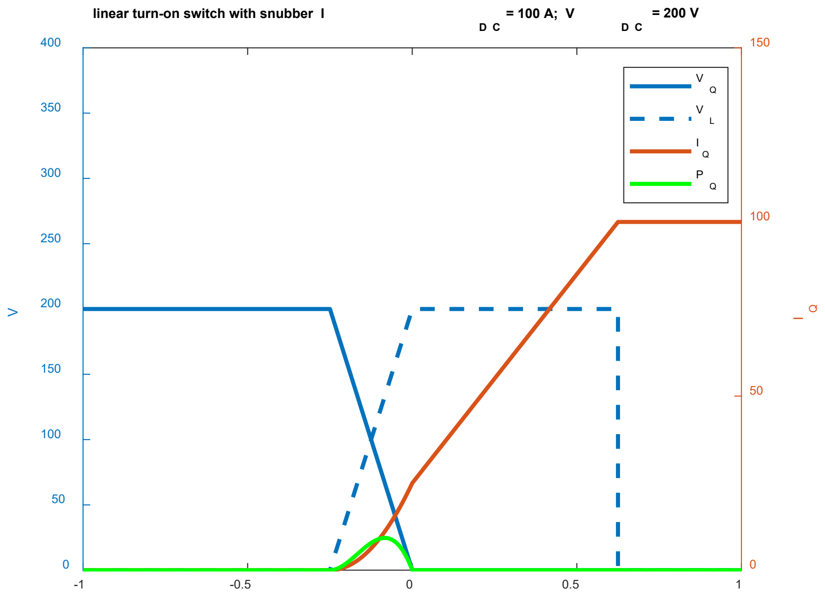
i_L *continues increasing with* V_{IN}*:* $i_L(t) = \frac{1}{L}\int_{t_1}^{t} V_{IN}\,dt' + \frac{V_{IN}t_1}{2L} = \frac{V_{IN}t}{L} - \frac{V_{IN}t_1}{2L}$

i_L *stops increasing when* $i_L(t_2) = I_{OUT} \Rightarrow t_2 = \frac{LI_{OUT}}{V_{IN}} + \frac{t_1}{2} \Rightarrow$ *Diode turns off*

Switch Turn-on Loss Reduction: Series Q with Inductor

$$\mathcal{E}_{QS} = \int_{t_{ON}}^{t_{OFF}} v_Q(t) i_Q(t) dt$$

$$= \int_0^{t_1} \frac{V_{IN} t^2}{2Lt_1} V_{IN}[1 - (t/t_1)]\, dt = \frac{(V_{IN} t_1)^2}{24L}$$



If L is selected such that $i_L(t_1) = \alpha I_{OUT}$

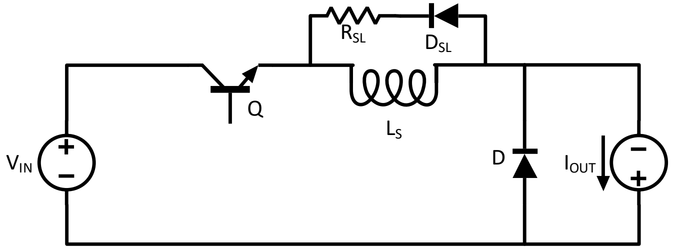
$$\mathcal{E}_{QS} = \frac{V_{IN} t_1}{2LI_{OUT}} \frac{I_{OUT} V_{IN} t_1}{12} = \frac{\alpha V_{IN} I_{OUT} t_1}{12}$$

We can make this $\ll \frac{V_{IN} I_{OUT}(t_1 + t_2)}{2} = \mathcal{E}_{Q0}$

Other consequences, considerations, and trade-offs:

- *Larger L reduces energy loss in the device*
- *Also increases the transition time at which the diode turns off.*
- *Additional current in L will create a voltage spike when Q turns off again*
  - *This voltage may challenge the instantaneous voltage and thermal limits of the device.*

Switch Turn-on Loss Reduction: Series Q with Inductor, Resistor, Diode



We now introduce a damping resistor R_{SC} to:

- *Limit voltage out of the series L_S into Q*
- *Dissipate energy stored in L_S*

We still want to keep the low impedance charging path to L_S, so we series R_{SC} with a diode D_{SC}. The value of R_{SC} is chosen

- *Large enough to limit voltage from L_S through Q: $I_{OUT}R_{SC} < V_{Q_{MAX}} - V_{IN}$*
- *Small enough to discharge L_S during t_{ON} of Q: $L_S/R_{SC} \ll t_{ON}$*

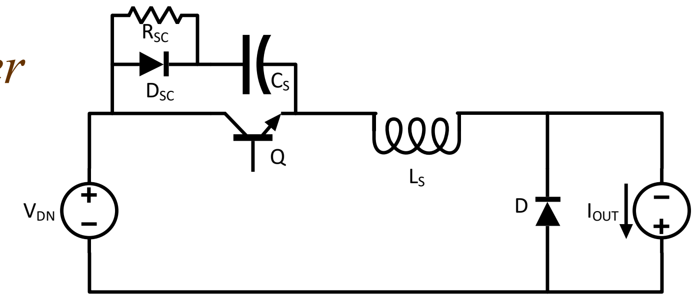
We now have two sources of energy that need to be dissipated

- $\mathcal{E}_{QS} = \frac{V_{IN}t_1}{2LI_{OUT}} \frac{V_{IN}I_{OUT}t_1}{12}$ *proportional to* $\frac{1}{L_S}$ *from the turn-on*
- $\mathcal{E}_L = \frac{1}{2}L_S I_{OUT}^2$ *proportional to L_S from the snubber*

L_S can be chosen such that $\mathcal{E}_{QS} + \mathcal{E}_L < \mathcal{E}_{Q0}$

Same mathematics for shunt turn-off (C) and series turn-on (L) snubbers

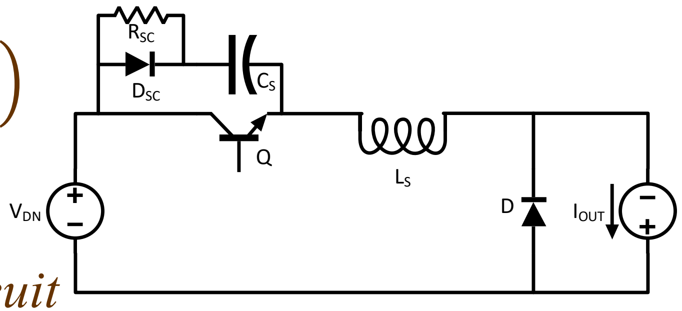
Combination Turn-on and Turn-off Snubber: Capacitor, Inductor, Resistor, Diode



We can reduce parts count in a combination snubber

Steps in the cycle:

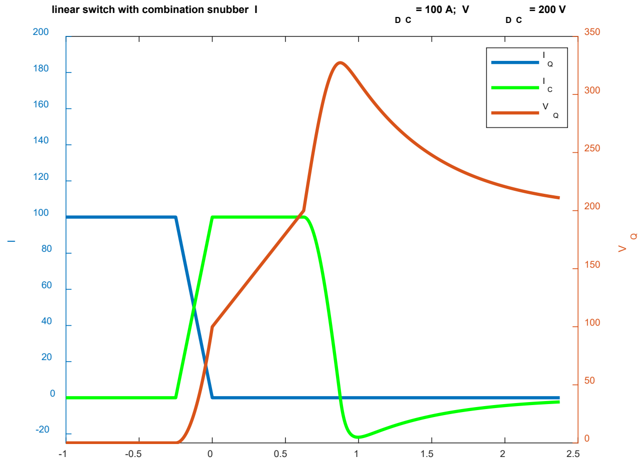
- *Q off:* $V_C = V_Q = V_{IN}$; $I_L = 0$;
- *Q turns on:*
  - *Same process as before: As* I_Q *increases,* $V_L = L\frac{dI_L}{dt} > 0 \Rightarrow V_Q$ *decreases faster*
- *Q on:* $V_C = V_Q = 0$; $I_L = I_{OUT}$
- *Q turns off:*
  - *Turn-off starts as before:* V_Q *increases* $\Rightarrow I_C = C_S\frac{dV_{C_S}}{dt} >$ $\Rightarrow I_Q$ *decreases faster than without* C_S
- *Difference at end of turn-off*



Diode turns on when $V_Q = V_{IN} \Rightarrow V_D = 0 \quad \left(\frac{dI_L}{dt} = 0\right)$

$I_L = I_{DC}$ *so this current needs to be dissipated*

$V_D = 0; \;\; V_{DSC} = 0 \Rightarrow I_L$ *flows in a series* $L - C$ *circuit*



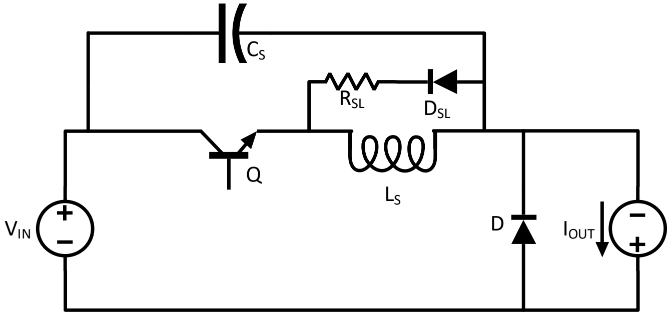
Recall dynamics of series resonant circuit

$$\omega_0 = \frac{1}{\sqrt{LC}}; \;\; Z_0 = \sqrt{\frac{L}{C}}$$

$$v_C(t) = V_{IN} + Z_0 I_{OUT} \sin \omega_0 t$$
$$i_L(t) = I_{OUT} \cos \omega_0 t$$

$v_C(t)$ *increases to* $V_{IN} + Z_0 I_{OUT}$ *when* $i_L = 0$

$i_L(t)$ *reverses sign,* D_{SC} *turns off and* $i_L(t)$ *exponentially damps through* R_{SC}



Disadvantage of this circuit is that $V_{Q_{BREAKDOWN}}$ *must be larger than* V_{IN}

Can also have combined snubber across L

Combination switcher

- When L and C are inserted in the circuit in series
    - Resonant behavior that causes voltage and current to ring in the circuit
    - Increased voltage and current stresses on semiconductor devices

We can further extend this concept

- Design circuit to further reduce losses
    - Configure $L - C$ circuit to ring through zero voltage or zero current
    - Turn switch on and off at zero crossings
    - Less loss per cycle enables circuit to operate at higher frequencies

Disadvantage

- Voltage and current stresses on the devices are much higher

Will work through zero voltage switching circuit; zero current switching is its dual

Resonant circuits have two natural parameters

$$\omega_0 = \frac{1}{\sqrt{LC}}; \quad Z_0 = \sqrt{\frac{L}{C}}$$

Behavior of circuits depends on initial conditions and sources

$$\begin{pmatrix} v_C(t) \\ i_L(t) \end{pmatrix} = \begin{pmatrix} \cos\omega_0 t & Z_0 \sin\omega_0 t \\ -\frac{\sin\omega_0 t}{Z_0} & \cos\omega_0 t \end{pmatrix} \begin{pmatrix} v_C(0) \\ i_L(0) \end{pmatrix}$$
$$+ \begin{pmatrix} (1-\cos\omega_0 t) & Z_0 \sin\omega_0 t \\ \frac{\sin\omega_0 t}{Z_0} & -(1-\cos\omega_0 t) \end{pmatrix} \begin{pmatrix} V_{IN} \\ I_{OUT} \end{pmatrix}$$

Coefficient signs depend on the orientations of the signals and sources.

Note the dual nature of $v_C(t)$ *and* $i_L(t)$.

Both are continuous since they evolve in t

- *From their initial values as* $\cos\omega_0 t$
- *With the initial value of the other variable as* $\sin\omega_0 t$

In soft switching applications our standard oscillator equations will evaluate to equations such as

$$v_C(t) = V_0 + Z_0 I_0 \sin \omega_0 t$$
$$i_L(t) = I_0 + (V_0/Z_0) \sin \omega_0 t$$

These equations will place conditions on values of V_0, I_0, Z_0 *that will allow soft switching, that is:* $v_C(t) = 0$ *and* $i_L(t) = 0$

Also recall (dual) linear charging relations:

Constant current charging a capacitor

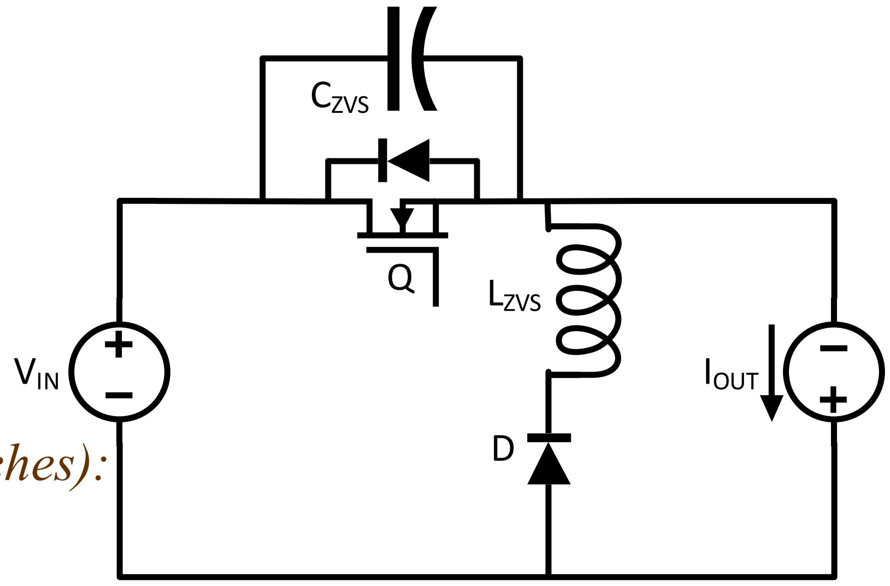
$$C \frac{dv_C(t)}{dt} = I_0 \Rightarrow v_C(t) = \frac{I_0}{C} t$$

Constant voltage charging an inductor

$$L \frac{di_L(t)}{dt} = V_0 \Rightarrow i_L(t) = \frac{V_0}{L} t$$

Power dissipation across semiconductor:

$$P_Q = f_{SW} \int_{t_{ON}}^{t_{OFF}} v_Q(t) i_Q(t) dt$$



Control Q to switch when $v_Q(t) = 0$

Theory of operation (assume perfect switches):

- Put series $L - C$ around transistor
- Let $v_Q(t)$ ring and turn on Q when $v_Q(t) = 0$ (note reverse diode across Q)

Four states of operation:

- Q on, D off: $v_Q = v_C = 0$; $v_D < 0$; $i_Q = I_{OUT}$; $i_C = i_L = 0$
- Q off, D off (charge v_C): $v_Q = v_C > 0$; $v_D < 0$; $i_C = I_{OUT}$; $i_Q = i_L = 0$
- Q off, D on (resonant state): $v_Q = v_C \neq 0$; $v_D = 0$; $i_Q = 0$; $i_C - i_L = I_{OUT}$
- Q on, D on (discharge i_L): $v_Q = v_C = 0$; $v_D = 0$; $i_C = 0$; $i_Q - i_L = I_{OUT}$;

Q on, D off; $v_Q = 0;\ i_Q = I_{OUT};\ v_O(t) = V_{IN};\ \ t_1$ *open*

Q off, D off; $V_Q \neq 0;\ \ i_C = I_{OUT}\ \ \Rightarrow$

$$v_C(t) = \frac{I_{OUT}}{C}t, 0 \le t \le \frac{CV_{IN}}{I_{OUT}} = t_2$$

$$v_O(t) = V_{IN} - v_C(t)$$

Q off, D on; $V_C = V_{IN} \Rightarrow V_D = 0 \Rightarrow$ *resonant circuit;*

$$\omega_0 = 1/\sqrt{LC};\ \ Z_0 = \sqrt{L/C};$$

$$v_C(0) = V_{IN};\ \ i_L(0) = 0;$$

$$v_C(t) = V_{IN} + (v_C(0) - V_{IN})\cos\omega_0 t$$

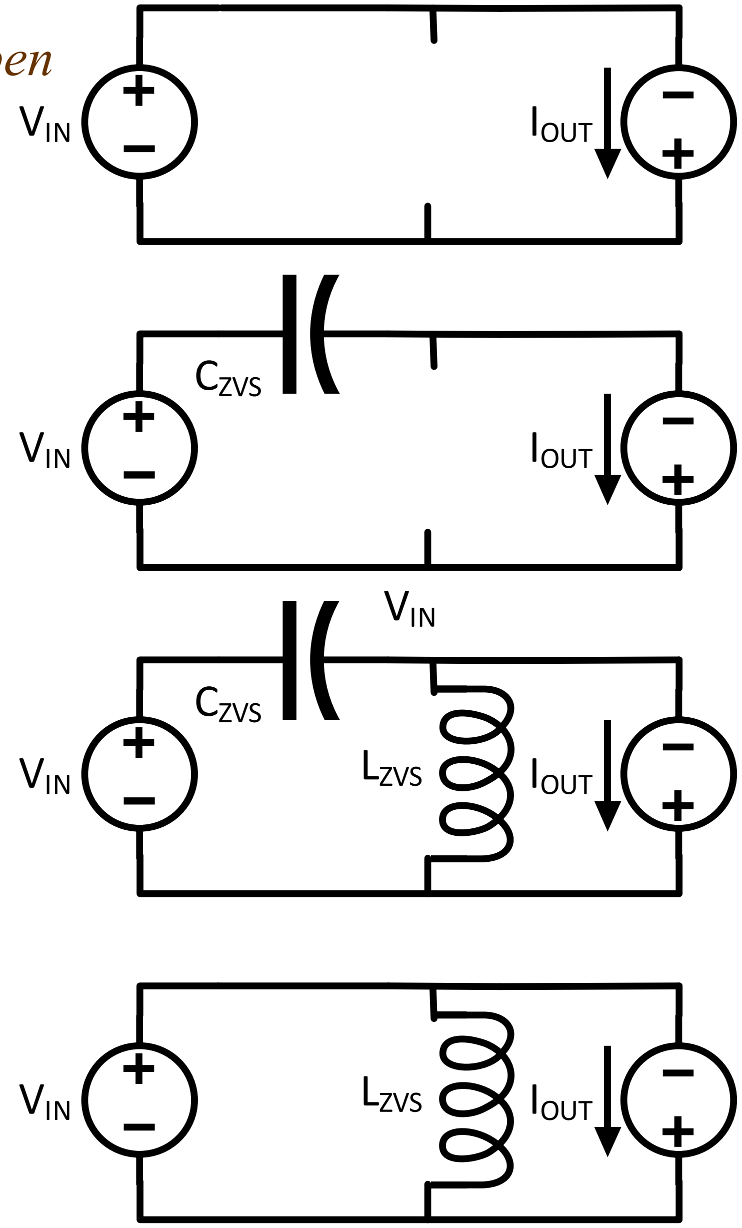
$$+Z_0 I_{OUT}\sin\omega_0 t$$

$$v_C(t) = V_{IN} + Z_0 I_{OUT}\sin\omega_0 t$$

Q on, D on;

$$i_L(t) = \frac{V_{IN} - v_C(0)}{Z_0}\sin\omega_0 t - (1 - \cos\omega_0 t)I_{OUT}$$

$$i_L(t) = (\cos\omega_0 t - 1)I_{OUT}$$



$v_C(t) = V_{IN} + Z_0 I_{OUT} \sin\omega_0 t\,;\;\; v_Q(t) = V_{IN} - v_C(t)$
$i_L(t) = (\cos\omega_0 t - 1) I_{OUT}$

$v_C(t) = 0$ *when* $\sin\omega_0 t = -V_{IN}/(Z_0 I_{OUT})$

Requires $Z_0 > \dfrac{V_{IN}}{I_{OUT}}$

Two zero crossings (t_{3i}, t_{3f}) *will occur:*

- *One on the way down and one on the way up*

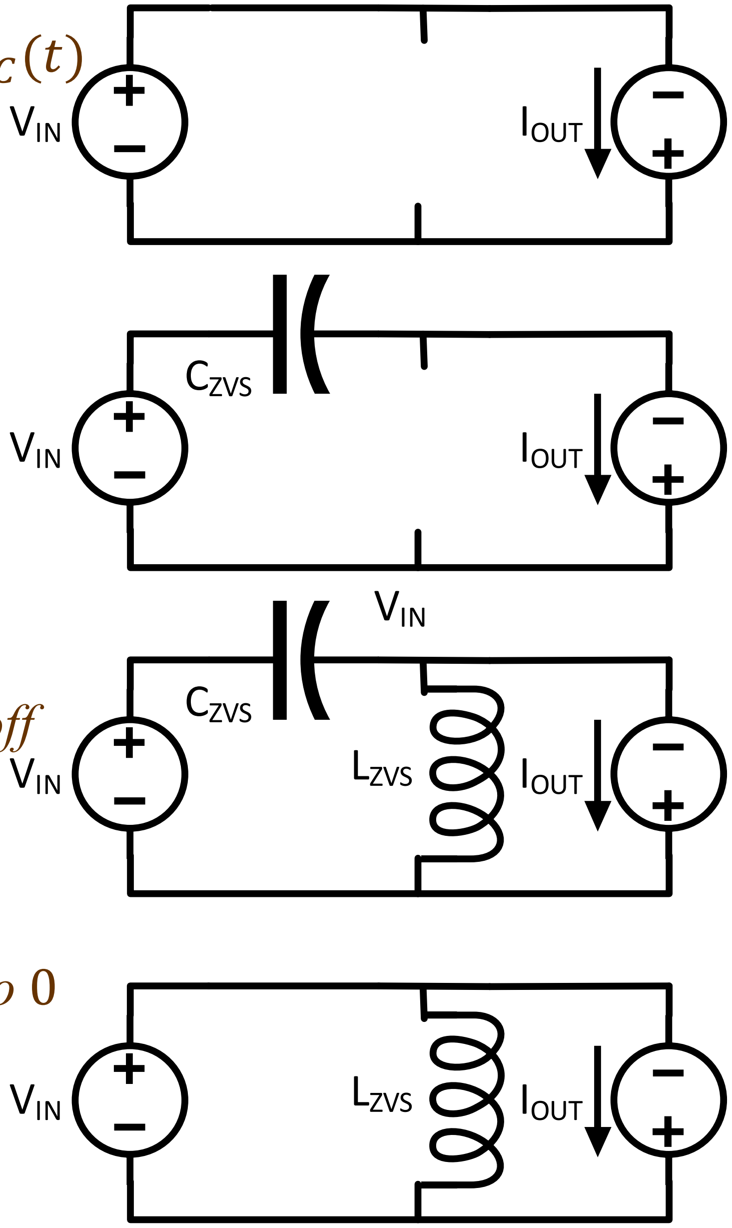
$t_{3i} = \omega_0^{-1}\{\pi - \arcsin[-V_{IN}/(Z_0 I_{OUT})]\}$

C_{ZVS} *across Q already has ZVS switching at turn-off (see snubber section above)*

Now turn on Q when $v_C(t) = 0;\;\; t_{3i} \le t \le t_{3f}$

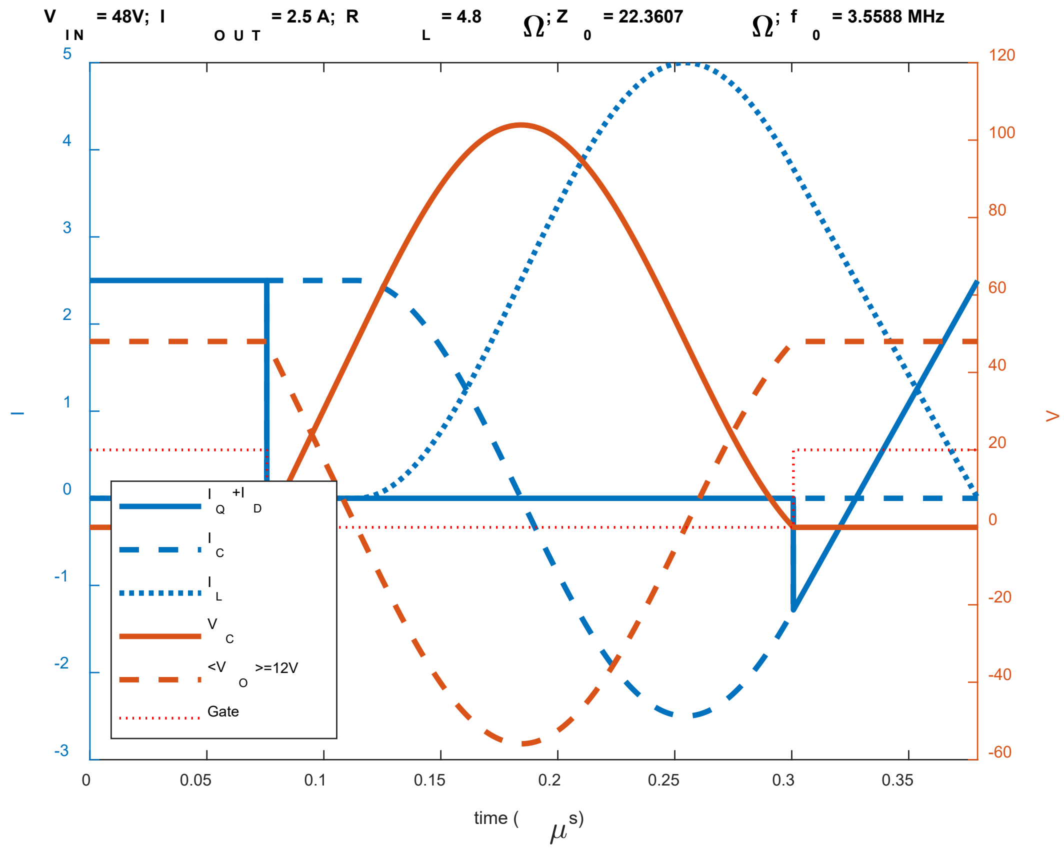
- *Note: Reverse diode across Q clamps voltage to* 0

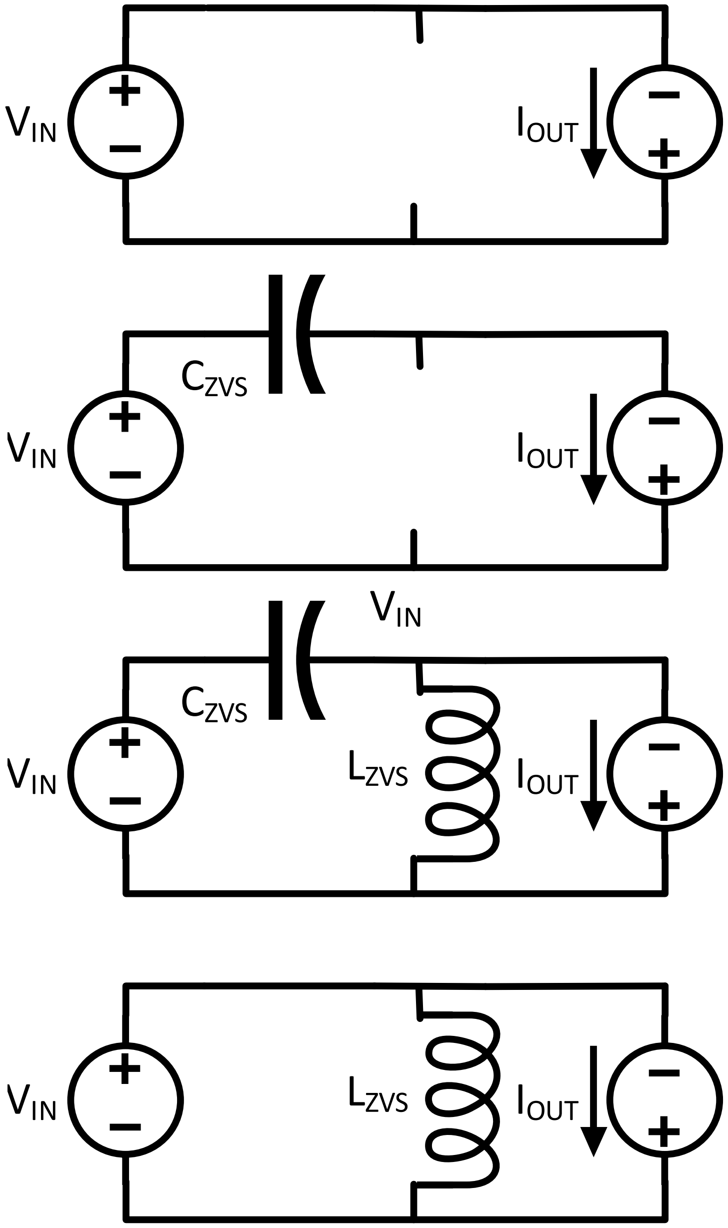
$i_L(t)$ *rings until* t_{3i}*, supplying* $C\dot{v}_C$ *and* I_{OUT}
$i_L(t_{3i}) = (\cos\omega_0 t_{3i} - 1) I_{OUT}$



Q *on;* D *on;* V_{IN} *discharges* $i_L(t)$ *until* $i_L(t) = 0$

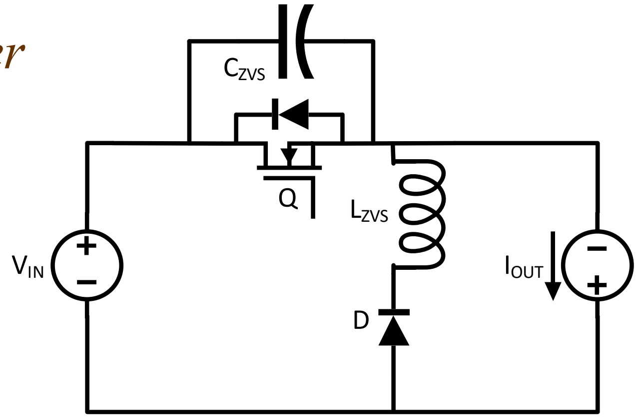
$$i_L(t) = i_L(t_{3i}) + (V_{IN}/L)t$$
$$= (\cos\omega_0 t_{3i} - 1)I_{OUT} + (V_{IN}/L)t$$
$$\Rightarrow t_4 = (1 - \cos\omega_0 t_{3i})(LI_{OUT}/V_{IN})$$
$$v_O(t) = V_{IN}$$





ZVS does not have a fixed period; t_1 *a free parameter*

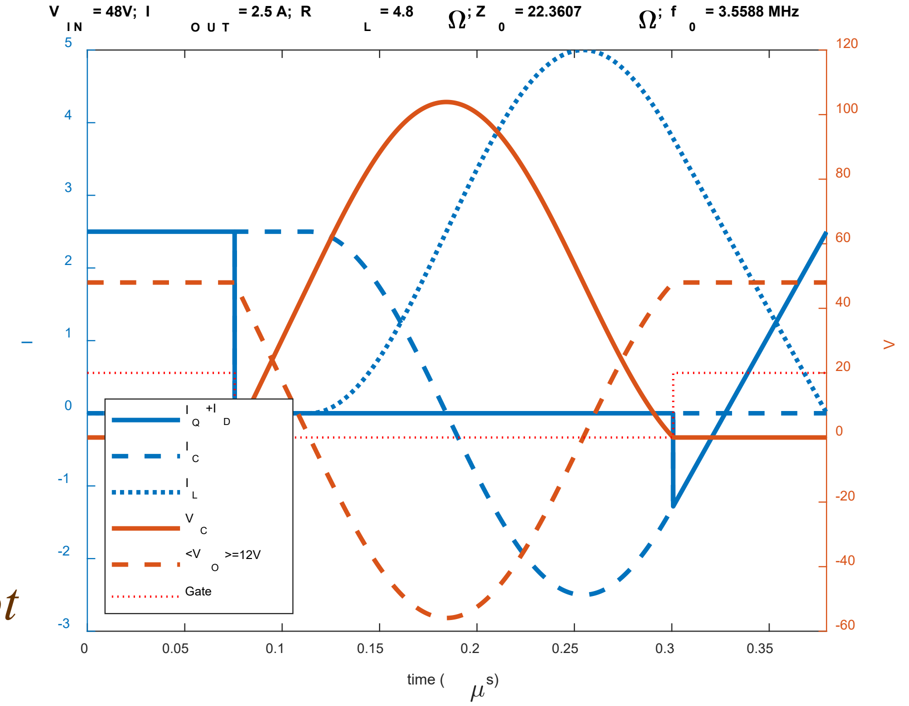
- *Off time determined by* $Z_0, \omega_0, V_{IN}, I_{OUT}$
- *On time determined by required* $V_{OUT} = R_L I_{OUT}$



V_{OUT} *decreases as* f_{ZVS} *increases*

$$< v_0 > = V_{IN} - \alpha_{ZVS}(Z_0, \omega_0, V_{IN}, I_{OUT}) f_{ZVS}$$

where α_{ZVS} *depends on the system.*

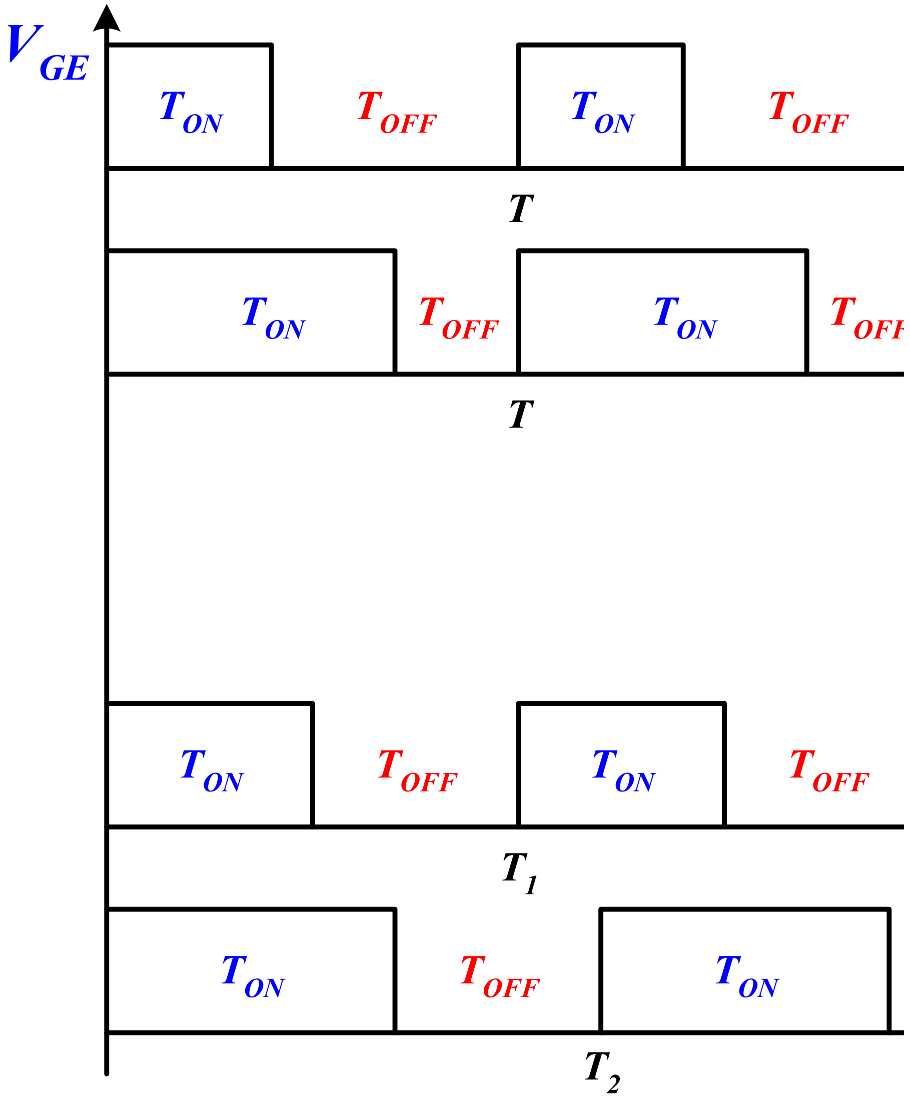


Disadvantages:

- (V, I) *stresses on* $(Q, D) > 2\times$
- *ZVS mode only works for a limited range of* V_{IN} *and* I_{OUT}
- *Losses still exist*

Most useful when voltage stresses are not an issue.

Reducing Switch Losses By Resonant Switching

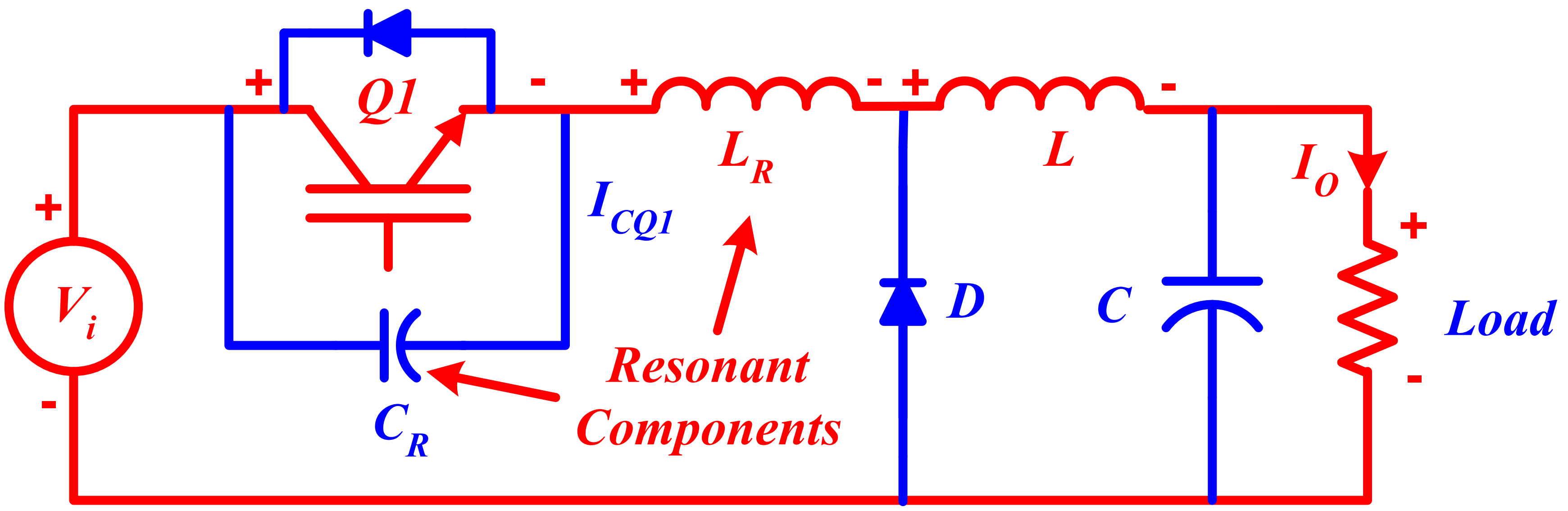


Fixed Frequency Switching

- T_{on} and T_{off} vary

ZVS Resonant Mode Switching

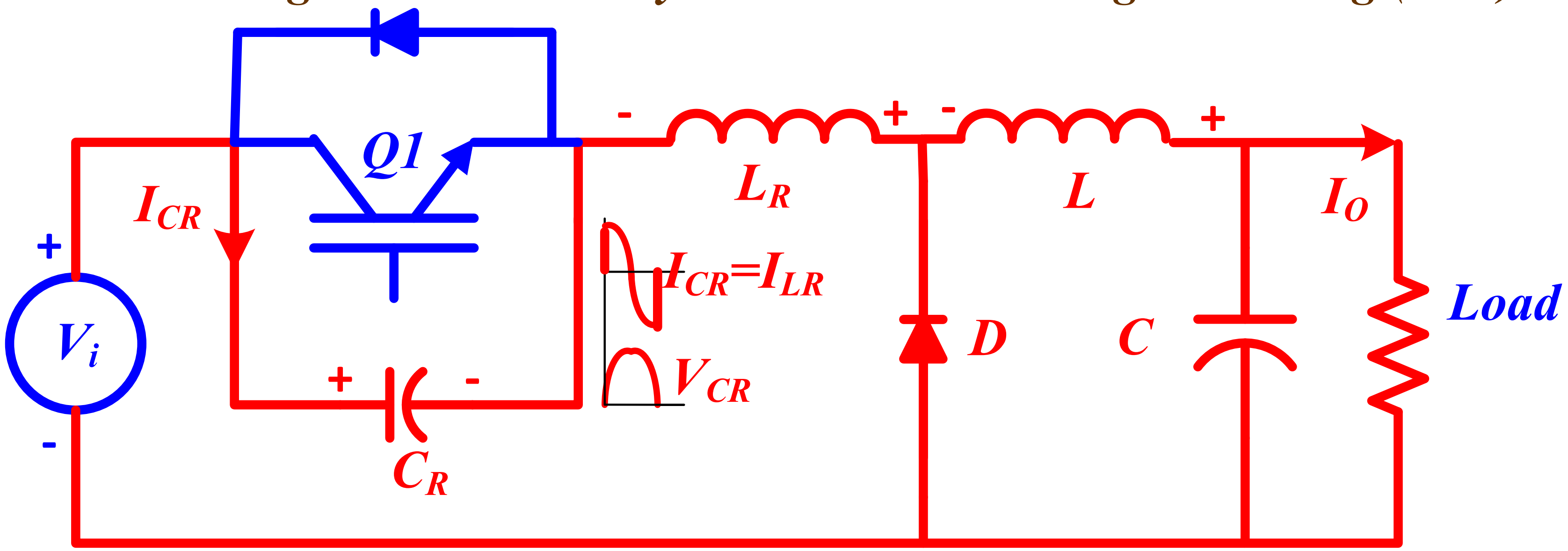
- Frequency varies
- T_{on} varies
- T_{off} fixed to accommodate resonant circuit
- Conversion frequency inversely proportional to load current



Time Interval 1

- Q1 has been closed and is carrying load current. D and C do not have current flow in this steady-state condition.
- V_{CR}=0 and I_{CR}=0 as it has been sinusoidally discharged
- Note that $V_{CR} = V_{CEQ1}$ and $I_{CQ1}=I_{LR}$

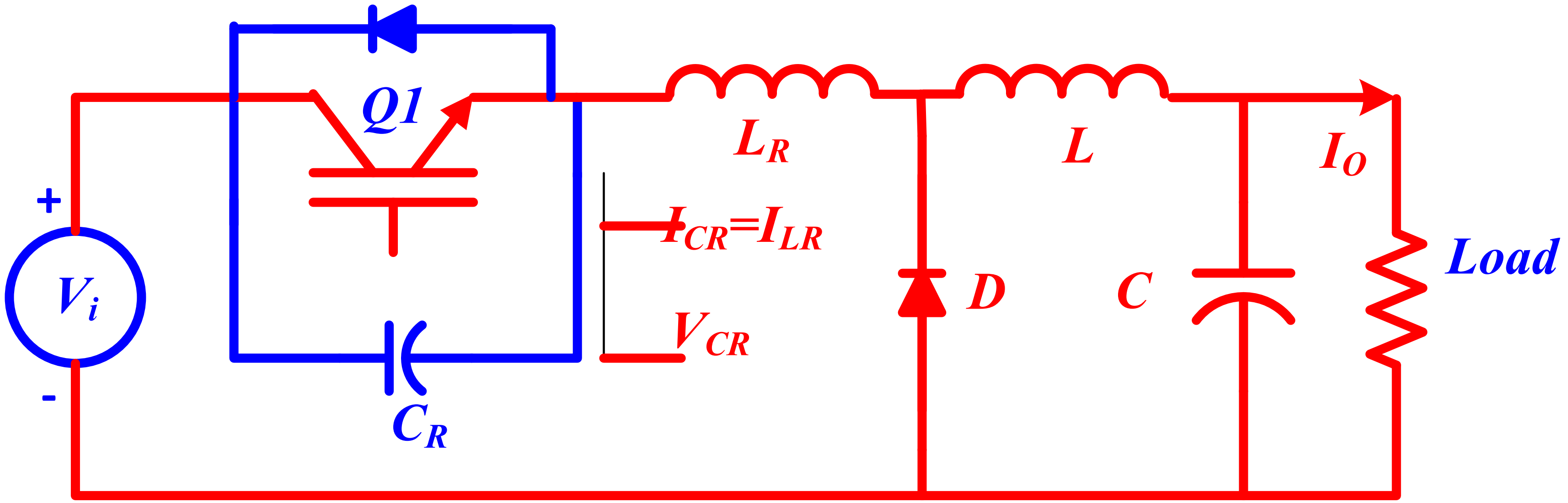
Reducing Switch Losses By Resonant Zero Voltage Switching (ZVS)



Time Interval 2

- Q1 is opened. Diode D conducts
- Current commutates (rushes) into C_R
- C_R charges and discharges sinusoidally with frequency determined by C_R and L_R . 1/2 sine wave occurs
- V_{CR} is sine wave , I_{CR} is cosine wave = C dV_{CR} / dt
- $V_{CEQ1} = V_{CR}$
- $I_{CR} = I_{LR}$

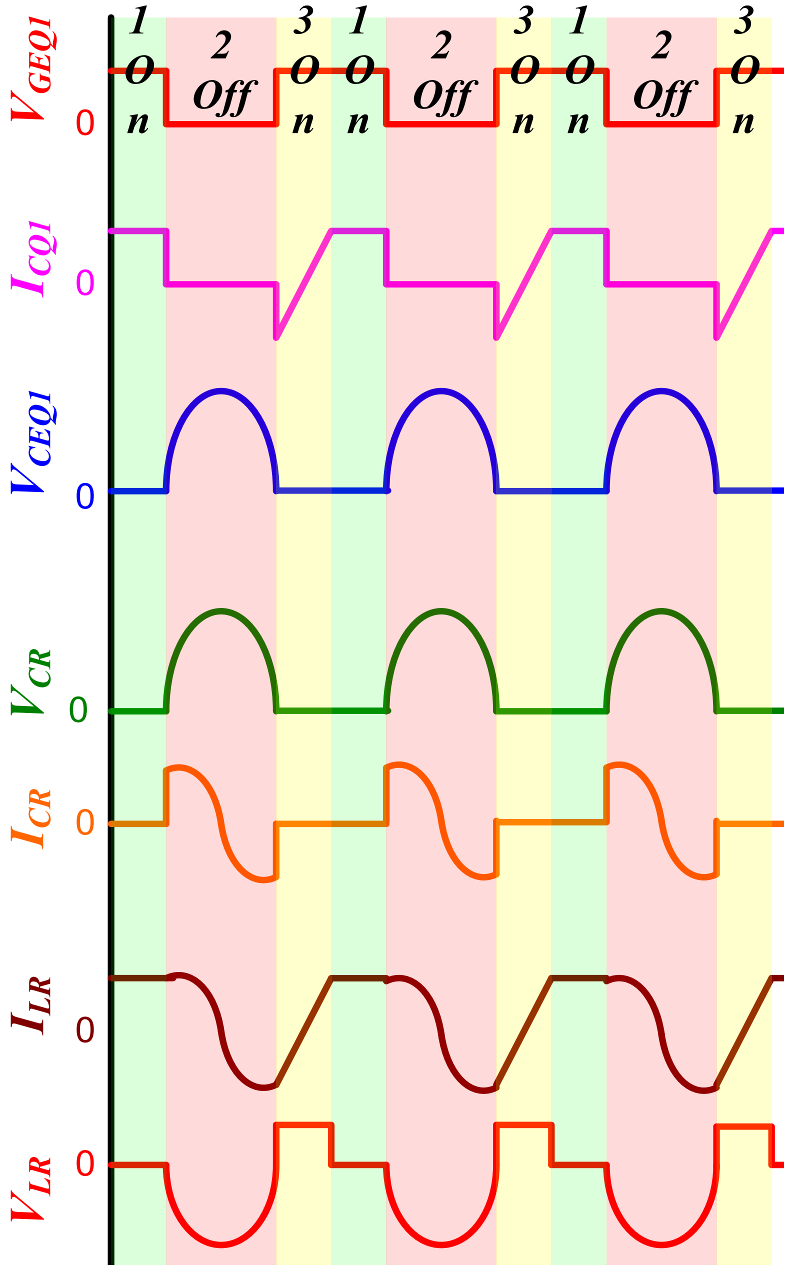
Reducing Switch Losses By Resonant Zero Voltage Switching (ZVS)

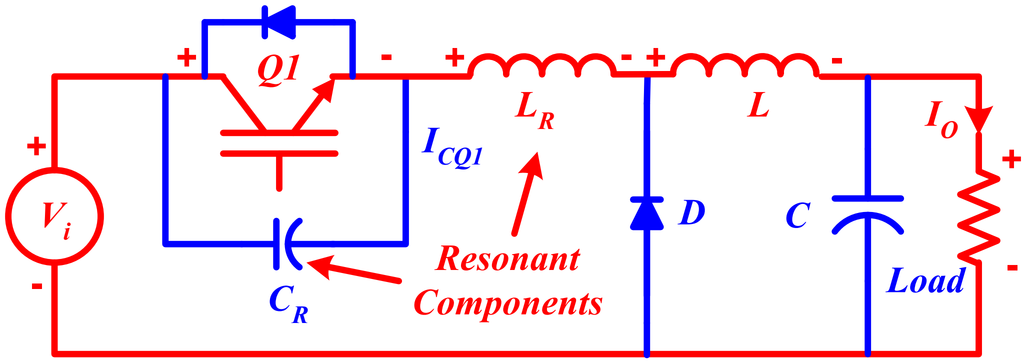


Time Interval 3

- *When V_{CR} discharges to 0 (V_{CEQ1}=0) , Q1 is re-closed.*
- $I_{CQ1} = I_{LR}$
- *There is a linear current buildup in Q1 due to L_R and L*

ZVS Waveforms





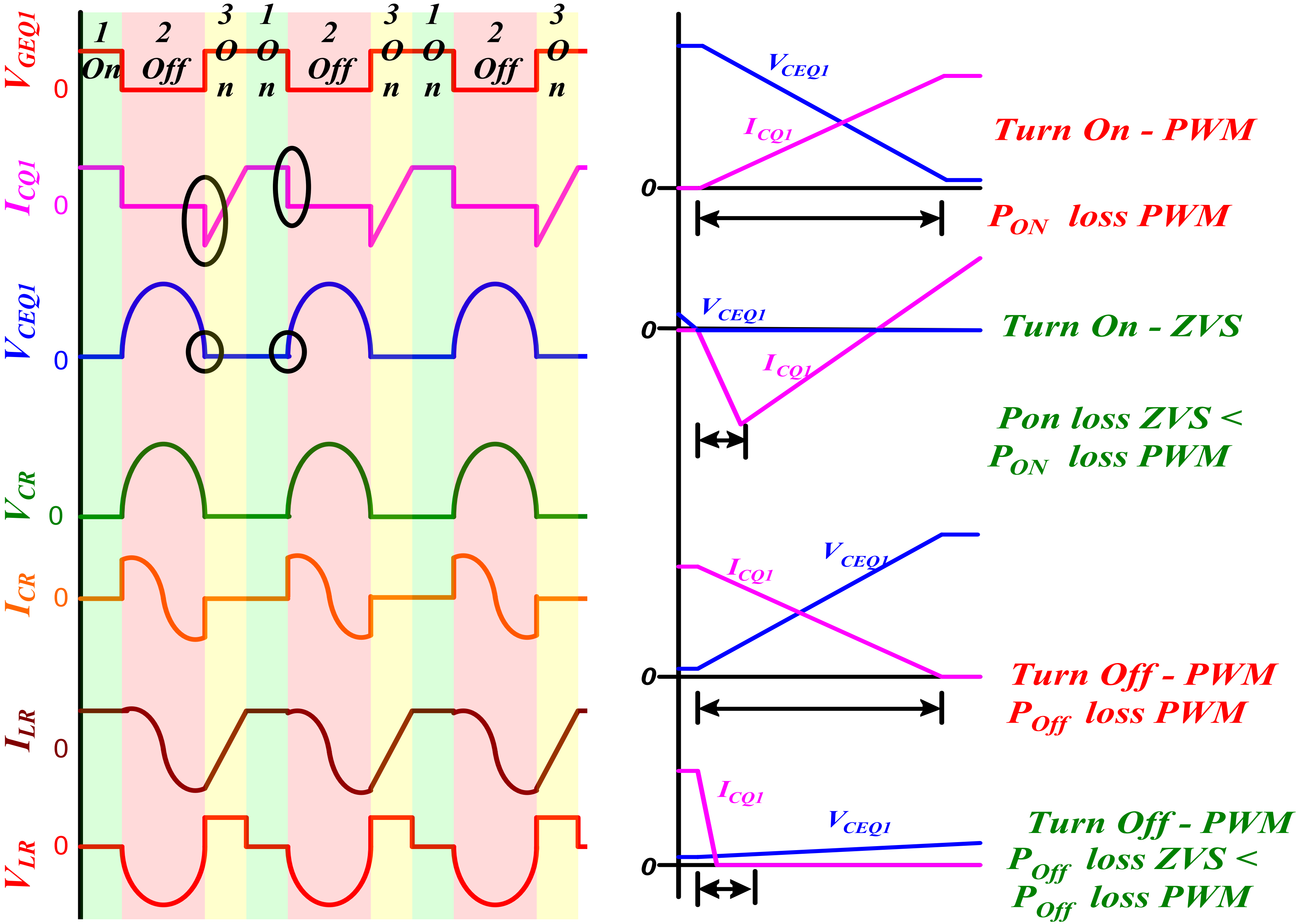
Q1 On

- $V_{CEQ1} = V_{CR} \approx 0$
- $I_{CQ1} = I_{LR}$
- $I_{CR}=0$
- $V_{LR}=L*dI_{LR} / dt=0$

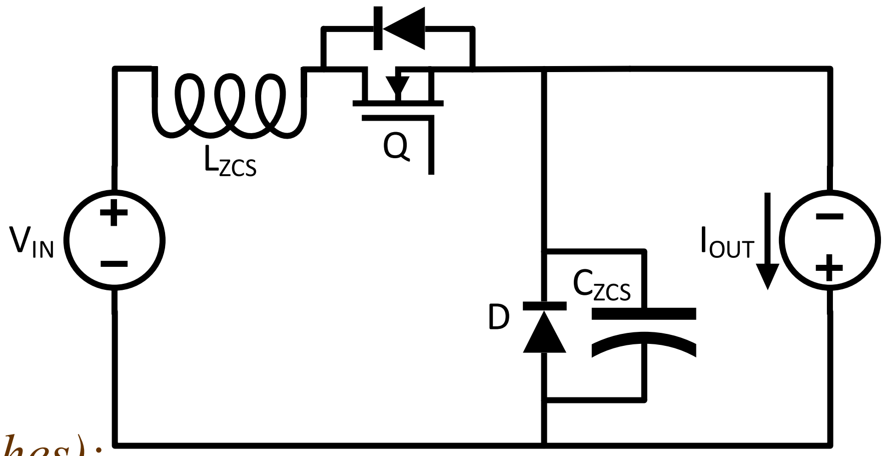
Q1 Off

- $V_{CEQ1} = V_{CR}$
- $I_{CR} = I_{LR}=C*dV_{CR} / dt$

ZVS Waveforms



Soft Switching: Zero Current Switching



Power dissipation across semiconductor:

$$P_Q = f_{SW}\int_{t_{ON}}^{t_{OFF}} v_Q(t)i_Q(t)dt$$

Control Q to switch when $i_Q(t) = 0$

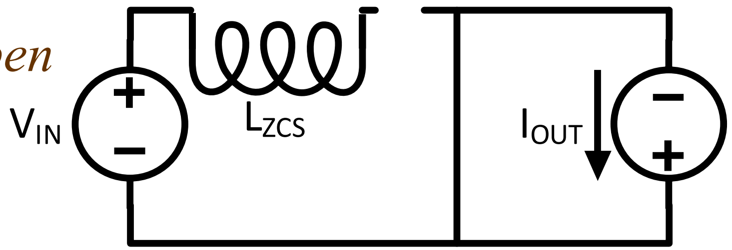
Theory of operation (assume perfect switches):

- *Put series* $L - C$ *around transistor*
- *Let* $i_Q(t)$ *ring and turn on Q when* $i_Q(t) \leq 0$ *(note reverse diode across Q)*

Four states of operation:

- *Q off, D on:* $v_Q = V_{IN};\ v_D = 0;\ i_D = I_{OUT};\ i_Q = 0;\ i_C = 0$
- *Q on, D on (charge* i_L*):* $v_Q = 0;\ v_D = 0;\ i_L = i_Q > 0;\ I_{OUT} = i_L + i_D$
- *Q on, D off (resonant state):* $v_Q = 0;\ v_D < 0;\ v_Q = 0;\ i_L = i_C + I_{OUT}$
- *Q off, D off (discharge* v_C*):* $v_Q = V_{IN};\ V_D < 0;\ i_C = -I_{OUT};$

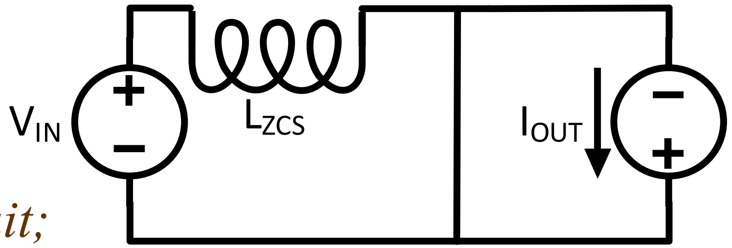
Q *off,* D *on;* $i_Q = 0;\ i_D = I_{OUT};\ \ v_O(t) = 0;\ \ t_1$ *open*



Q *on,* D *on;* $v_Q = 0;\ \ v_D = 0\ \Rightarrow$

$$i_L(t) = \frac{V_{IN}}{L} t, 0 \le t \le \frac{L I_{OUT}}{V_{IN}} = t_2$$

$$v_O(t) = 0$$



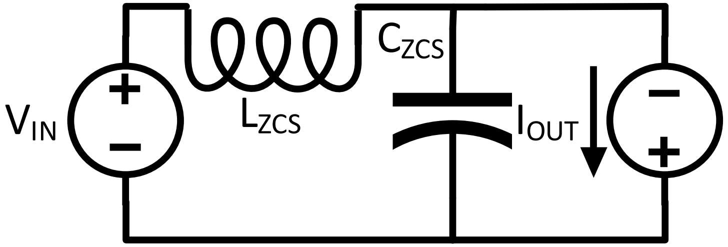
Q *on,* D *off;* $i_L = I_{OUT} \Rightarrow V_D < 0 \Rightarrow$ *resonant circuit;*

$$\omega_0 = 1/\sqrt{LC};\ \ Z_0 = \sqrt{L/C};$$

$$i_L(0) = I_{OUT};\ \ v_C(0) = 0;$$

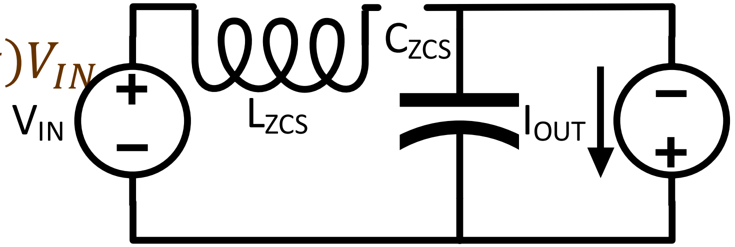
$$i_L(t) = I_{OUT} + (i_L(0) - I_{OUT}) \cos \omega_0 t$$
$$+ (V_{IN}/Z_0) \sin \omega_0 t$$

$$i_L(t) = I_{OUT} + (V_{IN}/Z_0) \sin \omega_0 t$$



$$v_C(t) = Z_0(i_L(0) - I_{OUT}) \sin \omega_0 t + (1 - \cos \omega_0 t) V_{IN}$$

$$v_C(t) = (1 - \cos \omega_0 t) V_{IN}$$



$i_L(t) = I_{OUT} + (V_{IN}/Z_0)\sin\omega_0 t$
$v_C(t) = (1 - \cos\omega_0 t)V_{IN};\;\; v_O(t) = v_C(t)$

$i_L(t) = 0$ *when* $\sin\omega_0 t = -Z_0 I_{OUT}/V_{IN}$

Requires $Z_0 < \frac{V_{IN}}{I_{OUT}}$ *(opposite of ZVS)*

Two zero crossings (t_{3i}, t_{3f}) *will occur:*

- *One on the way down and one on the way up*

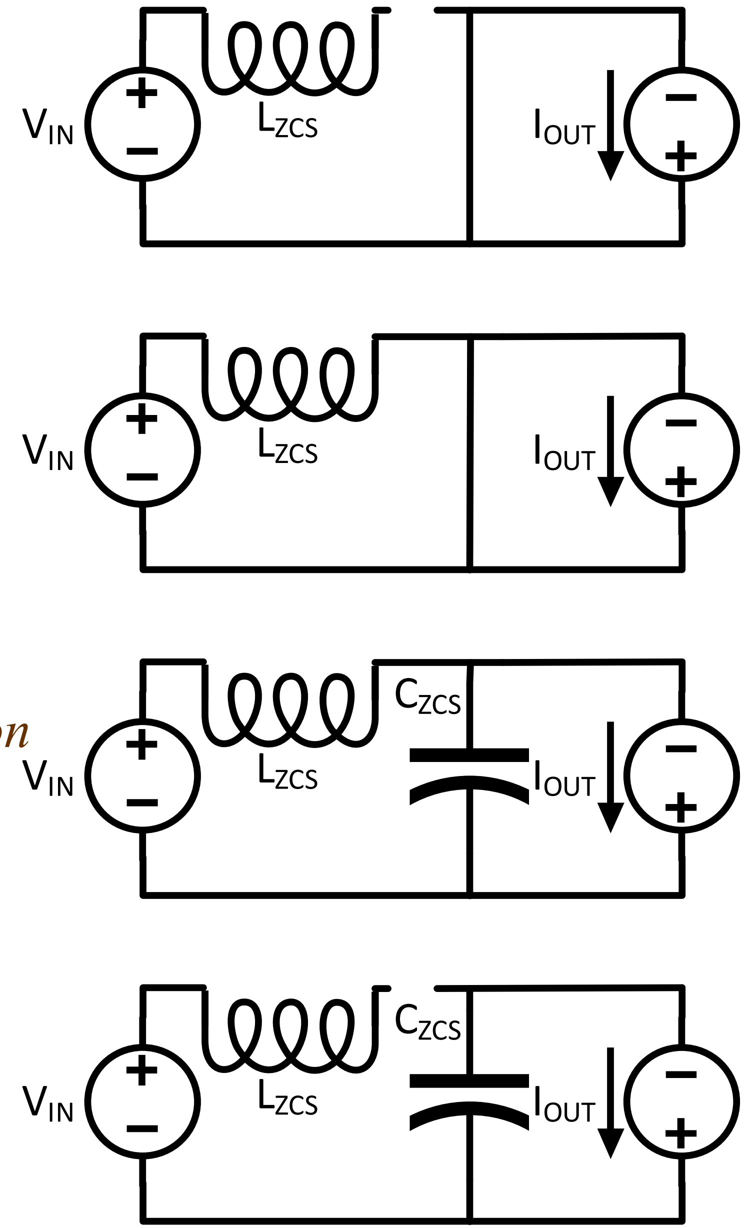
$t_{3f} = \omega_0^{-1}\{\arcsin[-Z_0 I_{OUT}/V_{IN}] + 2\pi\}$

I_{ZCS} *before Q already has ZCS switching at turn-on (see snubber section above)*

Now turn on Q when $i_L(t) \le 0;\;\; t_{3i} \le t \le t_{3f}$

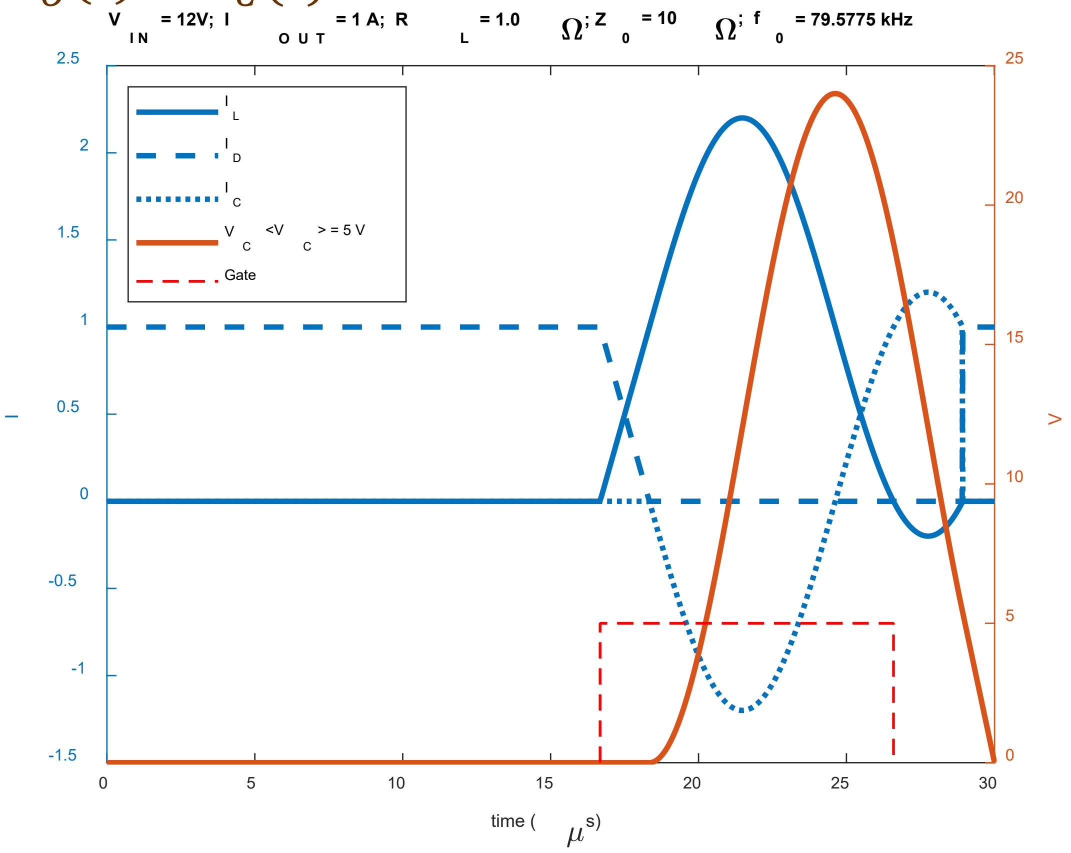
- *Note: Reverse diode allows* $i_L(t) \le 0$

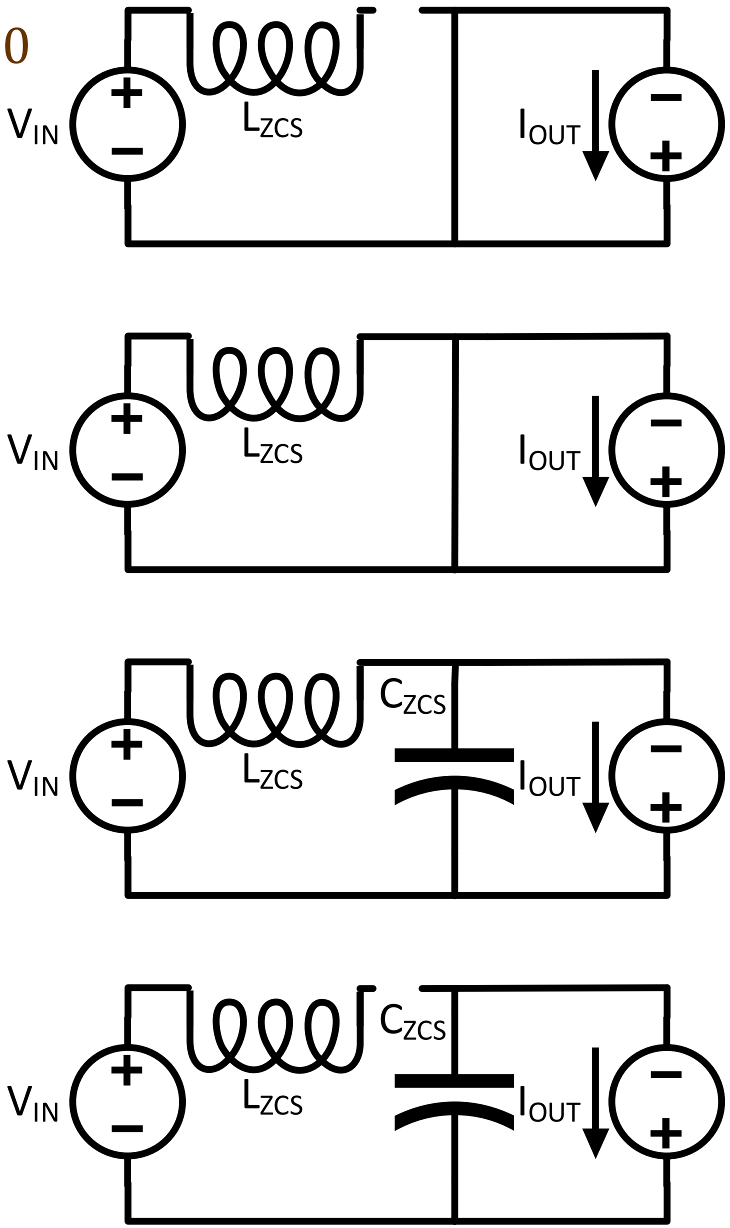
$v_C(t)$ *rings until* t_{3f}, $C\dot{v}_C = i_L - I_{OUT}$
$v_C(t_{3f}) = (1 - \cos\omega_0 t_{3f})V_{IN}$



Q *off*; D *off*; I_{OUT} *discharges* $v_C(t)$ *until* $v_C(t) = 0$

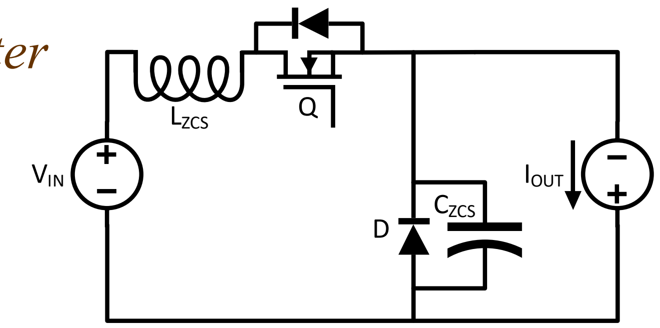
$$v_C(t) = v_C(t_{3f}) - (I_{OUT}/C)t$$
$$= (1 - \cos\omega_0 t_{3i})V_{IN} - (I_{OUT}/C)t$$
$$\Rightarrow t_4 = (1 - \cos\omega_0 t_{3i})(CV_{IN}/I_{OUT})$$
$$v_O(t) = v_C(t)$$





ZCS does not have a fixed period; t_1 a free parameter

- *On time determined by $Z_0, \omega_0, V_{IN}, I_{OUT}$*
- *Minimum period T_{ZCS}, maximum f_{ZCS} exist*
- *Off time determined by required $V_{OUT} = R_L I_{OUT}$*



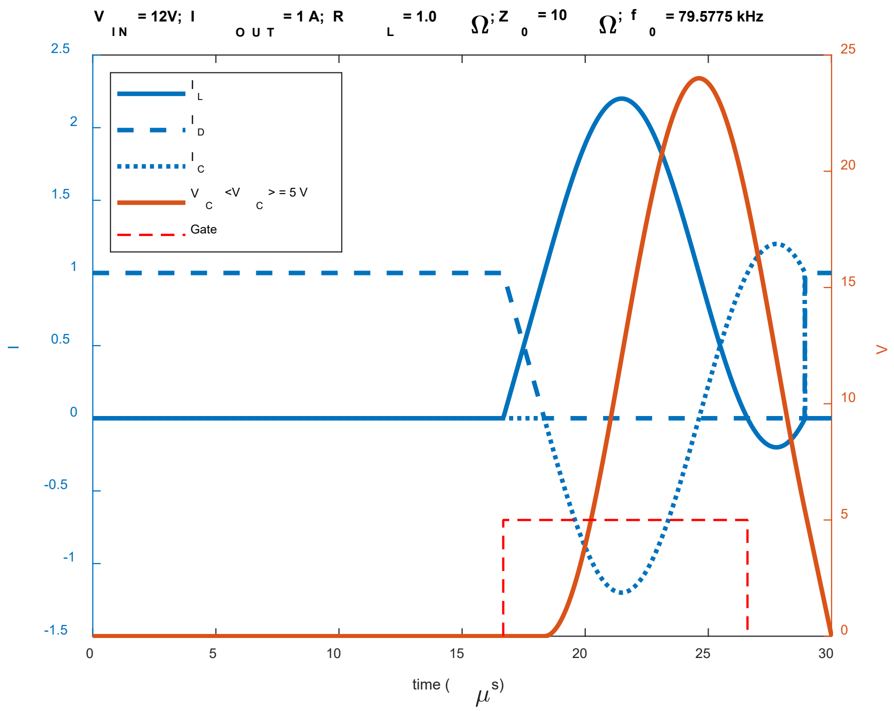
V_{OUT} increases as f_{ZCS} increases

$$< v_0 > = \alpha_{ZCS}(Z_0, \omega_0, V_{IN}, I_{OUT}) f_{ZCS}$$

where α_{ZCS} depends on the system.

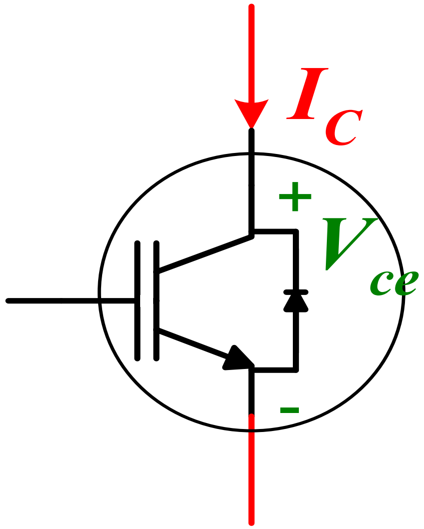
Disadvantages:

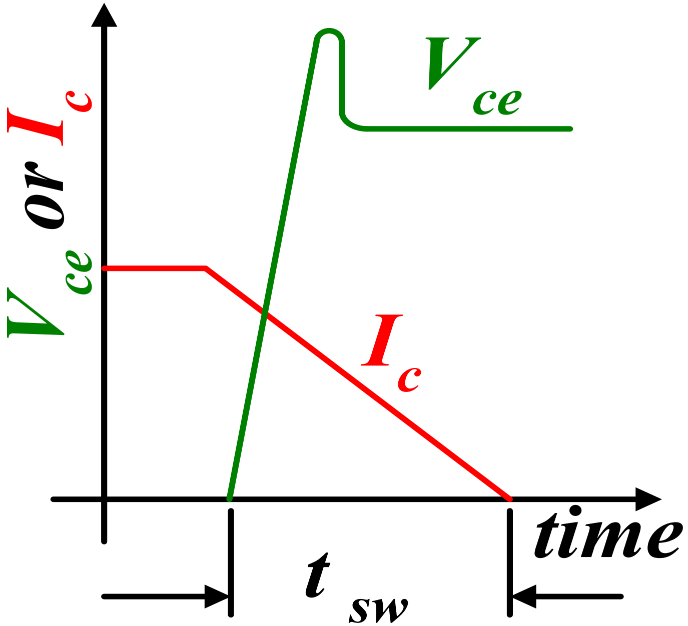
- *(V, I) stresses on $(D, Q) > 2\times$*
- *ZCS mode only works for a limited range of V_{IN} and I_{OUT}*
- *Losses still exist*



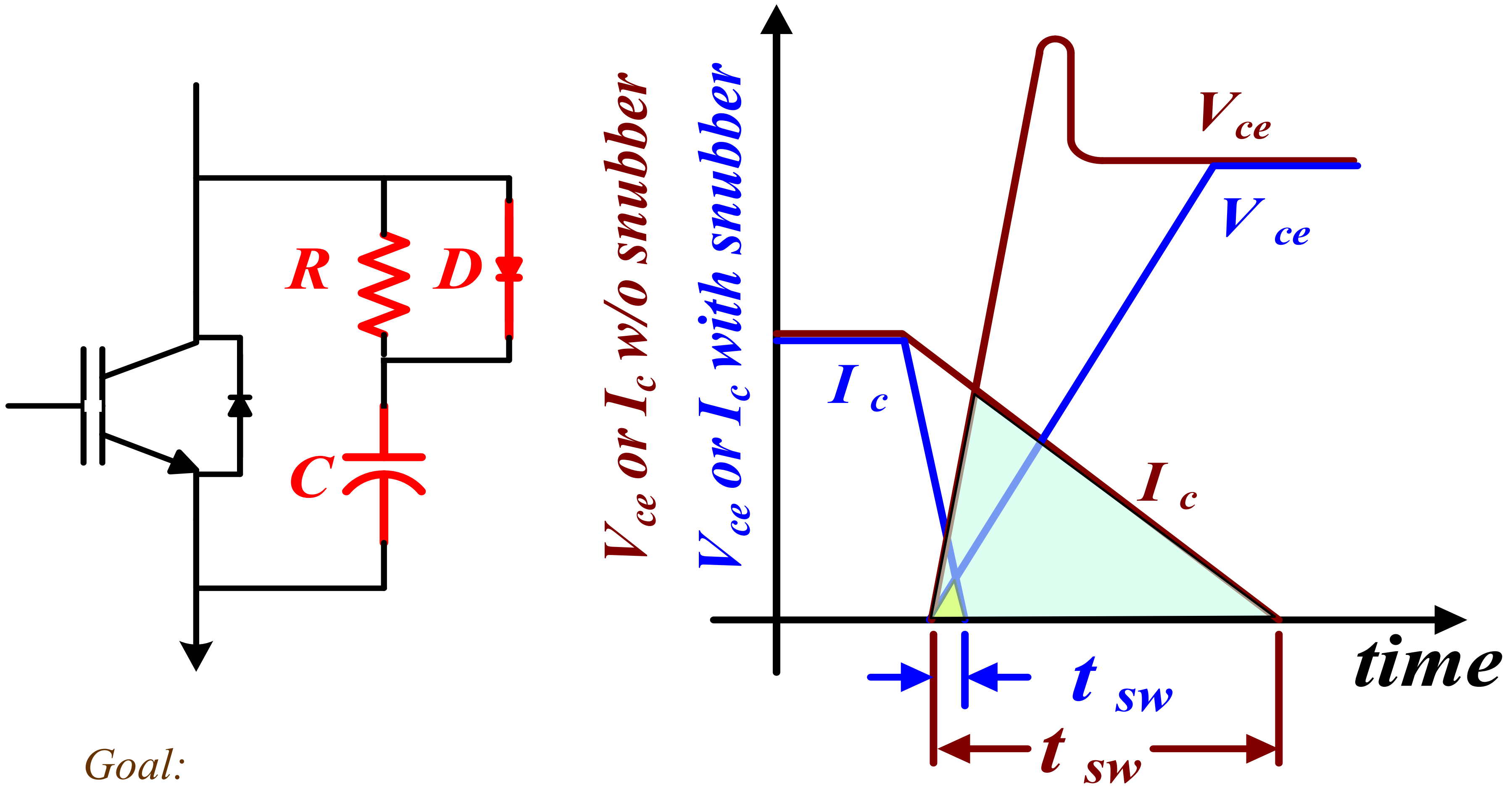
Most useful when component stresses are not an issue.

Switch Turnoff Loss Reduction By RCD Snubber



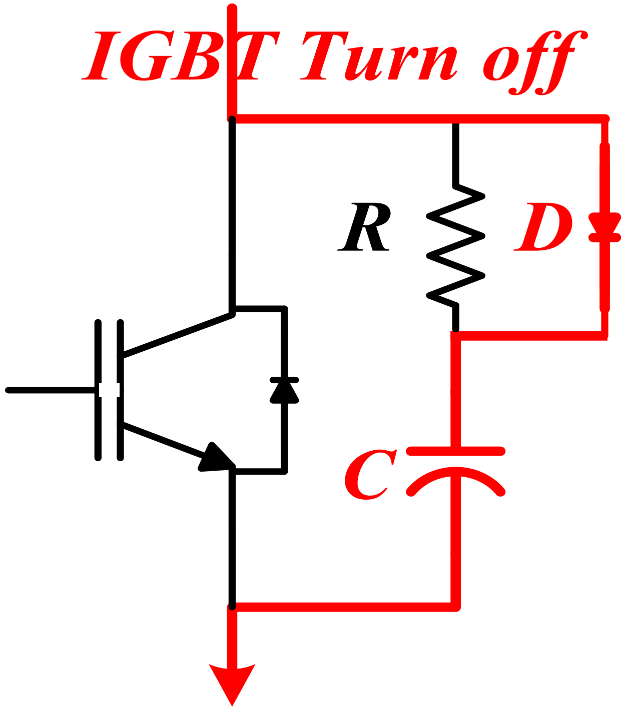


$$P_{SwOff} = \frac{1}{T} \int_0^{t_{Sw}} v_{CE}(t)\, i_C(t)\, dt$$

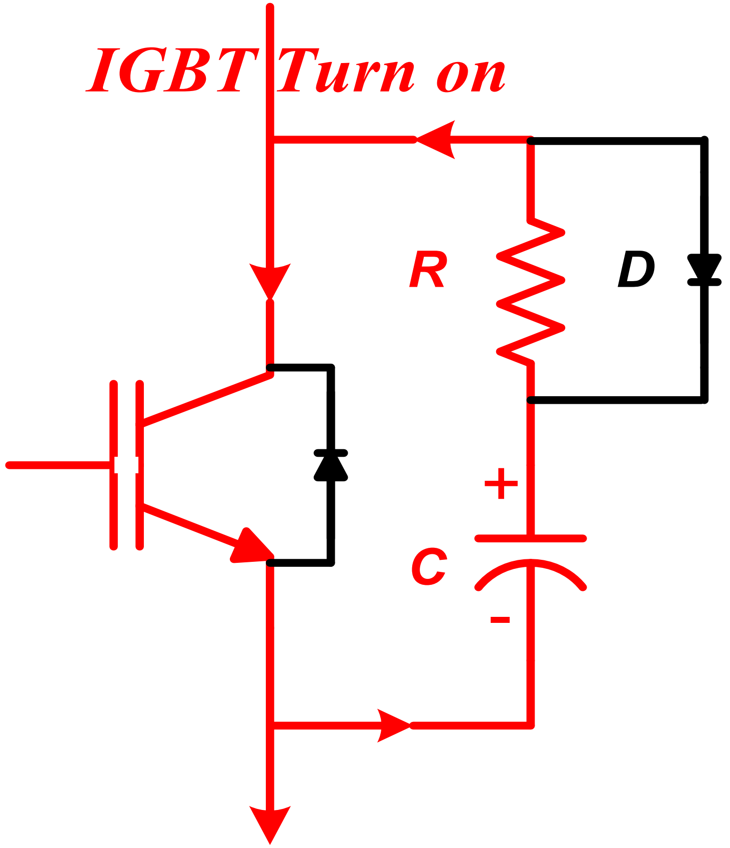


Goal:

- To increase the rate of decay of I_C during turnoff
- To decrease the rate of V_{CE} build up during turnoff
- To realize goal, add a resistor R, capacitor C, diode D snubber network

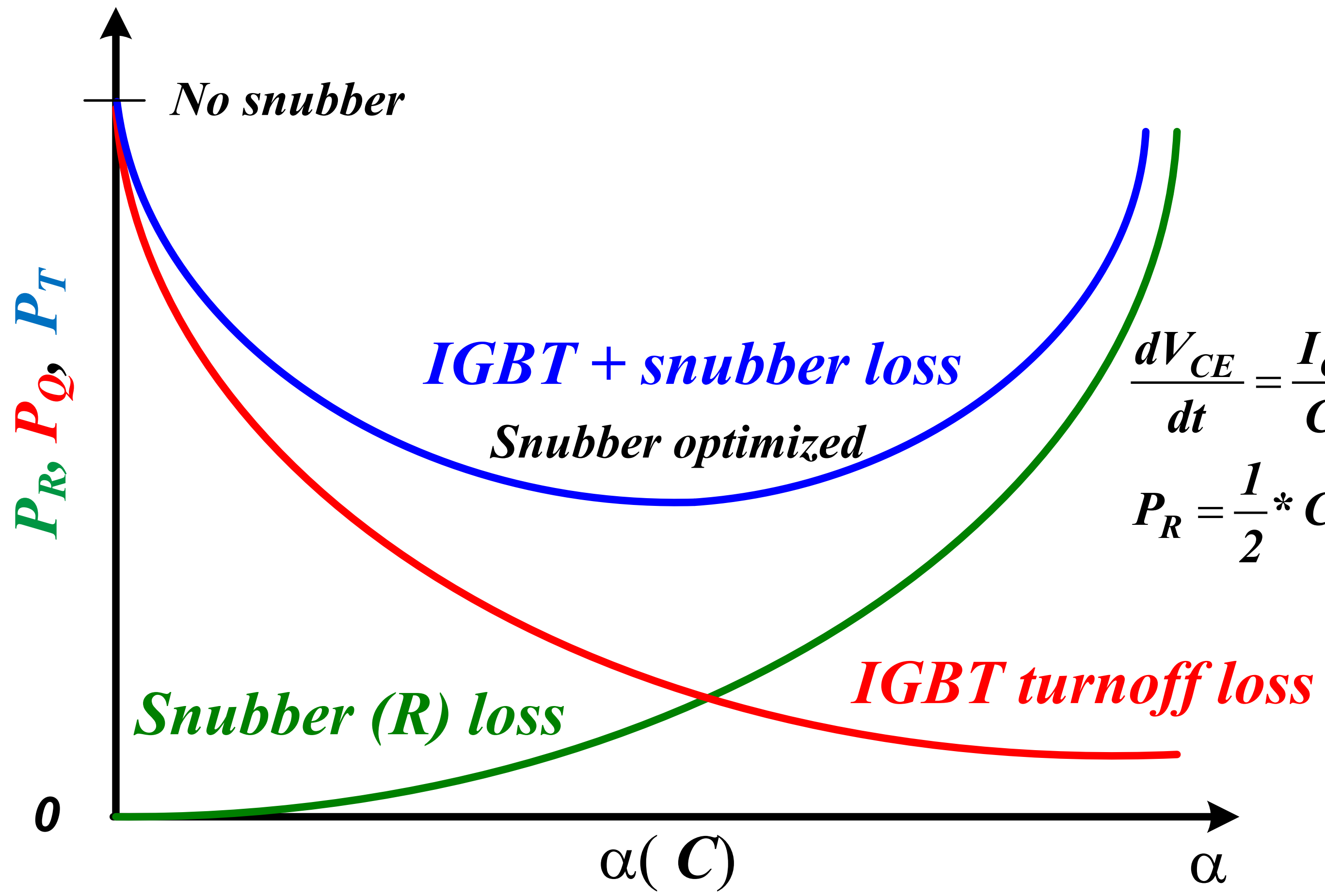


- *When the IGBT turns off, current commutates out of the IGBT into the capacitor, C via the diode D*
- *This aids fast I_C current decay*
- *C becomes linearly charged to the bus voltage*
- *dV_{CE} / dt inversely proportional to C – this slows V_{CE} recovery*



- When the IGBT turns on, the capacitor C, discharges through R and the IGBT

Switch Turnoff Loss Reduction By RCD Snubber



$$\frac{dV_{CE}}{dt} = \frac{I_C}{C}$$

$$P_R = \frac{1}{2} * C * V^2 * f$$

- Small C = fast dV_{CE}/dt, V appears with current still in the IGBT, have IGBT loss
- Large C means slow dV_{CE}/dt, current gone before voltage buildup but the resistor losses are high
- When the snubber circuit is optimized, the IGBT turnoff loss with snubber + snubber loss < IGBT loss w/o snubber !

Design criteria

- *R must limit discharge I through IGBT to < IGBT rating*
- $P_R \geq E_C / T = 1/2\, C\, V^2 f$
- *C ripple current rating* $\geq \Sigma$ *(ave charge + ave discharge currents)*
- *C must appreciably discharge each cycle, so R C < minimum expected IGBT on time*
- *D has to be rated to hold off the bus voltage and carry peak capacitor charging current*

Note: *Turn-on losses in the latest IGBTs have been reduced so that snubber circuits are no longer required in most applications*

Resonant Switching Attractions

- *Drastically reduce switch turn-on and turn-off losses*
- *Almost loss-less switching allows higher switching frequencies*
- *Reduce the electromagnetic interference (EMI) associated with pulse width modulation (PWM)*

Two Resonant Switching Methods

- *Zero current switching (ZCS)*
- *Zero voltage switching (ZVS)*
- *ZVS prevalent as disadvantages in ZCS*
- *Lets examine ZVS*

High Frequency Inductors and Transformers

Low and High Frequency Transformers Compared

	Low frequency	High frequency
Standards	Well defined by ANSI, IEEE, NEMA and UL	Not as well defined Insulation standard followed
Operation	60 Hz Sine wave 3 phase	10 kHz to 100 kHz Square wave – transformers Triangular wave – inductors Single phase
Core material	3 to 100 mil laminations of steel or Fe	0.5 to 3 mil laminations of Fe or Si-Fe Powdered Fe Powdered ferrites, Ni-Zn, Mn-Zn
Winding material	Single-strand Cu wire Layer or bobbin-wound	Multi-strand Cu Litz wire Cu foil, layer wound

The power rating of a transformer is dependent upon the kollowing factors

$$V * A = K_1 * K_2 * f * A_C * A_E * J * B_M$$

where

$V * A$ = power rating of the transformer (V*A)

K_1 = waveshape factor (sine or square wave)

K_2 = copper fill factor (0 to 1)

f = excitation frequency (Hz)

A_C = core area (m^2)

A_E = winding area (m^2)

J = conductor current density $\left(\frac{A}{m^2}\right)$

B_M = peak flux density $\left(\frac{Wb}{m^2}\right)$ where a Weber = 1*volt*sec

$$\text{The transformer area product} = A_C * A_E \propto \frac{V * A}{B_M * f * J}$$

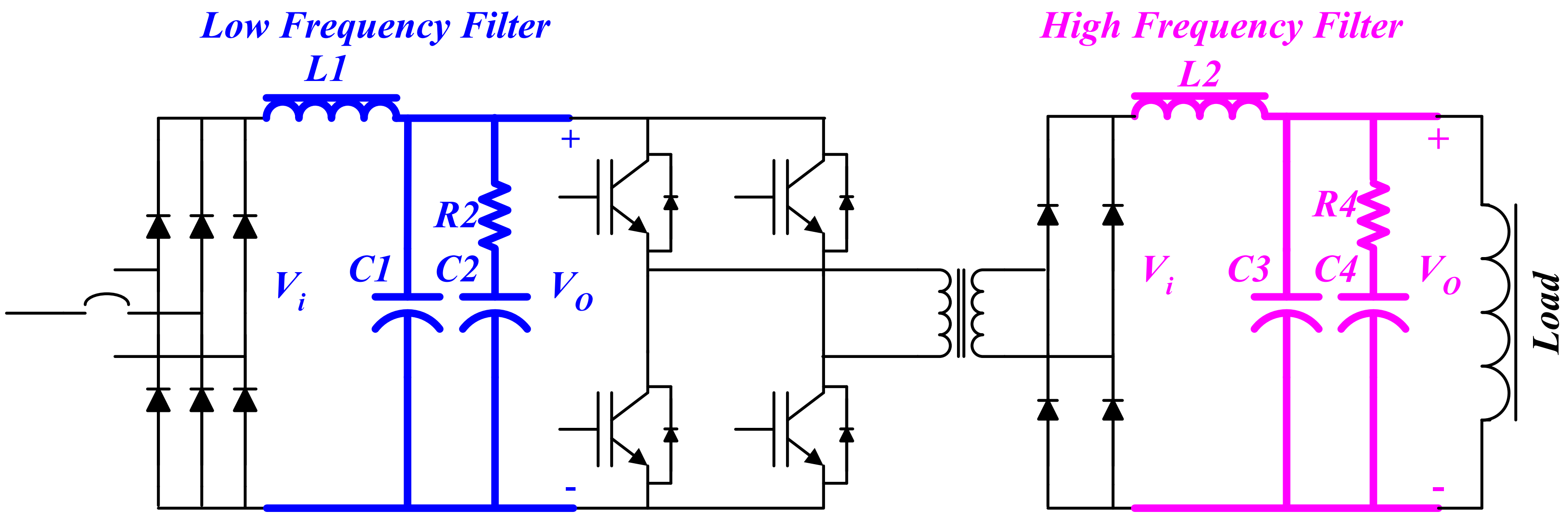
An example of a 10kVA, 480V : 208V Transformer ***At 60Hz the volume and weight would be***					
f	*f ratio to 60Hz*	*Volume (in ^{3})*	*Volume ratio to 60Hz*	*Weight (lb)*	*Weight ratio to 60Hz*
60 Hz	*1*	*18 X 18 X 18 = 5832 (in ^{3})*	*1*	*100*	*1*
20 kHz	*333*	*6H X 5.25W X 3.37D 118 (in ^{3})*	*1 / 50*	*5*	*1 / 20*

Some Parameters For HF Inductor Specification

- *Inductance*
- *Ripple current frequency*
- *Peak current*
- *RMS value of AC current*
- *DC current*
- *Saturation DC current*
- *Resonant frequency (an order of magnitude > ripple frequency)*

Ripple Filters

Ripple Filters



Low Frequency	High Frequency
Pass DC – reject f > 60 Hz	Pass DC – reject f > switching frequency
Large L1 to reduce On inrush & high PF	Large L2 to reduce inrush and prevent discontinuous current
R2 C2 for "critical" damping	R4 C4 for "critical" damping

Time Domain $y(t) = f(t) \otimes x(t)$ where $\otimes$ implies the convolution operation

- *Difficult computations, particularly transient calculations, requires solution of differential or difference equations*

Frequency Domain $Y(f) = F(f) * X(f)$ where * implies multiplication

- *Easier computations, all calculations for steady-state or transient conditions that look algebraic in nature.*

Transfer Function

- *Relates the output response of a circuit/system to the input stimulus*

- *Form is $T(f) = Y(f) / X(f)$ where $X(f)$ is the input stimulus and $Y(f)$ is the output response* $Y(f) = X(f) * T(f)$

The "s" Operator

- *s is used in the frequency domain and in La Place analysis*
- $s = j\,\omega = j\,2\,\pi\,f \qquad j = \sqrt{-1}$

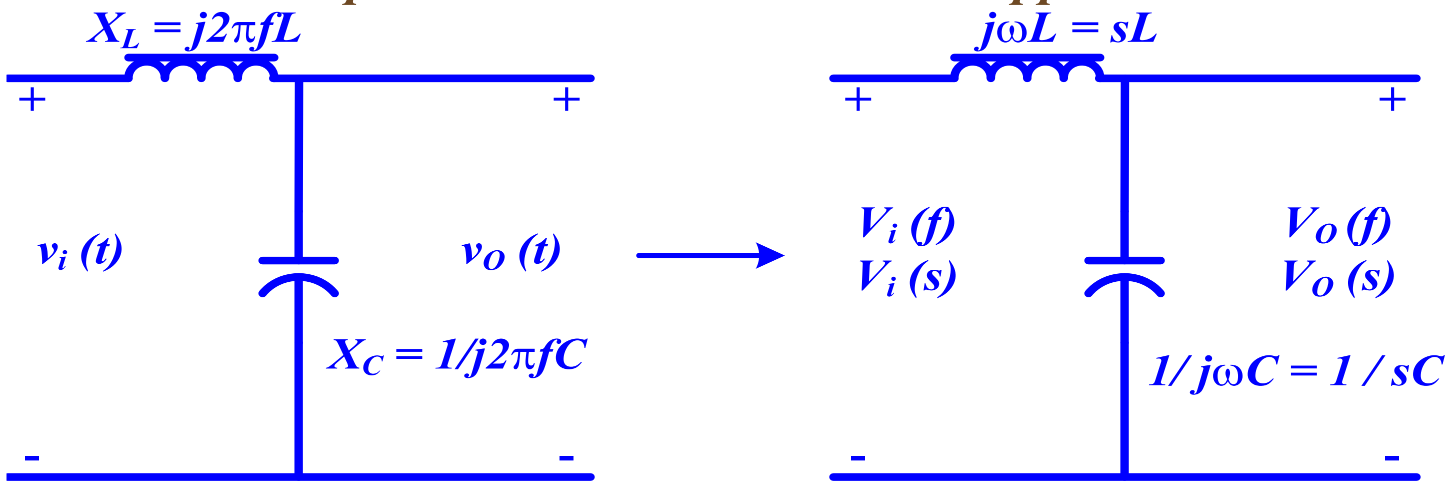
Poles and Zeros

- $Zero = 0 \qquad Pole = \infty$

- *Zeros occur at frequencies that cause the transfer function to go to zero. Transfer function = 0 is caused by a zero numerator and or an infinite denominator* $T(s)=0\,/\,X(s)=0$ *or* $T(s)= Y(s)\,/\,\infty =0$

- *Poles occur at frequencies that cause the transfer function to become infinite. Transfer function =* ∞ *is caused by an infinite numerator or a zero denominator* $T(s) = \infty\,/\,X(s)= \infty$ *or* $T(s) = Y(s)\,/\,0 = \infty$

A Simple Second Order Low Pass Ripple Filter



By voltage divider law $\quad V_o = V_i * \dfrac{\frac{1}{sC}}{\frac{1}{sC} + sL} \qquad T = \dfrac{1}{s^2 LC + 1}$

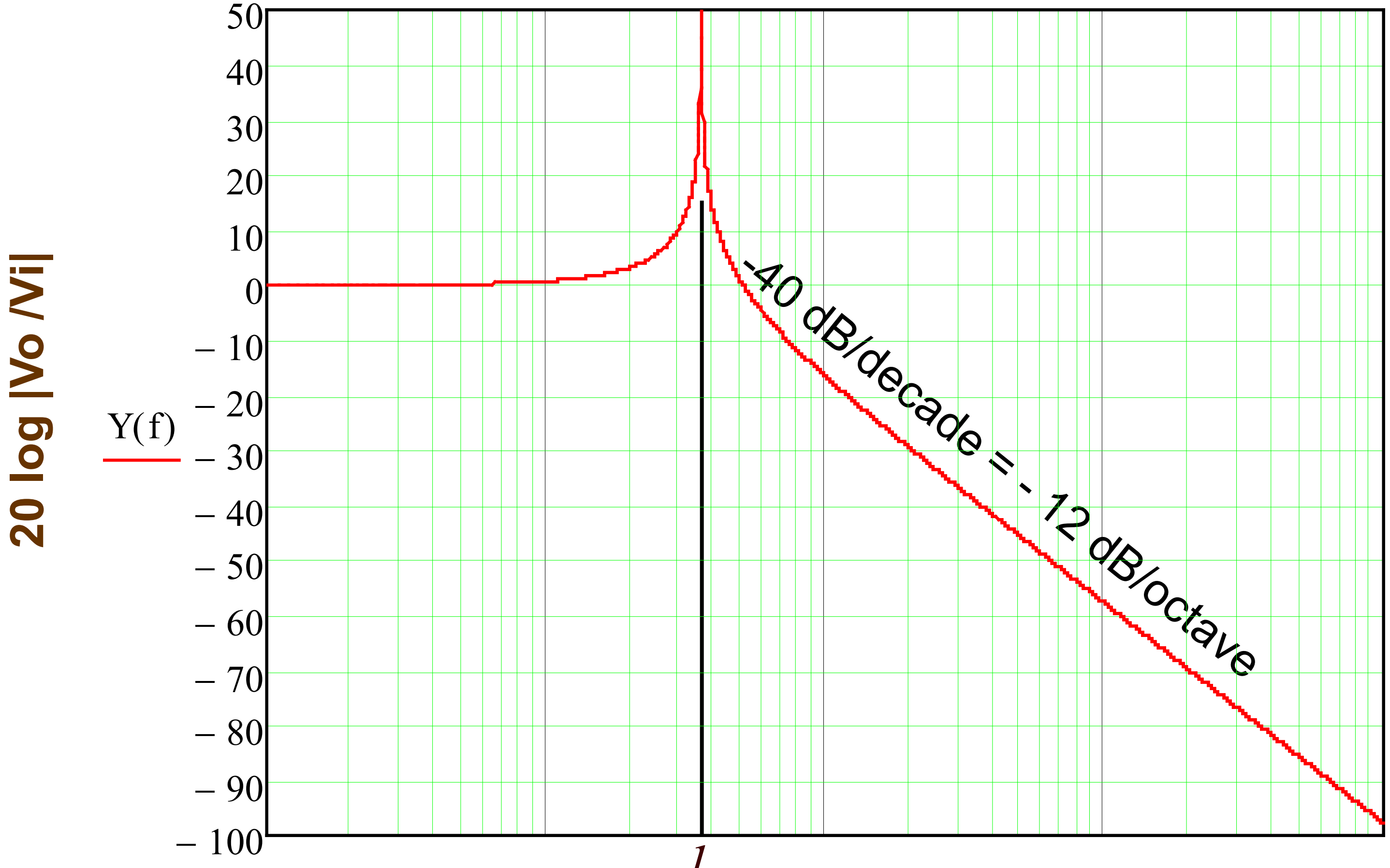
Pole $\quad s^2 LC + 1 = 0 \qquad (j2\pi f_p)^2 LC + 1 = 0$

Resonant frequency (pole) $\quad f_p = \dfrac{1}{2\pi\sqrt{LC}}$

Zero occurs at $\quad (j2\pi f_p)^2 LC + 1 = \infty$

Zero frequency at $\quad f_z = \infty$

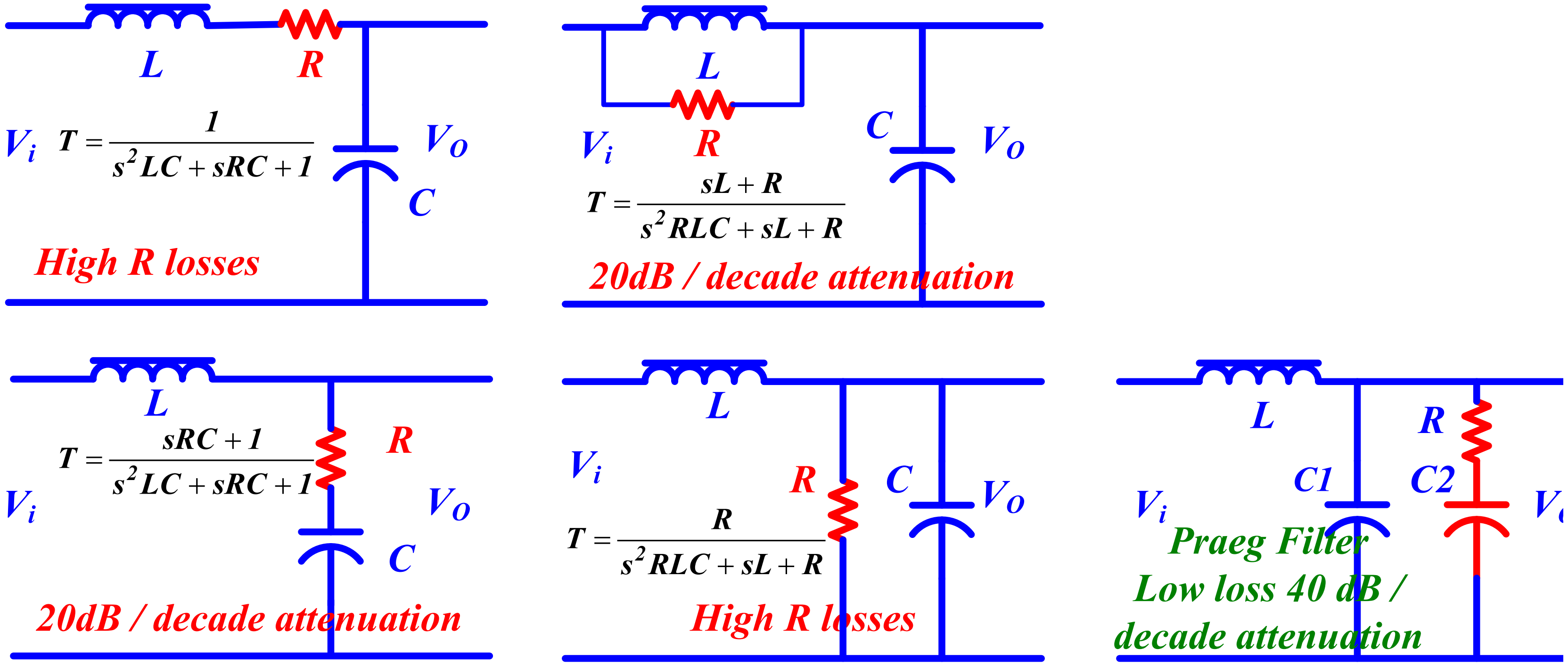
A Simple Low Pass Filter



$$f_p = \frac{1}{2\pi\sqrt{LC}} \qquad \longrightarrow \quad f_z = \infty$$

- *Resonant frequency (pole) at* f_p *will cause problems !*
- *At* $f = \infty$ *, the output goes asymptotically to zero*

The Praeg Low Pass Ripple Filter

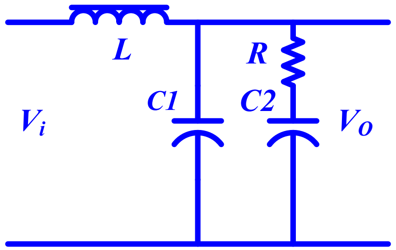


Why important:

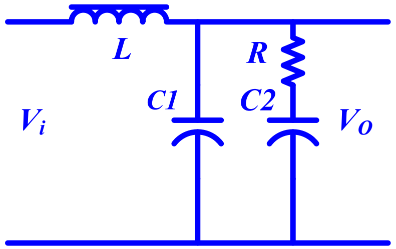
- *Used as low and high frequency filters in virtually every power supply*
- *Provides the filtering of the previous* 2^{nd} *order filter*
- *Essentially critical damped*
- *No DC current in R, C2*

Component Selection Criteria

- *L and C1 must be chosen to yield the desired breakpoint frequency (1/10 of the ripple frequency for 40 dB attenuation)*
- *C1 and C2 must be rated for the rectifier working and surge voltages*
- *C1 and C2 must be rated to carry the ripple current at the rectifier output frequency and at the switching frequency*
- *L must be large enough to offset the leading PF introduced by main filter capacitor, C1*
- *L must be large enough to limit the inrush current caused by rapid charge of C1 during power supply turn-on to an acceptable level*
- *L must be rated to carry the DC load current without overheating or saturating*
- $C2 \geq 5 * C1$
- $R = (L / C1)^{1/2}$

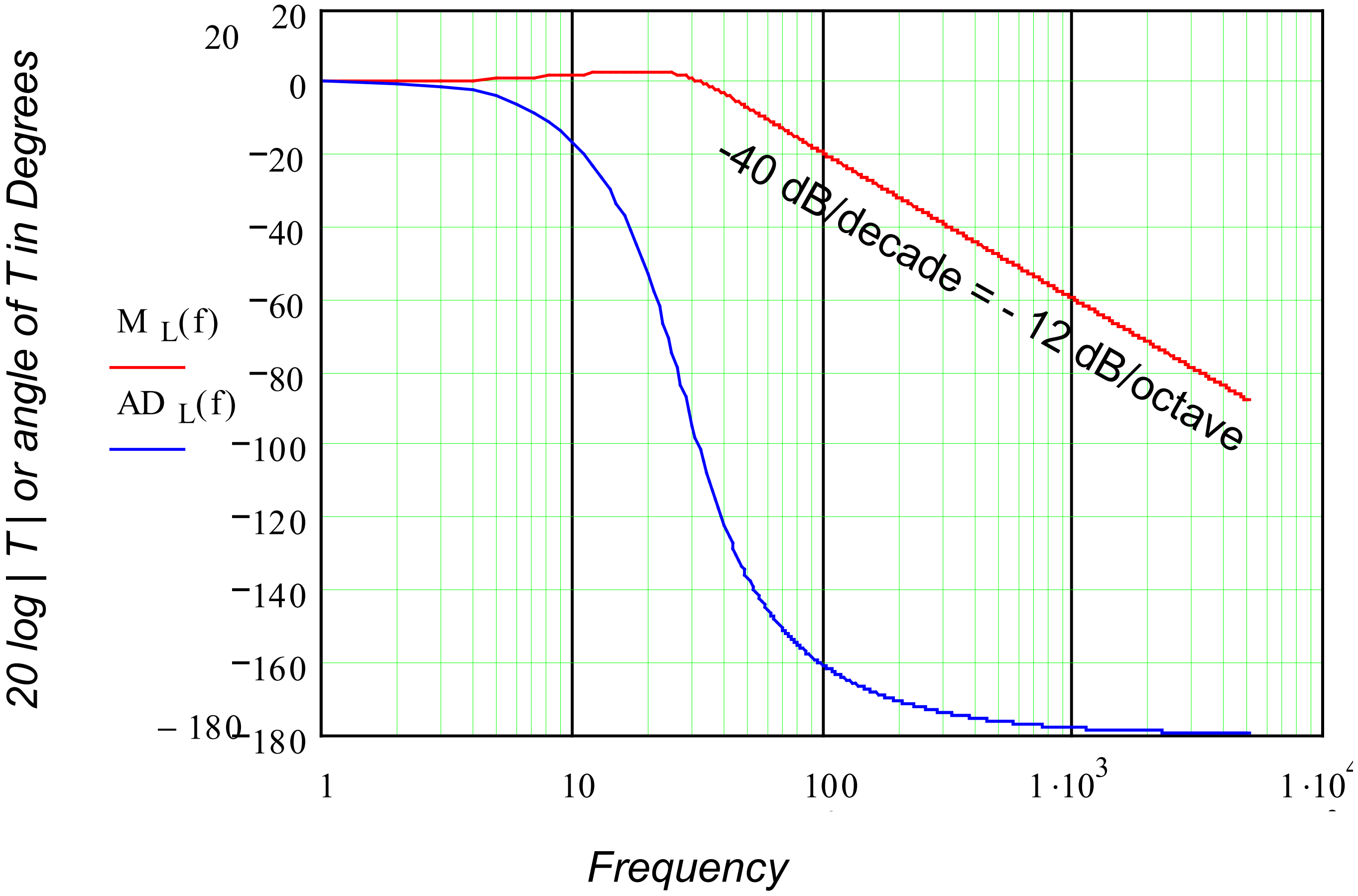


The Praeg Low Pass Ripple Filter



$$T = \frac{s R C_2 + 1}{s^3 R L C_1 C_2 + s^2 L (C_1 + C_2) + s R C_2 + 1}$$

$$C_2 \geq 5 * C_1 \qquad R = \sqrt{\frac{L}{C_1}}$$



360 Hz Praeg Filter

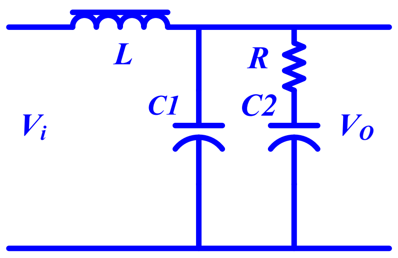
$f := 1 \cdot Hz, 2 \cdot Hz .. 1000 \cdot Hz$ $\quad s(f) := j \cdot 2 \cdot \pi \cdot f$ $\quad L := 1.5 \cdot 10^{-3} \cdot H$ $\quad f_r := 36 \cdot Hz$ $\quad C_1 := \dfrac{1}{4\pi^2 \cdot L \cdot f_r^2}$ $\quad C_1 = 0.0130\,F$

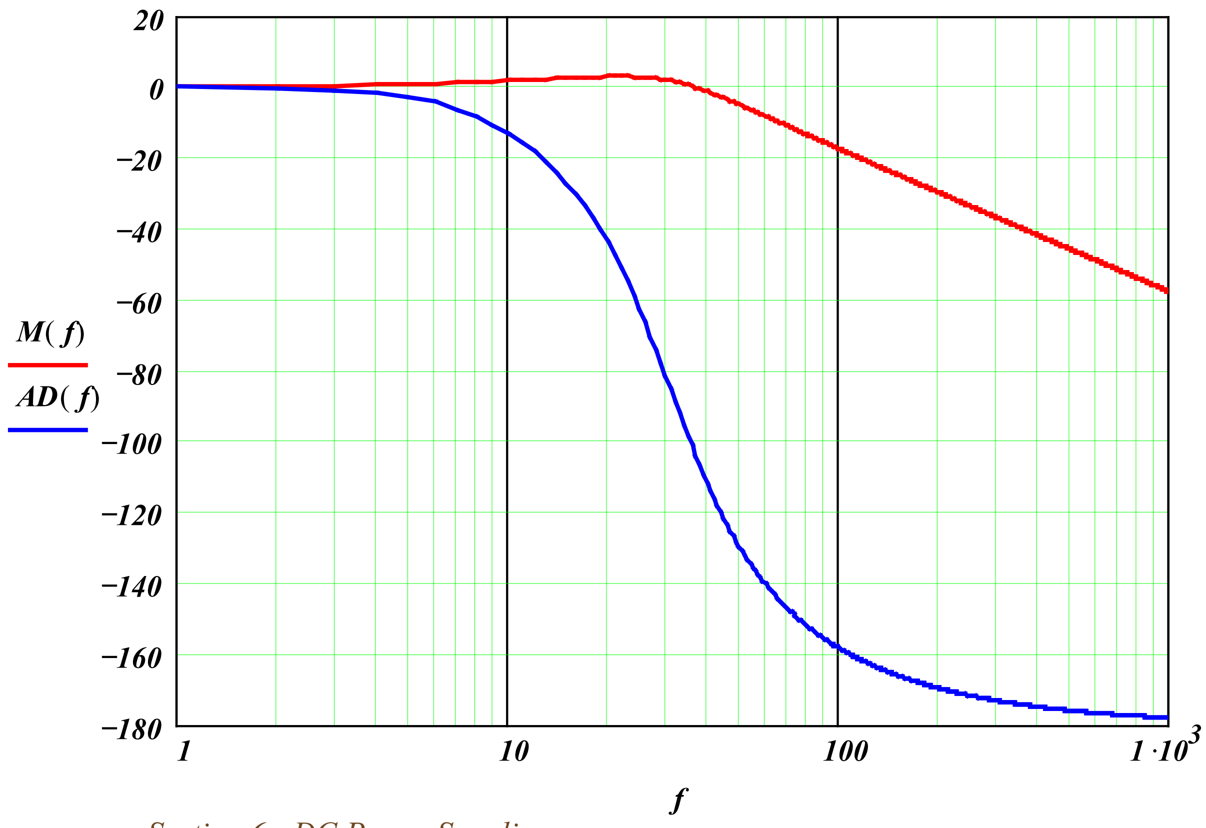
$R := \sqrt{\dfrac{L}{C_1}}$ $\quad R = 0.34\,\Omega$ $\quad C_2 := 5 \cdot C_1$ $\quad C_2 = 0.065\,F$

$$T(f) := \frac{s(f) \cdot R \cdot C_2 + 1}{s(f)^3 \cdot R \cdot L \cdot C_1 \cdot C_2 + s(f)^2 \cdot L \cdot (C_1 + C_2) + s(f) \cdot R \cdot C_2 + 1}$$

$M(f) := 20 \cdot \log(|T(f)|)$ $\quad AR(f) := \arg(T(f))$

$AD(f) := AR(f) \cdot 57.3$





Higher Frequency Operation Means a Smaller Filter

$$f_{r1} = \frac{1}{2\pi\sqrt{LC}}$$

$$\text{Let } f_{r2} = nf_{r1} = \frac{n}{2\pi\sqrt{LC}}$$

$$nf_{r1} = \frac{1}{2\pi\sqrt{\frac{L}{n}\frac{C}{n}}}$$

L is smaller by the factor n

C is smaller by the factor n

36 kHz Praeg Filter

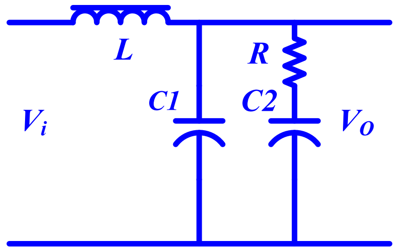
$f := 10 \cdot Hz, 20 \cdot Hz .. 100000 \cdot Hz \quad s(f) := j \cdot 2 \cdot \pi \cdot f$ $\quad L := 1.5 \cdot 10^{-5} \cdot H \quad C_1 := 0.00013 \cdot F \quad f_r := \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C_1}}$

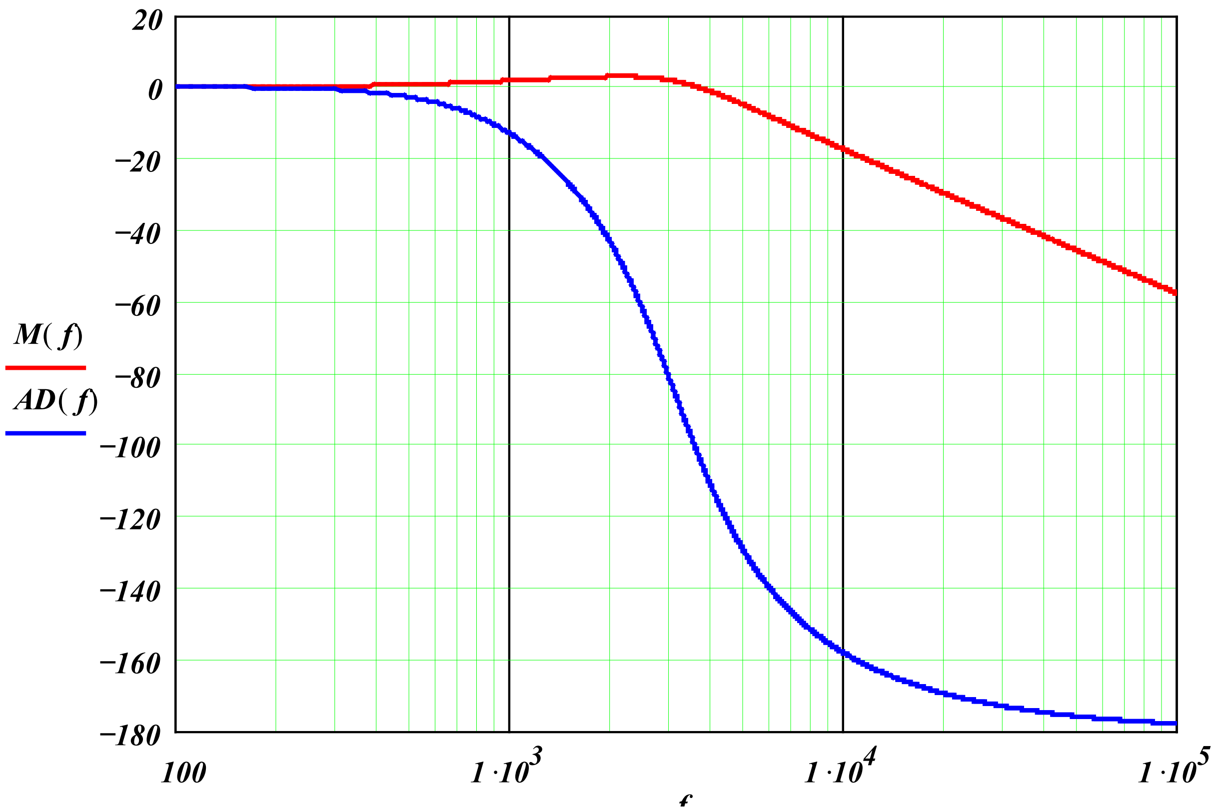
$R := \sqrt{\frac{L}{C_1}} \quad R = 0.34\,\Omega \quad C_2 := 5 \cdot C_1 \quad C_2 = 6.5 \times 10^{-4}\,F \quad f_r = 3604\,Hz$

$$T(f) := \frac{s(f) \cdot R \cdot C_2 + 1}{s(f)^3 \cdot R \cdot L \cdot C_1 \cdot C_2 + s(f)^2 \cdot L \cdot (C_1 + C_2) + s(f) \cdot R \cdot C_2 + 1}$$

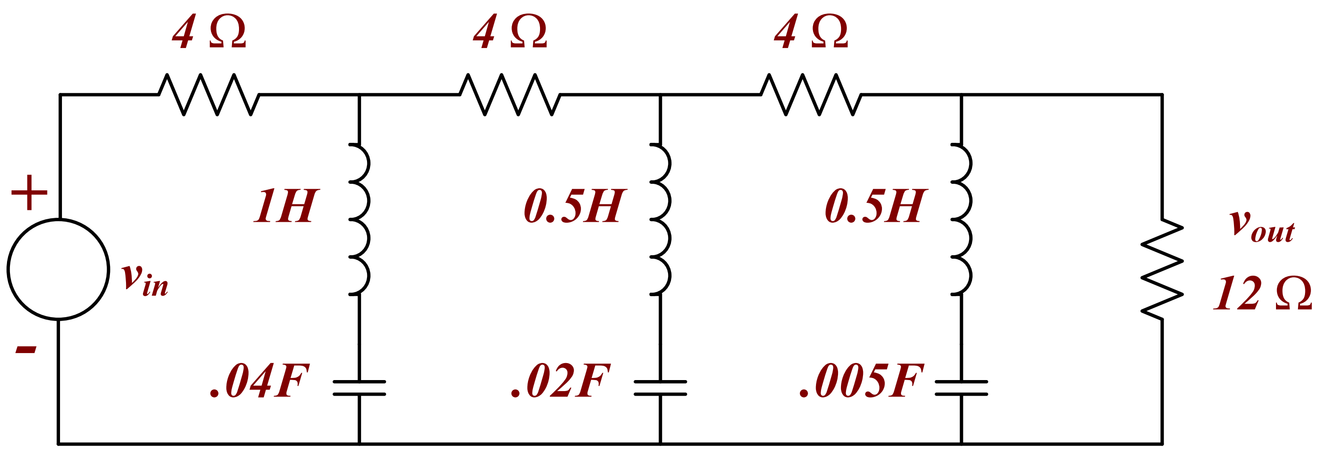
$M(f) := 20 \cdot \log(|T(f)|) \quad AR(f) := \arg(T(f))$

$AD(f) := AR(f) \cdot 57.3$





Given the circuit below:



$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$$

•*Remember that* $s=j\omega$

Sketch $|H(j\omega)|$ *versus* ω

Other Design Considerations And Power Supply Costs

Other Design Considerations - Heat Loading Into Building Air

$$All\ equipment = \sum (P_{switchgear} + P_{transformer} + P_{AC\,cables} + P_{PS} + P_{DC\,cables})$$

- $Switchgear\ effiency \geq 98\%$ $\quad Switchgear\ losses = P_O * (\frac{1 - Eff}{Eff})$

- $Transformer\ efficiency \geq 97\%$ $\quad Transformer\ losses = P_O * (\frac{1 - Eff}{Eff})$

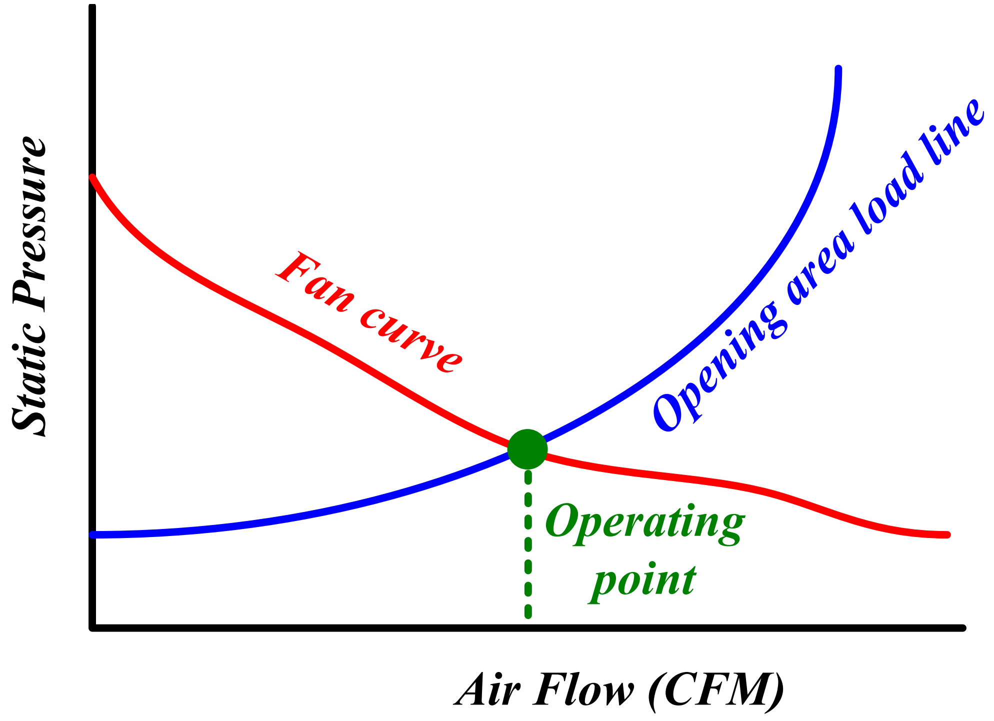
- $P_{AC\,cables} = \sum_j i^2_{j\,RMS} * \frac{R_j}{ft} * Length_j$

- $Power\ supply\ losses = \sum_j (P_{in\ j} - P_{out\ j})$

- $P_{DC\ output\ cable} = \sum_j i^2_{j\,DC} * \frac{R_j}{ft} * Length_j$

Other Design Considerations - Rack Cooling

- *Thermal radiation from rack surface*
- *Electronics – maximum 50C inside rack*
- *Max rise in rack = 50C – $T_{ambient\ max}$*
- *Size openings, back pressure drops $Bp=(CFM / (k*Opening\ Area))^2$*
- *Fan vs load curve – junction is operating flow point*



Power supply heat loss to water $= \sum$ *electrical losses of all water-cooled components*

Heat lost (dissipated) by PS water cooled components = Heat gained by cooling water system

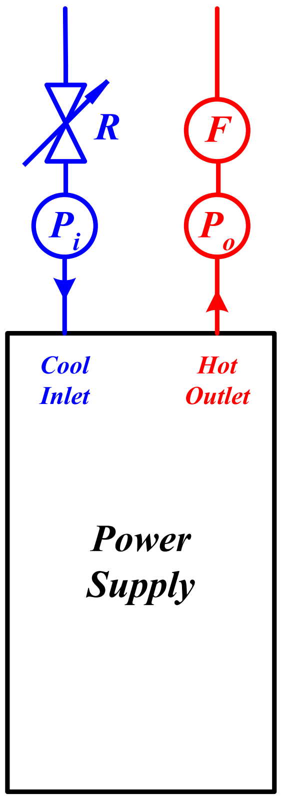
$$Q = M * c * \Delta T \qquad cal = gm * \frac{cal}{gm * {}^{O}C} * \left({}^{O}C_{Outlet} - {}^{O}C_{Inlet} \right)$$

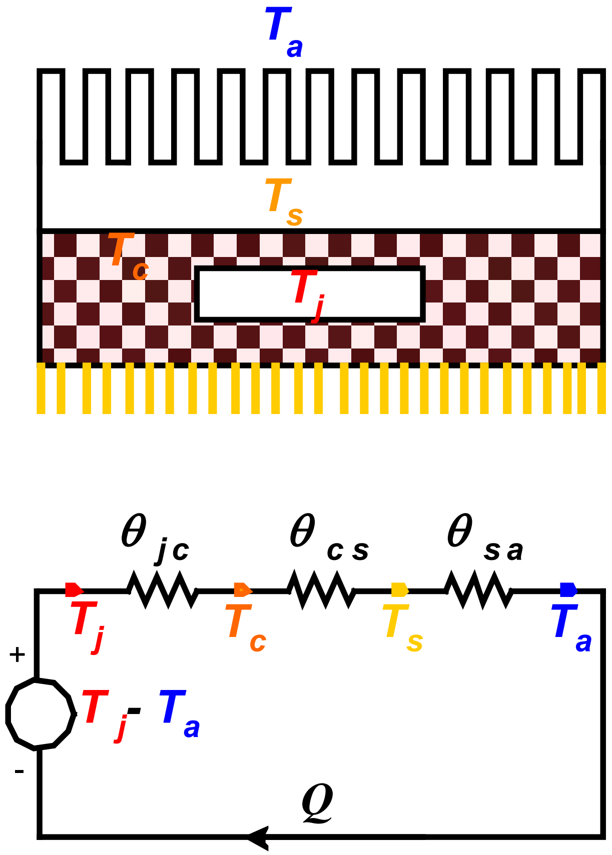
$$q = m * c * \Delta T \qquad watt = gpm * \frac{264\,watt}{gpm * {}^{O}C} * \left({}^{O}C_{Outlet} - {}^{O}C_{Inlet} \right)$$

Usually the power loss and the inlet and maximum allowable outlet temperatures are known. The mechanical group will usually ask for an estimate of the water flow requirements. So solving for the flow yields

$$m = \frac{q}{c * \Delta T} = \frac{watt}{\frac{264\,watt}{gpm * {}^{O}C} * \left({}^{O}C_{Outlet} - {}^{O}C_{Inlet} \right)}$$

The system pressure drop is $\Delta P = \sum_i P_i$





Q = Power that can be removed by the air or cooling water (W)

T_j = Device junction temperature (^oC)

T_c = Device case temperature (^oC)

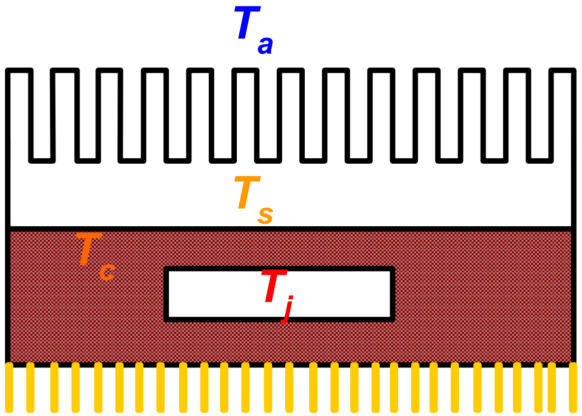
T_s = Heatsink temperature (^oC)

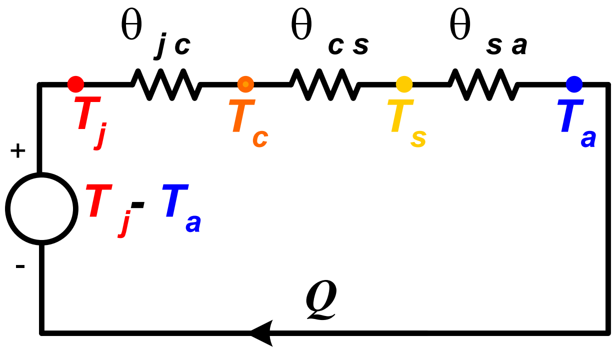
T_a = Ambient air or cooling water inlet temperature (^oC)

θ_{jc} = junction to case thermal resistance ($^oC / W$)

θ_{cs} = case to heatsink thermal resistance ($^oC / W$)

θ_{sa} = Heatsink to ambient air or cooling water thermal resistance ($^oC / W$)





Calculate $Q = \dfrac{T_j - T_a}{\theta_{jc} + \theta_{cs} + \theta_{sa}}$

Q is heat that can be pulled out of the ambient air or cooling water

If calculated Q > q

q is the power disspiated by the device

then all of the device dissipation will be removed by the air or water

*Calculate the actual air or water temperature rise from q=m*c*ΔT*

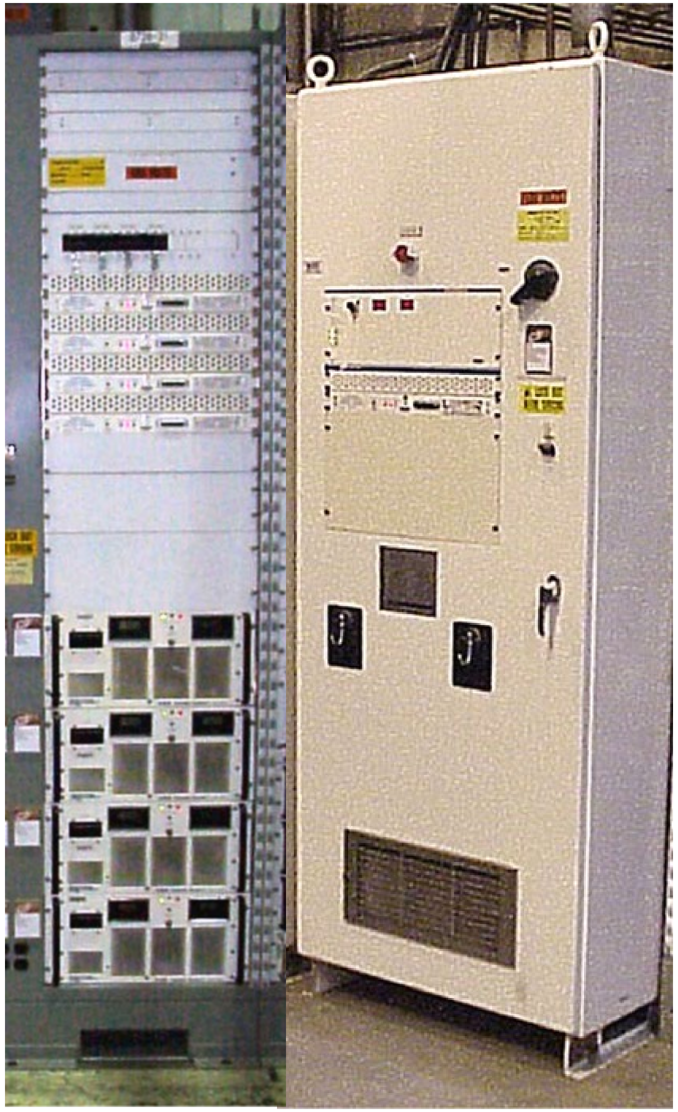
$$\Delta T = \frac{q}{m^*c} = \frac{watts}{gpm * \dfrac{264watt}{gpm * {}^{O}C}}$$

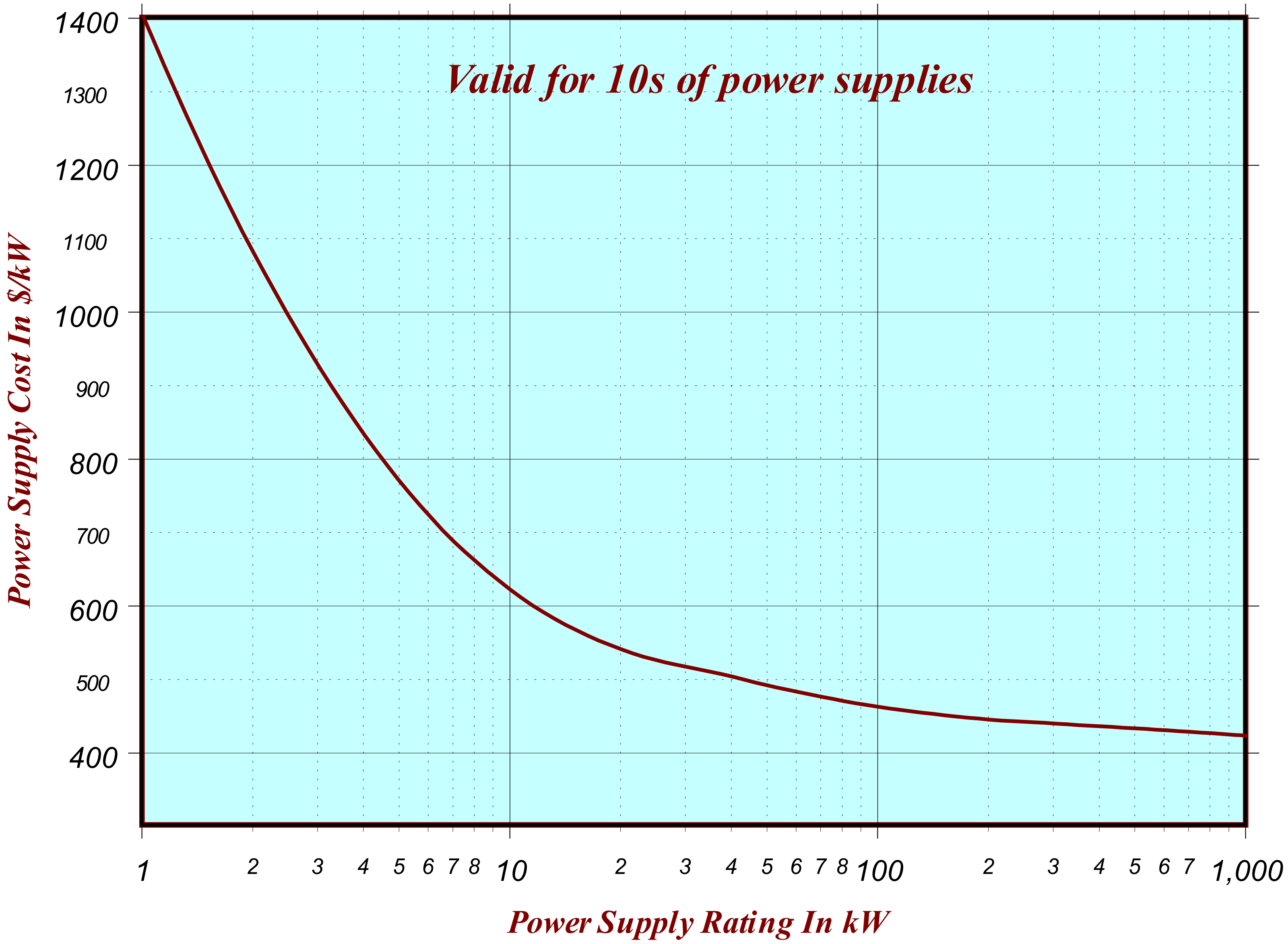
$\Delta T \leq$ *the maximum allowable temperature rise*

Power Output Vs Mounting / Input Voltage / Cooling Considerations

	Input AC (V)				Cabinet		Cooling	
Power Output	1 ϕ 120	3 ϕ 208	3 ϕ 480	3 ϕ 4160	RM	FS	AC	WC
< 2 kW	X				X		X	
2 kW → 5 kW		X			X		X	
> 5 kW → 40 kW			X		X		X	
> 40 kW → 100 kW			X			X	X	
> 100 kW → 1 MW			X			X	X	X
> 1 MW				X		X	X	X

RM = Rack mounted   FS = Freestanding
AC = Air-cooled   WC = Water-cooled



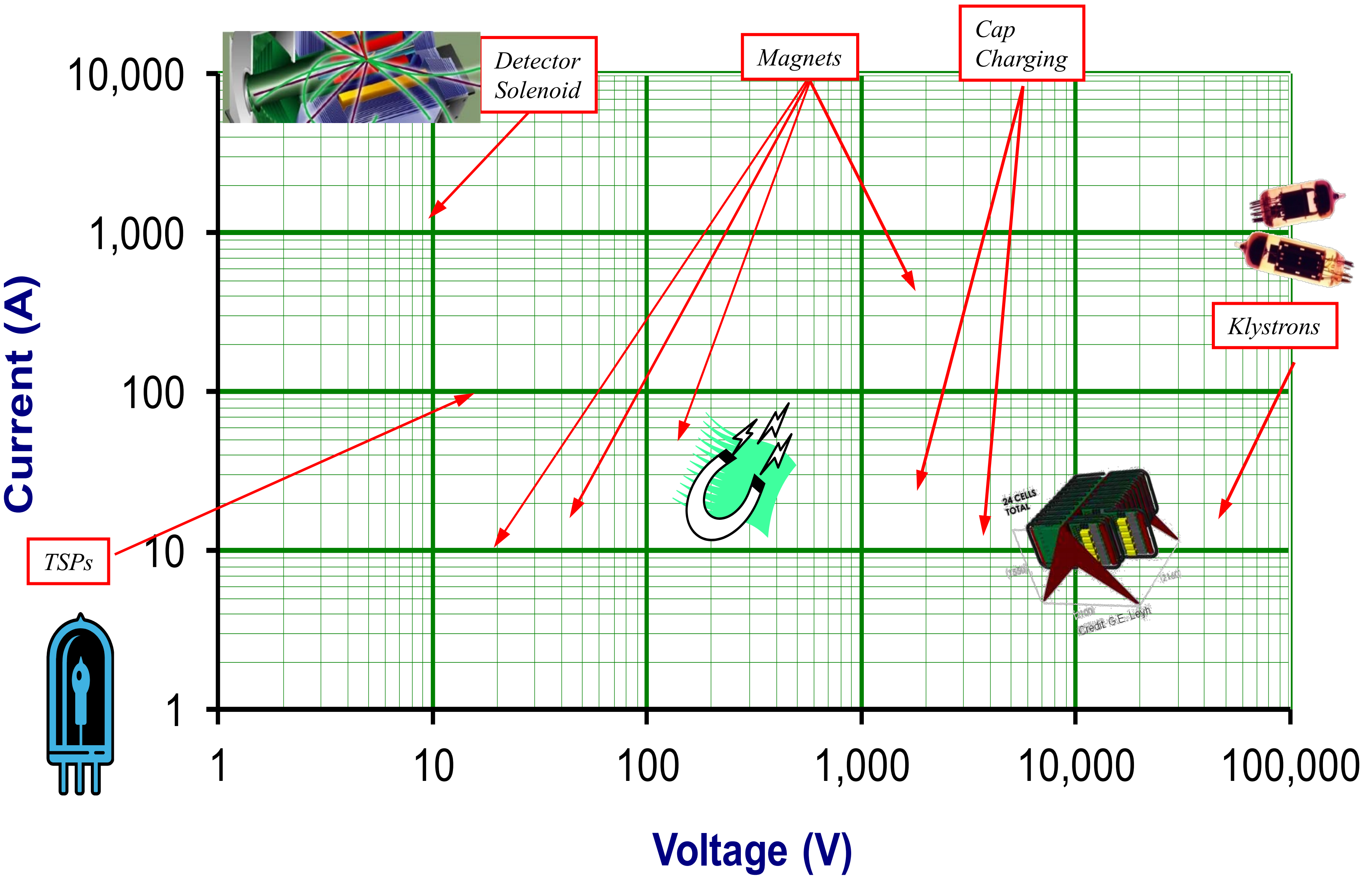


Valid for 10s of power supplies
Power Supply Cost In $/kW
1400
1300
1200
1100
1000
900
800
700
600
500
400
1
2
3
4
5
6
7
8
10
100
1,000
Power Supply Rating In kW

Other Design Considerations - Homework Problem # 11

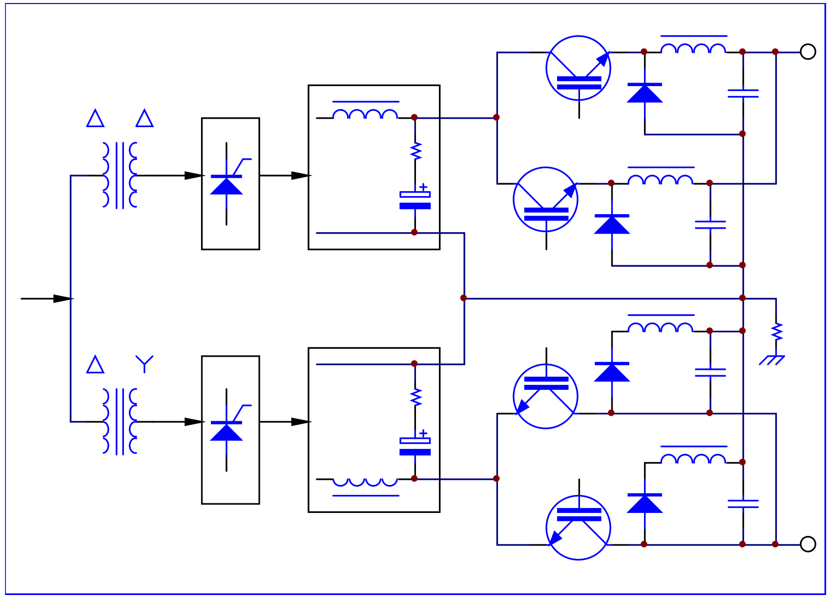
A 100kW power supply is 80% efficient. Approximately 50% of the power supply heat loss is removed by cooling water.

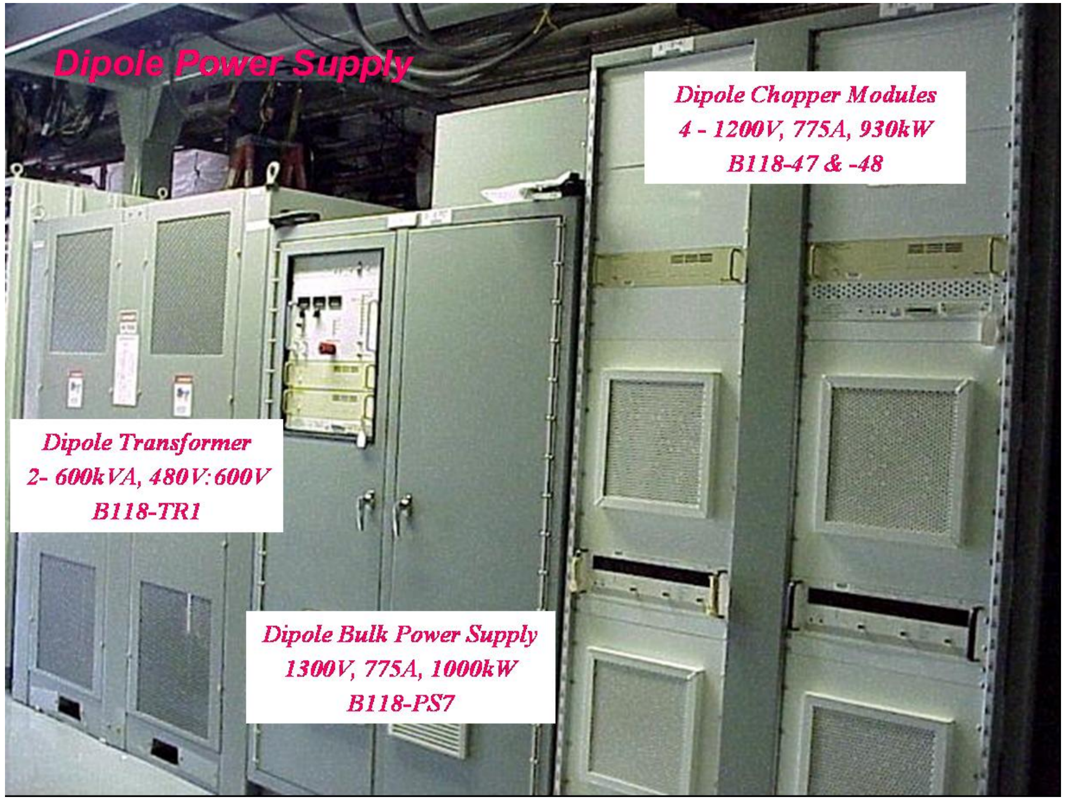
- *How much heat is dissipated to building air and how much heat is removed by the water system.*
- *Calculate the water flow rate needed to limit the water temperature rise to 8^{O}C maximum.*


Detector Solenoid
Magnets
Cap Charging
Klystrons
TSPs
10,000
1,000
100
10
1
Current (A)
1
10
100
1,000
10,000
100,000
Voltage (V)
24 CELLS TOTAL
Credit: G.E. Leyh

PEP-II and SPEAR3 Dipole Power Supplies

- 1200 VDC, 800 Amperes, 960 KW
- Powers largest magnet string at Spear3, 36 ring bend magnets in series
- Requires 50 PPM (full scale) current regulation, 0.1% voltage regulation
- Requires 600 VAC, 6-Phase AC Input





Storage Ring of the Diamond Project

- *The power converter comprises of 8 paralleled modules*
- *Each module is a non-isolated step down PWM switching regulator operating at a fixed frequency of 2 kHz*
- *IGBT devices are used as the switching element*
- *The 8 PWM drives are phase shifted by 360/8° to achieve a 16 kHz output ripple frequency*
- *1 quadrant operation*

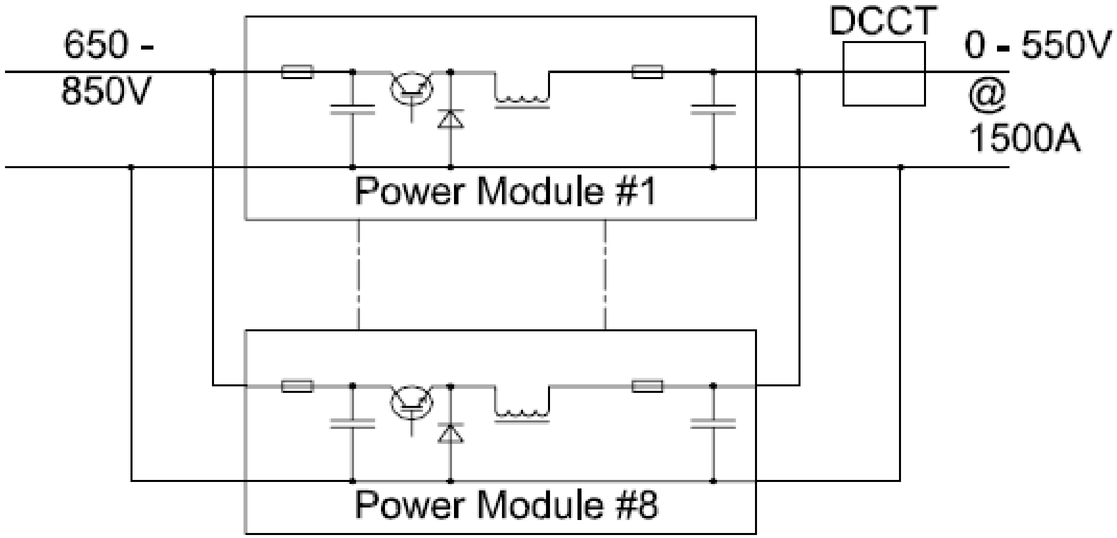


Figure 1: Dipole Converter Topology.

Diamond Booster Magnet Power Converters

- *Booster operates at 5 Hz to accelerate the electrons: 100 MeV to 3 GeV.*
- *Power converters produce an off-set sine wave current with high repeatability at 5 Hz*
- *To avoid disturbance on the ac distribution network, the dipole and quadrupole power converters were designed to present a constant load despite having high circulating energy: 2 MVA in the case of the dipole*
- *Redundancy was introduced wherever this was economically feasible.*
- *Plug-in modules are used to simplify and speed up repairs.*
- *Component standardization and de-rating across all power converters was an additional design goal*

- *Booster dipole PC is rated at peaks of 1000A and 2000V*
- *Three units are sufficient to produce the required output. The fourth is redundant*
- *Each unit is made up of a boost circuit and a 2-quadrant output regulator that produces the required offset sine wave current.*
- *The boost circuit regulates the voltage on the main energy storage capacitor and is controlled to draw constant power from the ac network.*
- *Displaced 4 kHz switching frequency*

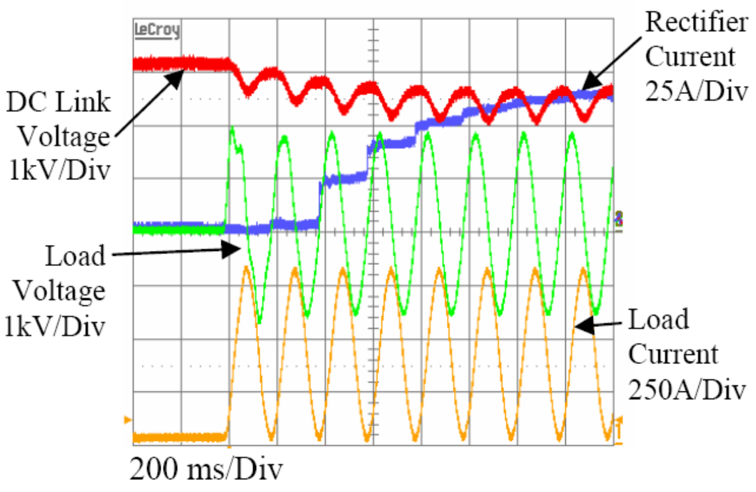


Figure 4: First few cycles after turn on.

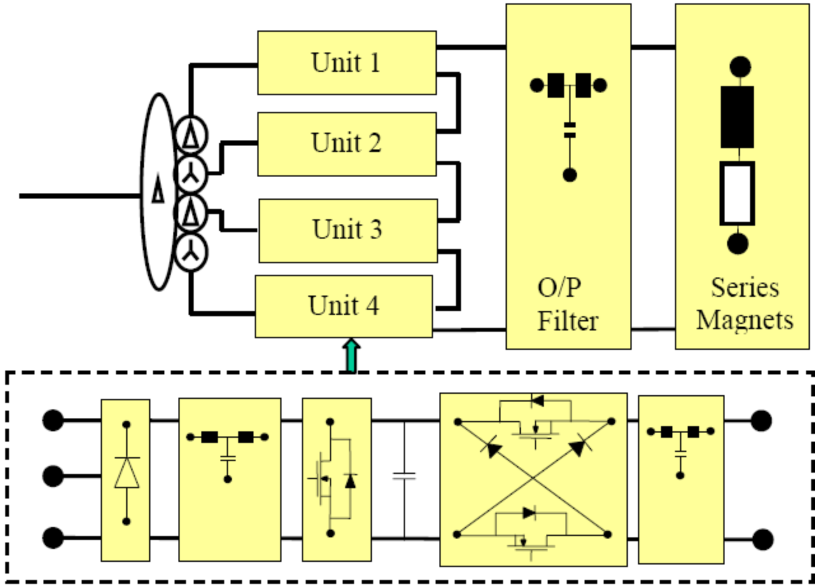
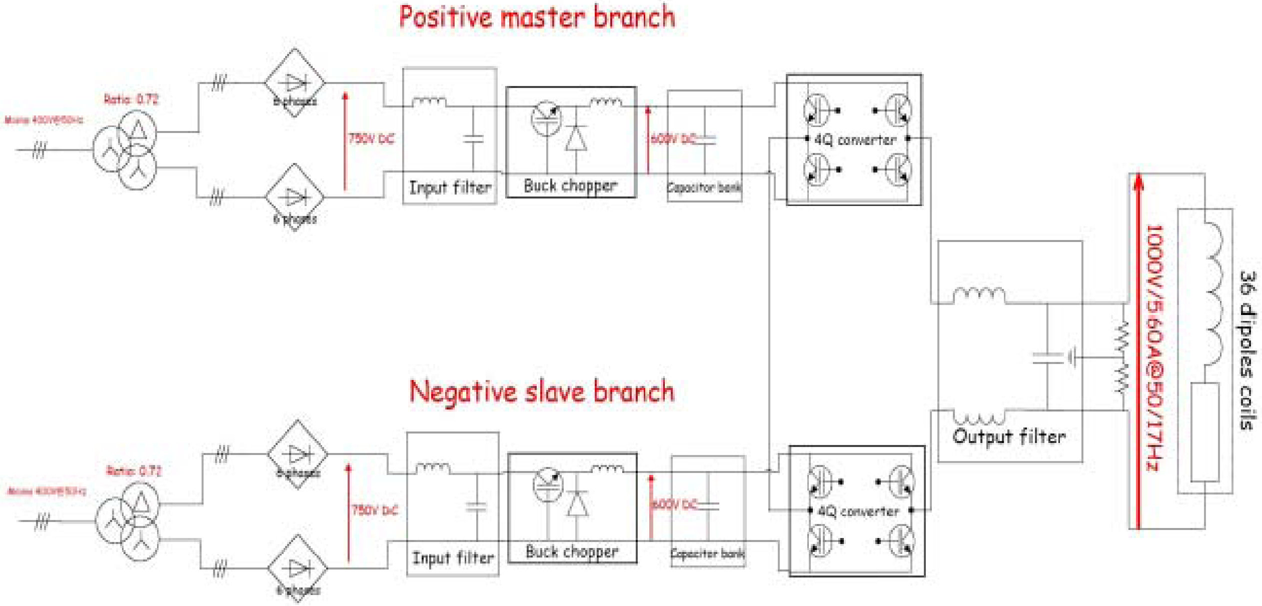


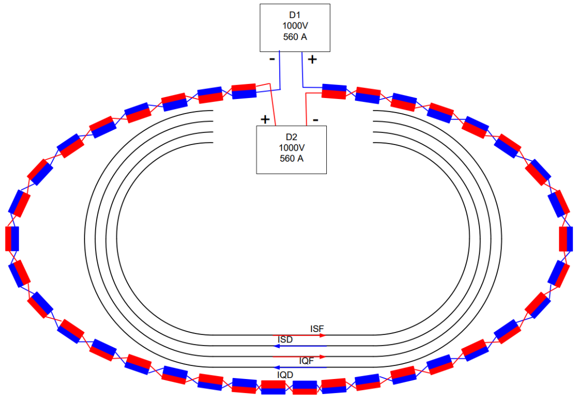
Figure 1: Booster dipole power circuit.

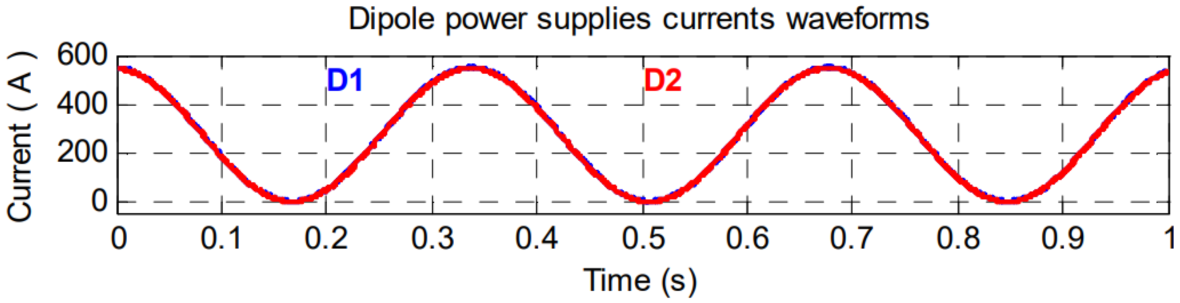
THE 3HZ POWER SUPPLIES OF THE SOLEIL BOOSTER

Table 1: Major booster parameters

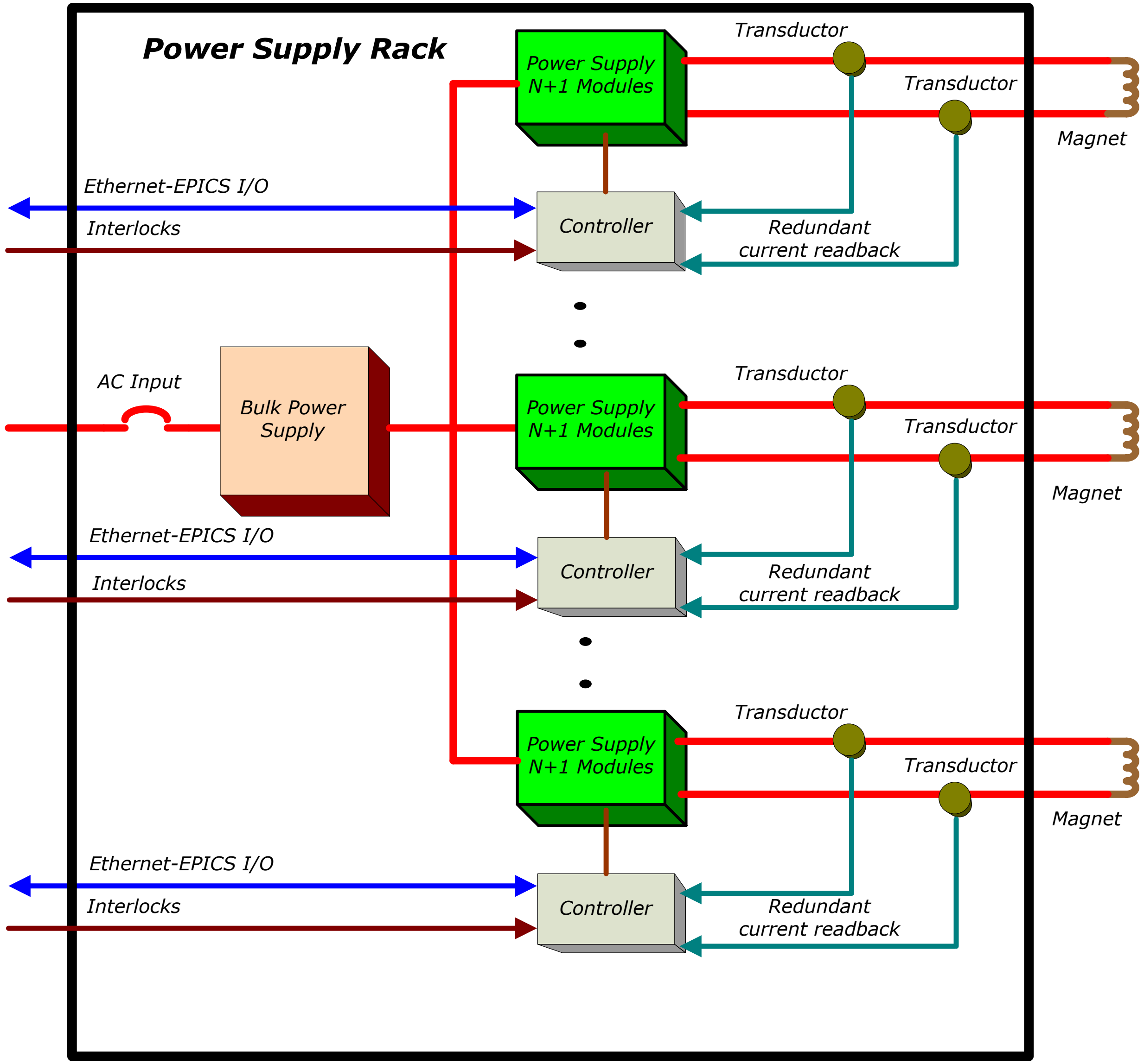
Injection energy	110	M
Extraction energy	2.75	G
Number of dipoles	36	
Dipole magnetic length	2.16	m
Dipole gap	22	m
Dipole field @2.75GeV	0.74	T
Dipoles inj. current	19.7	A
Dipoles ext.current	541	A
Dipoles load resistance	400	m
Dipoles load inductance	156	m







Power Supplies for the ATF2



CNAO STORAGE RING DIPOLE MAGNET POWER CONVERTER 3000A / ±1600V

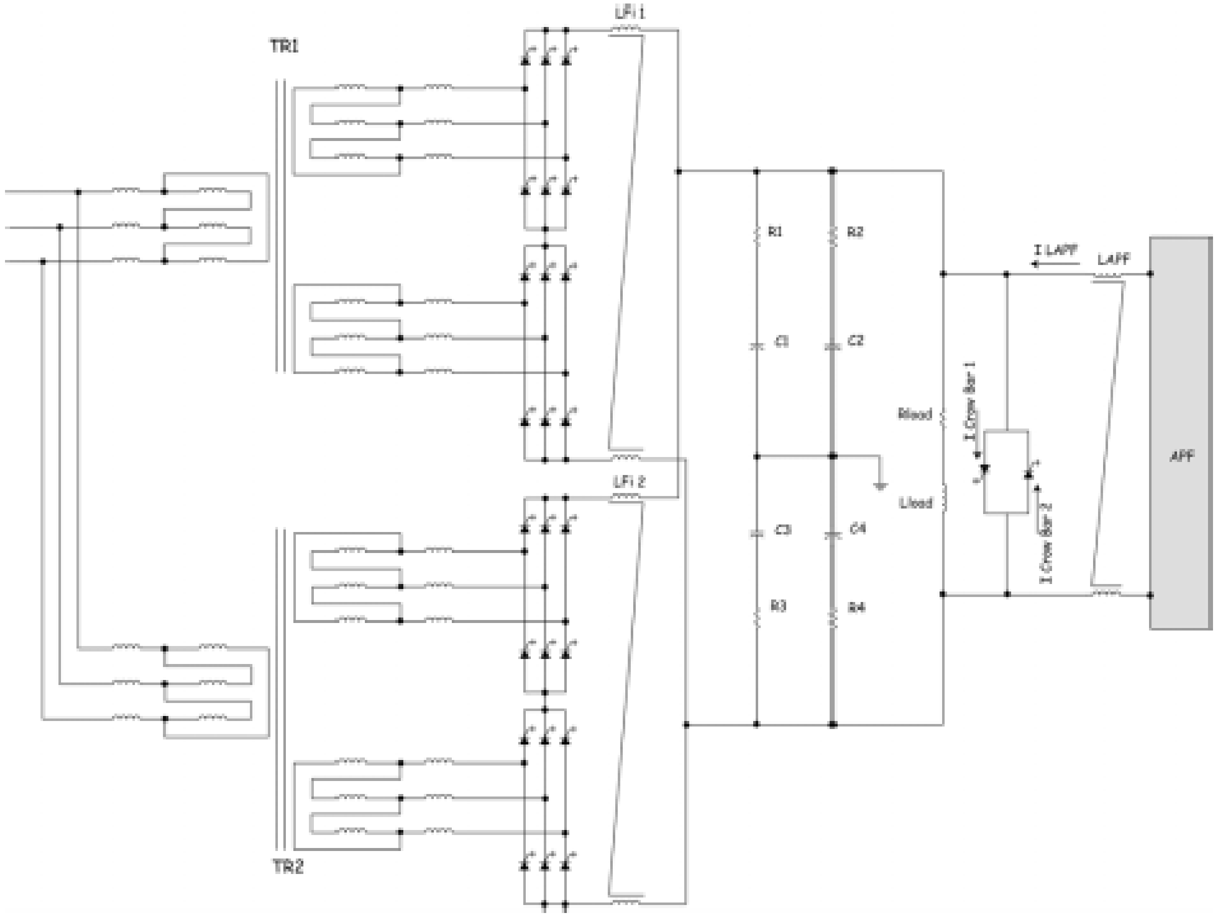


Figure 2: Topology of CNAO synchrotron power supply.

Bipolar Power Supplies at SPEAR3 and LCLS (480W, ±40V, ± 12A)

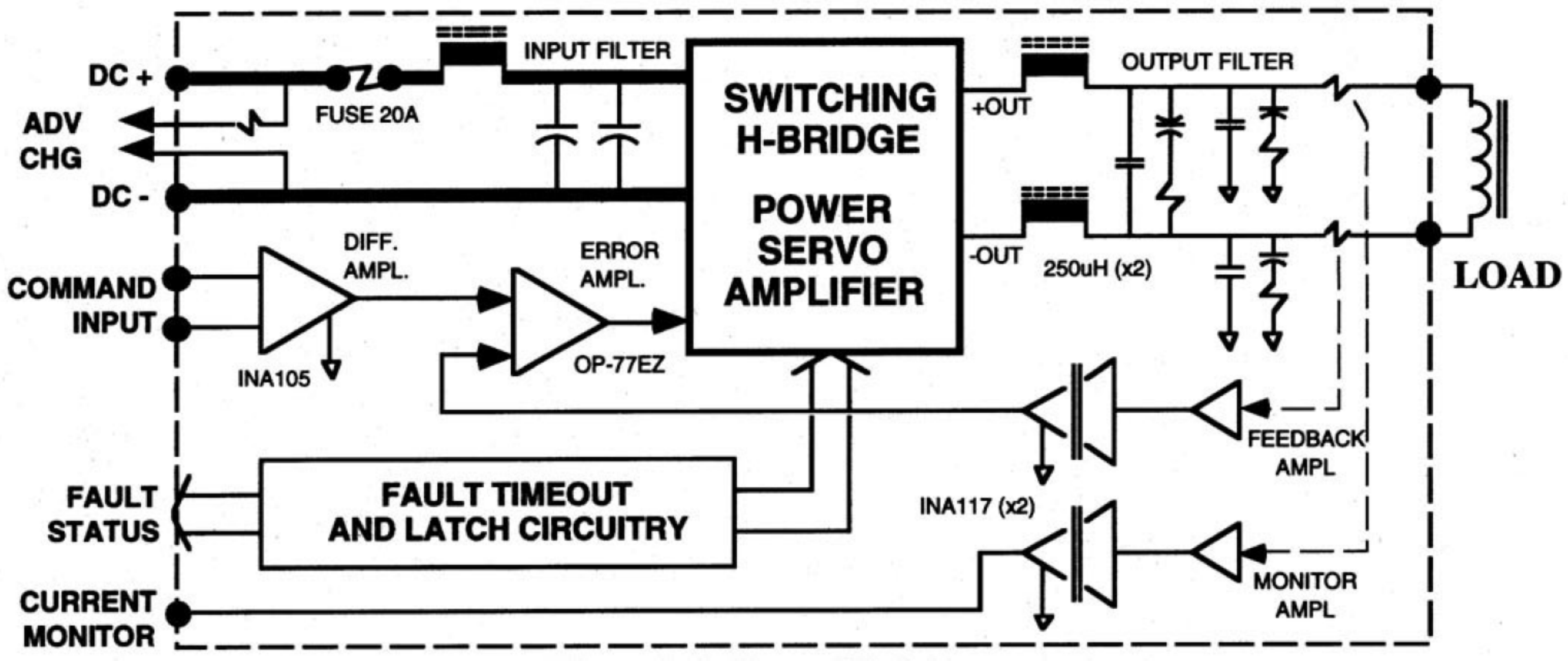
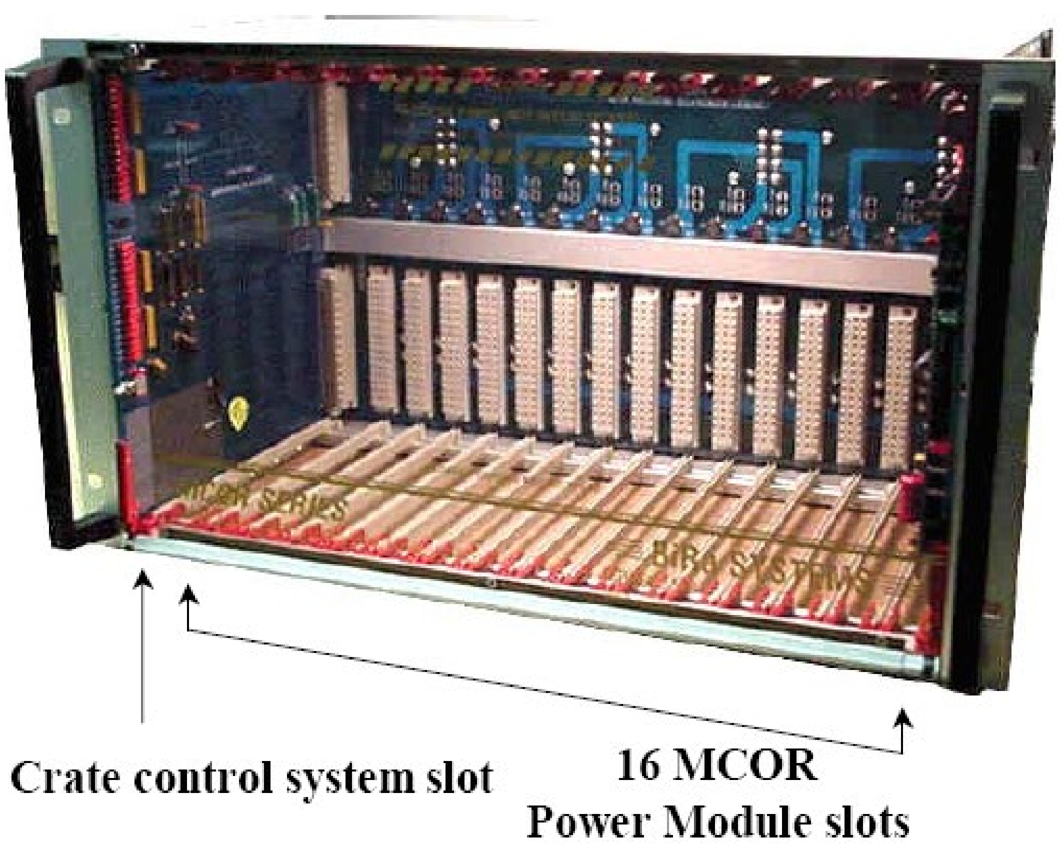


Figure 1.3. MCOR12 Block Diagram.

Bipolar Power Supplies at SPEAR3 and LCLS

Figure 1.1. A typical MCOR installation



NEW MAGNET POWER SUPPLY FOR PAL LINAC

Table 1: Development specifications of MPS

	Bipolar	Unipolar	
Size (W x H x D)	435x135×450	435×178×450	mm
Input	1ϕ 220V	3ϕ 30V	V
Output	±10/20	50/50	A/V
Output stability	±50ppm	±20ppm	< 1 hour
	±100ppm	±50ppm	> 10 hours
Output resolution	16		bit
Topology	Full-Bridge 4-Q DC/DC converter		
Switch freq.	50		kHz
Output Filter Cut-off freq.	< 5		kHz

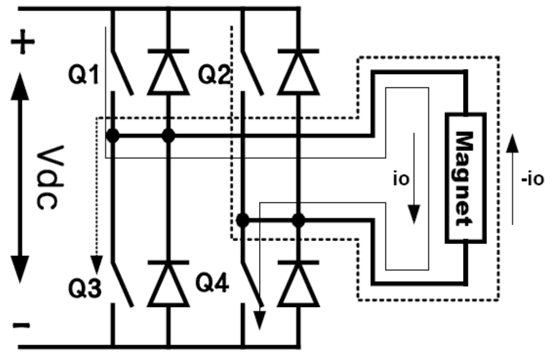


Figure 1: Bipolar MPS operation of full-bridge four-quadrant DC/DC converter.

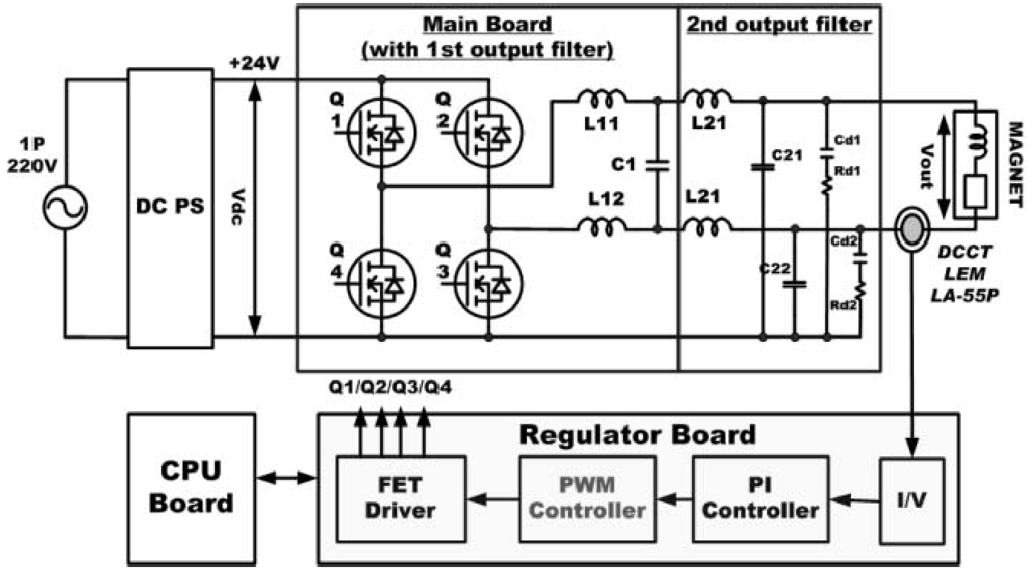
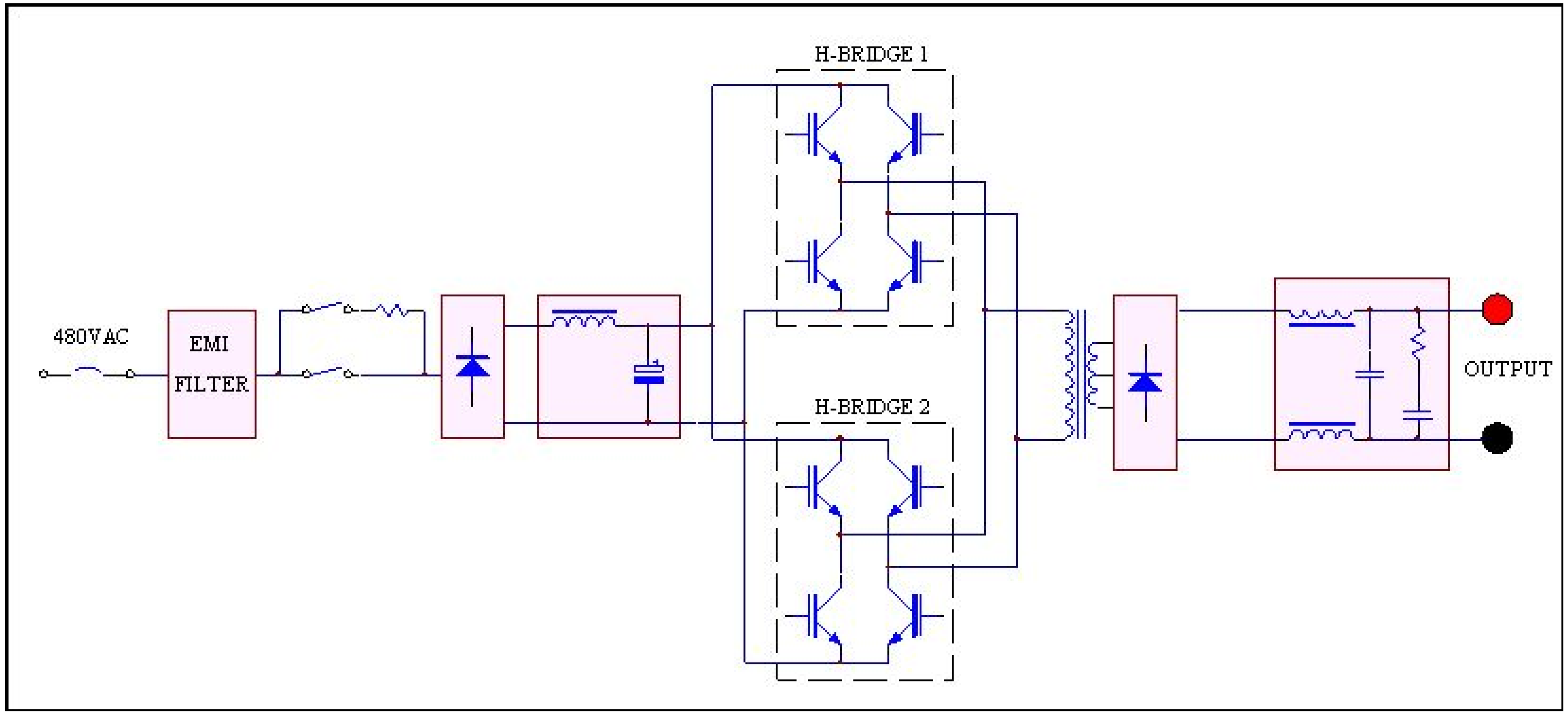


Figure 4: Circuit diagram of bipolar MPS.

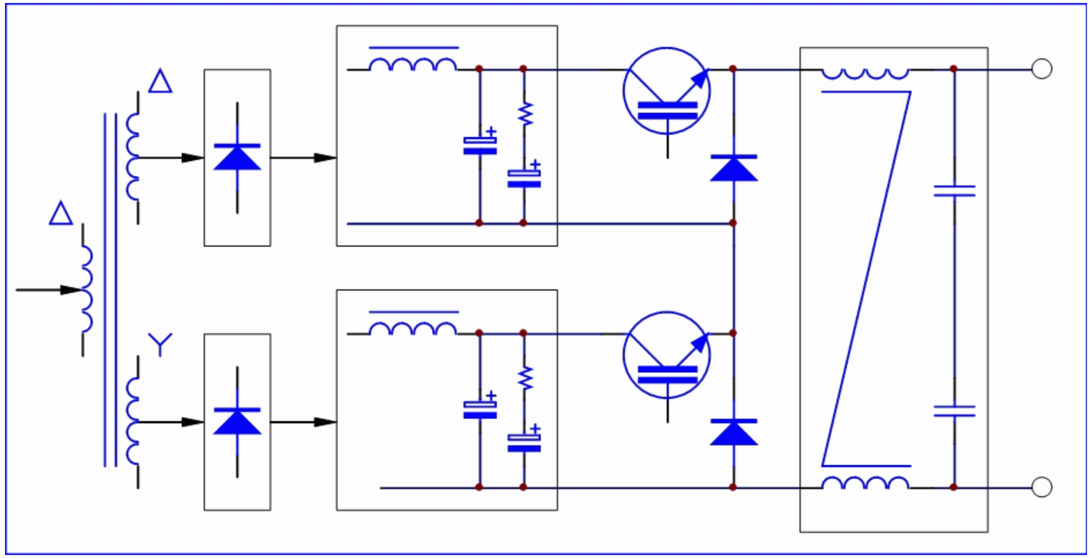
Table 1: LGPS ratings.

LGPS	V	I	P (kW)	Qty
BV1/2	80	900	72	1
QF2L/R	80	1250	100	2
QF5L/R	253	750	190	2
QD4L/R	200	1350	270	2



SPEAR3 Large Power Supplies

- Line-isolated
- 32 kHz Switch-output ripple
- High efficiency
- Fast output response
- Stability better than ±10 ppm
- 100A to 225A
- 70kW to 135kW
- Low cost: US$ 0.26 – 0.39/W



Section 7 – Superconducting Magnet Power Systems

Rationale For Using Superconducting Magnets

- ***Problem***
  - *Contemporary high energy physics questions require much higher beam energies*
    - *Higher energies mean larger magnets, larger facilities (size goes like bend radius which increases with energy).*
    - *Conventional magnets consume lots of electrical power, iron cores saturate at about 2T*
  - *Synchrotron light sources require high field insertion devices (undulators, wigglers)*
    - *Permanent magnet pole pieces also have limited magnetic fields*
- ***Superconducting Magnets***
  - *Are smaller (possess high current density $\Rightarrow$ compact windings, high gradients)*
  - *Consume much less power (primarily refrigeration power), consequently lower power bills*
  - *Can generate greater magnetic fields (typically to 10T and more). Greater magnetic fields mean smaller bend radius, smaller accelerator and rings, reduced capital expense. Furthermore, no expensive iron core*

Example – Superconducting solenoid

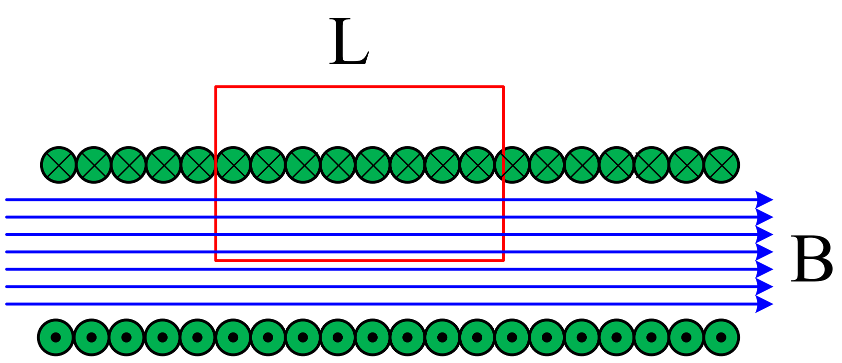
From Ampere's Law

$\oint H \cdot dl = NI_0 \qquad B = \mu_0 H$

$\mu_0 \oint H \cdot dl = \mu_0 NI_0$

$BL = \mu_0 NI_0$ *or*

$B = \mu_0 NI_0 / L$



B is in units of Tesla, T

$\mu_0 = 4\pi 10^{-7}\,\mathrm{T{-}m/A}$

N = number of solenoid turns (t)

I_0 *= amperes carried per turn (A/t)*

L or dl = solenoid length (m)

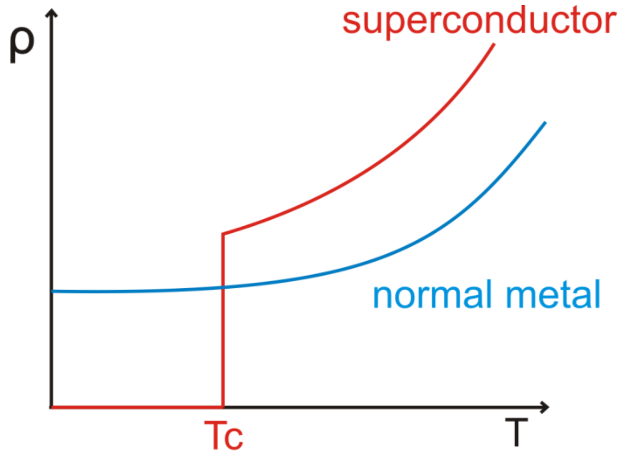
Assume a 3 m *solenoid with* 2500 *turns, each turn carrying* 5000 *A/t*

$$B = \frac{\mu_0 NI_0}{L} = \frac{4\pi 10^{-7}\,\mathrm{T{-}m/A} \cdot 2500\ \mathrm{t} \cdot 5000\ \mathrm{A/t}}{3} = 5.24\ \mathrm{T}$$

Normal conductors follow Drude's model

- *Electrons move freely in metal, accelerated by external* $\vec{\mathbf{E}}$ *field*
- *After a time* τ *the electron interacts with the lattice of the solid and gives up its energy*
- *Steady state average value of velocity* $\vec{\boldsymbol{v}} = -e\vec{\boldsymbol{E}}\tau/m$
- *Steady state value of current,* $\vec{\boldsymbol{j}} = -ne\vec{v} = (ne^2\tau/m)\vec{\boldsymbol{E}} = \sigma\vec{\boldsymbol{E}}$
- *This defines the conductivity* σ
- *Better conductors have longer times between interactions*
- *"Perfect" conductor has* $\sigma \rightarrow \infty$
- *Resistance of normal metal decreases to finite non-zero value as temperature decreases*

Superconducting Metals



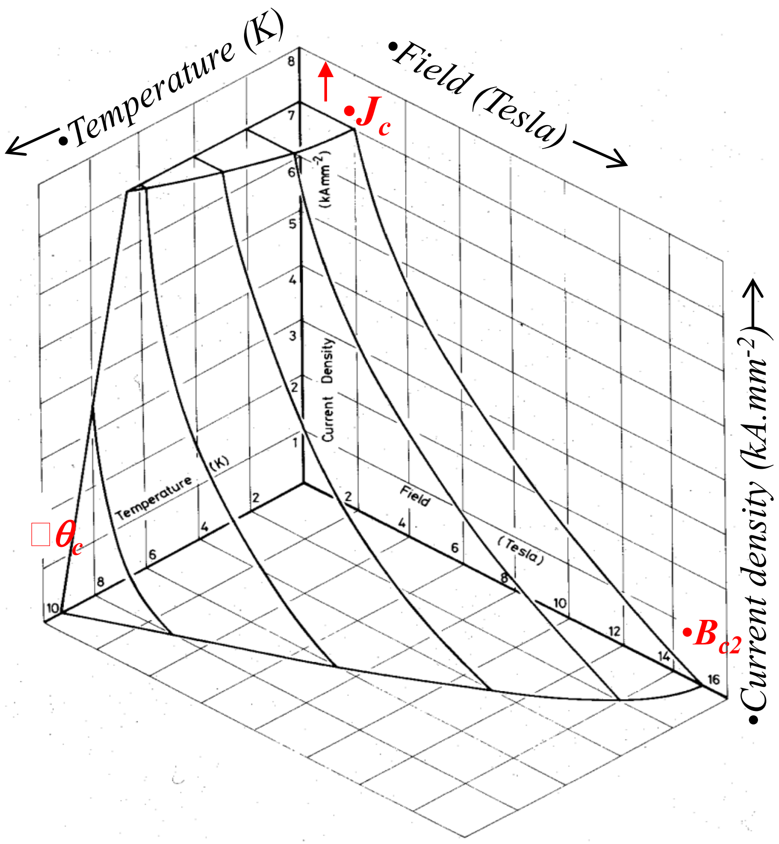
- *Superconducting metal resistance drops to zero at T_C*
- *Superconductors also exhibit Meissner effect*
  - *Excludes $\vec{\mathbf{H}}$ from the center of the SC*

BCS theory (Bardeen, Cooper, Schrieffer, 1957) explains superconductivity

- *Quantum mechanical many body problem*
- *In the presence of a solid-state lattice, conduction electrons can form "Cooper pairs" that lower the energy of the system*
- *Two phase system exists – normal and SC phases (most electrons are "normal")*
- *Band gap forms and Cooper pairs can carry current with no lattice interactions that cause energy loss. The resistance drops to zero.*

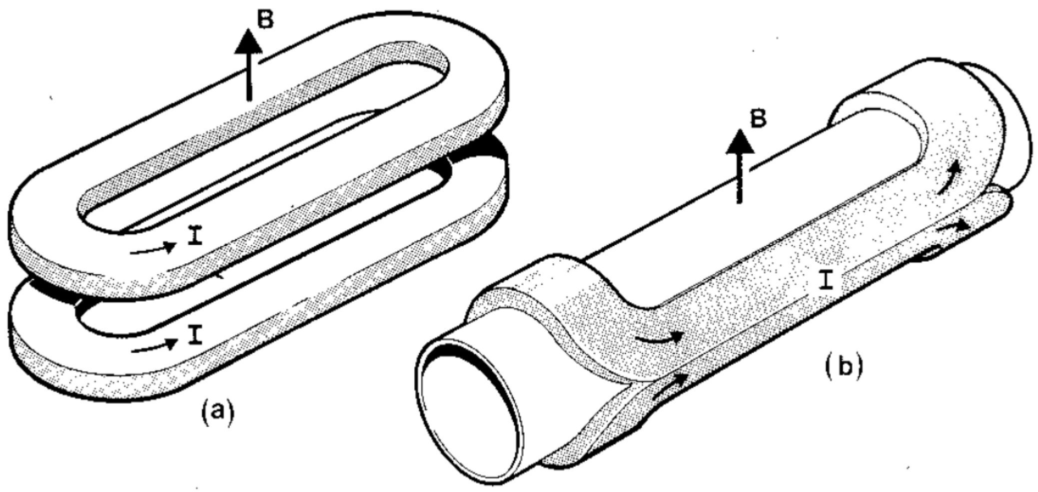
SC current capacity is dependent on the number of SC pairs

Superconductor Critical Surface Diagrams



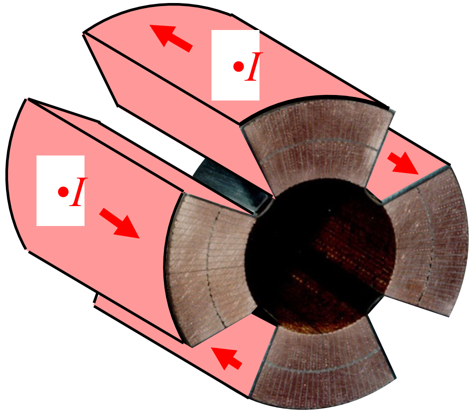
- *Exclusion of $\vec{H}$ (Meissner effect) increases system free energy.*
- *SC state is a very fragile system*
- *Sufficiently large $\vec{H}$ raises free energy of SC state above that of normal conductor and "quenches" SC state*
- *Many, but not all metals and alloys can exhibit SC behavior*
- *Different materials have different values of T_C, H_C, and J_C.*
- *Niobium or one of its alloys is the most common commercially used SC material*
- *Graphic shows the 3 dimensional space **critical surface**, which is the boundary between superconducting and normal conducting phases*
  - *Superconducting phase below surface*
  - *Normal conducting above*

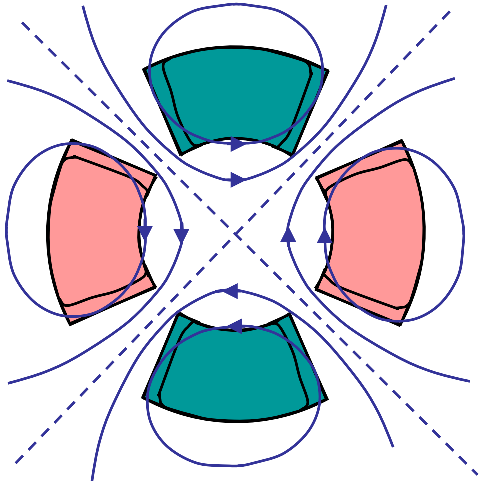
Dipole Magnet

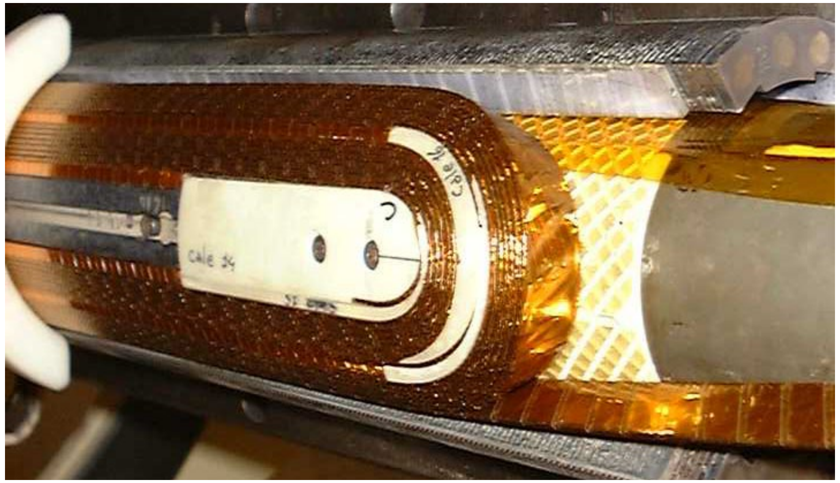


- *Conventional magnet is typically "iron-dominated"*
- *Iron pole pieces shape the field*
- *SC magnets are made from superconducting cable*
- *Winding location shapes the field according to Ampere's Law*
- *Windings must have the correct cross section*
- *Also need to shape the end turns*

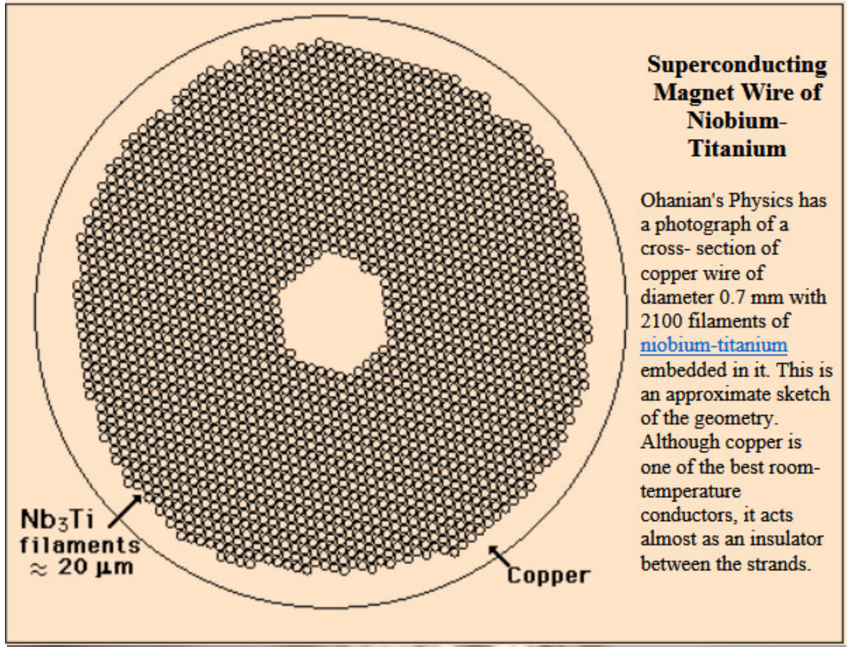
- *Quadrupole windings, gradient fields produce focusing*

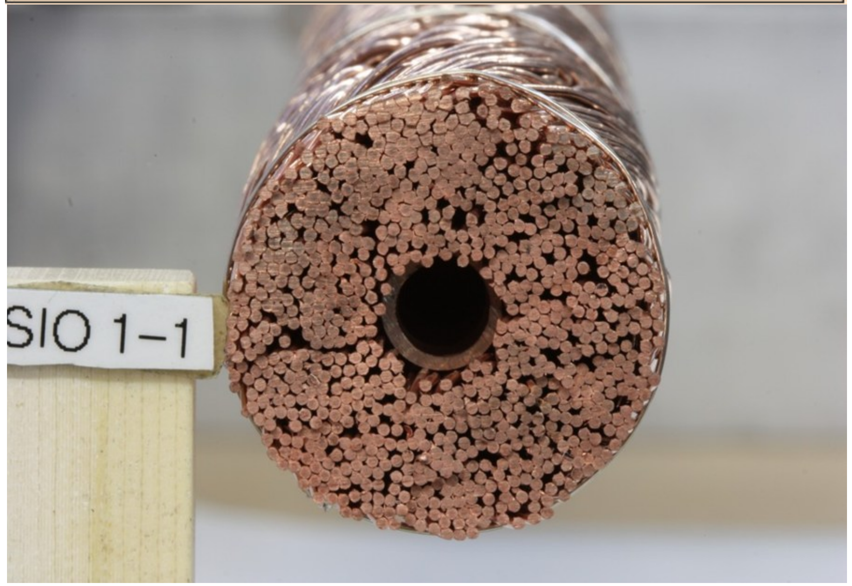






Winding Construction

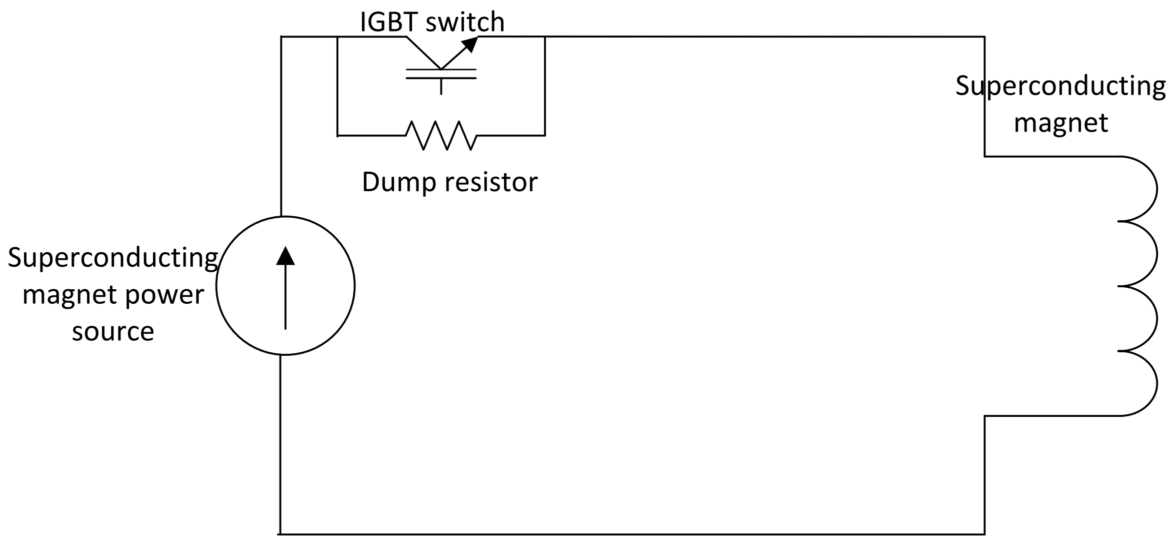




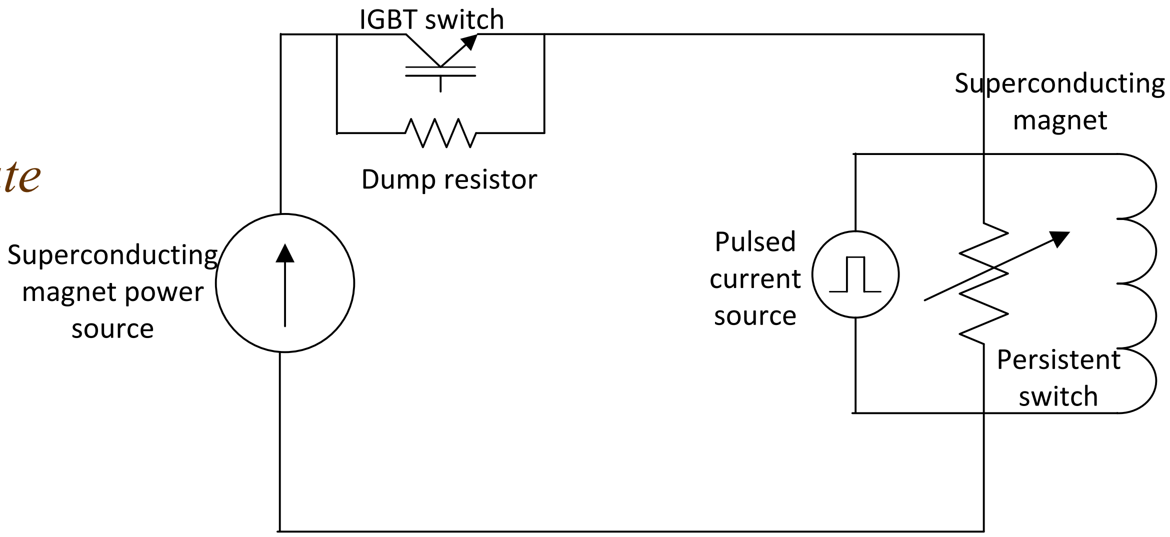
- *The superconductor is made in the form of fine filaments embedded in a matrix of copper. Copper here is the insulator. The filament diameter is typically* $10 - 60\ \mu m$.
- *These form a wire of diameter* $0.3\ - 1.0$ mm. *A typical wire is shown in the upper left.*
- *The composite wires are twisted like a rope as shown in the lower left to make one conductor.*
- *The choice of the filament material is a trade-off between* T_C, B_{Crit}, *and ductility*
- *Other filament materials have higher critical temperatures and yield higher fields, but only NbTi (*T_C*=10°K) is ductile*

Two Modes of operation

1. Power Supply Mode
   Source always drives magnet



2. Persistent Mode
   Source supplies required current
      Persistent switch heated to NC state
   When current is at its desired value
      Pulsed current source turned off
      Persistent switch becomes SC
      Current flows in small loop
   Source can be turned off

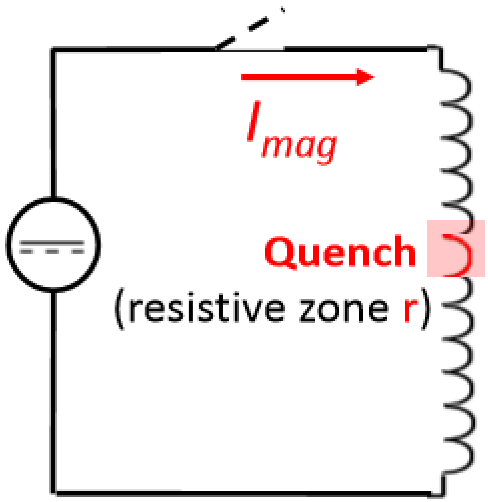


Quenches

- *Occur if the limits (T, P, B) of the critical surface are exceeded. The affected magnet coil changes from a superconducting to a normal conducting state.*
- *The resulting drastic increase in electrical resistivity causes Joule heating, further increasing the temperature and spreading the normal conducting zone through the magnet.*
- *High temperatures can destroy the insulation material or even result in a meltdown of superconducting cable*
- *The excessive voltages can cause electric discharges that could destroy the magnet*
- *In addition, high Lorentz forces and temperature gradients can cause large variations in stress and irreversible degradation of the superconducting material, resulting in a permanent reduction of its current-carrying capability.*

Quench in a Large Magnet

A formation of an unrecoverable normal zone within a superconductor. Quenching will convert energy supplied by the current source AND magnet stored energy into heat.



$$P = I_{mag}^2 r$$

$$E = \frac{LI^2}{2}$$

- *When quench occurs, energy release is localized and the heat changes that zone to a normal conductor, further increasing the heat!*
- *If that zone is small in volume, the quench may lead to unrepairable damage of the magnet windings or other electrical infrastructure (splices, current leads, etc).*
- *Quench protection is an array of techniques used to prevent such damage from occurring.*

Quench protection sequence:

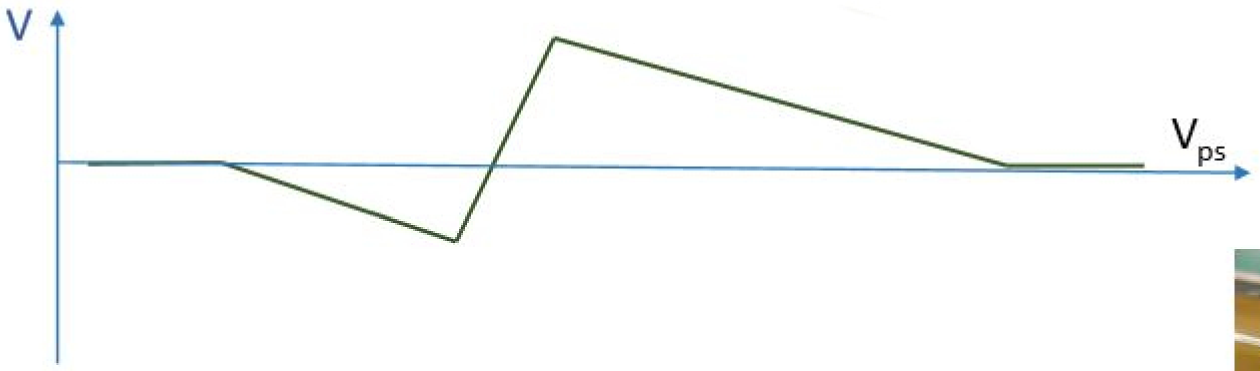
Detect → *De-energize* → *Extract*

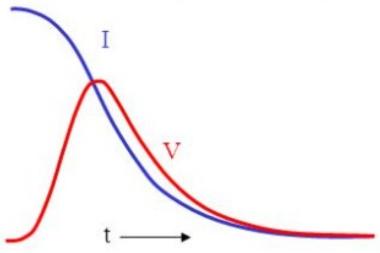
Slide courtesy M. Marchevsky, LBL – USPAS 2017

Quench Parameters

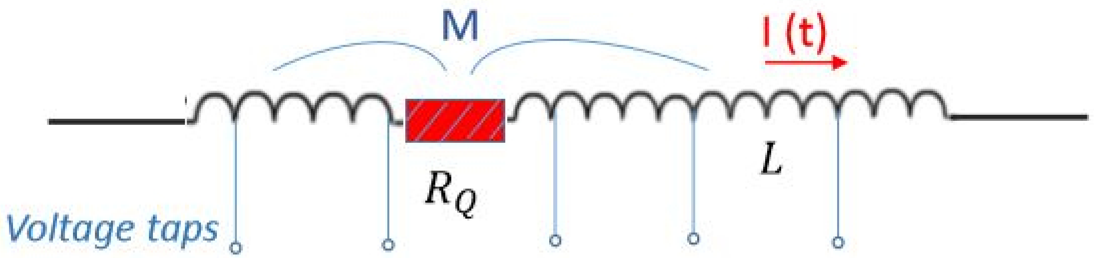
Parameter	*Values*
Detection Time	5 to 20 milliseconds
Resistance	10s to 100s of nanohms
Voltage	10s to 100s of microvolts
Energy	10 to 100s of microjoules
Energy Extraction Time Constant	10 to 100s seconds

Voltage Distribution During A Quench



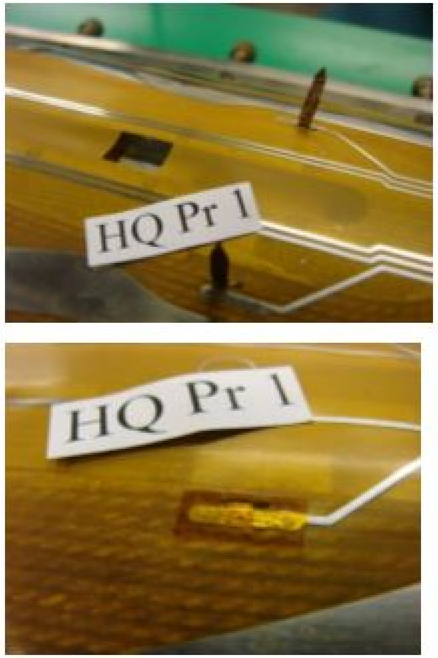


$$V_Q(t) = I(t)R_Q(t) - M\frac{dI(t)}{dt} \qquad L\frac{dI(t)}{dt} \cong I(t)R_Q(t)$$



$$V_Q(t) = I(t)R_Q(t)(1 - \frac{M}{L})$$

$V_Q(0) = V_Q(\infty)$ =0 => peaks during the quench

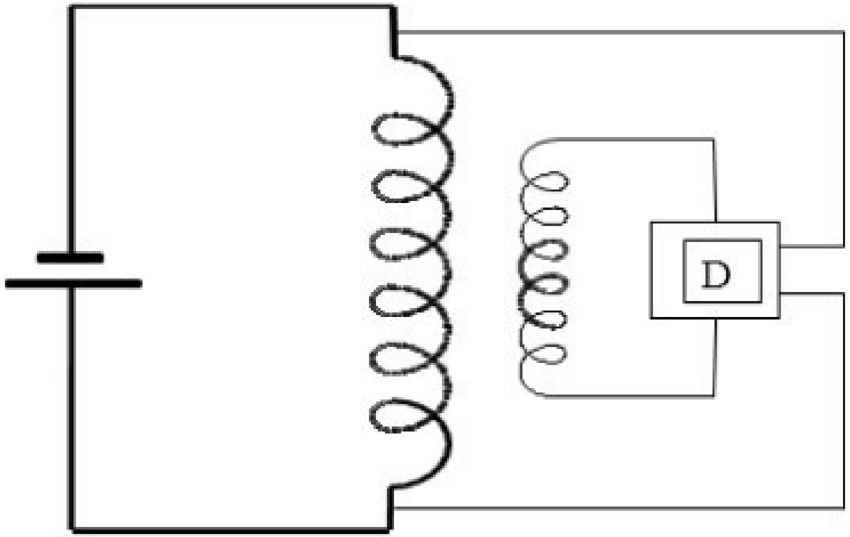


Voltage taps examples

Internal magnet voltage during quench may reach several hundreds of volts!

Slide courtesy M. Marchevsky, LBL – USPAS 2017

Quench Detection Methods - Mutual Inductance

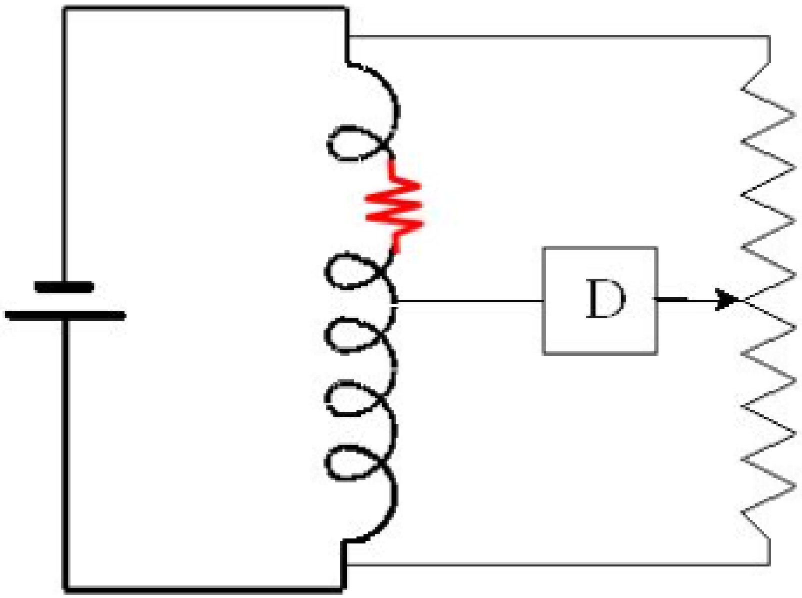


- *Detector subtracts voltages to give*

$$V = L\frac{di}{dt} + IR_Q - M\frac{di}{dt}$$

- *Adjust detector to make M=L*
- *M can be a toroid linking the current supply bus, but must be linear, which means no iron*

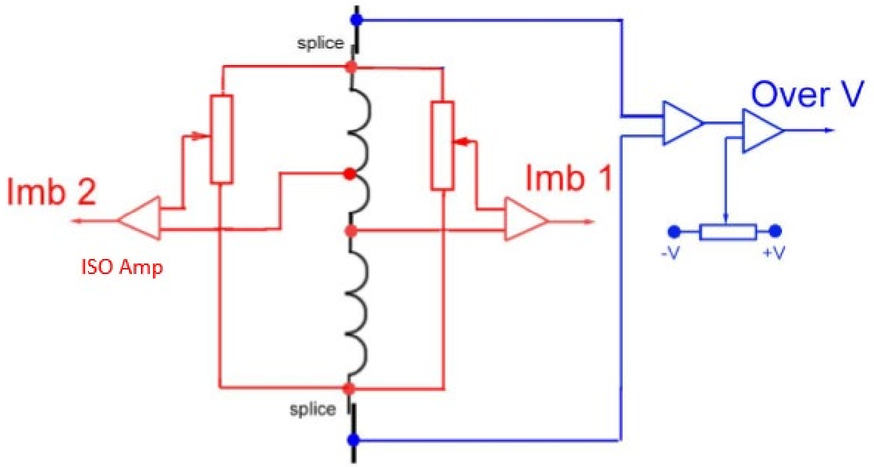
Martin Wilson, Cockroft Institute Jan 2013



- *Adjust for balance when not quenched*
- *Imbalance of resistive zone seen as voltage across detector, D*
- *If there is concern about symmetrical quenches, connect a second detector at a different point*

Martin Wilson, Cockroft Institute Jan 2013

Quench Detection For Symmetrical Quenches



- ***Imbalance bridge*** *circuit detects resistive voltage in any branch of the coil winding by comparing potential of a preselected voltage tap to that provided by a resistive divider. Several (at least 2) imbalance circuits are used in order to detect symmetric quenches. Typical Imb. threshold is ~100 mV for research magnets. Quench is detected when either of the detector circuits outputs voltage above pre-set threshold. A time interval over which voltage rises above the threshold is often called "detection time" (td).*
- ***Overvoltage Detector*** *senses voltage across coil compensated for the inductive component. Often includes resistive junctions (splices).*

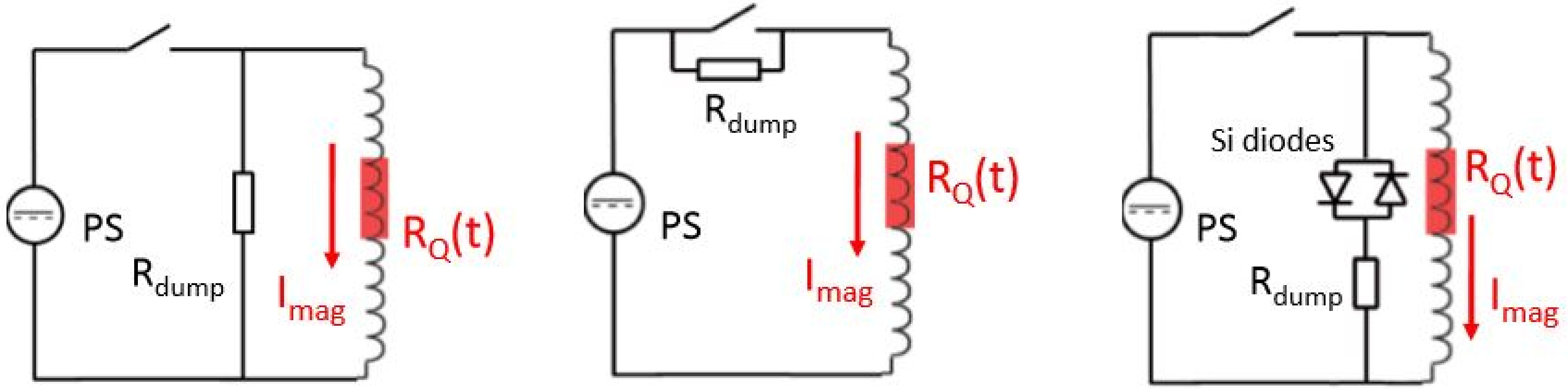
Slide courtesy M. Marchevsky, LBL – USPAS 2017

Protection Using An External Dump Resistor

By adding an external resistor in parallel or series with the quenching magnet, part of the magnet energy can be extracted outside of the cryostat.

Efficiency of energy extraction depends on $R_Q(t)/R_{dump}$ *. At most, 50 – 60% of the magnet energy is extracted outside of the cryostat using these methods.*

$$L\frac{dI(t)}{dt} = I(t)R_Q(t) + I(t)R_{dump}$$



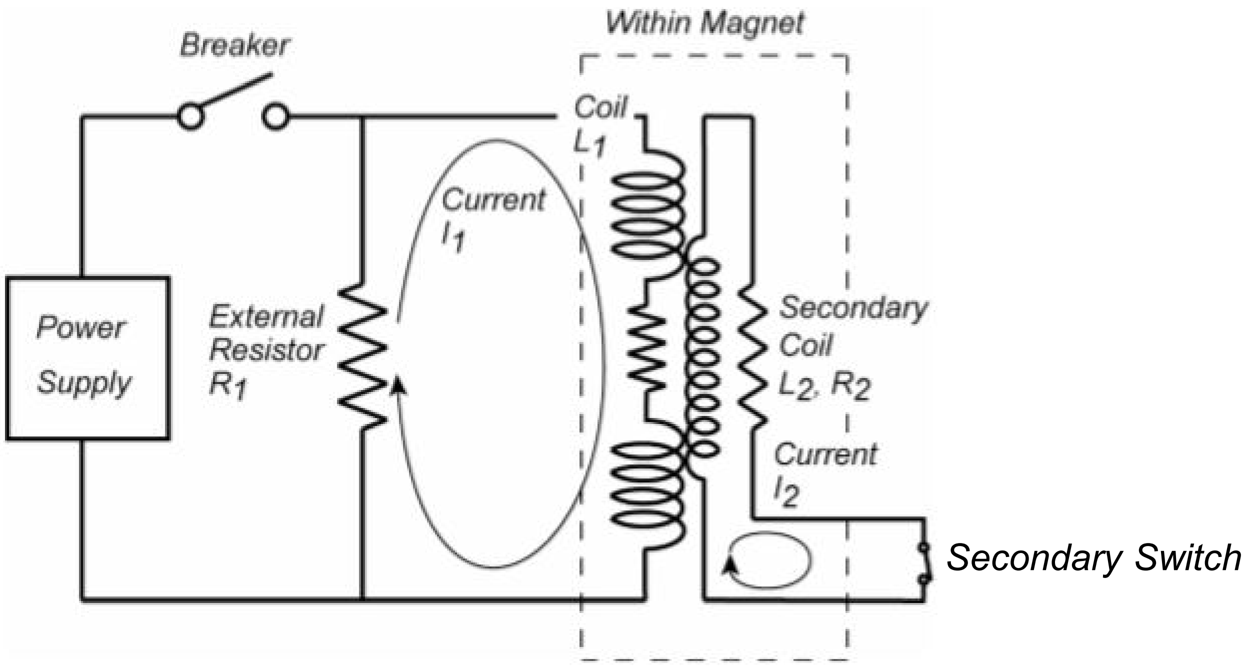
Standard scheme

Modified schemes 1 and 2 – current ramping not limited by the dump resistor

Drawback is, $V_{mag\ max} = I_{mag}R_{dump} = 2 \cdot \frac{1}{2}LI_{mag}^2 \cdot \frac{R}{I_{mag}L} = \frac{2E}{I_{mag}\tau}$ *appears across the magnet terminals*

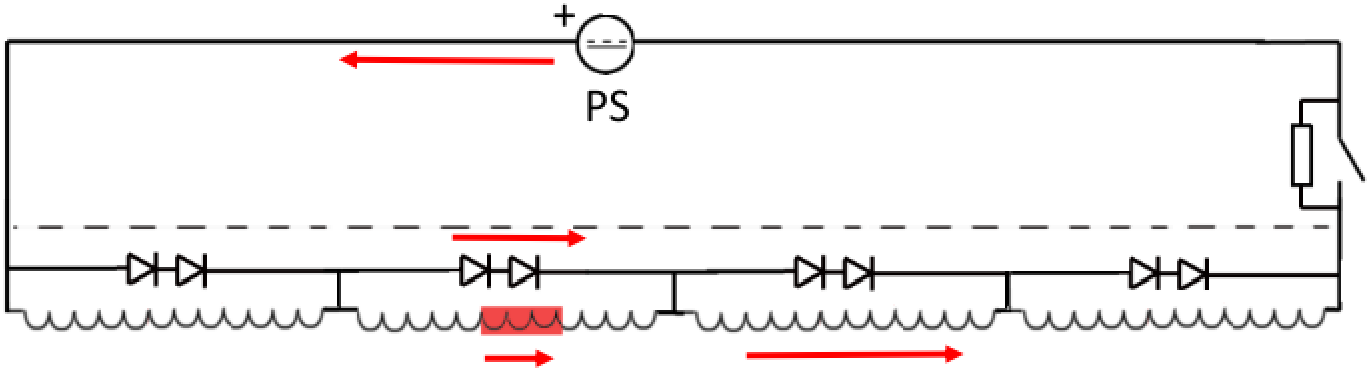
The extraction time constant is determined by L/R_{dump} *, since* $R_{dump} > R_Q$

Slide courtesy M. Marchevsky, LBL – USPAS 2017



- *Breaker is closed, secondary switch is open when magnet current I_1 is ramped for operation.*
- *When quench is sensed, breaker is opened and secondary switch is closed.*
- $L\frac{di}{dt}$ *in current decay induces a current in L_2 and R_2. R_2 heats and normalizes the entirety of L_1 very quickly. The quench voltage is spread over the entire magnet*
- *τ is reduced quickly, reducing magnet damage possibility*

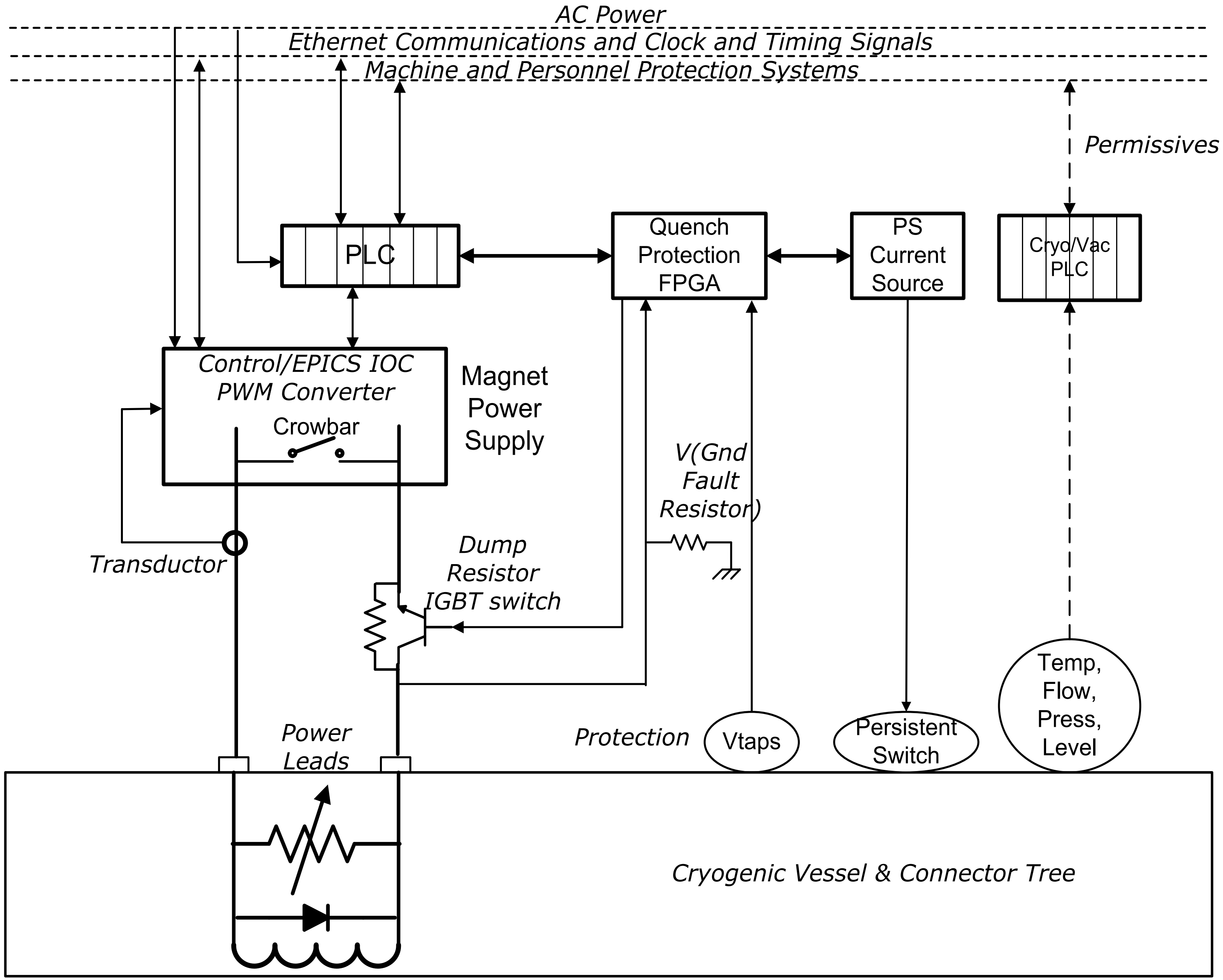
Protection Of A Magnet String



- Strings of silicon diodes are added in parallel to each magnet.
- Diodes start to conduct at ~2-5 V of bias at liquid helium temperature, and therefore are not carrying any current during ramping or normal magnet operation.
- As quench occurs, voltage across the magnet rises above, its diodes become conductive and so the chain current is bypassed through them
- This decouples the magnet energy and rundown time from the string energy and run-down time, reducing heat dissipation
- Same scheme can be used for protection of multi-coil magnets (quadrupoles, sextupoles). A complete accelerator can be also split in several chains, depending on its size.

A Powerex R7HC1216xx
Diode rated at 1600 A

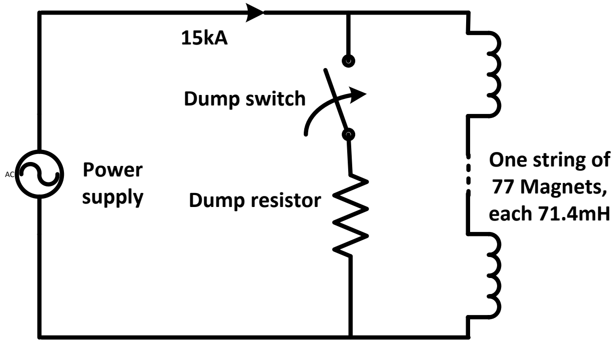
Slide courtesy M. Marchevsky, LBL – USPAS 2017


AC Power
Ethernet Communications and Clock and Timing Signals
Machine and Personnel Protection Systems
Permissives
PLC
Quench Protection FPGA
PS Current Source
Cryo/Vac PLC
Control/EPICS IOC
PWM Converter
Crowbar
Magnet Power Supply
V(Gnd Fault Resistor)
Transductor
Dump Resistor IGBT switch
Temp, Flow, Press, Level
Power Leads
Protection
Vtaps
Persistent Switch
Cryogenic Vessel & Connector Tree

Superconducting Magnet Power System Homework Problem #12

A collider has several equal strings of 77 superconducting magnets, each with 71.4mH inductance, carrying 15kA of current. If one, or more magnets in a string quenches, all the energy from the other magnets in the string will dissipate their energies into the quenched magnet, thus destroying it. Design a switched dump resistor to discharge the string current at a maximum rate, dI/dt, of 150A/s to prevent damage to the superconducting magnet in the event of a quench in the string. Refer to the circuit diagram below.

1. *What is the energy stored in each magnet and in the string when running at its design value?*
2. *What is the total inductance of the string?*
3. *Write the equation that describes the resistor current after closing the switch.*
4. *Find the resistor value to limit the maximum rate of decrease of current in the magnets to 150A/s*
5. *What is the maximum voltage generated across the resistor?*
6. *What is the time constant of this circuit?*
7. *Design a steel dump resistor that has little thermal conductance to the outside world (adiabatic system). Calculate how much steel mass (weight) will limit the temperature increase of the resistor to 500°K. (Steel gets structurally soft at 538°C and melts at 1510°C.)*



Help

$Q = M\ C_p\ \Delta T$

$Q =$ *heat (energy) into the system expressed in joules*

M= mass or weight of the resistor

$C_p =$ *specific heat of material* $= 0.466 \dfrac{J}{gm * {}^oK}$ *for steel*

$\Delta T =$ *Temperature rise of the resistor*

From information in "CERN LHC Magnet Quench Protection System, L. Coull, et. al, 13th International Conference on Magnet Technology, Victoria, Canada, 1993

Section 8 – Pulsed Power Supplies

Outline

- *For the study of pulsed power systems*
  - *Need to understand basics of transmission lines*
  - *Once we know the basics, we can follow simple rules to apply them*
- *If we just state the rules*
  - *It may sound like black magic and take away the intuition*
- *Therefore we derive the rules to help in understanding the basics of transmission lines*

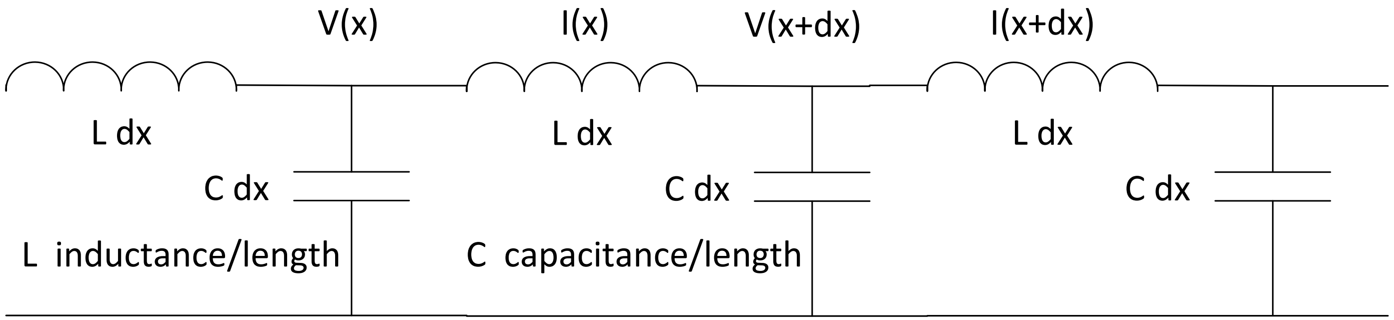
Impedance Matching

- *Pulsed Power systems differ from low power electronics; it is expensive to produce high power signals*
  - *High voltages*
    - *Semiconductors (and other devices) must be able to withstand voltages across their terminals*
    - *Circuits must be rated to prevent breakdown*
  - *High currents*
    - *Circuit elements must be able to handle current*
  - *High power*
    - *Generated heat must be dissipated*
- *The system requirements give us the minimum power required at the load*
- *By properly designing our circuits, matching impedances, we can minimize the required system power, and therefore the cost and complexity of our systems*

Transmission Line Basics

- *A transmission line is a "controlled impedance" device, usually consisting of two conductors.*
- *Its geometry and material properties determine the electric and magnetic field distributions between the conductors.*
  - *The voltage between the conductors is determined by the integral of the electric field between them (Coloumb's law)*
  - *The current along the conductors determines the integral of the magnetic field around the conductor (Ampere's law)*
- *Transmission lines support the propagation of fixed velocity waves in both directions (forward and backward) along the line.*
- *Transmission lines guide transverse electro magnetic (TEM) waves, TE or TM waves are guided by waveguides*

Transmission Line Equations



$$V(x+dx,t) - V(x,t) = -Ldx\frac{\partial I(x,t)}{\partial t}$$

$$I(x+dx,t) - I(x,t) = -C\text{dx}\frac{\partial V(x,t)}{\partial \text{t}}$$

$$\frac{\partial V(x,t)}{\partial x} = -L\frac{\partial I(x,t)}{\partial t}; \; \frac{\partial I(x,t)}{\partial x} = -C\frac{\partial V(x,t)}{\partial t}$$

$$\frac{\partial^2 V(x,t)}{\partial x^2} = -L\frac{\partial^2 I(x,t)}{\partial x \partial t} = LC\frac{\partial^2 V(x,t)}{\partial t^2} \Rightarrow \frac{\partial^2 V(x,t)}{\partial x^2} - LC\frac{\partial^2 V(x,t)}{\partial t^2} = 0$$

$$\frac{\partial^2 I(x,t)}{\partial x^2} - LC\frac{\partial^2 I(x,t)}{\partial t^2} = 0$$

Both solve the "Telegrapher's Equation"

- *General solution of the second order wave equation is a combination of two terms, both with velocity* $v = 1/\sqrt{LC}$

$$V(x,t) = V_+(x - vt) + V_-(x + vt)$$

- V_+ *is a forward traveling wave*
- V_- *is a backward traveling wave*
- V_+ *and* V_- *are determined by initial conditions*

Often can be determined from conservation of energy and momentum

Change variables to $\phi = x - vt; \psi = x + vt$.

Then for any function $f(x - vt)$ *(forward) and* $g(x + vt)$ *(backward)*

$$\frac{\partial f(x-vt)}{\partial x} = \frac{\partial \phi}{\partial x}\frac{df(\phi)}{d\phi} = \frac{df(\phi)}{d\phi}; \quad \frac{\partial g(x+vt)}{\partial x} = \frac{\partial \psi}{\partial x}\frac{dg(\psi)}{d\psi} = \frac{dg(\psi)}{d\psi}$$

$$\frac{\partial f(x-vt)}{\partial t} = -v\frac{df(\phi)}{d\phi}; \frac{\partial g(x+vt)}{\partial t} = v\frac{dg(\psi)}{d\psi}$$

We can rewrite the two terms in the circuit equations

$$\frac{\partial V}{\partial x} = \frac{\partial V_+(x-vt)}{\partial x} + \frac{\partial V_-(x+vt)}{\partial x} = \frac{\mathrm{d}V_+(\phi)}{d\phi} + \frac{dV_-(\psi)}{d\psi}$$

$$\frac{\partial I}{\partial t} = \frac{\partial I_+(x-vt)}{\partial t} + \frac{\partial I_-(x+vt)}{\partial t} = -v\frac{dI_+(\phi)}{d\phi} + v\frac{dI_-(\psi)}{d\psi}$$

Therefore separating the circuit equation $\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t}$ *into its two components means*

$$\frac{dV_+(\phi)}{d\phi} = Lv\frac{dI_+(\phi)}{d\phi} = \text{ and } \frac{dV_-(\psi)}{d\psi} = -Lv\frac{dI_-(\psi)}{d\psi}$$

Recalling that $v = 1/\sqrt{LC},\ Lv = \sqrt{L/C}$, *integrate to obtain, with* $Z = \sqrt{L/C}$

$$V_+(x - vt) = \sqrt{L/C}\ I_+(x - vt) = ZI_+(x - vt)$$

$$V_-(x + vt) = -\sqrt{L/C}I_-(x + vt) = -ZI_-(x + vt)$$

(The integration constant is zero for waves.)

This gives the definition of the transmission line impedance Z as the ratio of the voltage wave to the current wave (taking direction of travel into account)

$$\vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t; \qquad \vec{\nabla} \times \vec{H} = \partial \vec{D}/\partial t$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon r} e(z,t)\hat{\boldsymbol{r}}; \quad \vec{\mathbf{H}} = \frac{I}{2\pi r} h(z,t)\hat{\boldsymbol{\theta}}$$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \vec{\boldsymbol{r}} & \vec{\boldsymbol{\theta}} & \hat{\mathbf{z}} \\ \partial/\partial r & 1/r\, \partial/\partial\theta & \partial/\partial z \\ E_r & rE_\theta & E_z \end{vmatrix} = \frac{\lambda}{2\pi\epsilon r} \frac{\partial e(z,t)}{\partial z} \hat{\boldsymbol{\theta}}$$

$$\vec{\nabla} \times \vec{H} = -\frac{I}{2\pi r} \frac{\partial h(z,t)}{\partial z} \hat{\boldsymbol{r}}$$

$$\frac{\partial e(z,t)}{\partial z} = -\frac{\epsilon I}{\lambda} \mu \frac{\partial h(z,t)}{\partial t}; \quad \frac{\partial h(z,t)}{\partial z} = -\frac{\lambda}{I\epsilon} \epsilon \frac{\partial e(z,t)}{\partial t}$$

Differentiate w.r.t z and use second equation to get

$$\frac{\partial^2 e(z,t)}{\partial z^2} - \mu\epsilon \frac{\partial^2 e(z,t)}{\partial t^2} = 0$$

$$\frac{\partial^2 h(z,t)}{\partial z^2} - \mu\epsilon \frac{\partial^2 h(z,t)}{\partial t^2} = 0$$

This is the telegrapher's equation with

$$v = 1/\sqrt{\mu\epsilon} = 1/\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0} = c/\sqrt{\mu_r \epsilon_r}$$

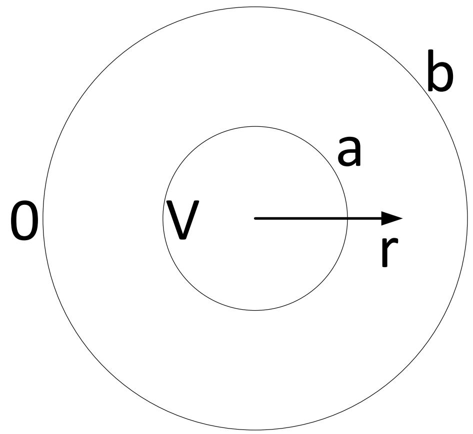
Capacitance/length (voltage between conductors)

$$C_l = \frac{Q}{Vl} = \frac{\lambda}{V} = \frac{\lambda}{-\int_b^a \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{x}}} = \frac{\lambda}{\int_a^b \frac{\lambda}{2\pi r\epsilon} dr} = \frac{2\pi\epsilon}{\log(b/a)}$$

Inductance/length (flux between conductors)

$$L_l = \frac{1}{Il} \iint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}} = \frac{1}{Il} \int_0^l \int_a^b \frac{\mu I}{2\pi r} dr\, dl = \frac{\mu}{2\pi} \log(b/a)$$

- $Z = \sqrt{\frac{L_l}{C_l}} = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu}{\epsilon} \frac{1}{2\pi}} \log\left(\frac{b}{a}\right)$
- $v = \frac{1}{\sqrt{L_l C_l}} = \frac{1}{\sqrt{\mu\epsilon}}$



- *Our wave equation has two solutions,* V_+, V_-
- *We are working with circuit equations, but with the proper identification with EM sources and fields we can use common conservation laws of physics to determine* V_+ *and* V_-
  - $V \sim \vec{\boldsymbol{E}}$
  - $I \sim \vec{\boldsymbol{H}}$

The energy of the electromagnetic fields in a volume is

$$\mathcal{E} = \frac{1}{2}\iiint \left(\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{D}} + \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{H}}\right) d^3x$$

$$= \frac{1}{2}\iiint \left(\frac{\lambda}{2\pi\epsilon r}\frac{\lambda}{2\pi r} + \frac{\mu I}{2\pi r}\frac{I}{2\pi r}\right) r\, dr\, d\theta\, dz$$

$$= \frac{1}{2}\frac{2\pi}{(2\pi)^2} l(\lambda^2/\epsilon + \mu I^2) \int_a^b \frac{r}{r^2} dr$$

$$= \frac{1}{2}\left[(\lambda l)^2 \frac{1}{2\pi\epsilon l} \log(b/a) + \frac{\mu l}{2\pi} \log(b/a)\, I^2\right]$$

$$\mathcal{E} = 1/2\, (Q^2/C + LI^2) = 1/2\, CV^2 + 1/2\, LI^2$$

Energy in Transmission Line

$$\mathcal{E} = \frac{1}{2}CV^2 + \frac{1}{2}LI^2 = \frac{1}{2}\left[CV^2 + L\left(\frac{V}{Z}\right)^2\right]$$

$$= \frac{1}{2}\left(CV^2 + \frac{L}{Z^2}V^2\right) = \frac{1}{2}\left(CV^2 + \frac{L}{L/C}V^2\right) = CV^2 = LI^2$$

In a wave, the EM energy is equally distributed.

- *Half of the energy is in the electric field.*
- *Half is in the magnetic field.*

Transmission Line Types

- *Coaxial transmission lines*
  - *Voltage between two coaxial conductors*
  - *Currents of equal magnitude and opposite sign are carried on the conductors*
  - *Conductors separated by air or dielectric*
  - *Transverse electromagnetic (TEM) transmission line media*
  - *Ideally non-dispersive (propagates all frequency components equally), with no cutoff frequency*
  - *No external electric or magnetic fields*

The power flow of fields is determined by the Poynting vector $\vec{\mathbf{P}} = \vec{\mathbf{E}} \times \vec{\mathbf{H}}$. *For the coaxial line*

$$\vec{\mathbf{P}} = \frac{\lambda}{2\pi\epsilon r}\hat{\mathbf{r}} \times \frac{I}{2\pi r}\hat{\boldsymbol{\theta}} = \frac{V}{r\log(b/a)}\frac{\mathrm{I}}{2\pi r}\hat{\mathbf{z}}$$

where we have used $V = \frac{\lambda}{2\pi\epsilon}\log\frac{b}{a}$

Power flow along the line is

$$P = \int_S \vec{\mathbf{P}} \cdot d\vec{\mathbf{s}} = \frac{VI}{2\pi\log(b/a)}\int_0^{2\pi} d\theta \int_a^b \frac{dr}{r} = VI$$

The momentum of an EM field is $\vec{\mathbf{p}} = \vec{\mathbf{P}}/c^2$ *so the momentum flow is* VI/c^2 *(with direction* $\pm$*)*

Energy of line of length d statically charged to voltage V

$\mathcal{E} = \frac{1}{2}(Cd)V^2$ *(C capacitance/length)*

Energy of two co-moving waves

$(V = V_+ + V_-)$; $V_+ = V_- = V/2$

$$\mathcal{E} = \frac{1}{2}[(Cd)V_+^2 + (Ld)I_+^2] + \frac{1}{2}[(Cd)V_-^2 + (Ld)I_-^2]$$

$$= \frac{1}{2}[(Cd)V_+^2 + (Cd)(ZI_+)^2] + \frac{1}{2}[(Cd)V_-^2 + (Cd)(ZI_-)^2]$$

$$= [(Cd)V_+^2 + (Cd)V_-^2] = 2(Cd)V_+^2$$

$$= 2(Cd)\left(\frac{V}{2}\right)^2 = \frac{1}{2}(Cd)V^2$$

Calculated energy the same in both cases

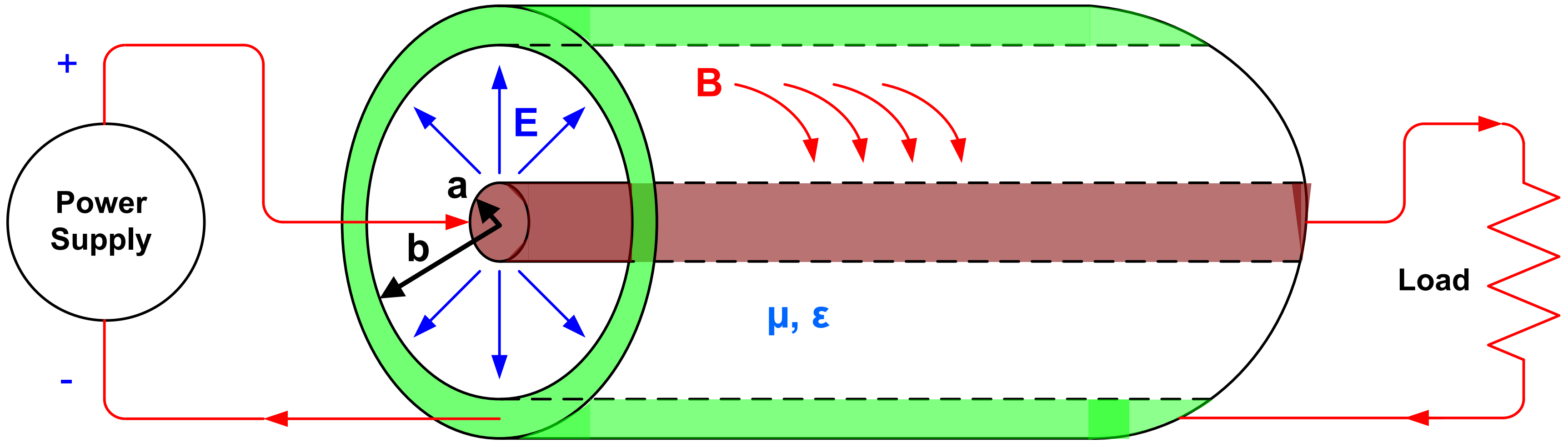
- *Momentum of EM field on line of length d statically charged to voltage V*
  - $(I = 0) \Rightarrow (P = 0) \Rightarrow (\vec{\boldsymbol{p}} = 0)$
- *Momentum of two co-moving waves*
  - *Power* $V_+ I_+$ *propagating in positive direction*
  - *Power* $V_- I_-$ *propagating in negative direction*
  - *Total momentum*

$$\vec{\boldsymbol{p}}_{\boldsymbol{T}} = \vec{\boldsymbol{p}}_+ + \vec{\boldsymbol{p}}_- = V_+ I_+ - V_- I_-$$

$$= 1/Z[(V/2)^2 - (V/2)^2] = 0$$

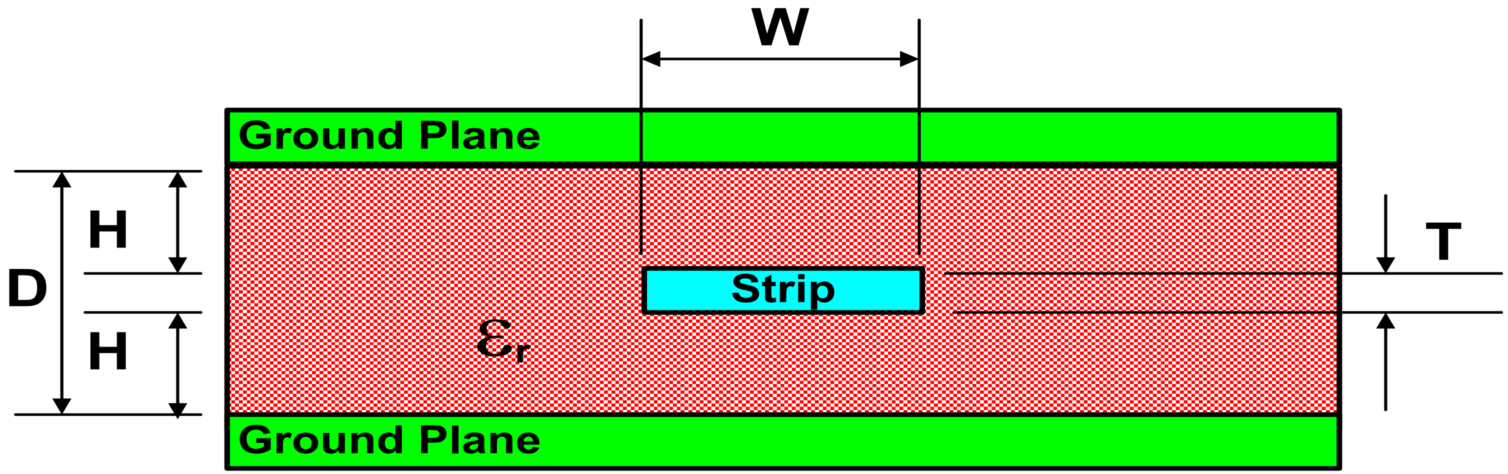
- *Calculated momentum the same in both cases*

- *Coaxial transmission lines and cables*



$$Z_0 = \frac{\ln {}^{b}\!/_{a}}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}$$

- *Planar transmission line - Stripline consists of a single strip buried in a dielectric separated from two or more ground planes*

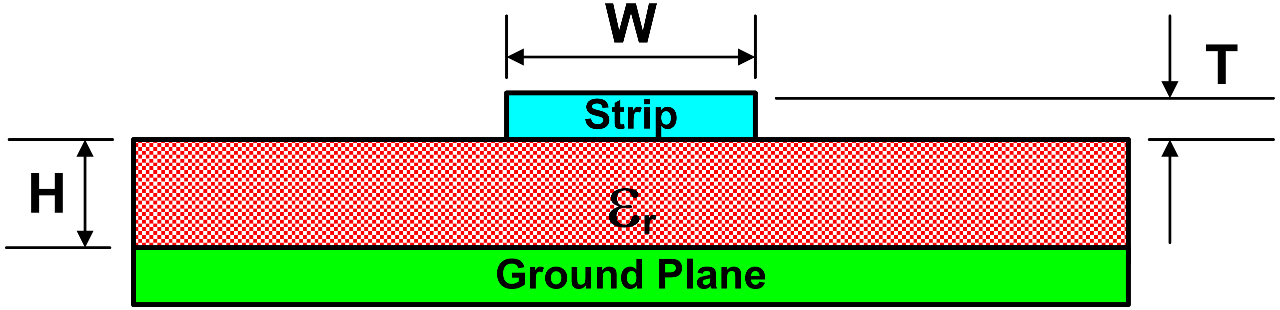


Characteristic Impedance

$$Z_O = \frac{60}{\sqrt{\varepsilon_r}} \ln\left[\frac{4H}{0.67\pi W\left(0.8+\frac{T}{D}\right)}\right] \text{ ohms}$$

Transmission Line Types

- *Planar transmission line - Microstrip line consists of a single strip on dielectric separated from a ground plane*



when $\left(\frac{W}{H}\right) < 1$ *Effective Dielectric Constant*

$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[\left(1 + 12\left(\frac{H}{W}\right)\right)^{-1/2} + 0.04\left(1 - \left(\frac{W}{H}\right)\right)^2\right]$$

Characteristic Impedance

$$Z_o = \frac{60}{\sqrt{\varepsilon_e}} \ln\left(8\frac{H}{W} + 0.25\frac{W}{H}\right) \quad \text{ohms}$$

when $\left(\frac{W}{H}\right) \geq 1$ *Effective Dielectric Constant*

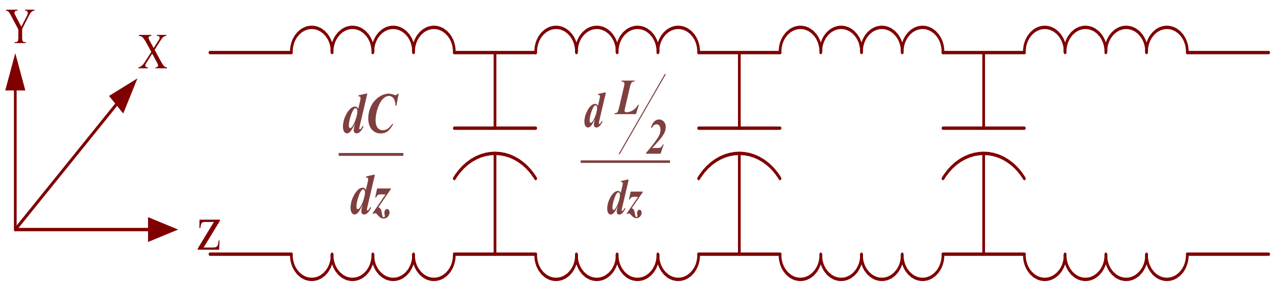
$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[\left(1 + 12\left(\frac{H}{W}\right)\right)^{-1/2}\right]$$

Characteristic Impedance

$$Z_o = \frac{120\pi}{\sqrt{\varepsilon_e}\left[\frac{W}{H} + 1.393 + \frac{2}{3}\ln\left(\frac{W}{H} + 1.444\right)\right]} \quad \text{ohms}$$

Transmission Line Types

- *Lumped element transmission lines*
  - *Combination of series inductors, shunt capacitors*
  - *Single inductor-capacitor combination is a resonant circuit*
  - *Series of an infinite combination of series L, shunt C turns into an ideal transmission line*
  - *Electric fields of the transmission lines are stored in shunt capacitors*
  - *Magnetic fields of the transmission lines are stored in series inductors*

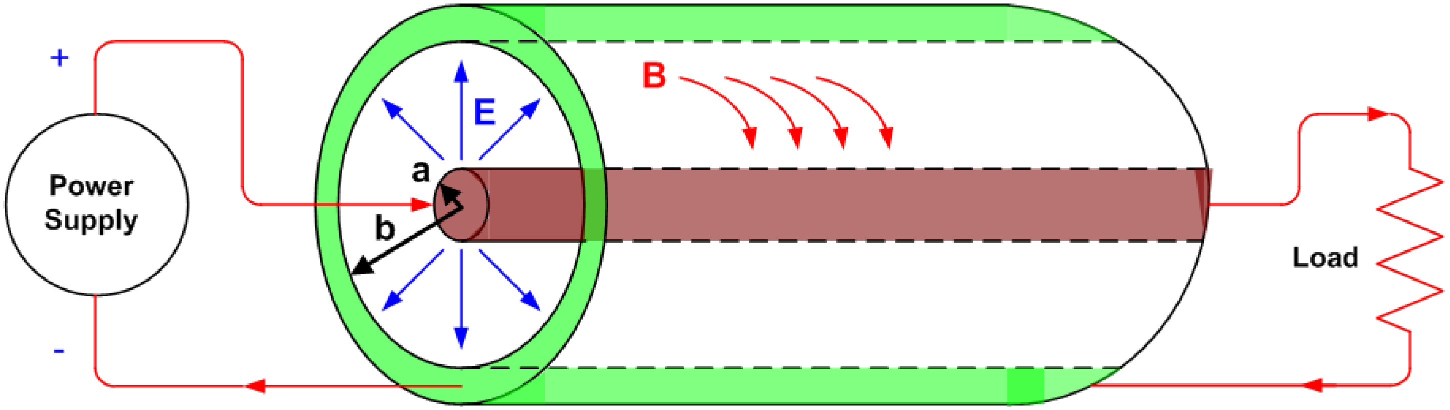


$E = \hat{y}E_y \quad H = \hat{x}H_x \qquad Z_0 = \sqrt{\frac{L}{C}}$ *Characteristic impedance - 377 ohms for air (free space)*

for air (and most dielectrics) $\mu_r = 1$, *for air* $\varepsilon_r = 1$ *(most other dielectrics* $\varepsilon_r > 1$, $n =$ *number of sections*

$Z_0 = \frac{\ln b/a}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}$ *For coaxial line,* $50\Omega \le Z_0 \le 80\Omega$

$v = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$ *= wave velocity wavelength* $\lambda = \frac{v}{f}$ *time delay=* $t_d = n * \sqrt{LC}$



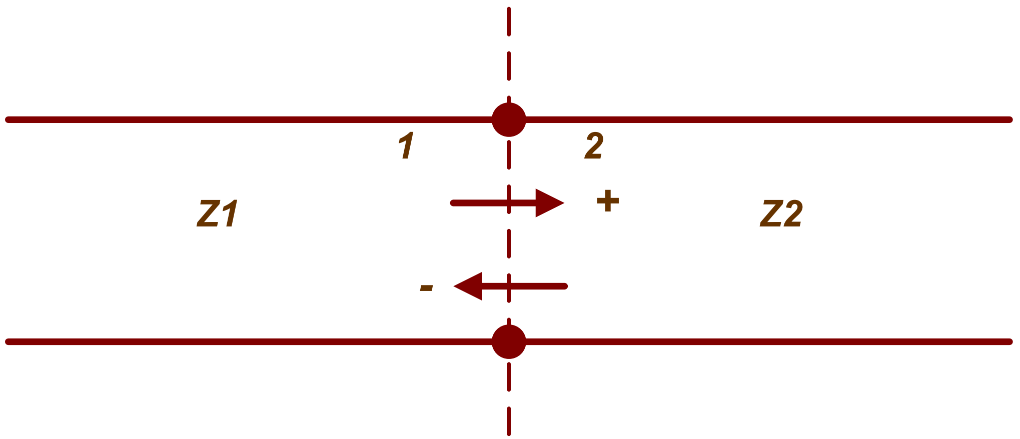
Transmission Line Boundary Conditions

- *Join two transmission lines together*
  - *If the impedances of both lines are the same, the electric and magnetic fields (voltage and current) can propagate without interruption.*
  - *If not, the boundary conditions on the fields force a reflection of part of the signal*

- *The general situation at an interface between two transmission lines of impedance* Z_1 *and* Z_2
- *A source generates an incident voltage and current,* (V_1^+, I_1^+) *moving forward on Line 1, with*

$$V_1^+ = Z_1 I_1^+$$

- (V_1^+, I_1^+) *at the interface causes a transmitted voltage and current,* (V_2^+, I_2^+)*, moving forward on Line 2, with* $V_2^+ = Z_2 I_2^+$
- (V_1^+, I_1^+) *at the interface, in general, also causes a reflected voltage and current,* (V_1^-, I_1^-)*, moving backward on Line 1, with* $V_1^- = Z_1 I_1^-$



The voltages on each side of the interface must be equal

$$V_1^+ + V_1^- = V_2^+$$

Current must be conserved at the interface

$$I_1^+ = I_1^- + I_2^+$$

$$\frac{V_1^+}{Z_1} = \frac{V_1^-}{Z_1} + \frac{V_2^+}{Z_2}$$

Substitute for V_2^+

$$\frac{V_1^+}{Z_1} = \frac{V_1^-}{Z_1} + \frac{V_1^+}{Z_2} + \frac{V_1^-}{Z_2}$$

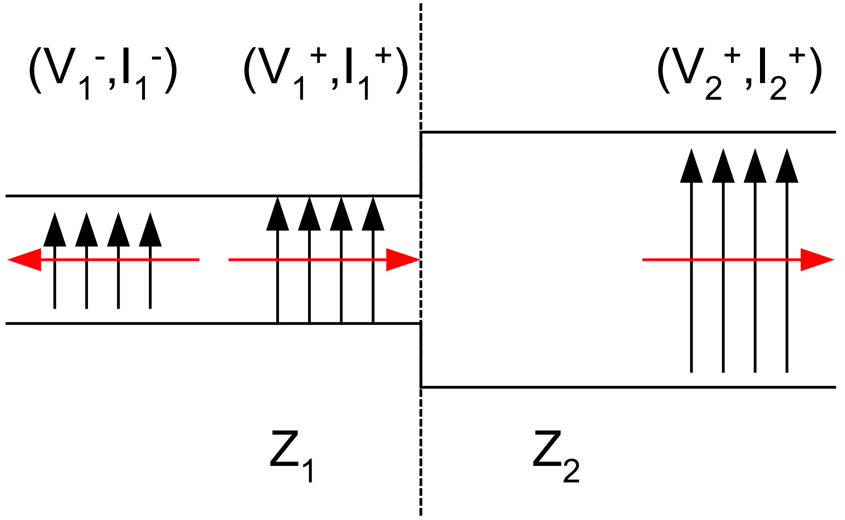
$$(Z_1^{-1} - Z_2^{-1})V_1^+ = (Z_1^{-1} + Z_2^{-1})V_1^-$$

We define and obtain the reflection coefficient, Γ

$$\Gamma \equiv \frac{V_1^-}{V_1^+} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

The transmission coefficient, T*, is defined as*

$$T \equiv \frac{V_2^+}{V_1^+} = \frac{V_1^+ + V_1^-}{V_1^+} = 1 + \Gamma = \frac{Z_2 + Z_1 + Z_2 - Z_1}{Z_2 + Z_1} = \frac{2Z_2}{Z_2 + Z_1}$$



The flow of energy $\left(P = \frac{d\mathcal{E}}{dt}\right)$ *is conserved at the interface*

$$P_{IN} = V_1^+ I_1^+ = \frac{(V_1^+)^2}{Z_1}$$

(assume all voltages and impedances are real)

$$P_T = \frac{(TV_1^+)^2}{Z_2} = \frac{4Z_2}{(Z_1 + Z_2)^2}(V_1^+)^2$$

$$P_R = \frac{(\Gamma V_1^+)^2}{Z_1} = \frac{(Z_2 - Z_1)^2}{Z_1(Z_2 + Z_1)^2}(V_1^+)^2$$

$$P_T + P_R = \frac{4Z_2Z_1 + (Z_2^2 - 2Z_2Z_1 + Z_1^2)}{Z_1(Z_2 + Z_1)^2}(V_1^+)^2 = \frac{(V_1^+)^2}{Z_1}$$

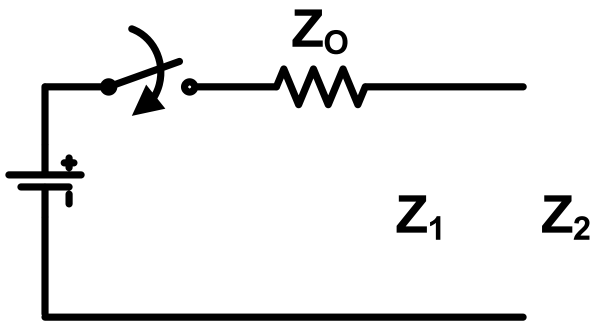
$$= P_{IN}$$

Note that the transmitted power $P_T \propto Z_2^{-1}$ *for* $Z_2 \gg Z_1$

Transmission Line Simple Examples

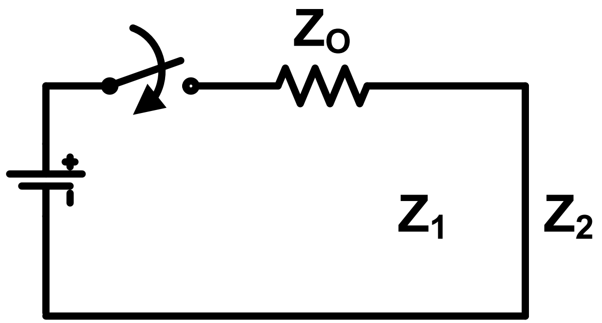
Open Line

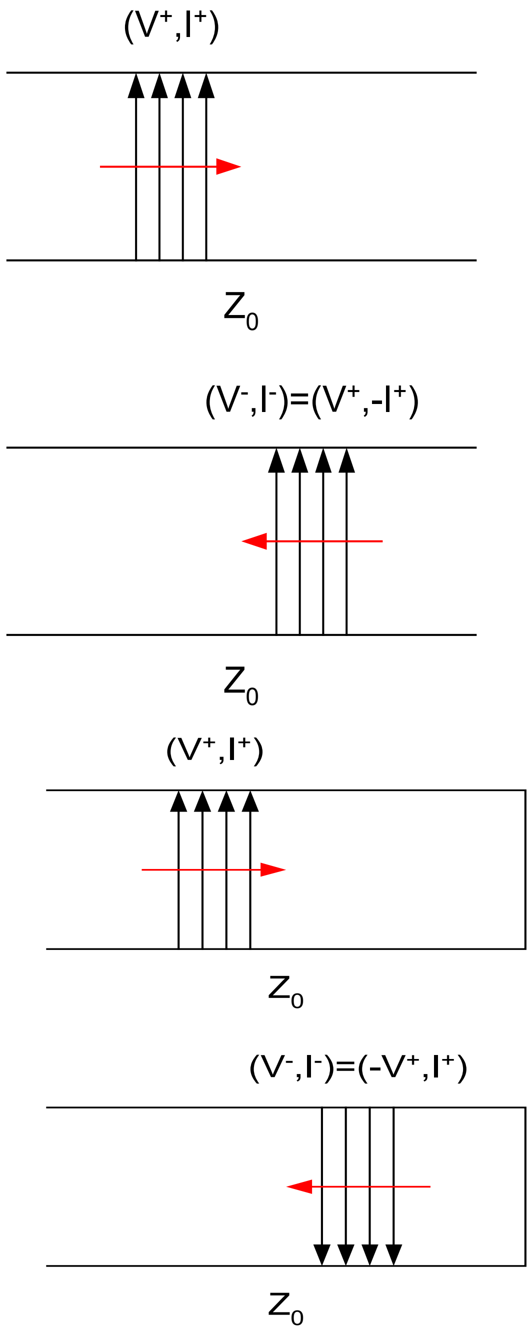
- $Z_1 = Z_0,\ Z_2 = \infty$
- $\Gamma = 1$
- $Z_2 = \infty \Rightarrow P_T = 0 \Rightarrow P_R = P_{IN}$
- *Voltage totally reflected without inversion*



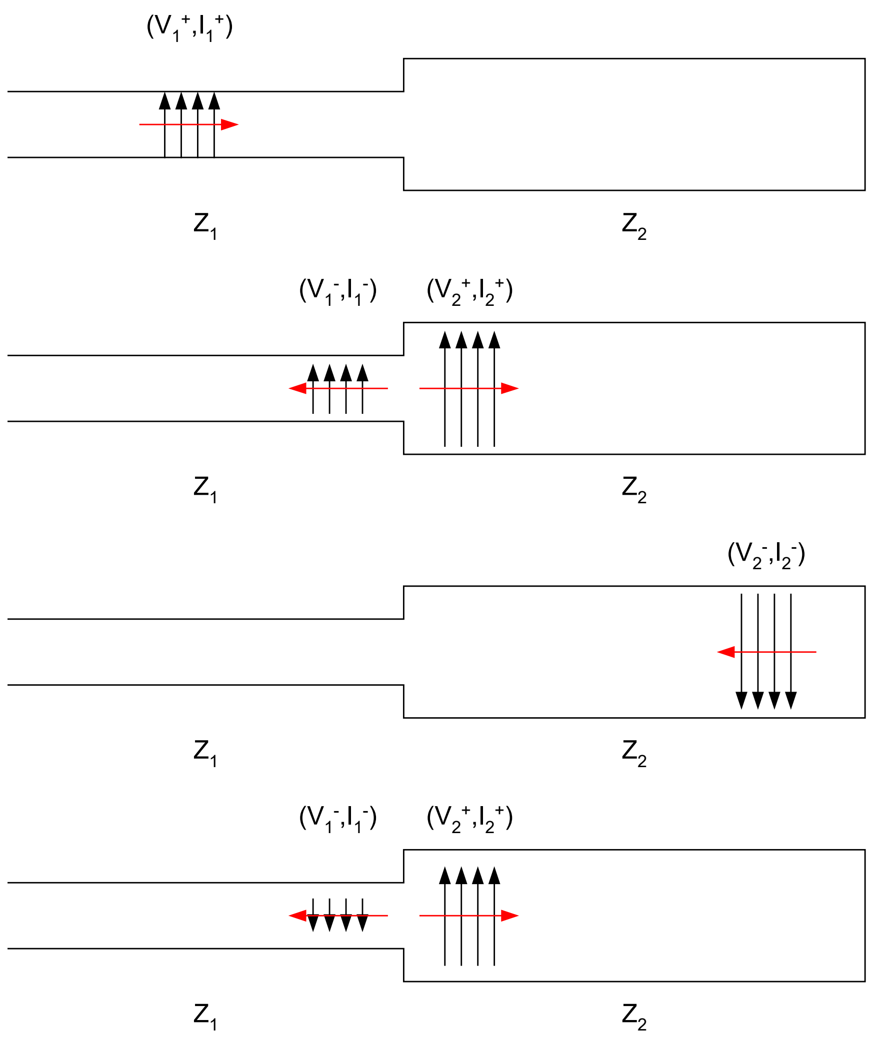
Shorted Line

- $Z_1 = Z_0,\ Z_2 = 0$
- $\Gamma = -1$
- $\Gamma = -1 \Rightarrow T = 0 \Rightarrow V_2 = 0$
- *Voltage totally reflected with inversion*





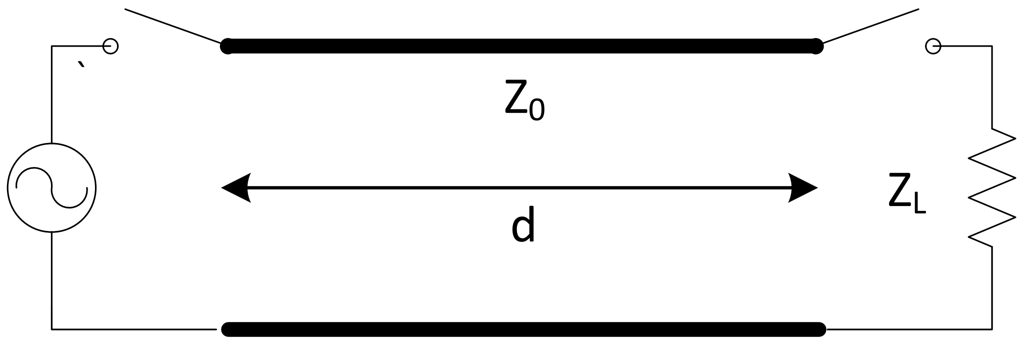
- *Pulse sent down line on controlled impedance*
- *First interface is with higher impedance device* $(Z_2 > Z_1)$
  - *–Transmitted pulse*
  - *–Reflected pulse*
- *Transmitted pulse reflects off short and inverts*
- *Reflected transmitted pulse reaches first interface*
- – *Transmitted pulse (inverted once) down original line*
- – *Reflected pulse (inverted twice) on second line*



Now apply this to a PFN

- *Charge the PFN to V*
- *Open charging switch*
- *Close discharge switch*
  - *Energy, momentum conserved*
  - V_+, V_- *waves with* $V_+ = V_- = V/2$
  - *Duration of pulse is time for a full round trip*

$$\tau = \frac{2d}{v} = 2d\sqrt{LC}$$



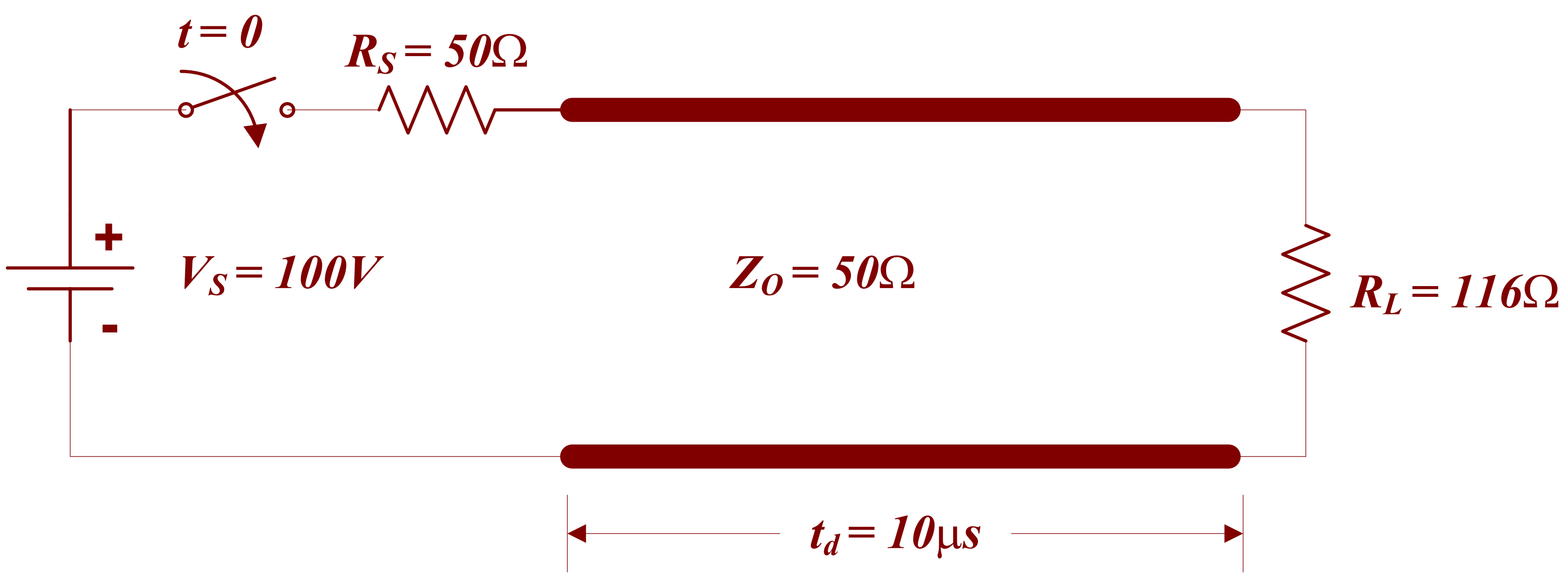
Transmission Line Homework Problem #13

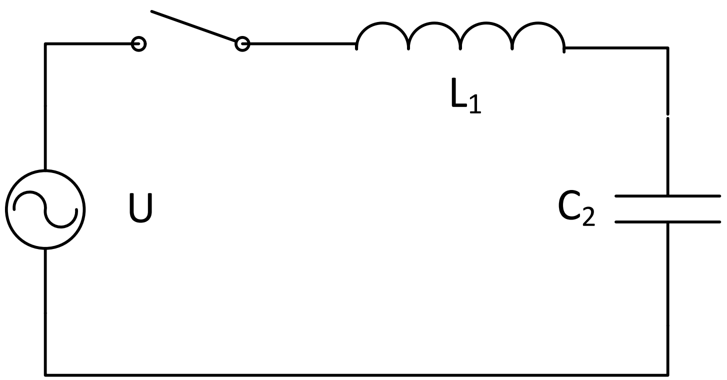
A. *A transmission line can be formed using lumped Ls and Cs. Calculate the delay of a line composed of 8 sections of inductances L=4mH per section and capacitance C=40pF per section.*

B. *The frequency of a signal applied to a two-wire transmission cable is 3GHz. What is the signal wavelength if the cable dielectric is air? Hint – relative permittivity of air is 1*

C. *What is the signal wavelength if the cable dielectric has a relative permittivity of 3.6?*

For the transmission line shown below, calculate the Reflection Coefficient Γ, the reflected voltage and the voltage and current along the line versus time.





$$KVL: \; U = L_1 \frac{di_1}{dt} + v_2$$

$$KCL: \; i_1 = C_2 \frac{dv_2}{dt}$$

$$\begin{pmatrix} \dot{i}_1 \\ \dot{v}_2 \end{pmatrix} = \begin{pmatrix} 0 & -1/L_1 \\ 1/C_2 & 0 \end{pmatrix} \begin{pmatrix} i_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} 1/L_1 \\ 0 \end{pmatrix} u$$

$$s \begin{pmatrix} I_1 \\ V_2 \end{pmatrix} - \begin{pmatrix} i_1(0) \\ v_2(0) \end{pmatrix} = A \begin{pmatrix} I_1 \\ V_2 \end{pmatrix} + BU$$

$$where \; \mathrm{A} = \begin{pmatrix} 0 & -1/L_1 \\ 1/C_2 & 0 \end{pmatrix} \; and \; \mathrm{B} = \begin{pmatrix} 1/L_1 \\ 0 \end{pmatrix}$$

In matrix notation

$$(sI - A)X = x(0) + BU$$

$$\mathrm{X} = (sI - A)^{-1}x(0) + (sI - A)^{-1}BU$$

$$(sI - A)^{-1} = \begin{pmatrix} s & 1/L_1 \\ -1/C_2 & s \end{pmatrix}^{-1} = \frac{1}{s^2 + 1/L_1C_2}\begin{pmatrix} s & -1/L_1 \\ 1/C_2 & s \end{pmatrix}$$

$$= \frac{1}{s^2 + 1/L_1C_2}\begin{pmatrix} s & -\dfrac{\omega_0}{\omega_0 L_1} \\ \dfrac{\omega_0}{\omega_0 C_2} & s \end{pmatrix} = \frac{1}{s^2 + \omega_0^2}\begin{pmatrix} s & -\omega_0/Z_0 \\ \omega_0 Z_0 & s \end{pmatrix}$$

$$\begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = \frac{1}{s^2 + \omega_0^2}\begin{pmatrix} s & -\omega_0/Z_0 \\ \omega_0 Z_0 & s \end{pmatrix}\left[\begin{pmatrix} i_{10} \\ v_{20} \end{pmatrix} + \begin{pmatrix} \dfrac{U_0}{Z_0}\dfrac{\omega_0}{s} \\ 0 \end{pmatrix}\right]$$

where $U(s) = U_0/s,\ \omega_0^2 = 1/L_1C_2,\ Z_0 = \sqrt{L_1/C_2} \Rightarrow \omega_0 L_1 = Z_0.$

Resonant Charging

Assume initial values of $(i_{10}, v_{20}) = (0,0)$, *then*

$$\begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = \frac{1}{s^2 + \omega_0^2} \begin{pmatrix} \omega_0/\omega_0 L_1 \\ \omega_0^2/s \end{pmatrix} U_0$$

$$I_1 = \frac{U_0}{Z_0} \frac{\omega_0}{s^2 + \omega_0^2} \Rightarrow i_1(t) = \frac{U_0}{Z_0} \sin(\omega_0 t)$$

$$V_2 = \frac{1}{s} \frac{U_0 \omega_0^2}{s^2 + \omega_0^2} = \left(\frac{1}{s} - \frac{s}{s^2 + \omega_0^2} \right) U_0$$

$$\Rightarrow v_2(t) = (1 - \cos \omega_0 t) U_0$$

At time $t = \pi/\omega_0, \cos(\omega_0 \pi/\omega_0) = -1$

Voltage doubles, $v_2(\pi/\omega_0) = 2U_0$

- *Use diode to prevent circuit ringing down*

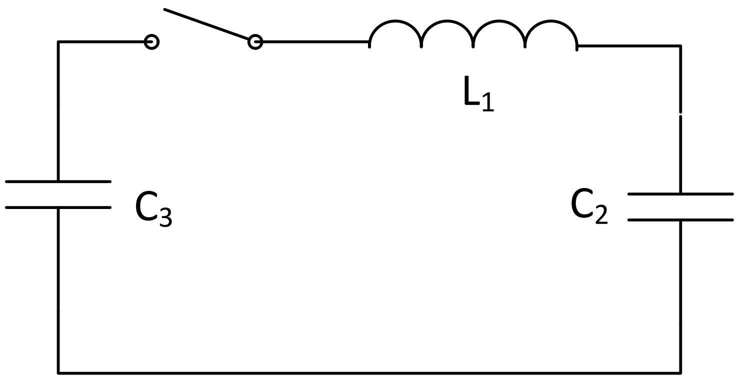
- *Second order undamped system implies oscillation*
  - *Resonant frequency* $\omega_0 = 1/\sqrt{LC}$
  - *Voltage and current across each reactive element* $\pi/2$ *out of phase* $\Rightarrow$ $\sin\omega_0 t, \cos\omega_o t$
  - *No step change of current across inductor allowed (requires infinite voltage)* $\Rightarrow i(t) = I_0 \sin\omega_0 t \,;\, v_C(t) = V_0 \cos\omega_0 t$
  - *Energy oscillates between inductor and capacitor* $\Rightarrow 1/2\, LI_0^2 = 1/2\, CV_0^2 \Rightarrow V_0 = \sqrt{L/C}\, I_0 = Z_0 I_0$
- *Output oscillates about "steady state" value* (U_0)
  - *Starts at* $v_C(0) = 0$
  - *Maximum value* $v_C(\pi/\omega_0) = 2U_0$

Two capacitors now in series

$C_S = C_2 C_3/(C_2 + C_3)$

$\omega_0 = 1/\sqrt{L_1 C_S}$

$Z = \sqrt{L_1/C_S}$



Initial conditions

$v_3(0) = U_0, i_1(0) = v_2(0) = 0$

There are several ways to calculate the final voltage on C_2.

1) *Integrate the current through L_1 for the time $(0, \pi/\omega_0)$*

$$Q_2 = \int_0^{\pi/\omega_0} i_1(t)dt = U_0 \sqrt{\frac{C_S}{L_1}} \int_0^{\pi/\omega_0} \sin \omega_0 t \, dt = U_0 \sqrt{\frac{C_S}{L_1}} \sqrt{L_1 C_S} \, 2$$

$$= 2C_S U_0 \Rightarrow v_2\left(\frac{\pi}{\omega_0}\right) = 2\frac{C_S}{C_2} U_0 = 2\frac{C_3}{C_3 + C_2} U_0$$

2) *Find the charge transfer necessary to change the voltage across the series capacitors from U_0 to $-U_0$.*

$$q_{30} = C_3 U_0$$

$$q_{3f}/C_3 - q_{2f}/C_2 = \left(q_{30} - q_{2f}\right)/C_3 - q_{2f}/C_2 = -q_{30}/C_3 = -U_0$$

$$\left(\frac{1}{C_2} + \frac{1}{C_3}\right) q_{2f} = \frac{C_3 + C_2}{C_3 C_2} q_{2f} = \frac{2}{C_3} q_{30} \Rightarrow q_{2f} = \frac{2C_2}{C_3 + C_2} q_{30}$$

$$\Rightarrow v_{2f} = [2C_3/(C_3 + C_2)] \cdot U_0; \;\; v_{3f} = [(C_3 - C_2)/(C_3 + C_2)] \cdot U_0$$

3) *Use conservation of energy and charge to find circuit equations*

$$\mathcal{E}_T = \mathcal{E}_0 = q_{30}^2/(2C_3); q_T = q_2 + q_3 = q_{30} = q_{2f} + q_{3f}$$

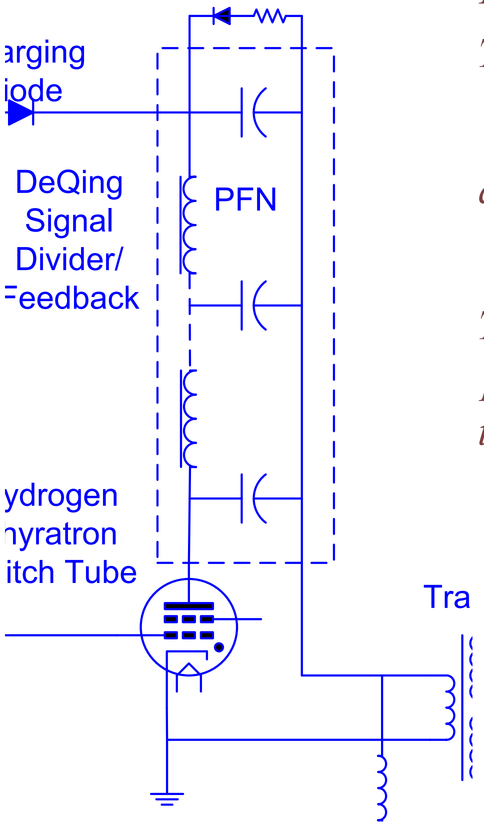
$$q_{3f}^2/(2C_3) + q_{2f}^2/(2C_2) = q_{30}^2/(2C_3)$$

$$\left(q_{30}^2 - q_{3f}^2\right)/(2C_3) = \left(q_{30} + q_{3f}\right) \cdot \left(q_{30} - q_{3f}\right)/(2C_3)$$

$$= \left(q_{30} + q_{3f}\right) \cdot q_{2f}/(2C_3) = q_{2f}^2/(2C_2)$$

$$q_{2f} = \left(q_{30} + q_{3f}\right)(C_2/C_3) = (2q_{30} - q_{2f})(C_2/C_3)$$

$$q_{2f} = [2C_2/(C_3 + C_2)] \cdot q_{30} \Rightarrow v_{2f} = [2C_3/(C_3 + C_2)] \cdot U_0;$$



Flatness is directly proportional to the number of LC meshes

Rise-time is determined by the LC of the mesh closest to the load

Pulse width T is twice the one way transit time t of the wave in the PFN

The one-way transit time is

$$t = n * \sqrt{L * C}$$

and the pulse width T is

$$T = 2 * n * \sqrt{L * C}$$

The load impedance and pulse width are usually specified. From these two parameters the PFN LC can be specified. The nominal L and C in each mesh is the total L and C divided by the number of meshes.

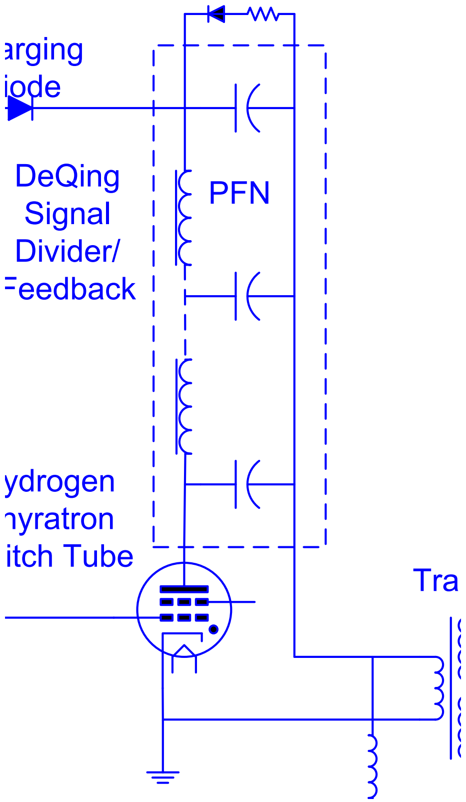
$$Z = \sqrt{\frac{L}{C}}$$

$$T = 2 * Z * C$$

$$C = \frac{T}{2 * Z}$$

$$L = \frac{T * Z}{2}$$

Since the PFN impedance is matched to the load impedance, all the PFN stored energy is dissipated in the load



The PFN is typically tuned to the impedance of the load in order to reduce voltage and current reflections. The effective output voltage at the load obeys the voltage divider law and is effectively

$$V_{load} = V_{pfn} * \frac{Z_{load}}{Z_{load} + Z_{pfn}}$$

$$V_{pfn} = V_{load} * \frac{Z_{load} + Z_{pfn}}{Z_{load}}$$

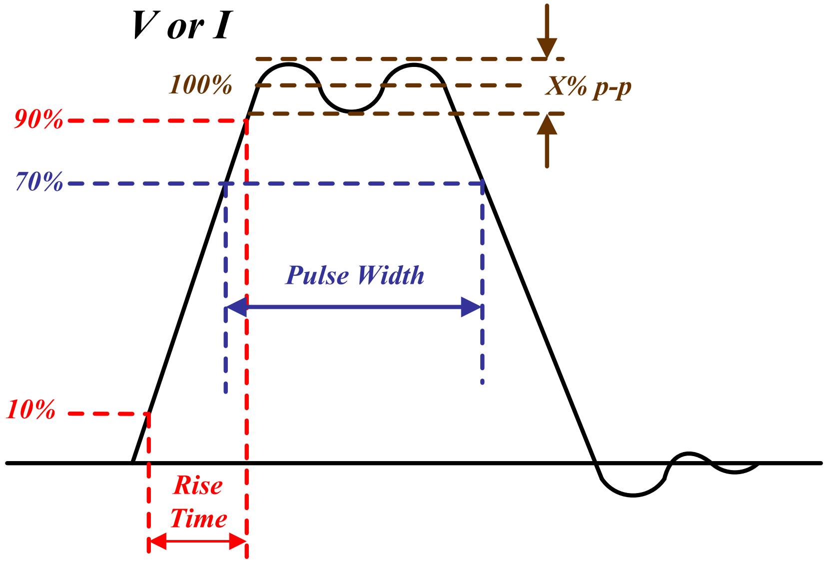
Because typically the PFN has the same impedance as the load,

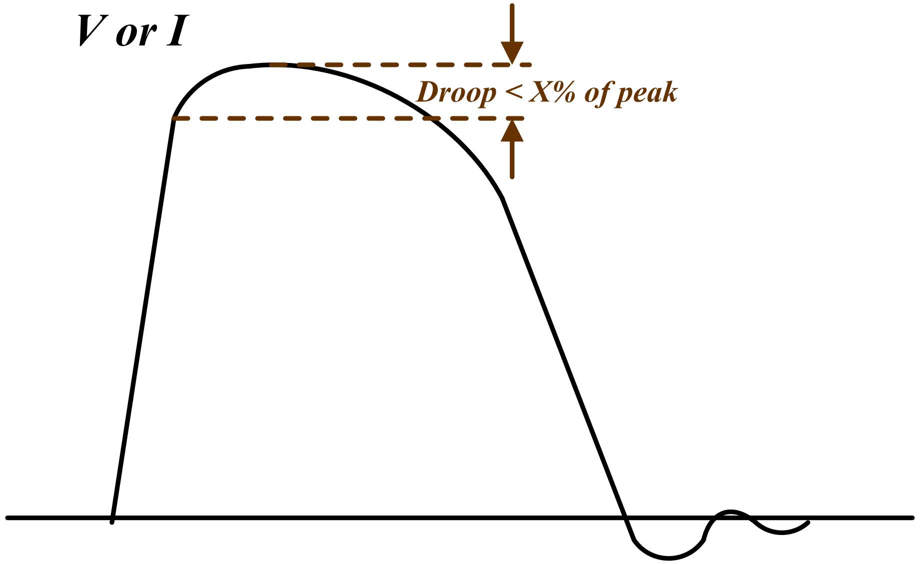
$$V_{pfn} = 2 * V_{load}$$

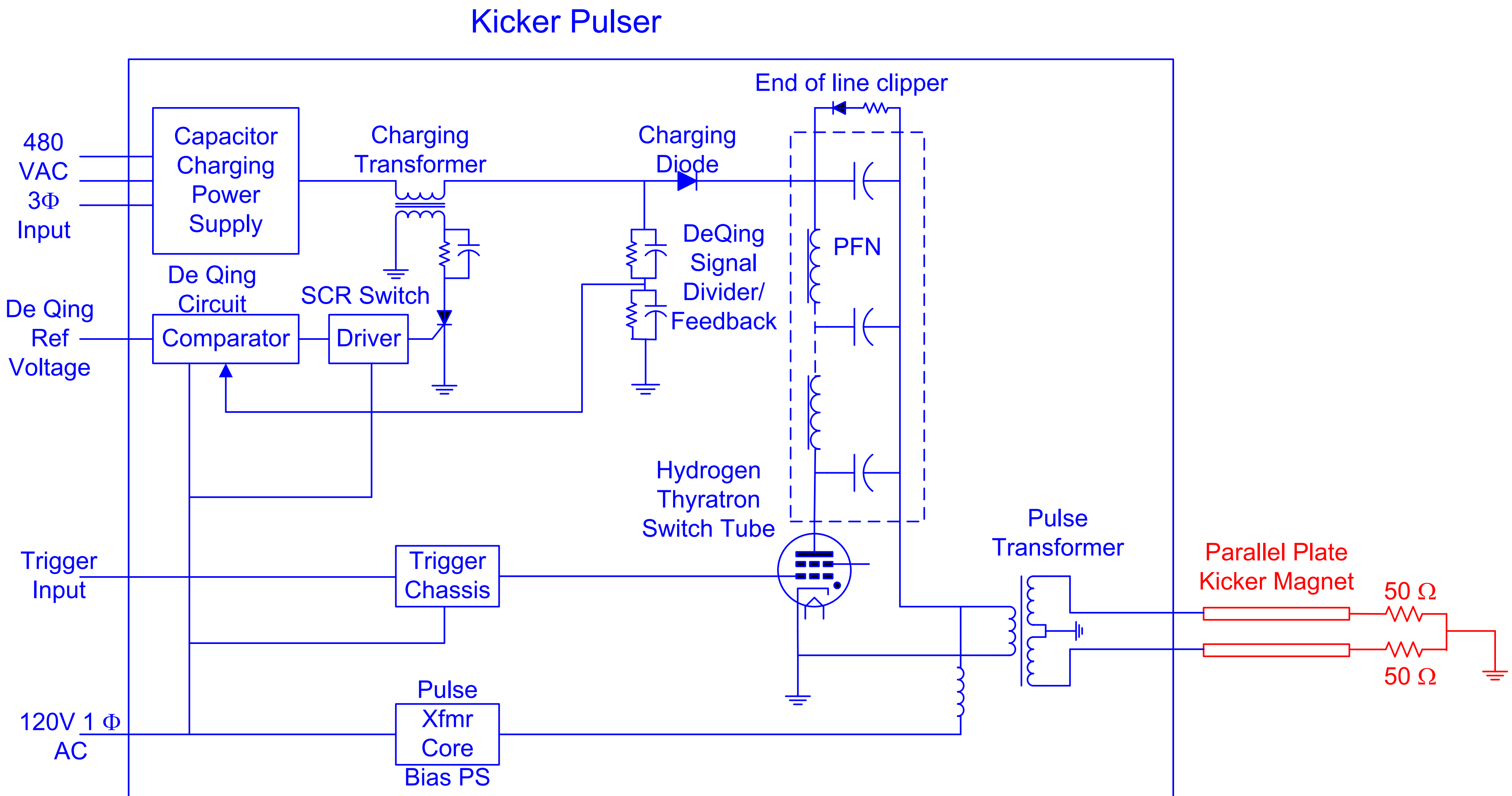
Therefore the PFN must be charged to twice the desired load voltage.

- *Open transmission lines are often used for Pulse Forming Networks (PFNs)*
  - *They are typically charged up from a high impedance source*
  - *Their open end is connected to a normally open switch that closes to connect the PFN to the load*
- *This situation can be viewed as a traveling wave reflecting back and forth off of two open ends*
  - *Total voltage on the line is the sum of the incident and reflected waves* ($V_{PFN} = 2V_{LOAD}$)
  - *Pulse has length* $2l/v$*, since the tail of the pulse must reflect off of the other open end before it reaches the load*
  - *Note:* $l =$ *the length of the open transmission line and* $v =$ *wave velocity*

Conventional Pulsers - The Pulse Forming Network (PFN)





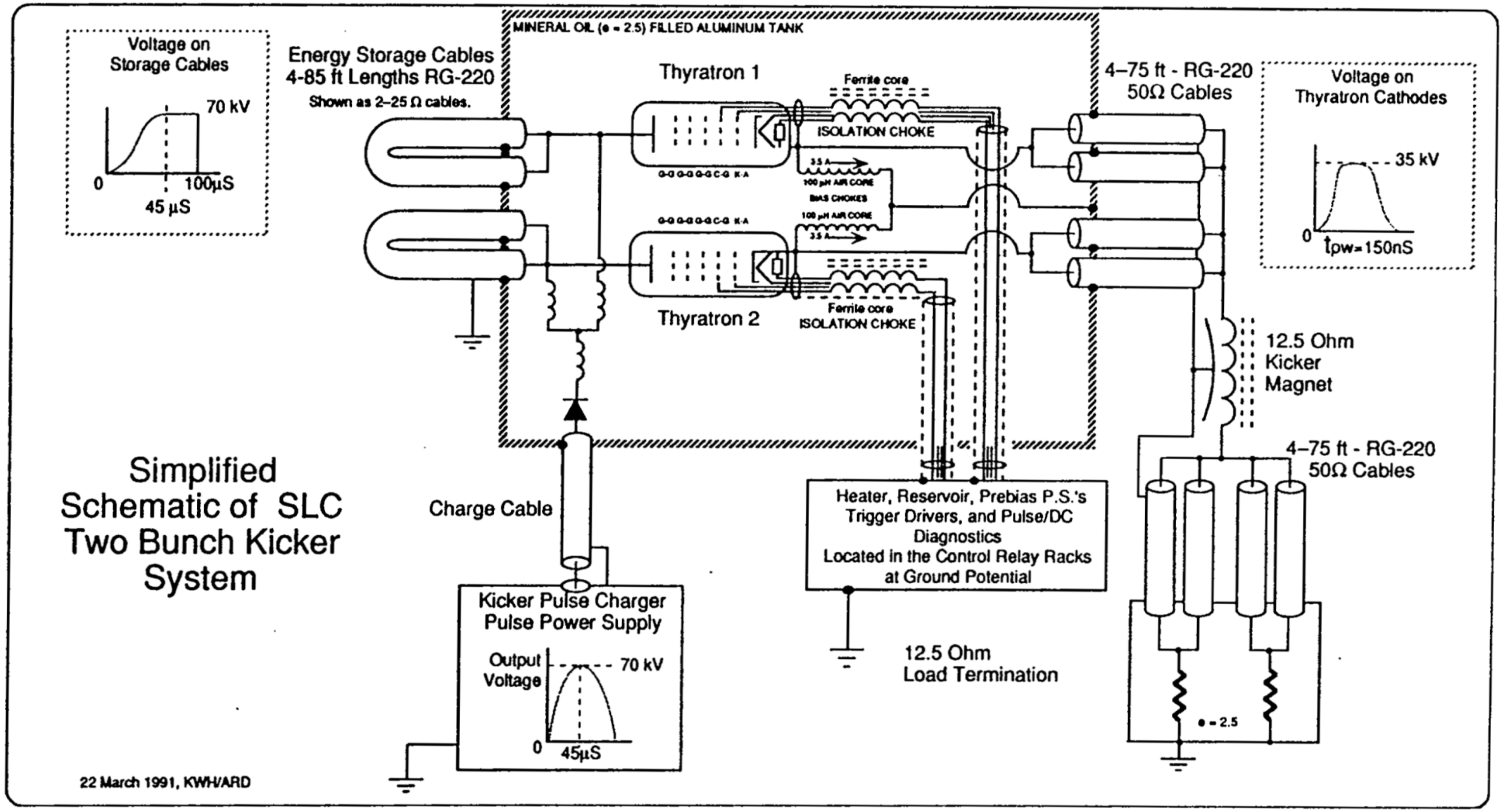

Kicker Pulser
480
VAC
3Φ
Input
Capacitor
Charging
Power
Supply
Charging
Transformer
Charging
Diode
End of line clipper
PFN
DeQing
Signal
Divider/
Feedback
De Qing
Circuit
SCR Switch
De Qing
Ref
Voltage
Comparator
Driver
Hydrogen
Thyratron
Switch Tube
Pulse
Transformer
Parallel Plate
Kicker Magnet
50 Ω
50 Ω
Trigger
Input
Trigger
Chassis
Pulse
Xfmr
Core
Bias PS
120V 1 Φ
AC

Conventional Pulsers - Kicker or Fast Modulator

- *Improve the rise time of modulator pulse using Cable PFN*
- *In line Switch with PFN*
- *Blumline with Shunt Switch*

Conventional Pulsers - Kicker Modulator

- *Conventional Inline Kicker Modulator*
- *Thyratron for switches*
- *Improve the rise time of modulator pulse using Cable PFN*



Conventional Pulsers - Why We Use a Pulsed Modulator to Drive a Klystron

$$Klystron\ perveance = P = \frac{I_{klystron}}{(V_{beam\ voltage})^{3/2}}$$

The perveance of 5045 klystron is 2 micropervs

The peak RF power from a 5045 is 65MW, the beam volatge is 350kV

$$I_{klystron} = P * (V_{beam\ voltage})^{3/2} = 2 * 10^{-6} * (350kV)^{3/2} = 414\,A$$

The power needed to achieve 65MW of RF $= V_{beam\ voltage} * I_{klystron}$

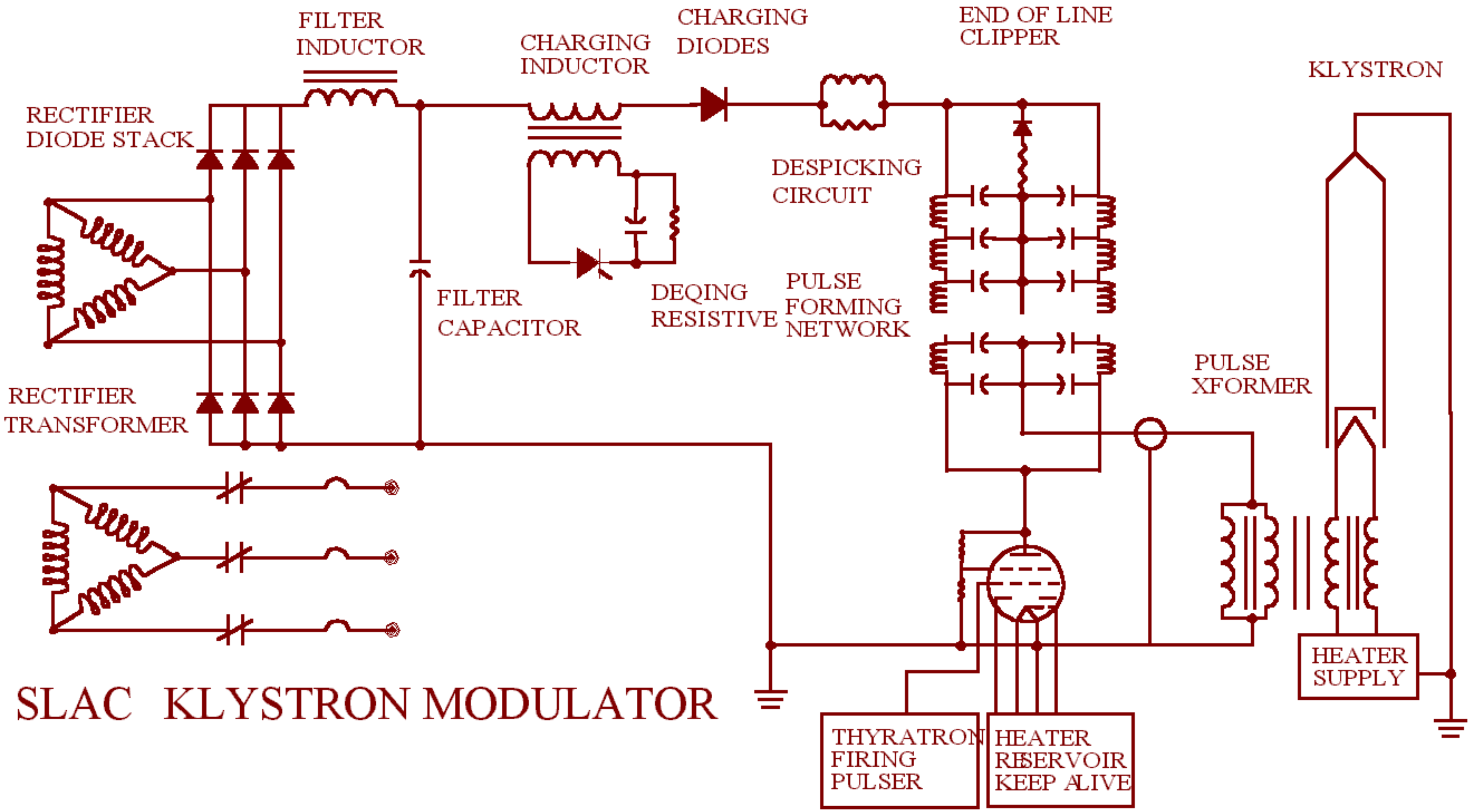
$$= 350kV * 414\,A = 144.0MW\ !$$

Pulsed power is the right approach

- Smaller power source
- Less cooling required (klystron efficiency is 45%)

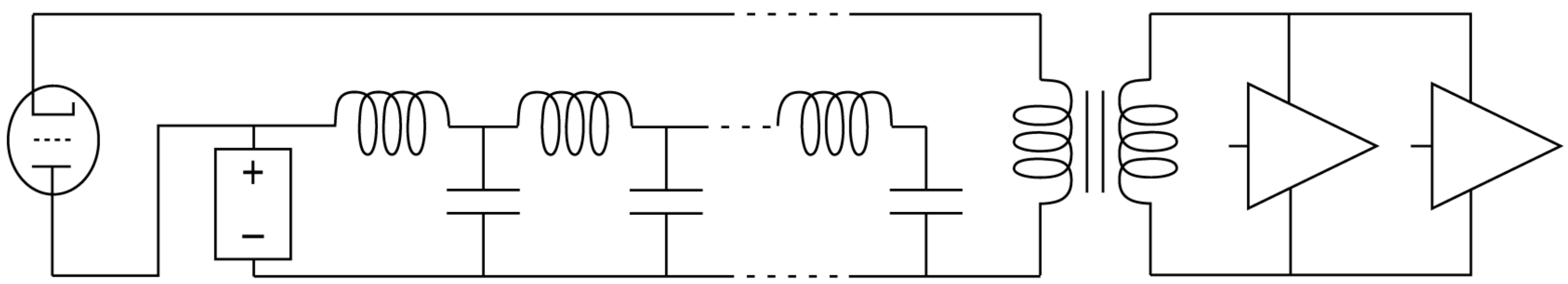
Average power = peak power *duty cycle(on-time*PRR)

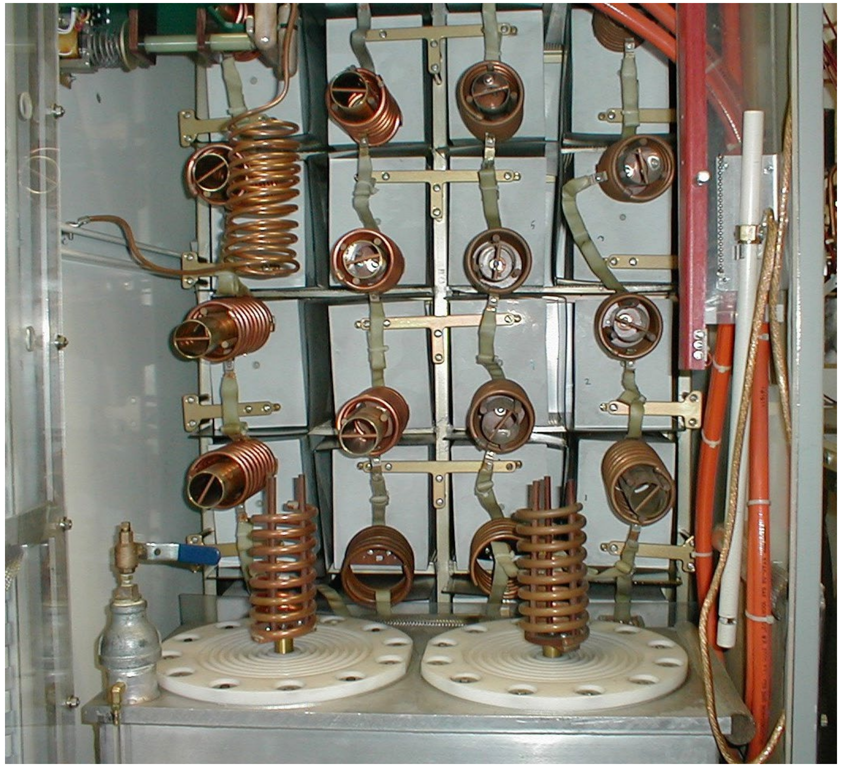
Average power = 144.9MW *5μS*60Hz=42.4kW much lower power



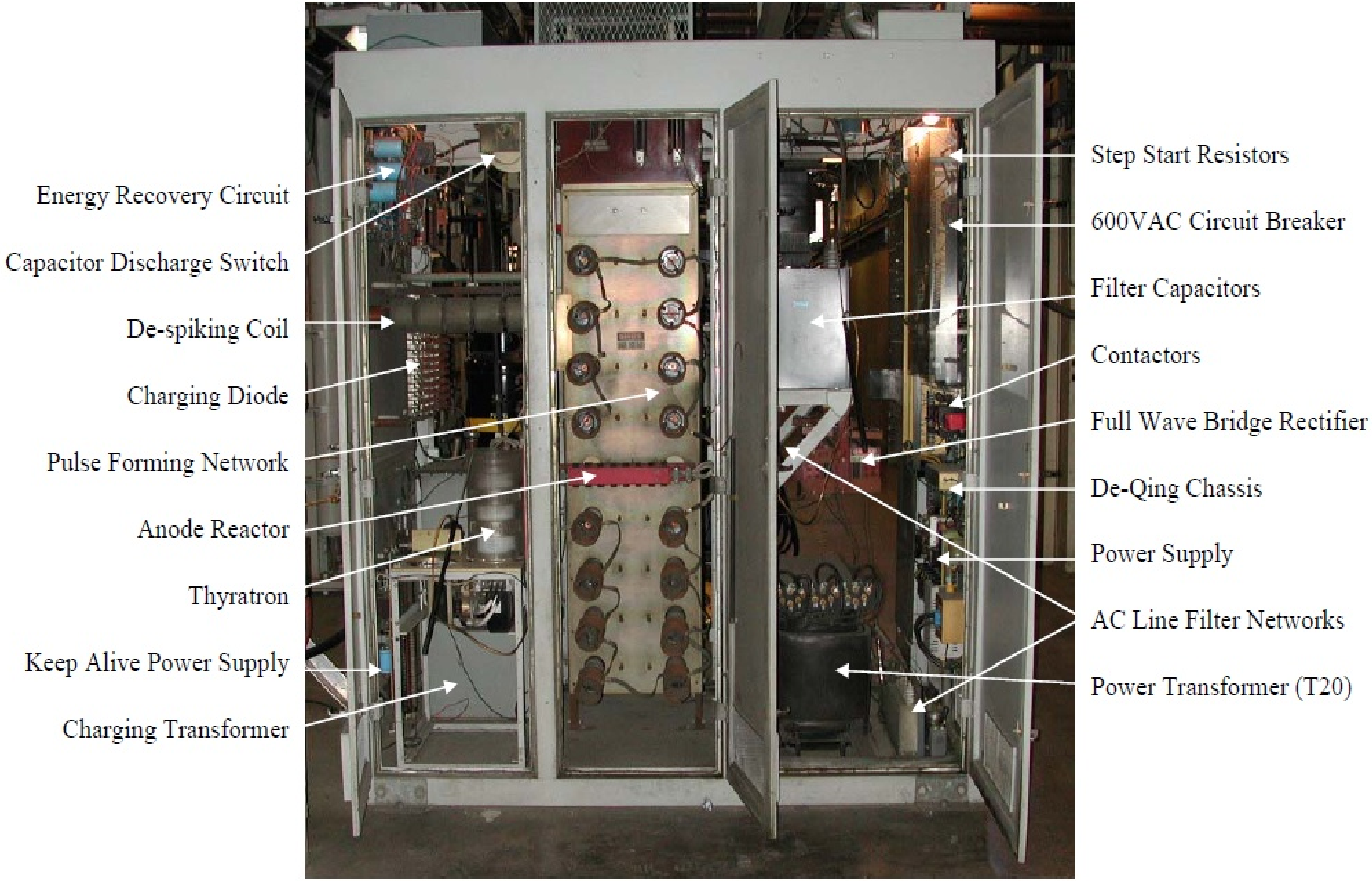
- *Primary VVT, with diode rectifier*
- *High voltage secondary with diodes and filter capacitor*
- *Protected against secondary faults*

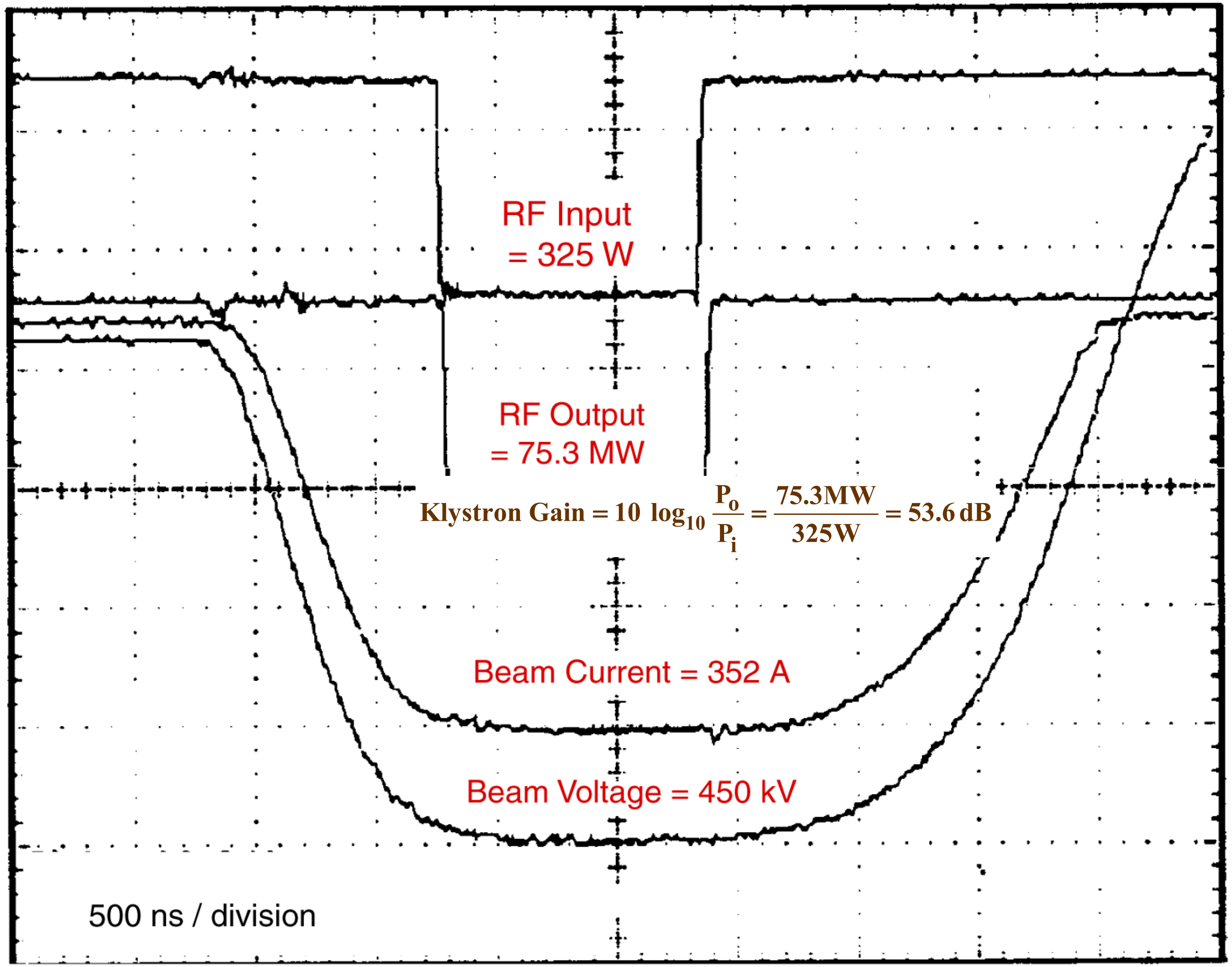
Conventional Pulsers - Klystron Modulator with PFN





Conventional Pulsers - Klystron Modulator PS – Cabinet Details




RF Input
= 325 W
RF Output
= 75.3 MW
Klystron Gain = 10 log$_{10}$ $\frac{P_o}{P_i}$ = $\frac{75.3MW}{325W}$ = 53.6 dB
Beam Current = 352 A
Beam Voltage = 450 kV
500 ns / division

Kicker Current Equations

Equations of Motion:

$$\frac{d\vec{p}}{dt} = \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\frac{dE}{dt} = q\vec{v} \cdot \vec{F}$$

where $\vec{p} = \gamma m\vec{v}$ *is the relativistic momentum,* $\gamma = 1/\sqrt{1-\beta^2}$,

and $\vec{\beta} = \vec{v}/c$.

For a system with only magnetic fields, $\vec{E} = 0$, *the energy E is constant*

$$\frac{dE}{dt} = q\vec{v} \cdot \left(\vec{v} \times \vec{B}\right) = 0$$

so we need to solve the differential equation

$$\frac{d\vec{v}}{dt} = \frac{q}{\gamma m}\vec{v} \times \vec{B}$$

since γ *is constant.*

We will choose our coordinate system so that the beam travels in the $\hat{z}$ *direction,* $\vec{v}(0) = \beta c\hat{z}$, *and we want to deflect the beam in the* $\hat{x}$ *direction. Therefore* $\vec{B} = B\hat{y}$.

Kicker Current Equations

Our coupled differential equations are

$$\frac{dv_x}{dt} = -\frac{qB}{\gamma m} v_z$$

$$\frac{dv_z}{dt} = \frac{qB}{\gamma m} v_x$$

Differentiating the first equation and substituting from the second equation we get

$$\frac{d^2 v_x}{dt^2} = -\left(\frac{qB}{\gamma m}\right)^2 v_x = -\omega_B^2 v_x, \qquad \omega_B = \frac{qB}{\gamma m}$$

This is the familiar harmonic oscillator equation with solutions

$$v_x(t) = v_{x0} \cos \omega_B t + \frac{\dot{v}_{x0}}{\omega_B} \sin \omega_B t$$

The beam is only traveling longitudinally in the vacuum chamber so that $v_{x0} = 0$

From the equations of motion $\dot{v}_{x0} = -\frac{qB}{\gamma m} \beta c = -\omega_B \beta c$. *Therefore*

$$v_x(t) = -\beta c \sin \omega_B t$$

$$x(t) = \frac{\beta c}{\omega_B} \cos \omega_B t + x_0$$

The beam follows a circular orbit in the constant magnetic field

$$x(t) = \rho \cos \omega_B t + x_0$$

where $\rho = \frac{\beta c}{\omega_B} = \frac{\gamma m \beta c}{qB}$ *is the radius of curvature of the particle trajectory through the magnet.*

We verify this relation from the equation for the magnitude of the momentum and re-express it in terms of the strength of the magnetic induction and the beam energy, speed, and charge.

$$|p| = \gamma m v = \gamma m \omega_B \rho = qB\rho$$

$$\rho = \frac{p}{qB} = \frac{cp}{cqB} = \frac{c\gamma m \beta c}{cqB} = \frac{\beta \gamma m c^2}{cqB} = \frac{\beta E}{cqB}$$

All of these equations have been written in MKS units. Accelerators use a mix of units. The unit of magnetic induction, B *is Tesla, but the unit of energy is* GeV. *The unit of* E/q *is the volt, which is also the ratio of an electron-Volt to the electron charge. Therefore this equation is unchanged if we measure* q *in units of electric charge and* E *in units of eV.*

$$1\ eV = 1.602 \times 10^{-19}\ J$$

$$1\ e^- = 1.602 \times 10^{-19}\ C$$

Inserting the units for a particle with a fundamental charge, the equation for the curvature in a dipole magnetic field is

$$\rho = \frac{\beta E(eV)}{c(m/s)B(T)} = \frac{\beta E(eV)}{2.998 \times 10^8 B(T)} = \frac{10^9 \beta E(GeV)}{2.998 \times 10^8 B(T)}$$

$$\rho = \frac{\beta E(GeV)}{0.2998 B(T)}$$

For ultra-relativistic beams, $\beta \approx 1$

$$E = 3\ GeV\ (electrons)$$

$$\gamma = \frac{3000}{0.511} = 5870.8$$

$$\beta = 0.9999999855$$

The kicker is designed to deflect the beam a certain angle θ*. If the B field is constant over the kicker length L, the angle* θ *that the beam will deflect during its pass through L is given by*

$$\rho \sin\theta = L$$

Substituting for ρ *from the equation above, we obtain the desired formula for the field strength*

$$BL = \frac{\beta E}{0.2998} \sin\theta$$

Example:

A 1 meter long kicker is required to deflect a 3 GeV electron beam by 2 mrad. Assuming a uniform field in the kicker, calculate the magnetic induction required for this deflection.

$$BL = \frac{\beta E}{0.2998} \sin\theta$$

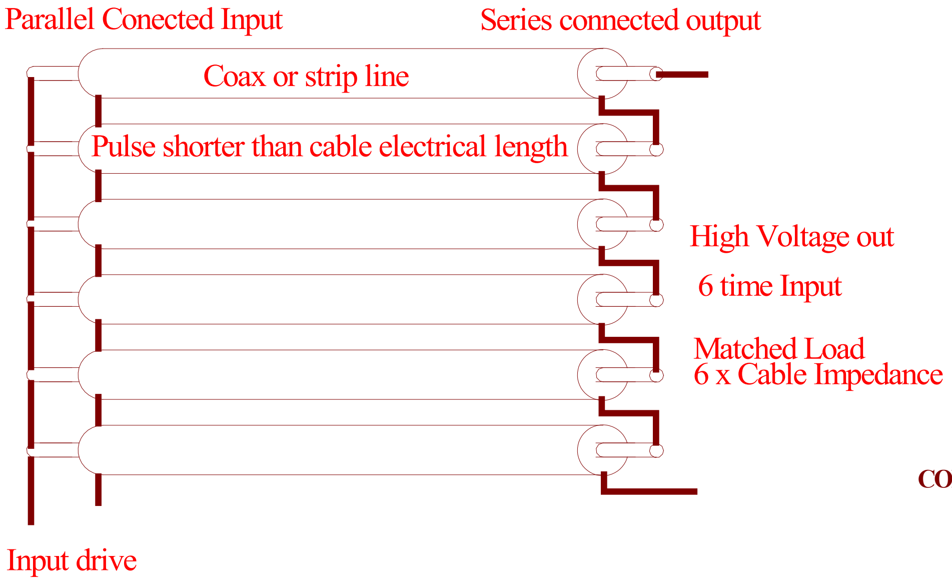
$$B = \frac{3}{0.2998} 2 \times 10^{-3} = 0.020\ T$$

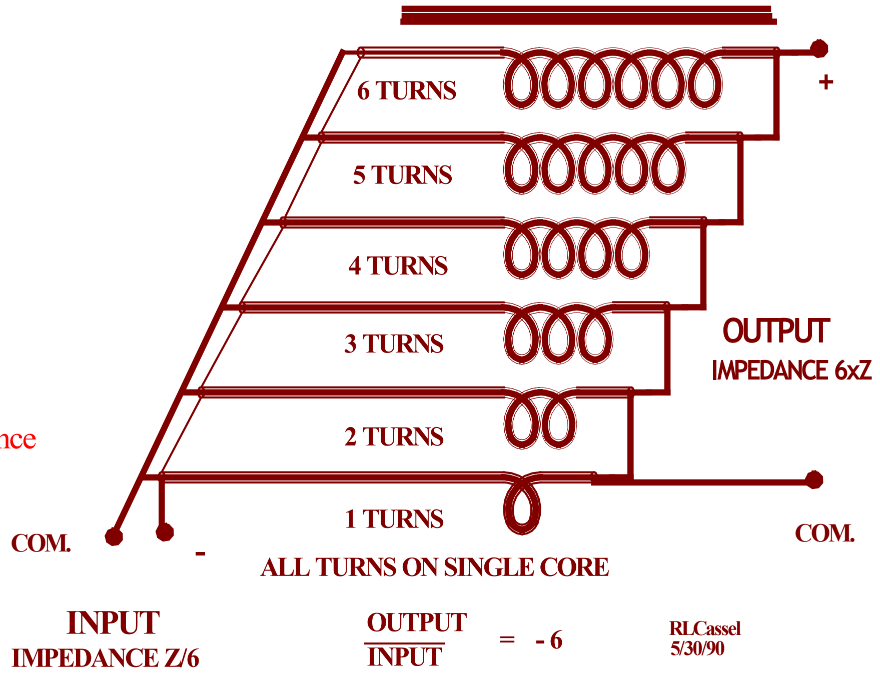
Assuming that the magnet has two conductors and the circumference of the loop of the magnetic field from each conductor passing through the beam trajectory is 0.150 meters $\left(\oint \overrightarrow{dl} = 0.15\right)$, *calculate the current that flows through each conductor.*

$$\oint \vec{H} \cdot \overrightarrow{dl} = 0.150 H_\theta = 2I = \frac{0.150B}{\mu_0}$$

$$I = \frac{0.150B}{2 \cdot 4\pi \times 10^{-7}} = \frac{0.150 \cdot 0.020}{2 \cdot 4\pi \times 10^{-7}} = 1194\ A$$

- *Cable Pulse Transformer parallels multiple cable inputs and series connects the outputs. The pulse length must be < 2X the electrical length of the cable and must drive a matched load.*
- *Fast rise time with simple transformer*
- *Disadvantage stray capacitance and floating cable return limits transformer usage*



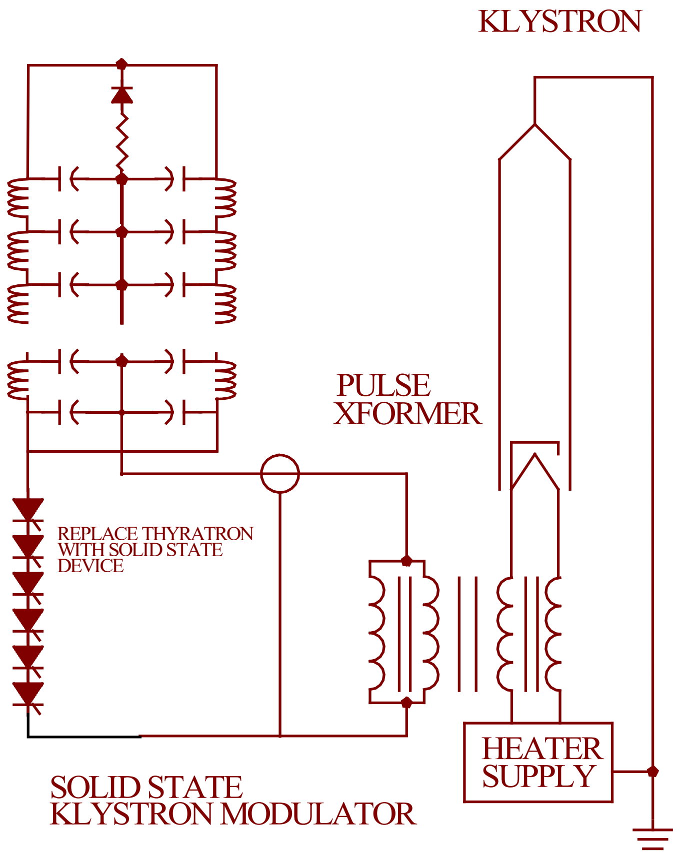


Comparison of Thyratron and Solid-State Pulser Parameters

Parameter	***Thyratron***	***Solid-state***
Control turn-on	*Yes*	*Yes*
Control turn-off	*No*	*Yes*
Pulse Shaping	*PFN*	*IGBT*
Output Voltage	*1/2 PFN voltage*	*Same as device voltage*

Solid-State Pulsers

- *Replace Thyratron with solid-state switch SCR, IGBT, MOSFET, etc*
- *Having a high enough di/dt capability is the problem*
- *For many applications IGBTs without PFNs are being used at the present time*



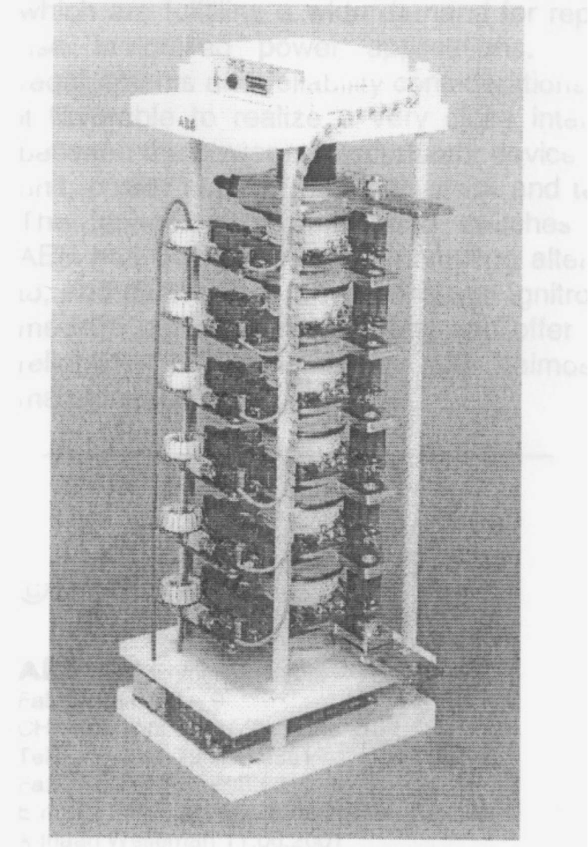
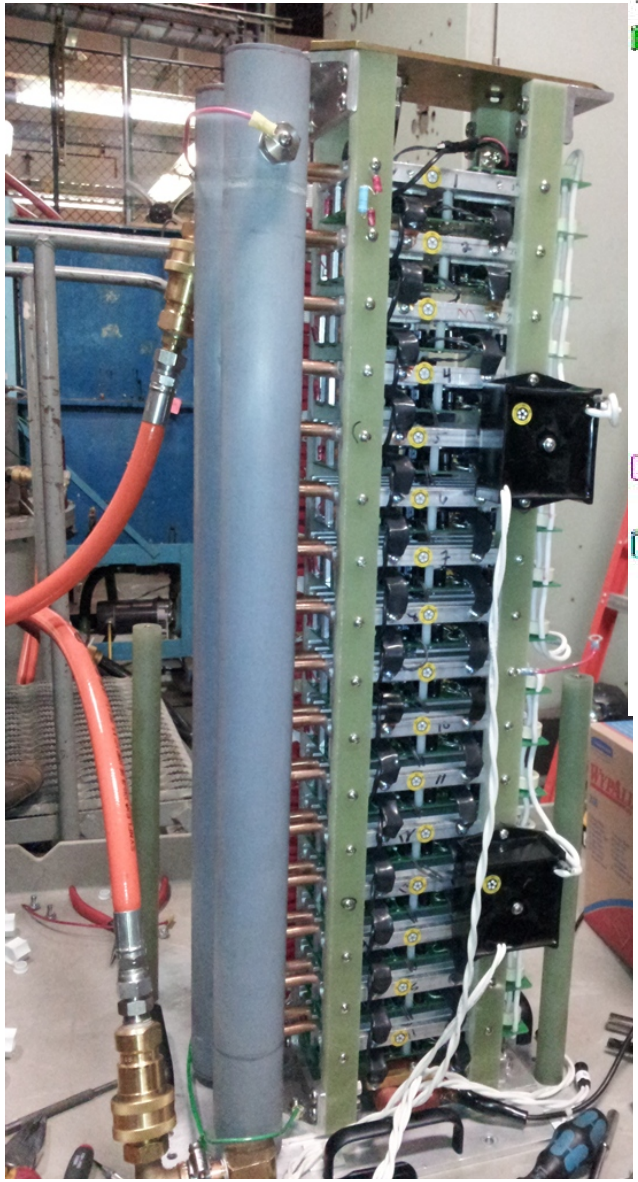
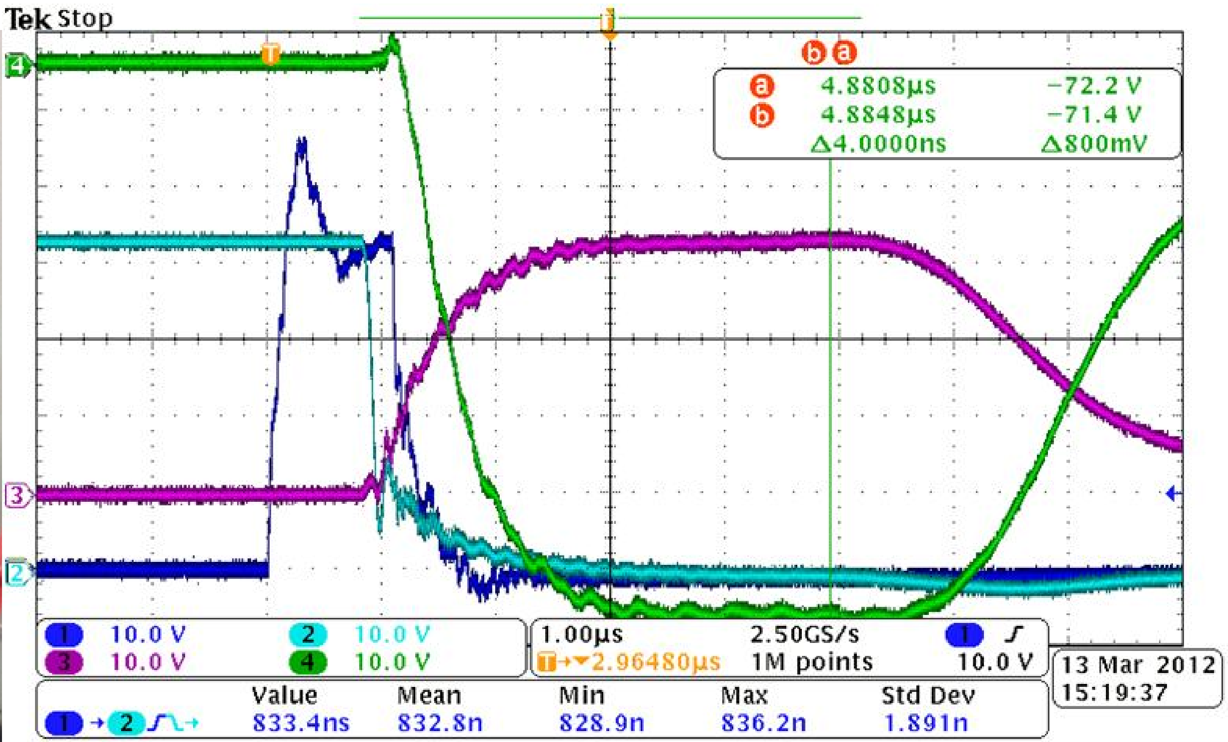


Fig. 11) Switch Assembly SPR-08F45-6-WC

ABB Semiconductor AG





Model S38
Thyristor Module

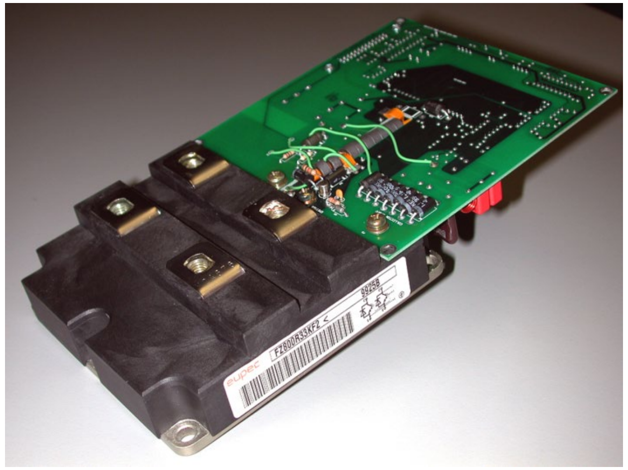
Features:

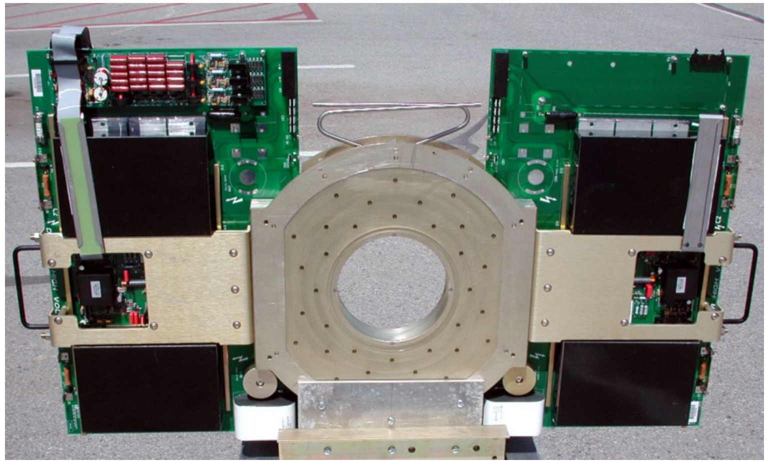
- 4700V Peak Off-State Voltage
- 14kA Peak Non-Repetitive Current
- 30kA/µS Maximum di/dt
- 100nS turn-on delay time
- Low Inductance

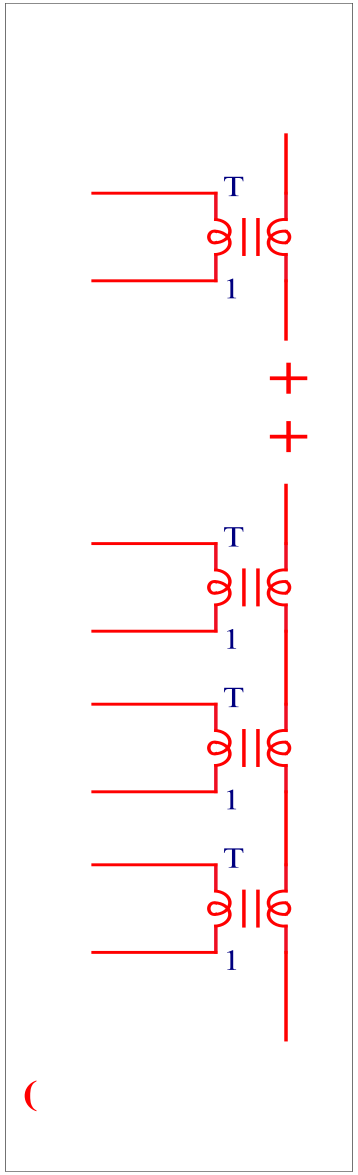
Solid-State Induction Modulators

- *Fractional turn pulse transformer*
  - *–Similar to a induction accelerator*
  - *–Multiple primaries driven in parallel*
  - *–The secondary connected in series*

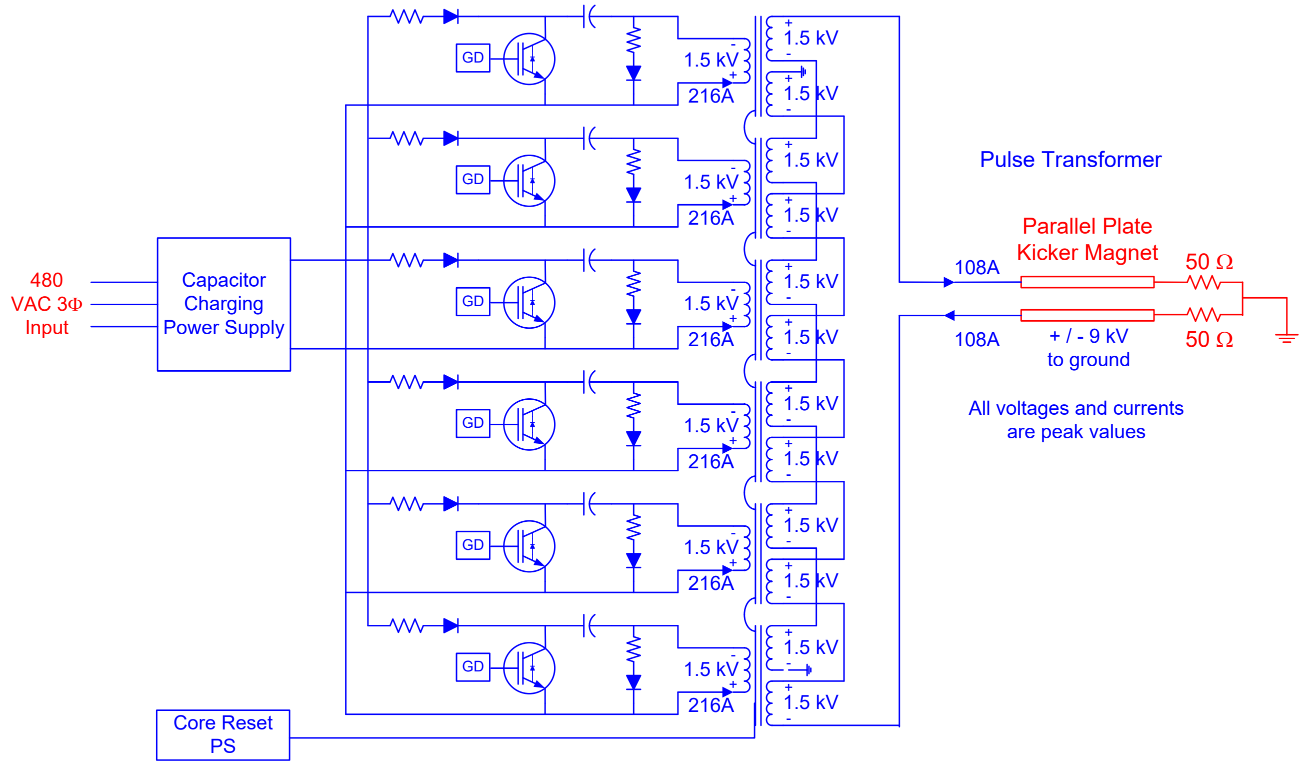
- *Solid-state driver consists of*
  - *– A solid state switch that turns on and off*
  - *– DC capacitor per primary winding*





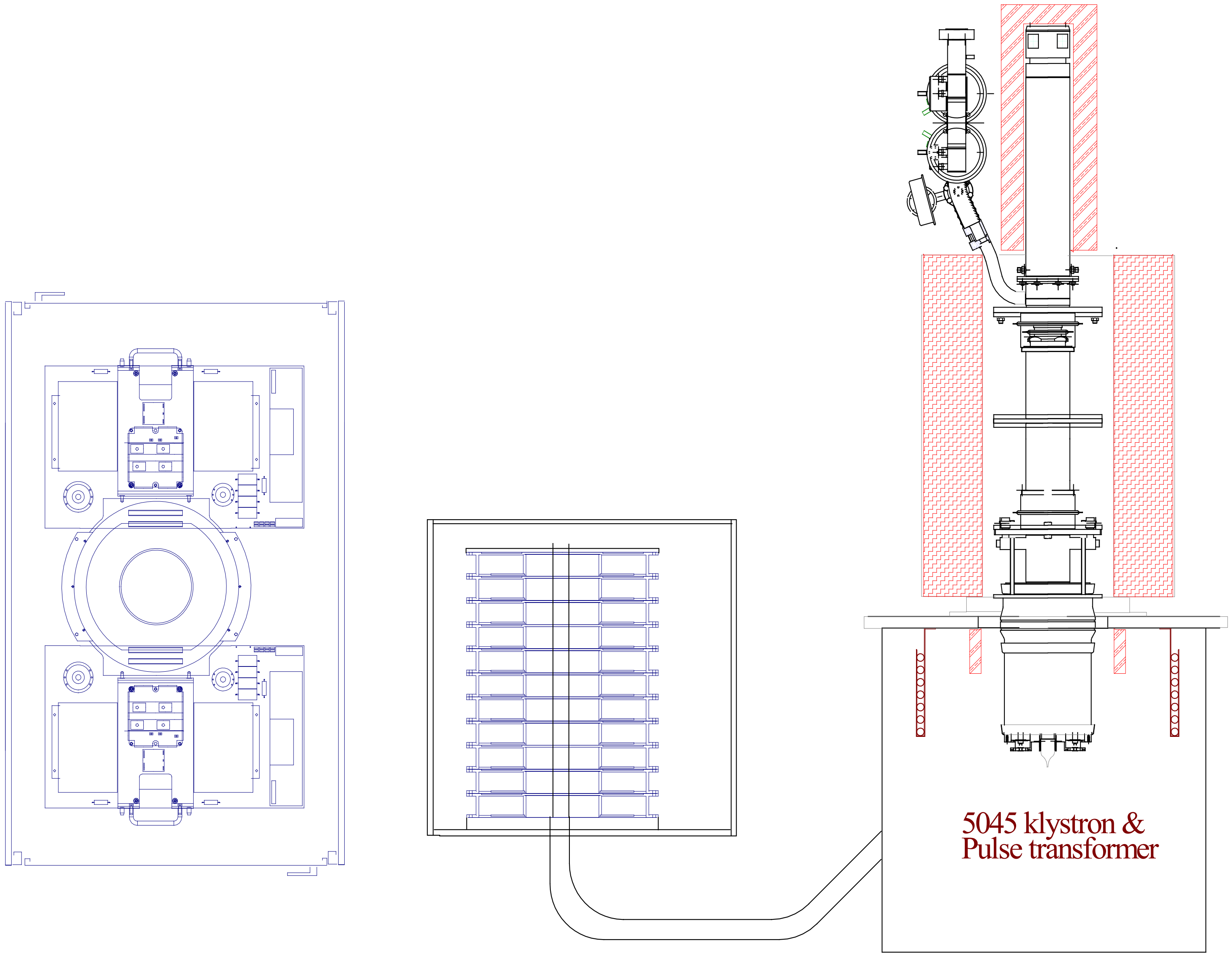


A Solid-State Turn-On Pulser



- *All pulse capacitors are pre-charged simultaneously*
- *IGBTs are all switched on together*
- *Capacitors are then simultaneously discharged producing sinusoidal V and I pulses in the pulse transformer and magnet. The secondary winding voltages are additive*
- *At the end of the pulse the IGBT is turned off. The magnet current decay causes a voltage reversal at the free-wheeling diode*
- *The freewheeling diodes conduct and the magnet current decays exponentially to zero*

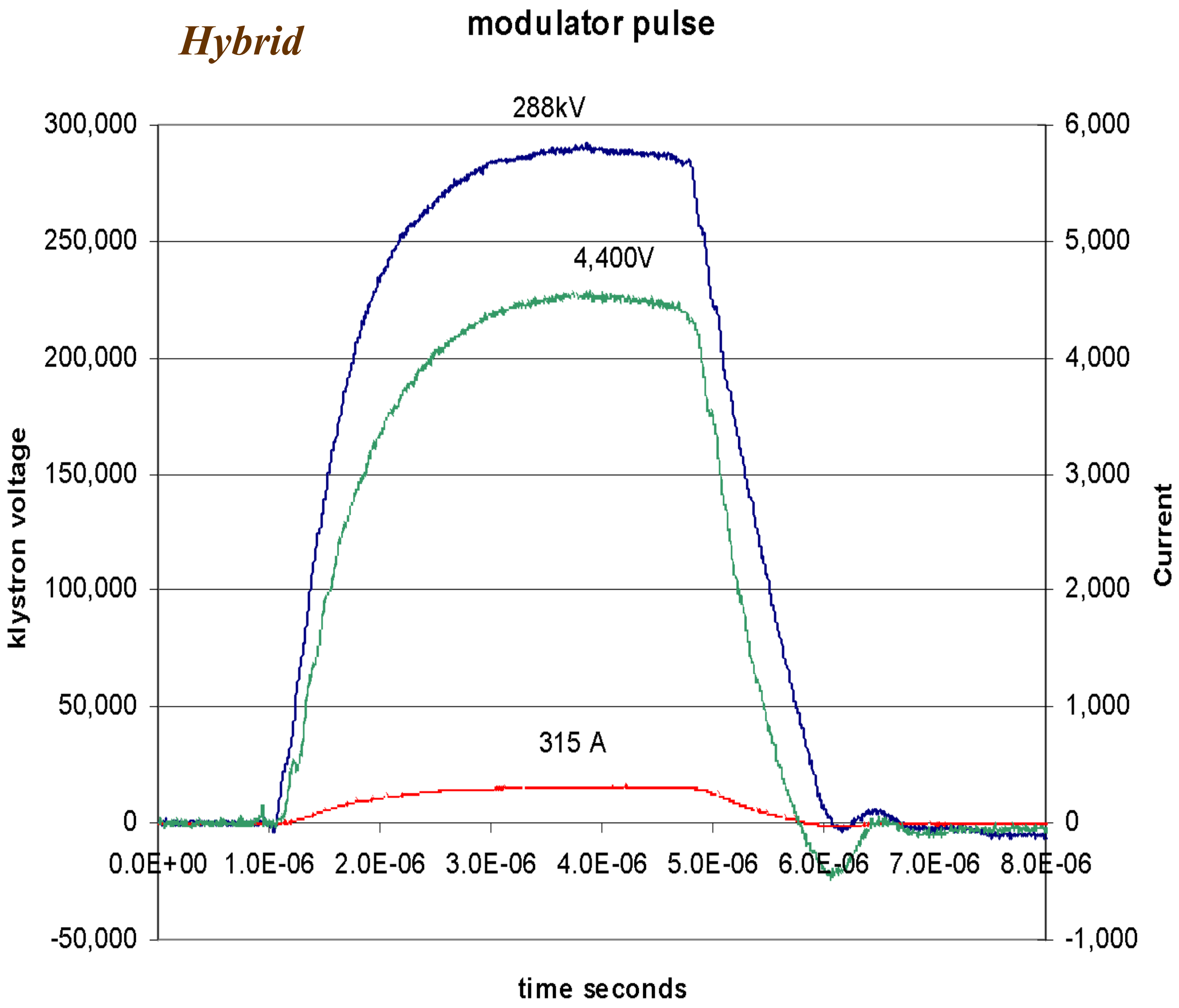
Solid-State Induction Klystron Modulator

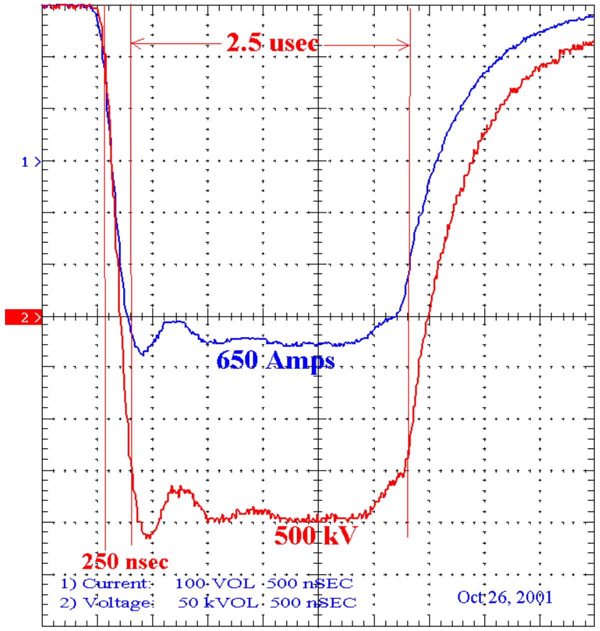


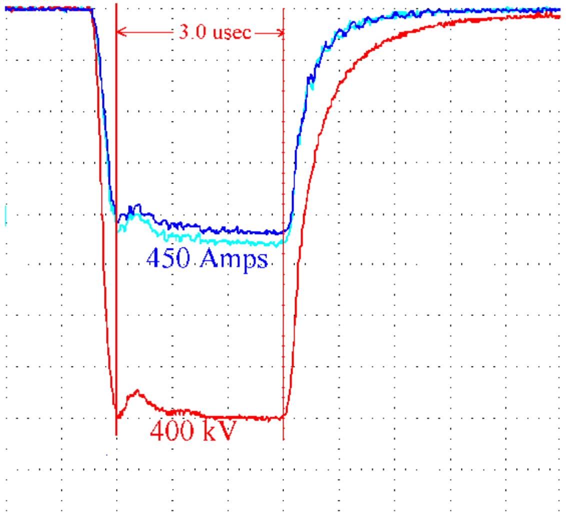
Hybrid

- *Solid-state 10 - stack installed alongside Gallery line-type PFN unit*
- *22 kV => 330 kV via 15:1 xfmr*
- *Prototype currently at 255 kV @ 2.2 μsec @ 120 PPS*

Hybrid


modulator pulse
288kV
4,400V
315 A
klystron voltage
Current
300,000
250,000
200,000
150,000
100,000
50,000
0
-50,000
6,000
5,000
4,000
3,000
2,000
1,000
0
-1,000
0.0E+00
1.0E-06
2.0E-06
3.0E-06
4.0E-06
5.0E-06
6.0E-06
7.0E-06
8.0E-06
time seconds





SOLID STATE DRIVERS

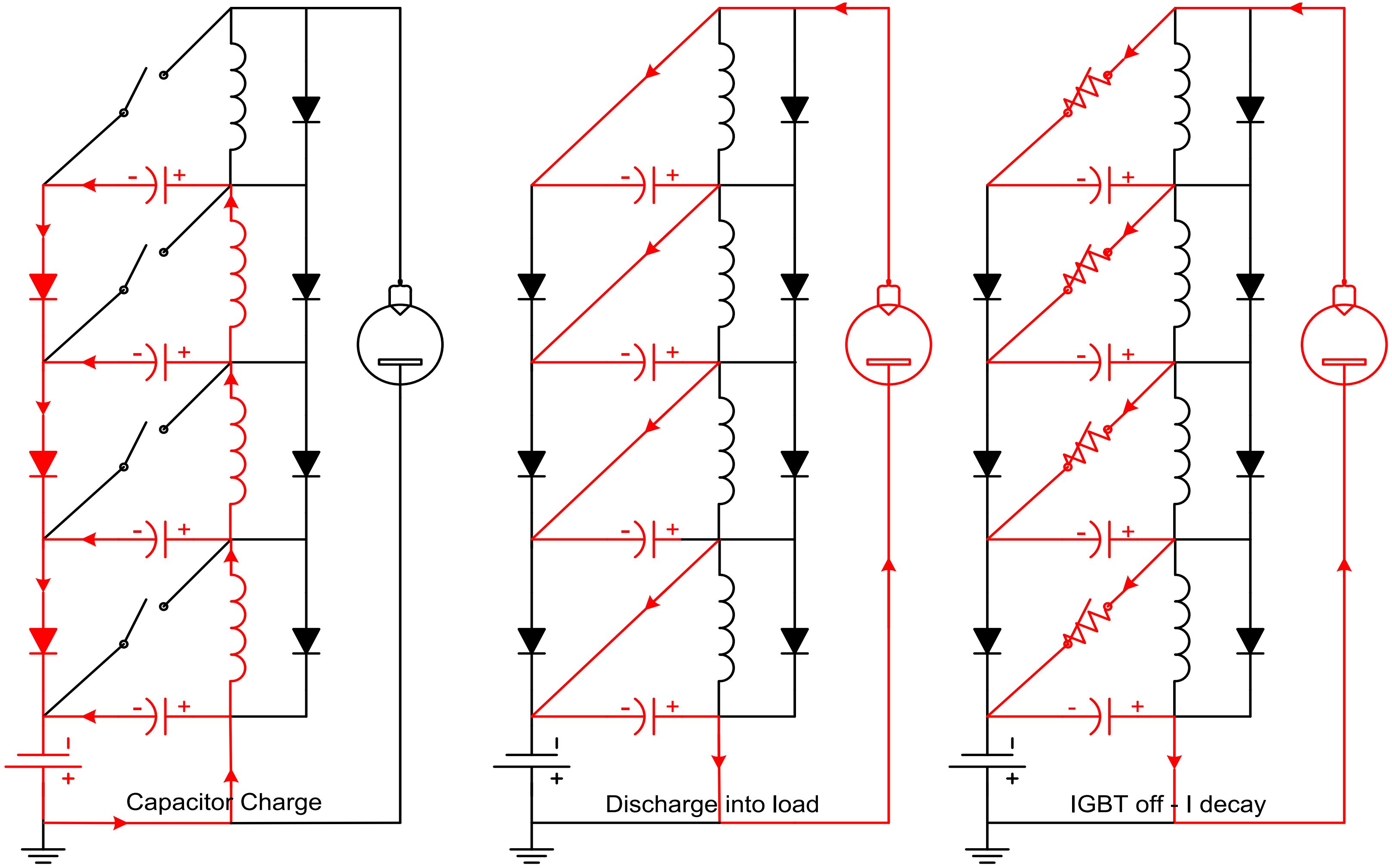
- 152 IGBT Drivers (two per each primary)
- 1800 Volts per IGBT
- 2700 Amps per Driver

CORES AND SECONDARY

- 76 Primaries @ 5400 A
- 3-Turns Secondary
- 400kV @ 1800A, 725MW for 3.2μs, 350kW Ave.

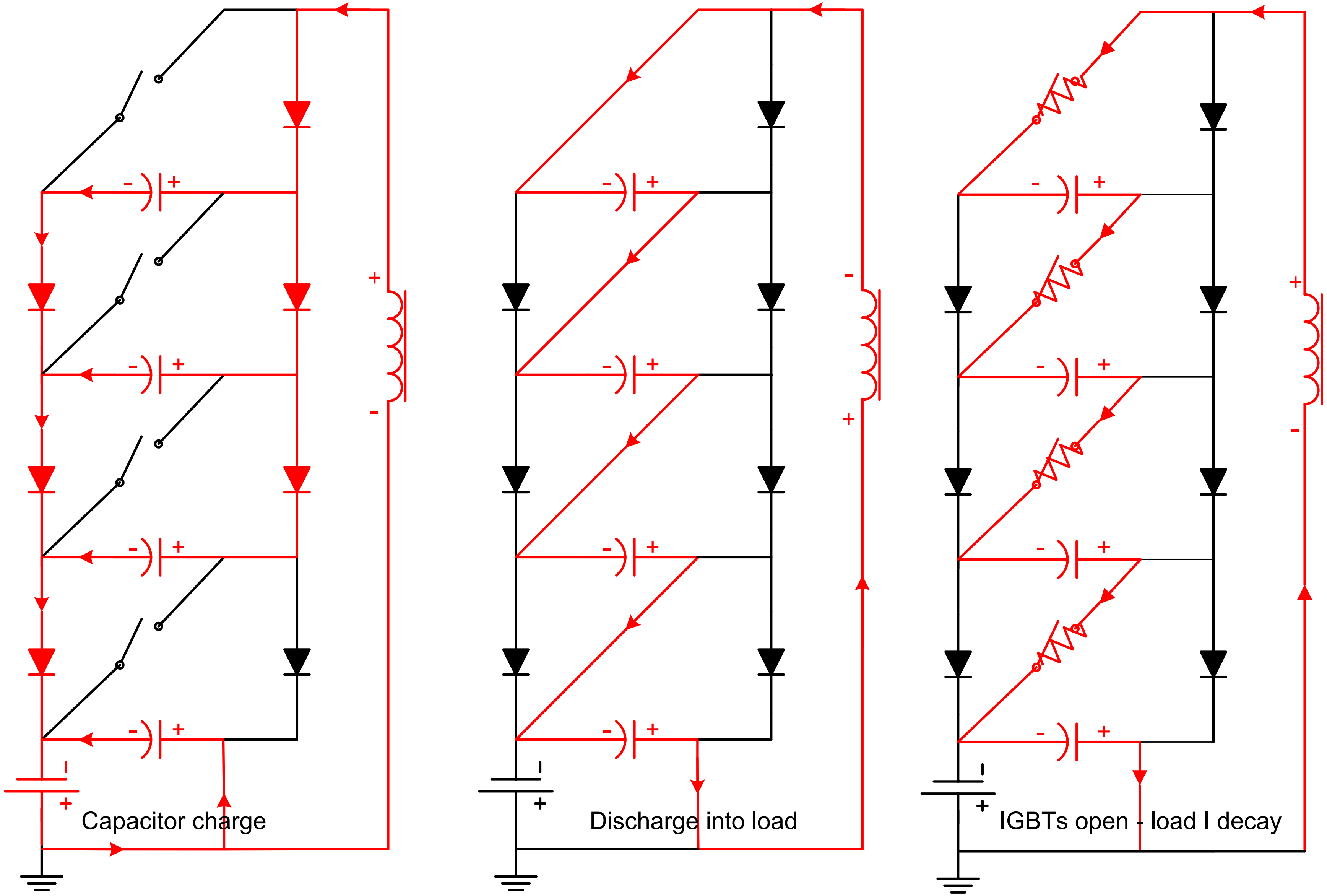
Solid-State Marx Generator for Modulators or Kickers

- *Marx Generator charges capacitors in parallel for quickness, discharges them in series for high output voltage. For long pulses, advantage is to avoid the need for large iron core transformers based on volt-second product*



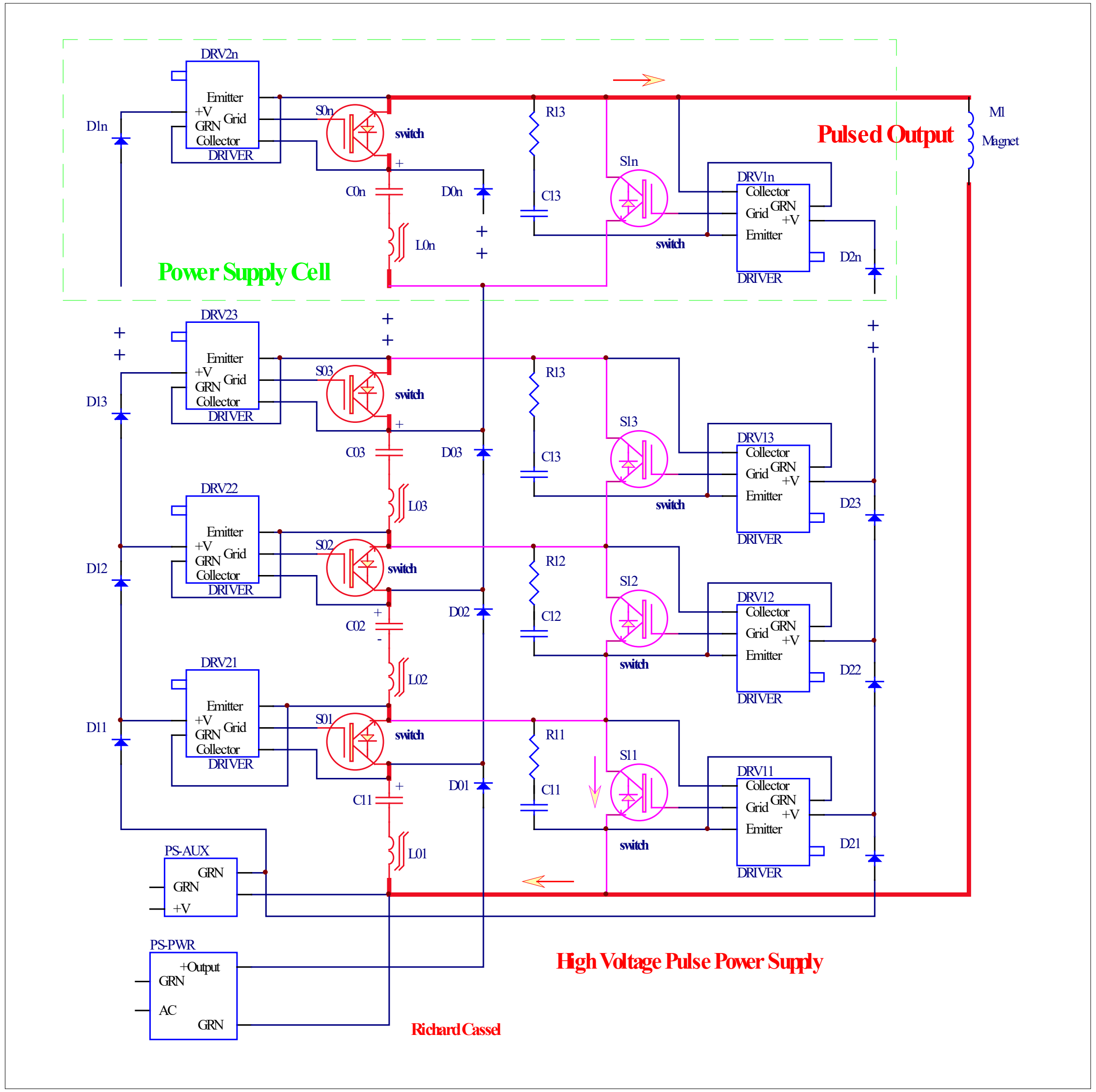
Solid-State Marx Generator for Modulators or Kickers

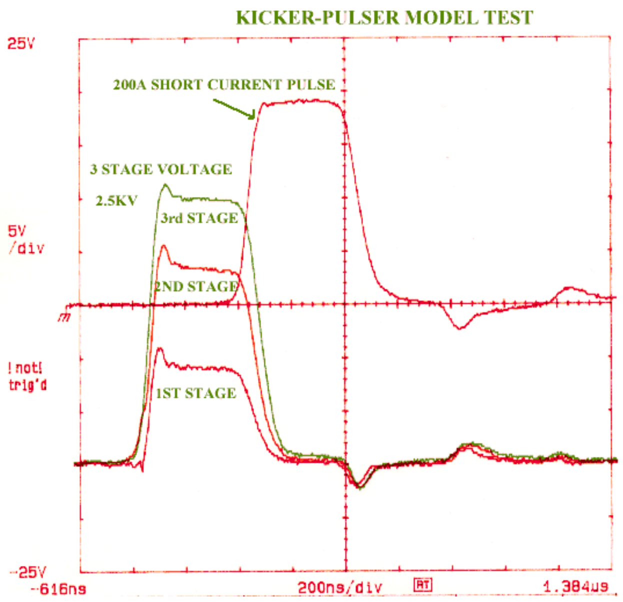
- *If the load is a magnet, the charging inductors are not required*



Solid-State Marx Generator for Modulators or Kickers

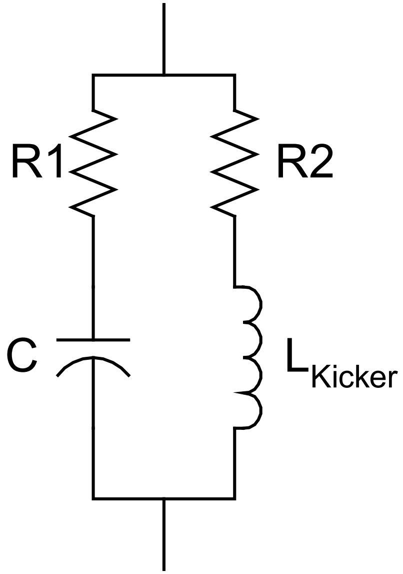
- *Another implementation, using solid-state switches in place of the charging inductors for smaller size and less diversion of capacitor current from load*





A controlled impedance transmission line often drives a kicker. The kicker is usually well modeled as an inductor. A matching circuit can be built around the kicker and its inductance so that this circuit, including the kicker magnet, has constant, frequency independent, impedance which is matched to the transmission line.

Assuming that the transmission line impedance is Z_0 and the kicker inductance is L_{Kicker} derive the values of R1, R2, and C necessary to make a frequency independent (constant) impedance Z_0.



A. What is the significance of the value $\sqrt{\dfrac{\mu_O}{\varepsilon_O}}$?

B. What is the significance of the values $\dfrac{1}{\sqrt{\mu_O \varepsilon_O}}$ and $\sqrt{L*C}$?

C. Calculate the speed of light in mediums with dielectric constants of:
$\varepsilon_r = 1$ $\varepsilon_r = 2$ $\varepsilon_r = 4$ $\varepsilon_r = 8$ $\varepsilon_r = 16$

Section 9

- *Magnetics*
  - *The Electric - Magnetic Equivalence*
  - *Field Due to a Current*
  - *Magnetic Units Including Turns*
  - *Cores and Materials*
  - *Transformer Design Issues*
  - *Inductors*

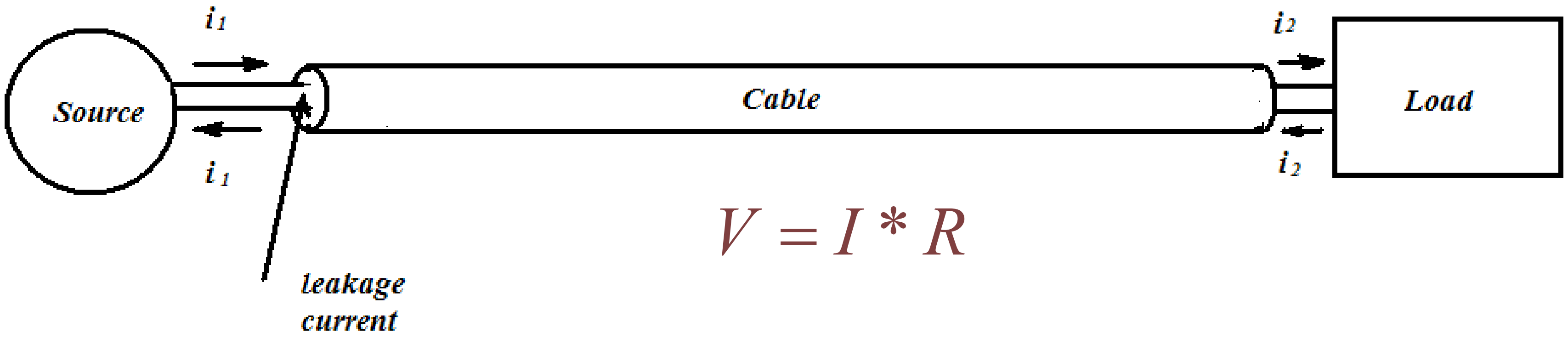
The Electric - Magnetic Equivalence

- *Various magnetic types, such as transformers and filter inductors, play a key role in many of the components used in power supplies*
- *Magnets are also extensively used in accelerators to guide, direct, steer, and focus beams. They are also used to correct chromatic aberrations.*
- *Magnetic circuits are analogous to electric circuits and are important for the analysis of magnetic devices. The equations for both electric and magnetic circuits show strong similarities*

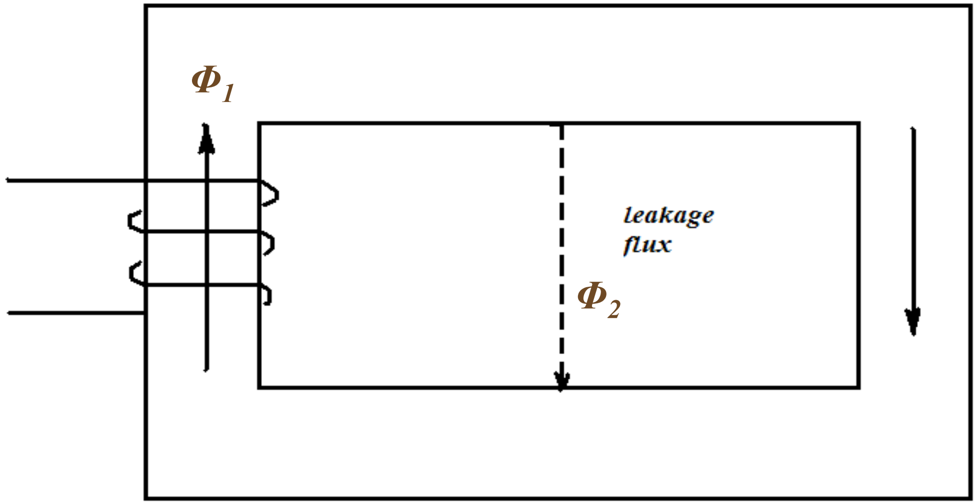
Electrical	***Closest Magnetic***
EMF (Volts)	*MMF (A*turn, F)*
Current (Amperes)	*Flux (Wb / turn, Φ)*
Resistance (ohms, Ω)	*Reluctance (A*turns / Wb, R)*
*Resistivity (ohm*m, ρ)*	
Conductance (mhos, σ)	*Permeance (Wb / A*turn, P)*
Conductivity (Siemens/m)	*Permeability (Henries / m, μ)*

Magnetic Units Including Turns

Symbol	Description	SI units	cgs units
N	Winding turns	turn (t)	t
H	Field intensity	(A·t)/m	Oersted (Oe)
B	Flux density	tesla (T)	gauss (G)
μ	Permeability	T·m/A or H/m	G/Oe
F	Magnetomotive force	A·t	gilbert (Gb)
Φ	Flux	weber/t (Wb/t)	maxwell
R	Reluctance	A·t/Wb	
P	Permeance	henry/t or (Wb/A*t)	Henry/t (H/t)
I	Current	ampere (A)	ampere (A)
L	Inductance	henry (H)	henry (H)

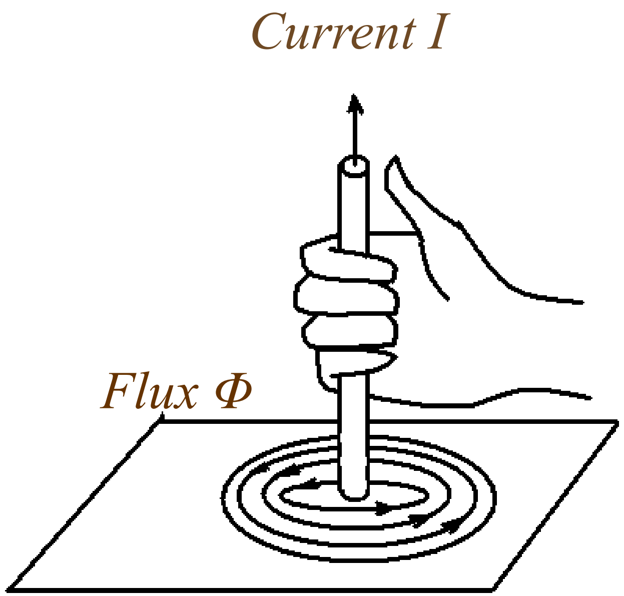

i1
i1
Source
Cable
i2
i2
Load
leakage
current

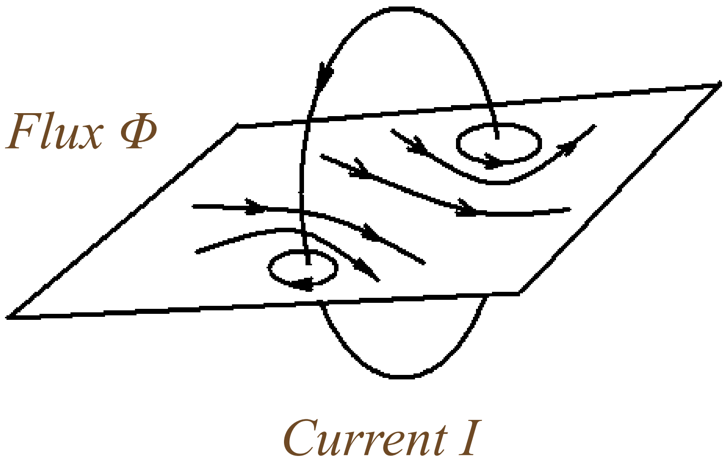
$$V = I * R$$


Φ1
leakage
flux
Φ2

$$F = \Phi * R$$

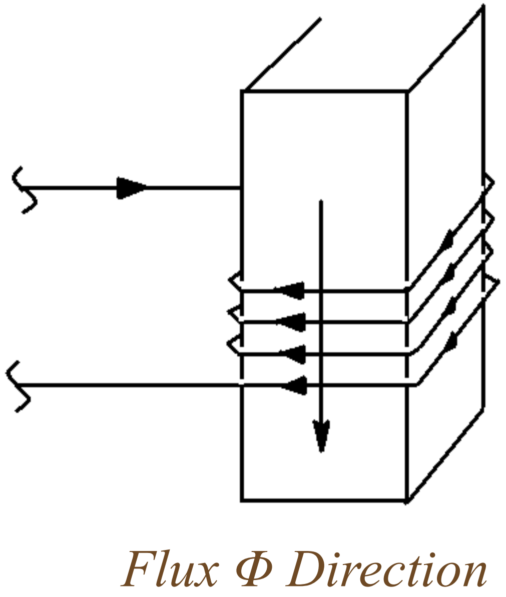
Field Due to a Current



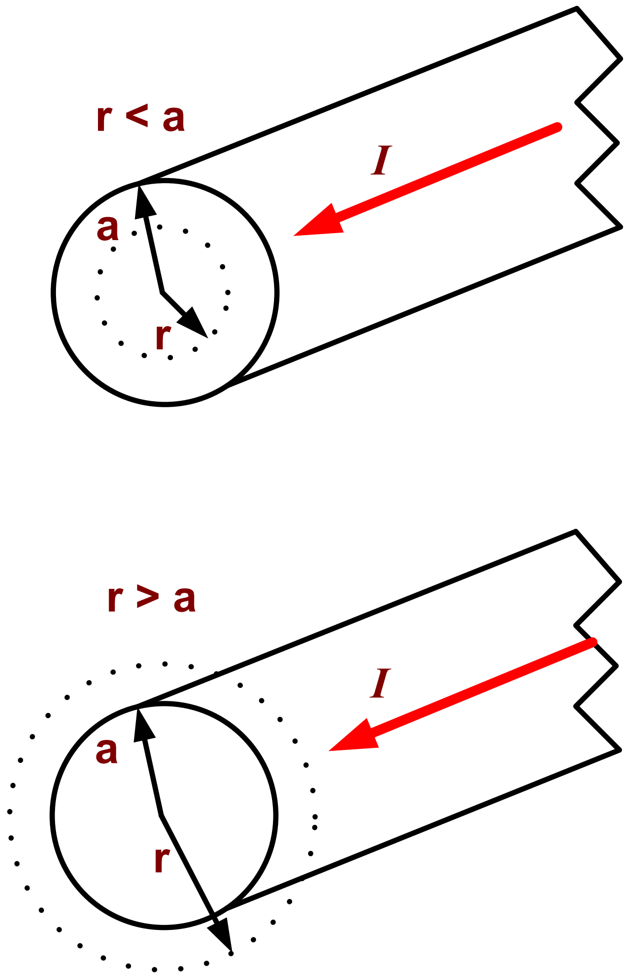


Right Hand Rule:

- *Thumb = Current*
- *Fingers Point in Direction of Magnetic Field*



H Field Around A Wire



$$I = \oint H \bullet dl$$

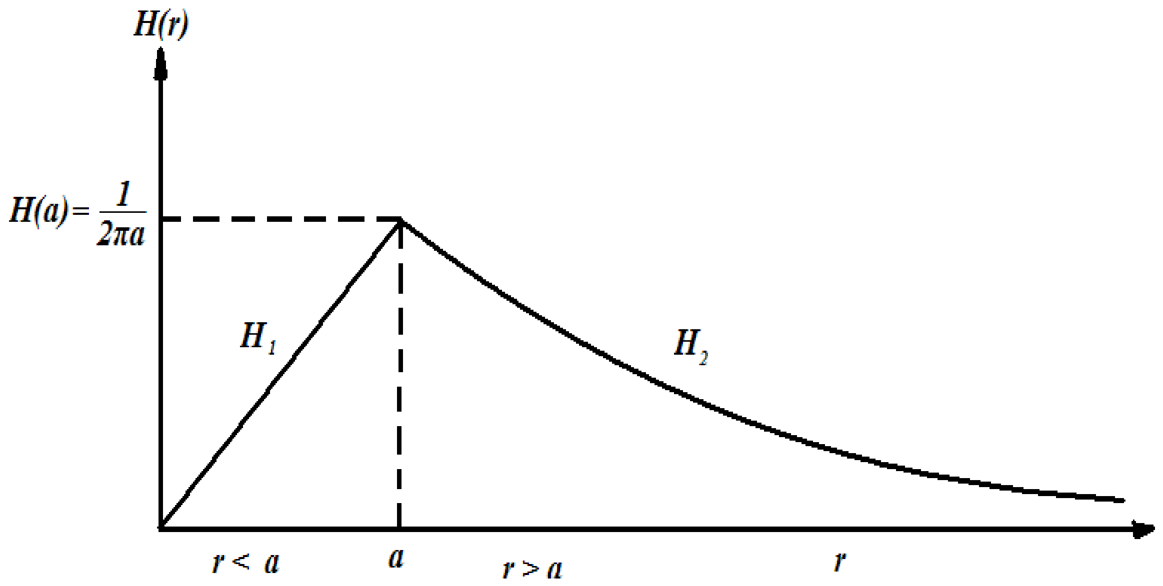
For uniform current density

$$H = \frac{I'}{l} \quad l = 2\pi r$$

$I' =$ *The fraction of the total current flow in the wire*

For $r \leq a \Rightarrow I' = \frac{r^2}{a^2} I \qquad H_1 = \left(\frac{I}{2\pi}\right)\frac{r}{a^2}$

For $r > a \Rightarrow I' = I \qquad H_2 = \frac{I}{2\pi r}$



Permeability Definitions

- μ_0 = permeability of vacuum = $4*\pi*10^{-7}$ H/m
- μ_r = relative permeability (dimensionless)
- μ_m = material permeability = B/H at any given point
- $\mu_m = \mu_0 * \mu_r$
- Permeability is an important core parameter
- Ferromagnetic materials used in transformer and inductor cores because of their high permeability

Core Materials
Air
Alloys of steel
Amorphous steel
Iron Powder
Manganese-Zinc Ferrite
Molybdenum Permalloy Powder
Nickel-Zinc Ferrite
Sendust (Fe, Si, Al)
Silicon Steel

Energy is power integrated over time, in this case extracted energy

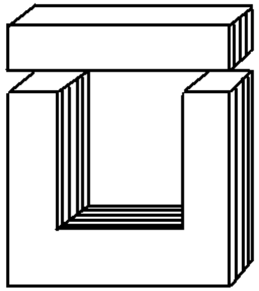
$$W = -\int_0^{t_1} VIdt \quad V = -nA\frac{dB}{dt} = -nA\mu\frac{dH}{dt} \text{ and } I = \frac{Hl}{n}$$

$$W = \int_0^{t_1} A\mu\frac{dH}{dt}Hldt = A\mu l\int_0^{H_1} H\,dH = Al\left(\frac{\mu H_1^2}{2}\right)$$

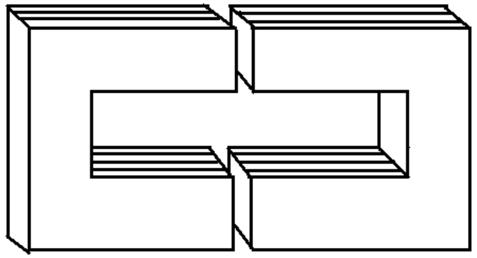
W is the magnetic energy stored in the volume, Al, and $\left(\frac{\mu H_1^2}{2}\right)$ *is the field energy density*

Core Shapes

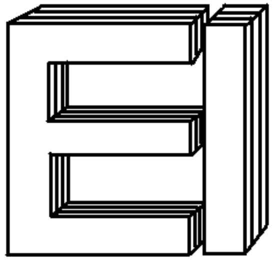
- *U-U, U-I cores*
- *E-E, E-I, ETD cores*
- *POT cores*
- *RM cores*
- *PQ and PM cores*
- *EP, EFD and ER cores*
- *Toroid*



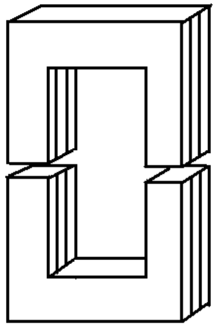
U-I



cut C core

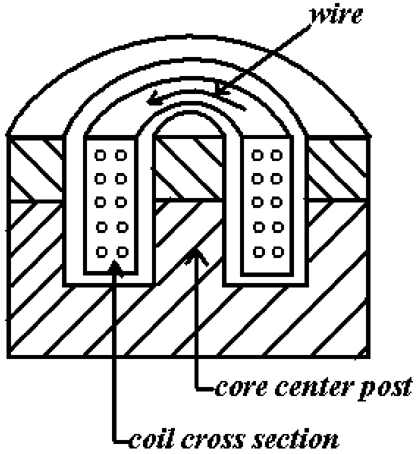


E Core

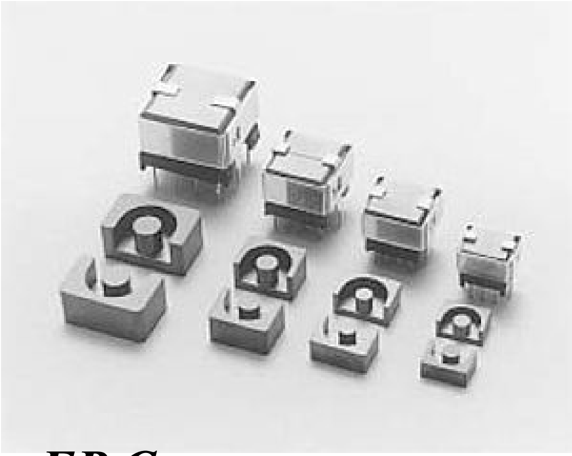


U-U

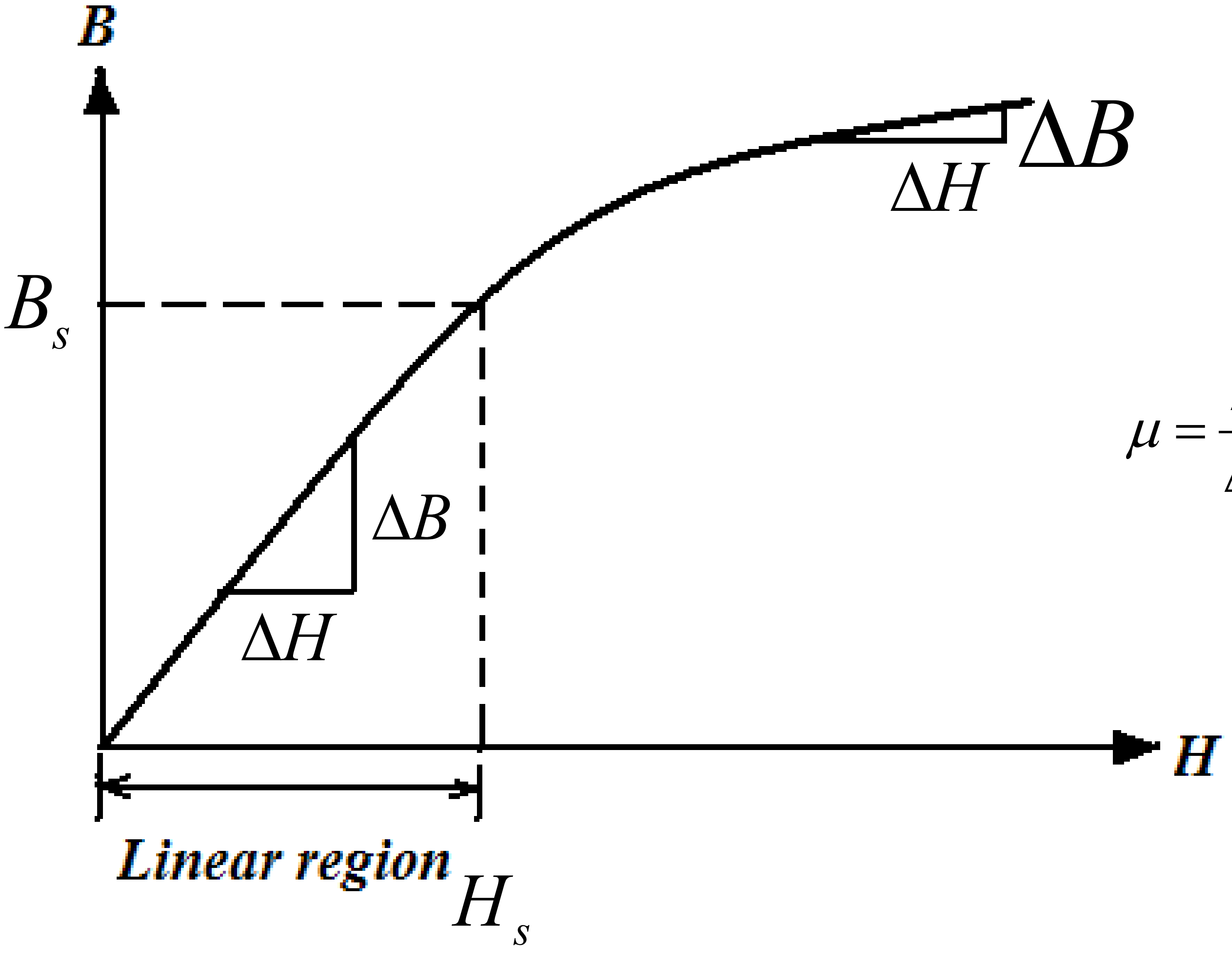
Pot cores



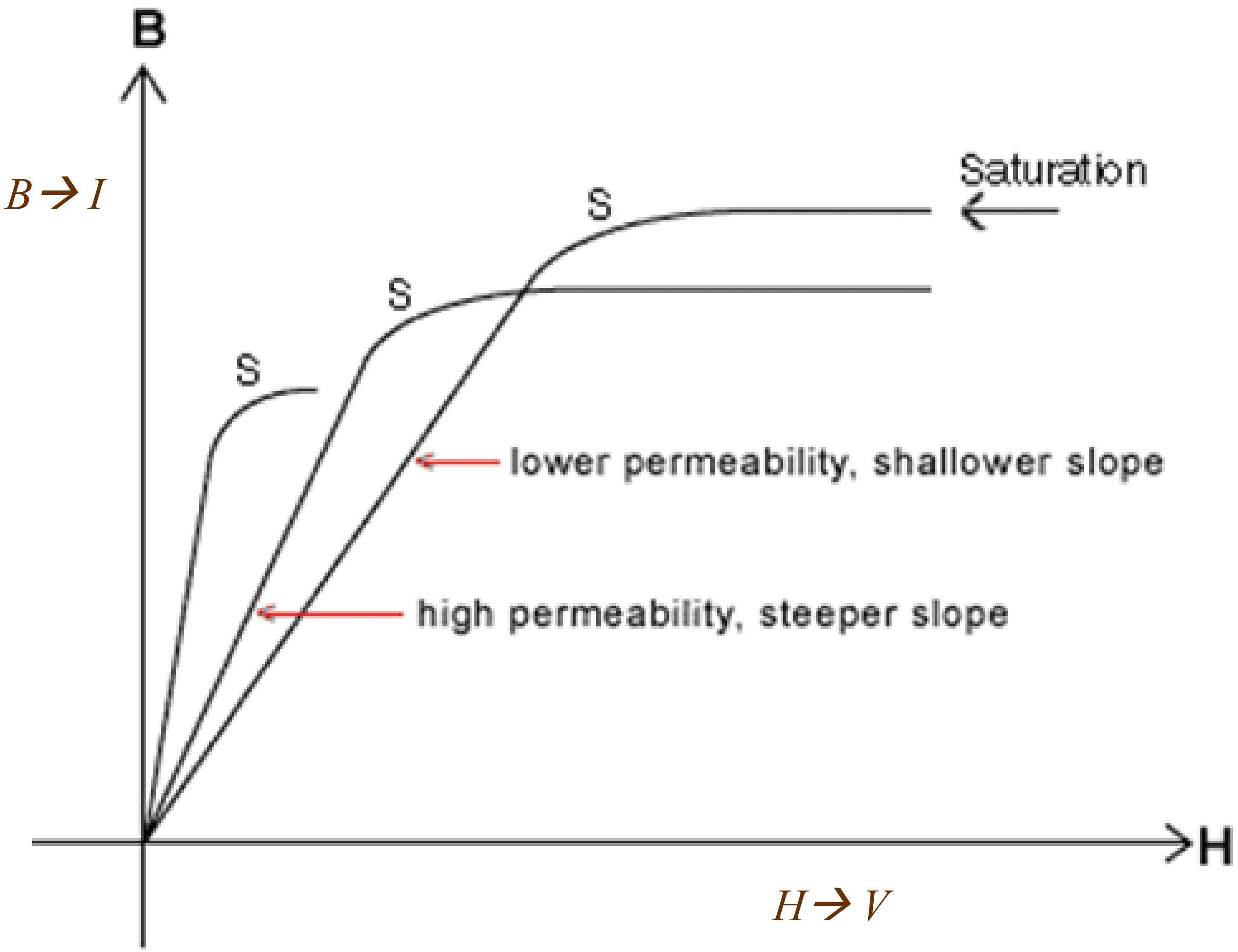
RM Core



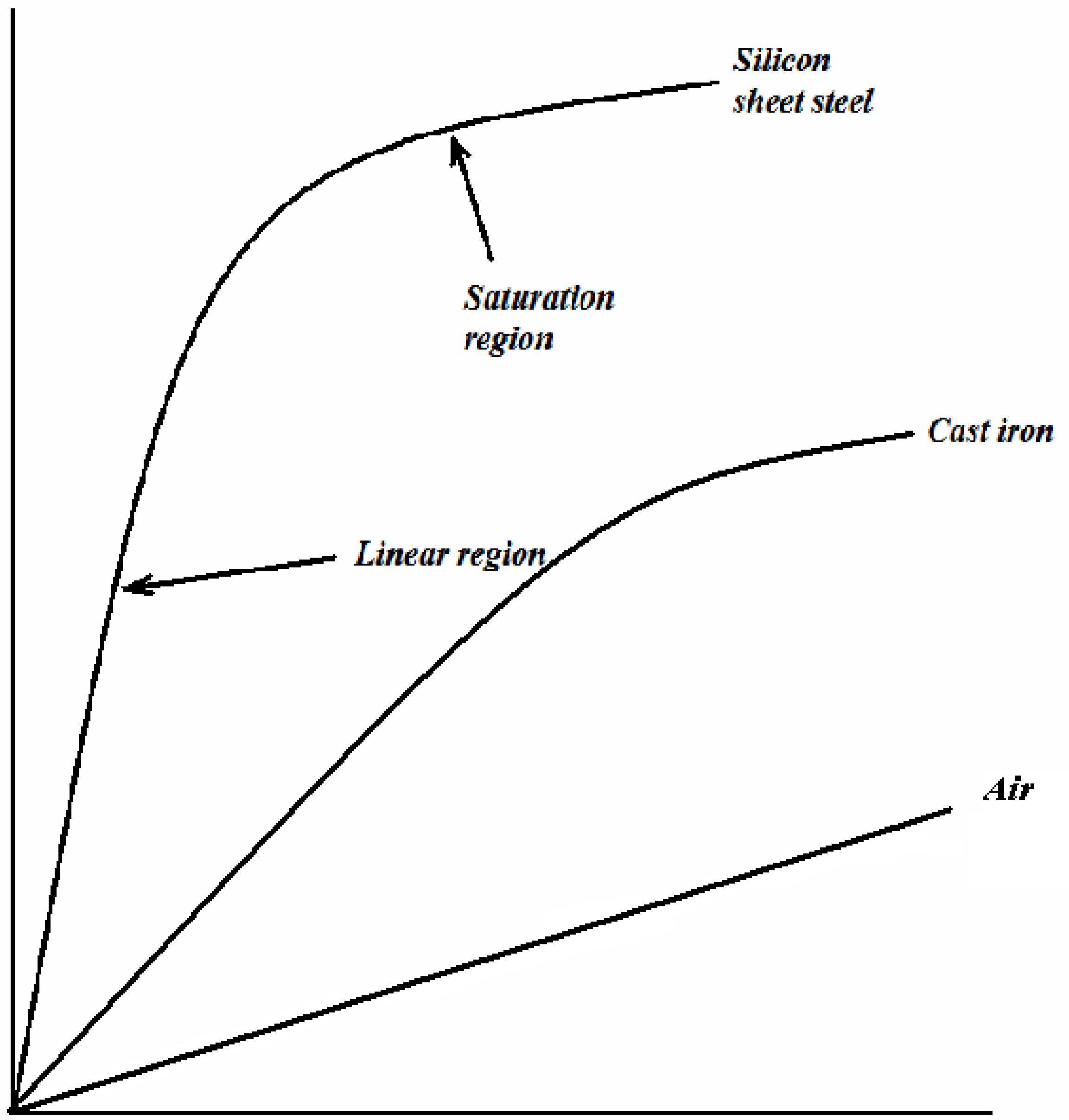
EP Core



$$\mu = \frac{\Delta B}{\Delta H} = \frac{B}{H}$$


B
B → I
Saturation
S
S
S
lower permeability, shallower slope
high permeability, steeper slope
H
H → V

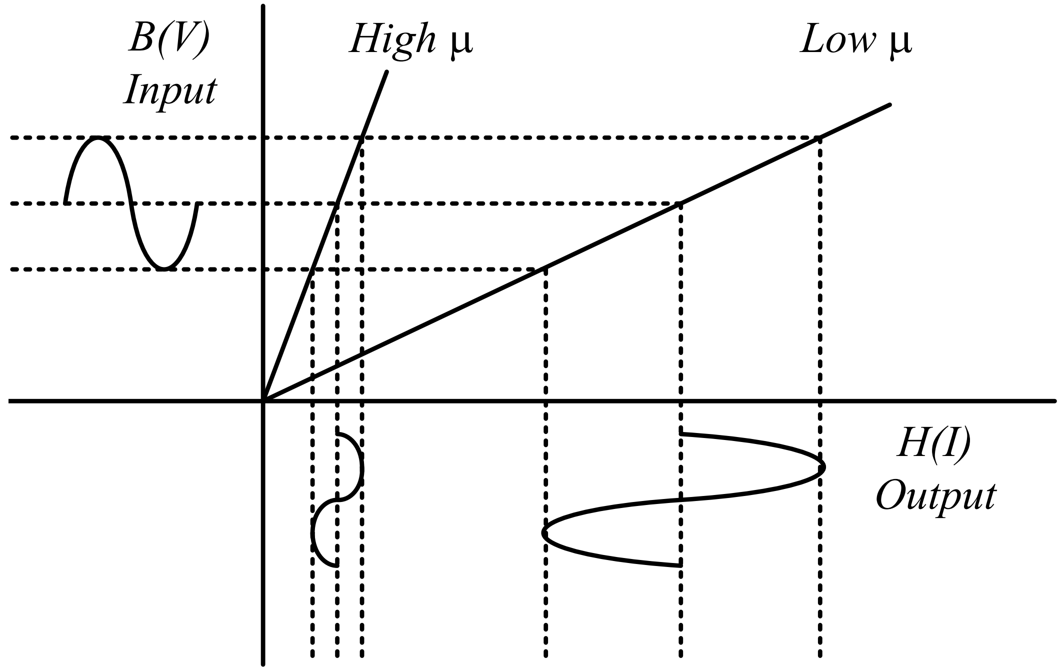
Material Comparison



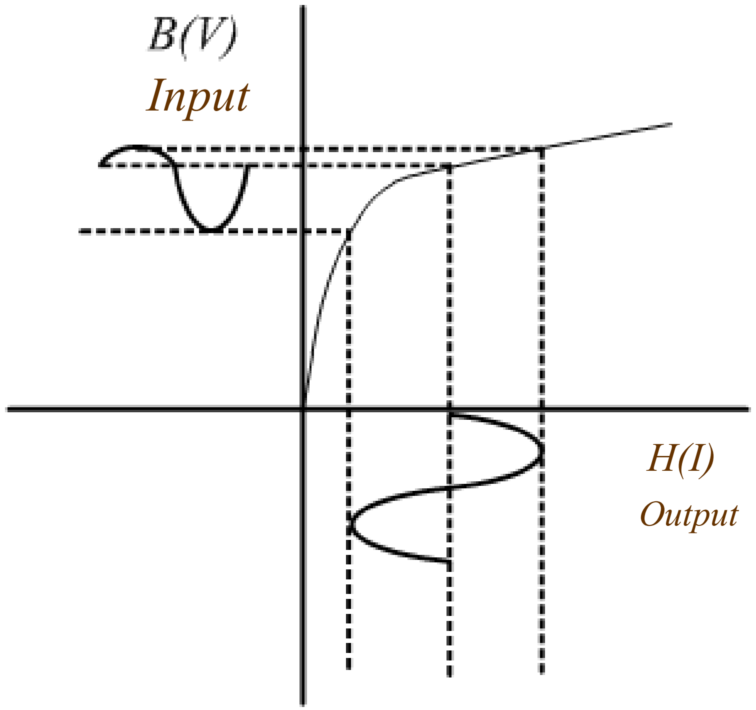
Core Material Guidelines

Material	Frequency Range	B_{sat}	Cost
Ferrites	Good to microwaves	0.2 T	Low
MPP (Moly Permalloy Powder)	200kHz	0.2 to 0.55 T	High
Powdered Fe	1MHz	0.4 to 1 T	Low
Laminated Si-Fe	2kHz	1T	Low
Laminated Electrical Steel	2kHz	0.5 to 1.8 T	Low
Ni-Fe Alloys	100kHz	0.5 to 1.8 T	High

Effect of permeability magnitude on transformer operation



Effect of permeability nonlinearity on transformer operation



Relationship Between v(t) and B(t)

$$v(t) = -\frac{d\Phi(t)}{dt} = V_{max}\cos 2\pi f t$$

$$v_t(t) = \frac{v(t)}{N_p} = \frac{V_{max}\cos 2\pi f t}{N_p}$$

$$v_t(t) = \text{volts per primary winding turn}$$

$$\Phi(t) = -\int v_t(t)dt = \int_s B(t)\bullet dA_c$$

$$A_c = \text{core crossectional area}$$

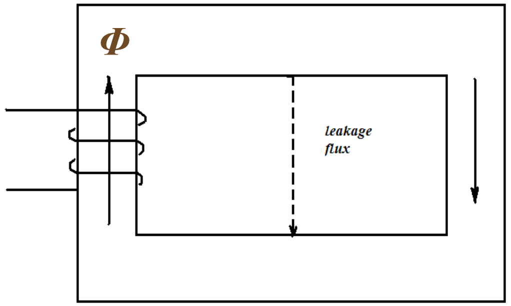
$$\int_s B(t)\bullet d\,A_c = -\int v_t(t)dt = -\int \frac{V_{max}\cos 2\pi f t}{N_p}dt$$

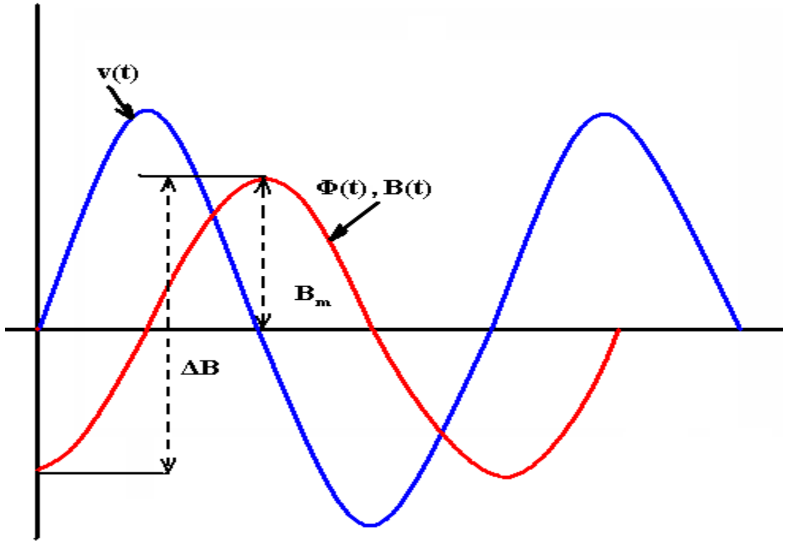
$$B(t)\,A_c = \frac{V_{max}\sin 2\pi f t}{2\pi f N_p}$$

$$B(t) = \frac{V_{max}\sin 2\pi f t}{2\pi f N_p A_c}$$

B_{max} occurs when $\sin 2\pi f t = 1$ and $V_{max} = \sqrt{2}\,V_{rms}$

$$B_{max} = \frac{\sqrt{2}\,V_{rms}}{2\pi f N_p A_c} = \frac{V_{rms}}{4.44 f N_p A_c}$$





Transformer Design – Ensure Sufficient Core Crossection

$$B_{max} = \frac{V_{rms}}{4.44 * f * A_c * N_p * 10^{-8}}$$

where

B_{max} = maximum allowable flux density in gauss

V_{rms} = voltage applied to the primary in volts

$4.44 = \frac{\sqrt{2}}{2\pi}$ converts peak AC to rms and ω to $f(Hz)$

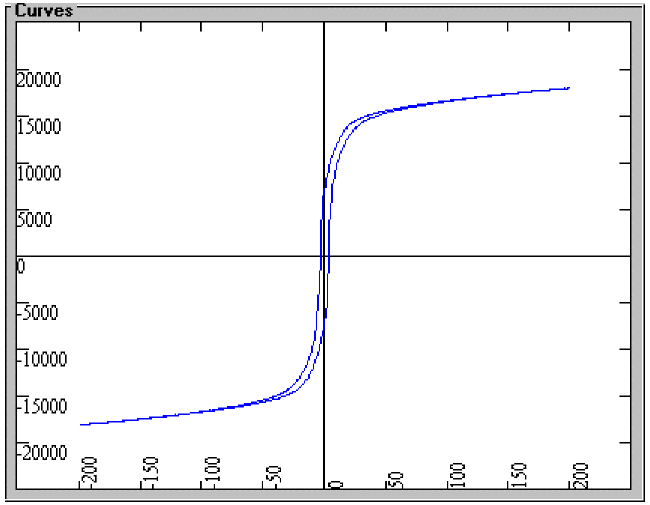
f = frequency of the applied voltage in hertz

A_c = Core crossectional area in cm^2

N_p = Number of primary winding turns

10^{-8} = conversion from engineering to SI units

B(gauss)



H(A/m)

Example for a 480V, 600kVA, laminated electrical steel core

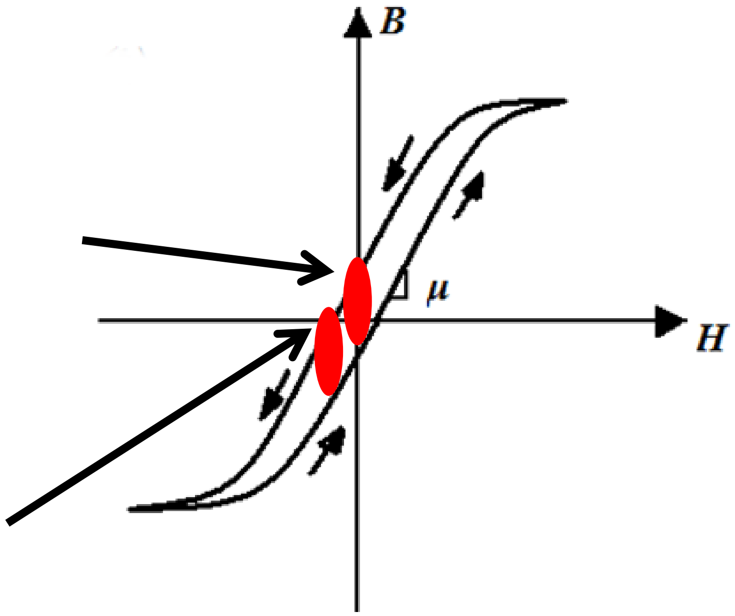
$$B_{max} = \frac{480V * 1.05(\text{voltage safety factor})}{4.44 * 60Hz * 300cm^2 * 60\,turns * 10^{-8}} = 10{,}510\ gauss$$

For square wave or rectangular wave excitation

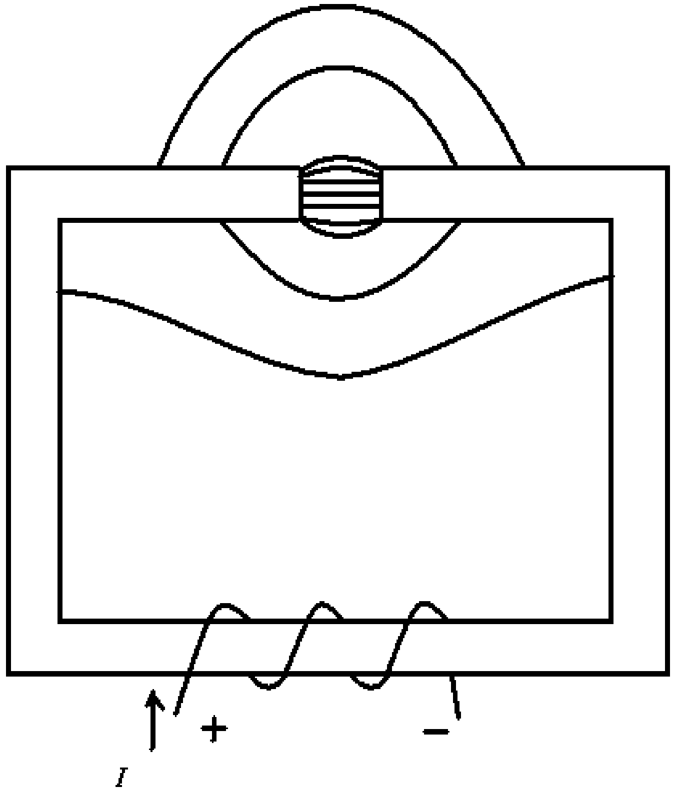
$$B_{max} = \frac{V_{peak}}{2\pi * f * A_c * N_p * 10^{-8}}$$

V_{peak} = peak applied voltage

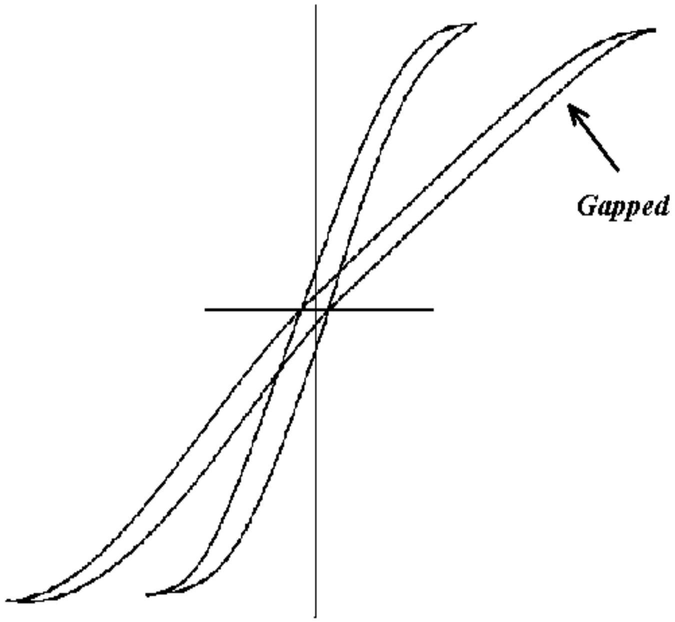
- *Four quadrant B-H curves are known as hysteresis curves. Note that the curve is open in the middle. This is a consequence of the magnetic microstructure.*
- ***Remanence*** *is defined as the absolute value of the magnetic field when the applied voltage is removed. The remnant field can cause inrush current problems when the transformer is re-energized*
- ***Coercive Force*** *- The amount of reverse magnetic field which must be applied to a magnetic material to make the magnetic flux return to zero.*



Effect of Air Gap



Inductor With An Air Gap



B-H Loop

Why We Use Air Gaps

- *They are unavoidable in many cores*
- *In an inductor they permit increased energy storage for a given B by reducing the effective permeability*
- *Air gaps also stabilize the inductance value for both bias and manufacturing variations*
- *In general gaps are undesired in transformers but very useful in inductors*
- *An air gap may be discrete or distributed*

For the 480V, 600kVA transformer

$$i_{max} = \frac{10^3 * h * A_c * ((B_r + 2 * B_{max}) - 130)}{3.2 * N_p * A_s}$$

i_{max} = maximum instantaneous current in amperes

h = the length of the coil in inches=40

A_c = the crossectional area of the core in sq inches=46.5

B_{max} = Maximum flux density=10,500G=1.05T=68 kilolines per square inch

B_r = residual flux density in kilolines (Maxwells) per square inch

= 60% of 1.05T, expressed as 41 kilolines per square inch

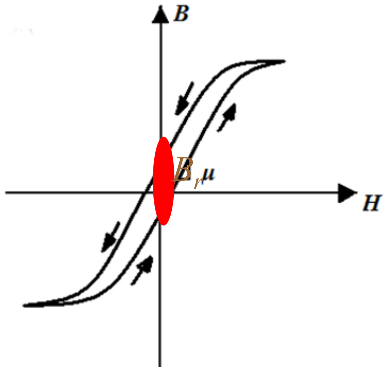
N_p = number of primary turns=60

A_s = effective square inches of the air-core magnetic field=69.4

Example $I_{fl} = \frac{600kVA}{\sqrt{3} * 480V} = 722\,A$, the inrush current is

$$i_{inrush} = \frac{10^3 * 40 * 46.5 * ((41 + 2 * 71) - 130)}{3.2 * 60 * 69.4} = 6.56\,kA$$

This is about 9X the transformer full load (operating) current



Reduce the inrush current by increasing the number of primary turns and/or increasing the effective area of the air-core magnetic field

Transformer Losses

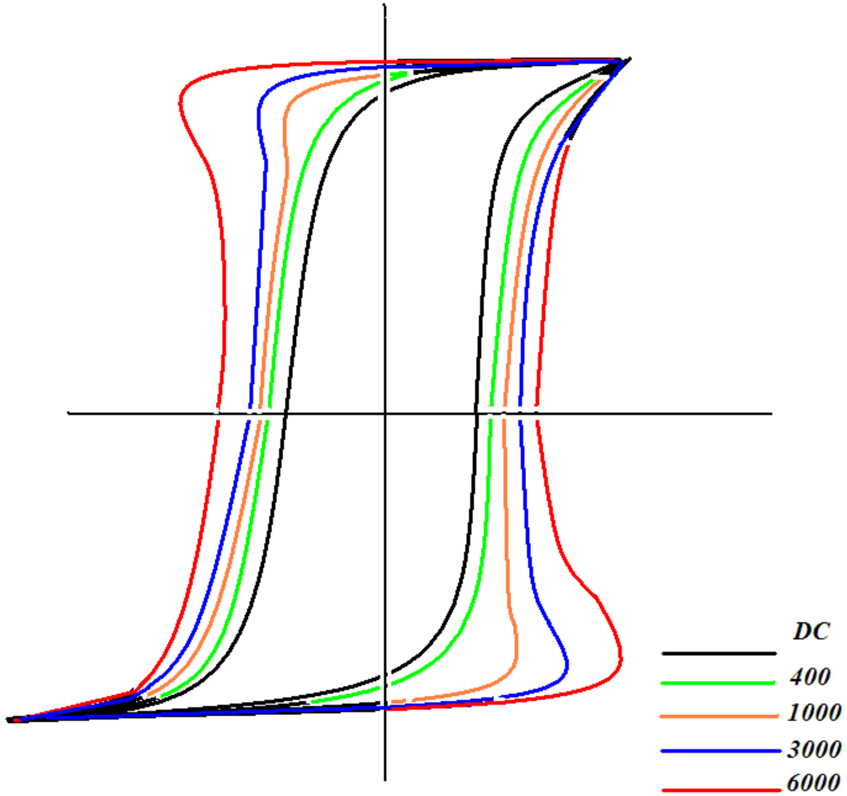
There are always energy losses in transformers. These energy losses generate heat in the form of core losses and winding losses. The losses are from the following sources:

1. *Hysteresis loss from sweeping of flux from positive to negative and the area enclosed by the loop is the loss. Hysteresis loss is due to the energy used to align and re-align the magnetic domains. The smaller the loop area, the smaller the energy loss per cycle*

2. *Eddy current loss from the circulating currents within the cores due to flux –generated voltages.*

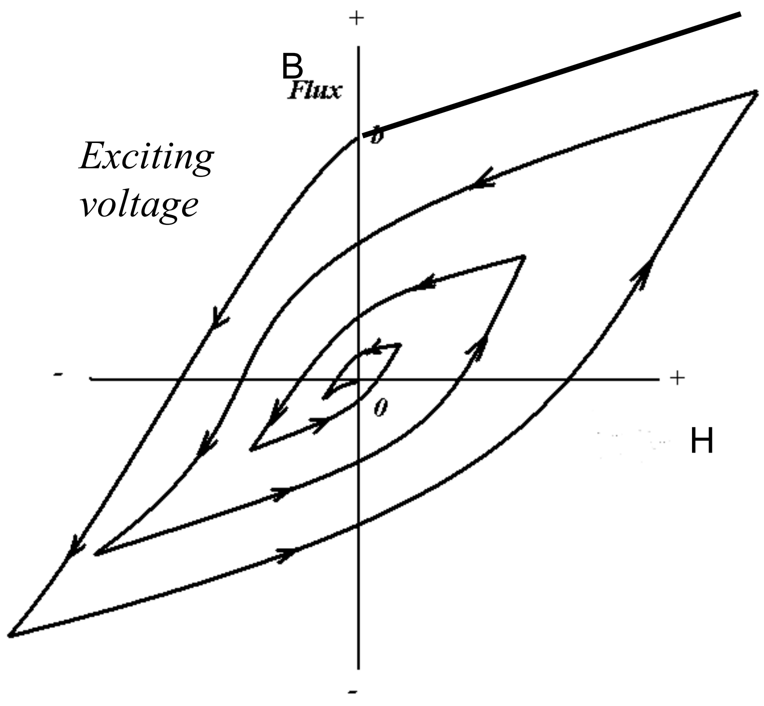
3. *Copper or winding loss. This is also dependent on the wire size, switching frequency, etc. Skin effect and proximity effect will contribute to this loss.*

Effect of Frequency on B-H Characteristics



Hysteresis increases as frequency increases

Demagnetization Or Degaussing



Removing residual magnetism from a ferromagnetic circuit by using decreasing excitation

Skin Effect

- *As the frequency of a given ac current in a conductor is increased, the power dissipation increases*
- *We ascribe this to an increase in ac resistance of the conductor but in actuality it is due to a rearrangement of the current distribution within the conductor*
- *The increase in loss is due to a tendency for the current to concentrate on the perimeter of the conductor rather than being uniform over the conductor area as it would be at dc*
- *This effect becomes more severe as frequency is increased*
- *This is called "skin effect"*

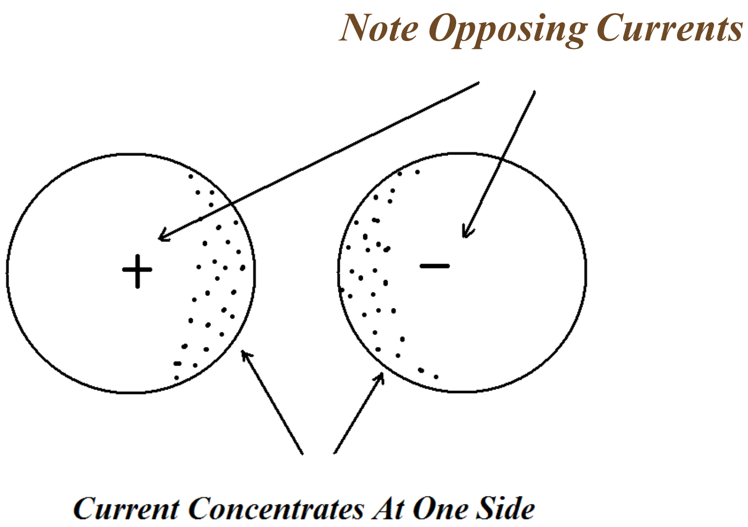
$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \quad \textbf{meters}$$

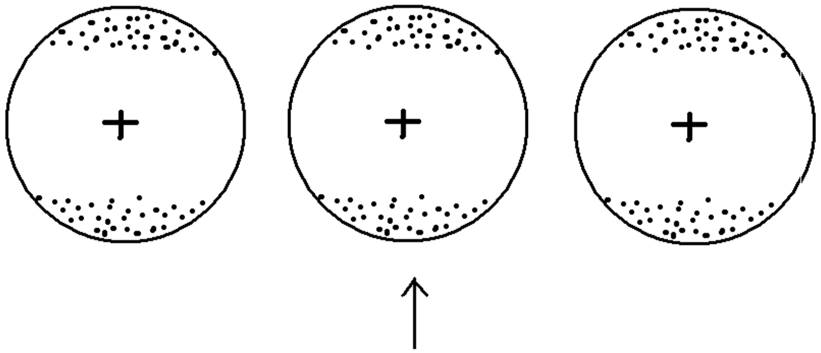
63% of the current is carried in this depth.

Proximity Effect

- *A current carrying conductor will generate a magnetic field*
- *This field can induce eddy currents in nearby conductors, increasing losses in addition to any skin effect. The eddy currents obey Lenz's Law. They flow in a direction that reduces the flux in the conductor*
- *This is referred to as "proximity effect"*
- *In a transformer or inductor, the inner windings operate in a field created by the outer windings*
- *This can also limit the conductor size*
- *As a general rule the wire diameter or the layer thickness is usually less than twice the skin depth at the operating frequency. For multi-layer windings wire diameters of less than 0.5 skin depth may be required.*

Proximity Effect





Proximity Effect - Multiple Parallel Wires

Purposes

- *Used as filters for smoothing power supply ripple*
- *Used as fault current limiting reactors in AC power currents*
- *Used to limit di/dt in certain pulsed circuits*

Requirements

- *Must carry high DC current*
- *Must select core size that is able to store the required magnetic energy (volt-seconds)*
- *An air gap is sometimes employed to extend DC current capability without saturating. Iron and Ferrites are manufactured with distributed air gaps.*

$$L = \frac{\mu_0 \mu_r N^2 A_c}{\mu_0 \mu_r l_g + l_c}$$

where
N = *the number of winding turns (dimensionless)*
A_c = *the core cross sectional area in* m^2
l_c = *the length of the magnetic path in the core in meters*
l_g = *the effective length of the air gap in meters*
$\mu_0 \mu_r$ = *core material permeability under the operating conditions*
μ_r *is dimensionless*

$$\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{H/m}$$

Section 10 - Controls

- *Kirchoff's current law: The sum of all currents into a node is zero.*
- *Kirchoff's voltage law: The sum of all voltage drops around a loop is zero.*
- *There are linear voltage-current relationships across each passive element*
  - *Resistor:* $v_R(t) = Ri_R(t)$
  - *Inductor:* $v_L(t) = L\frac{di_L(t)}{dt}$
  - *Capacitor:* $i_C(t) = C\frac{dv_C(t)}{dt}$
- *For a real magnet with both inductance and resistance add the voltages*
  - $v(t) = Ri(t) + L\frac{di(t)}{dt}$
- *If we represent the current and voltages as complex exponentials of a fixed frequency ω, then* $i(t) = Ie^{j\omega t};\ v(t) = Ve^{j\omega t}$ *and the voltage-current relationship across the magnet is*

$$Ve^{j\omega t} = RIe^{j\omega t} + Lj\omega Ie^{j\omega t} = (R + j\omega L)Ie^{j\omega t}$$

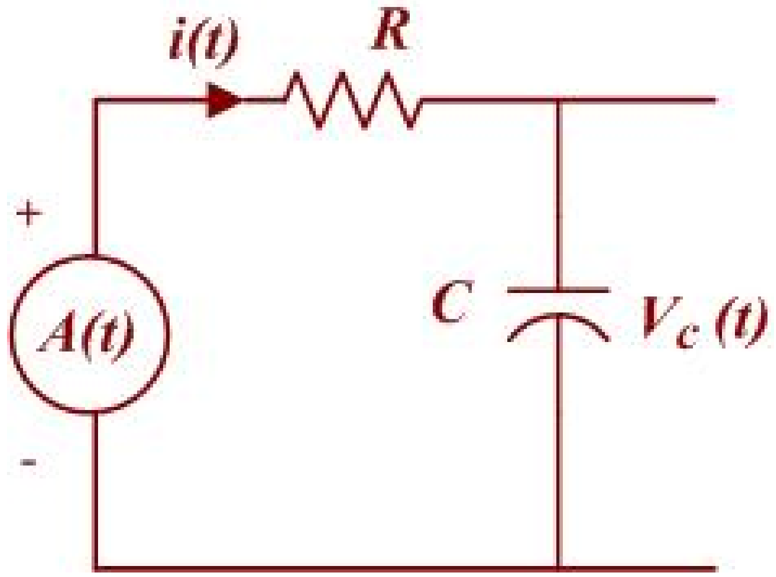
  - $Ie^{j\omega t}$ *is the eigenfunction*
  - $R + j\omega L$ *is the eigenvalue*
  - *For electrical circuits, this is the impedance* $Z(\omega)$

- *Apply Kirchoff's voltage law around the loop*

$$-A(t) + Ri(t) + v_C(t) = 0$$

- *From this, using* $i(t) = C\frac{dv_C(t)}{dt}$*, we get the system equation*

$$RC\frac{dv_C(t)}{dt} + v_C(t) = A(t)$$



- *Define the time constant* $\tau = RC$
  The solution to the system equation, for arbitrary $A(t)$ *is*

$$v_C(t) = v_C(0)e^{-t/\tau} + \tau^{-1}e^{-t/\tau}\int_0^t e^{u/\tau}A(u)du$$

  When $A(t) = A$*, a constant*

$$v_C(t) = v_C(0)e^{-t/\tau} + A\left(1 - e^{-t/\tau}\right)$$

- *The solution is in the form of a final value, A, and time varying terms, due both to the source and the initial condition of the capacitor. The time varying terms are the eigenfunctions of the system.*

- *Repeat the same problem using Laplace transforms*

$$C\frac{dv_c(t)}{dt} = \frac{[-v_c(t) + A(t)]}{R}$$

$$\frac{dv_C(t)}{dt} = \frac{1}{RC}[v_C(t) + A(t)]$$

- *Transform both sides*

$$[sV_c(s) - v_c(0)] = -\frac{1}{RC}V_c(s) + \frac{1}{RC}A(s)$$

$$\left(s + \frac{1}{RC}\right)V_c(s) = v_c(0) + \frac{1}{RC}A(s)$$

Again, setting $\tau = RC$

$$V_c(s) = \frac{1}{s + \tau^{-1}}v_c(0) + \frac{\tau^{-1}}{s + \tau^{-1}}A(s)$$

- *For the case when A is a constant,* $A(s) = \frac{A}{s}$

$$V_C(s) = \frac{1}{s + \tau^{-1}}v_c(0) + A\frac{1}{s}\frac{\tau^{-1}}{s + \tau^{-1}} = \frac{1}{s + \tau^{-1}}v_c(0) + A\left(\frac{1}{s} - \frac{1}{s + \tau^{-1}}\right)$$

- *The inverse transform gives us the same result as on the previous page*

$$v_C(t) = v_C(0)e^{-t/\tau} + A(1 - e^{-t/\tau})$$

- *Transfer Function*
  - *The transfer function is defined as the ratio of the system output to the system input after all initial conditions have died out.*
  - *We can immediately read off the system transfer function from the transform equation.*
  - *From the transform equation*

$$V_c(s) = \frac{1}{s+\tau^{-1}} v_c(0) + \frac{\tau^{-1}}{s+\tau^{-1}} A(s)$$

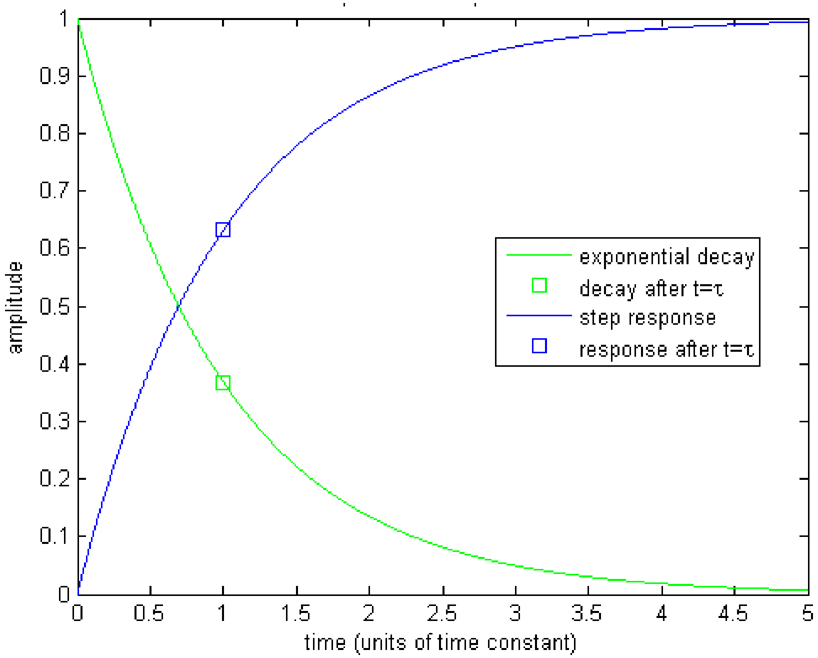
so that

$$\frac{V_C(s)}{A(s)} = \frac{\tau^{-1}}{s+\tau^{-1}}$$

- *We note that*
  - *The transforms of both the response to the initial condition and the transfer function have the same poles*
  - *Therefore, their time responses both have terms that behave like the system eigenfunctions*
  - *The transfer function may have other time responses that depend on the functional dependence of* $A(s)$

- *Dynamics are determined by the numerator and denominator of transfer function*
- *The values of s for which the numerator or denominator vanishes are called "zeroes" and "poles", respectively*
- *One pole circuits all have the same shape response and depend only on the time constant,* $\tau = RC$ *or* $\tau = L/R$
- *A one pole circuit rises to* 63% *or decays to* 37% *of its final value at* $t = \tau$

$$H(s) = \frac{\tau^{-1}}{s + \tau^{-1}}$$



- *Since we will analyze our systems primarily in the frequency domain, it is important to understand the properties of a one pole system as a function of frequency.*
- *We can calculate the transfer function using algebra on the system impedances*

$$H(j\omega) = \frac{Z_C(\omega)}{Z_R(\omega) + Z_C(\omega)}$$

$$= \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$= \frac{\tau^{-1}}{j\omega + \tau^{-1}}$$

$$= \frac{1}{1 + j\omega\tau}$$

- *Magnitude*

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega\tau)^2}}$$

$$|H(j\omega)|_{dB} = 20\log_{10}(|H(j\omega)|)$$

$$= -10\log_{10}(1+(\omega\tau)^2) \approx 0 \textit{ at low frequencies, } \omega\tau \ll 1$$

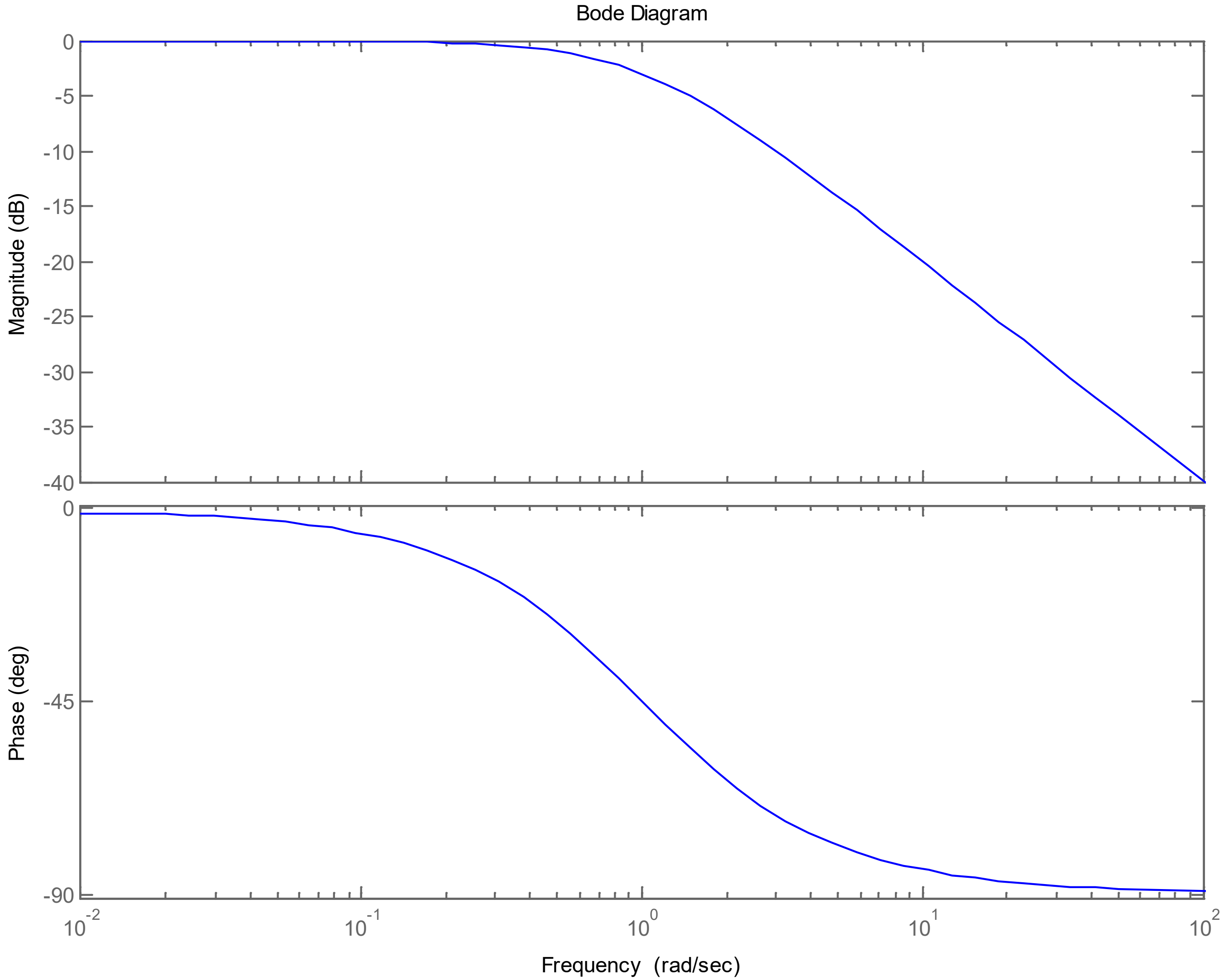
- *Half-power point*
  - *Power goes like* $|H(j\omega)|^2$
  - *Half-power when* $\omega\tau = 1$
  - $-10\log_{10}(2) = -3.01\text{ dB}$
- *High frequency behavior* $\omega\tau \gg 1$
  - $|H(j\omega)| \approx |\omega\tau|^{-1}$
  - $20\log_{10}(|H(j\omega)|) \approx -20\log_{10}\omega - 20\log_{10}\tau$
  - *The last term is a constant; the first decreases* 20 dB *per decade*
- *Phase*

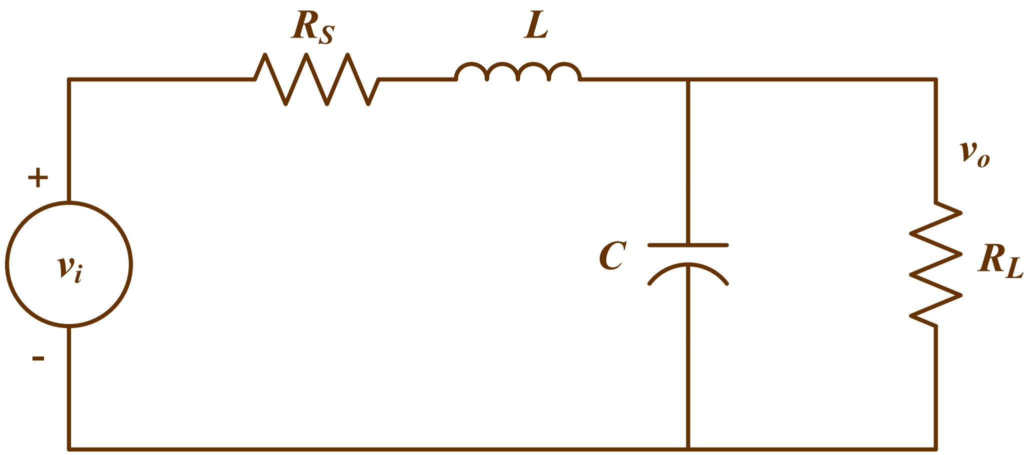
$$\angle H(j\omega) = -\arctan(\omega\tau)$$

$$\approx 0 \qquad \omega\tau \ll 1$$

$$= -45° \quad \omega\tau = 1$$

$$\approx -90° \quad \omega\tau \gg 1$$


Bode Diagram
Magnitude (dB)
0
-5
-10
-15
-20
-25
-30
-35
-40
Phase (deg)
0
-45
-90
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)



$$Z_S = R_S + j\omega L = R_S + sL$$

$$Z_L = \frac{\frac{R_L}{j\omega C}}{R_L + \frac{1}{j\omega C}} = \frac{\frac{R_L}{sC}}{R_L + \frac{1}{sC}}$$

- *Find transfer function of voltage divider*

$$H(j\omega) = \frac{\dfrac{\dfrac{R_L}{j\omega C}}{R_L + \dfrac{1}{j\omega C}}}{R_S + j\omega L + \dfrac{\dfrac{R_L}{j\omega C}}{R_L + \dfrac{1}{j\omega C}}} = \frac{R_L}{-R_L LC\omega^2 + j(R_L R_S C + L)\omega + (R_S + R_L)}$$

$$= \frac{1}{LC}\frac{1}{-\omega^2 + j\left(\frac{R_S}{L} + \frac{1}{R_L C}\right)\omega + \left(1 + \frac{R_S}{R_L}\right)\left(\frac{1}{LC}\right)} \quad \text{let } \omega_0^2 = \frac{1}{LC}$$

$$= \frac{\omega_0^2}{-\omega^2 + j\left(\frac{R_S}{L} + \frac{1}{R_L C}\right)\omega + \left(1 + \frac{R_S}{R_L}\right)\omega_0^2}$$

This has the form

$$H(s) = \frac{a_0}{s^2 + a_1 s + a_0} = \frac{a_0}{(s - s_1)(s - s_2)}$$

$$s_1 = -\frac{a_1}{2} + \sqrt{\left(\frac{a_1}{2}\right)^2 - a_0} \quad s_2 = -\frac{a_1}{2} - \sqrt{\left(\frac{a_1}{2}\right)^2 - a_0}$$

Often the second order low-pass transfer function is expressed in a form to make the characteristics of an underdamped system more transparent.
We set $\omega_0^2 = a_0$ *and* $2\zeta\omega_0 = a_1$ *and rewrite the transfer function*

$$H(s) = \frac{a_0}{s^2 + a_1 s + a_0} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

Now the poles are expressed as
$s_1 = -\zeta\omega_0 + j\omega_0\sqrt{1-\zeta^2}$ and $s_2 = -\zeta\omega_0 - j\omega_0\sqrt{1-\zeta^2}$

which lead to the eigenfunctions

$$e^{-\zeta\omega_0 t} e^{\pm j\omega_0\sqrt{1-\zeta^2}t}$$

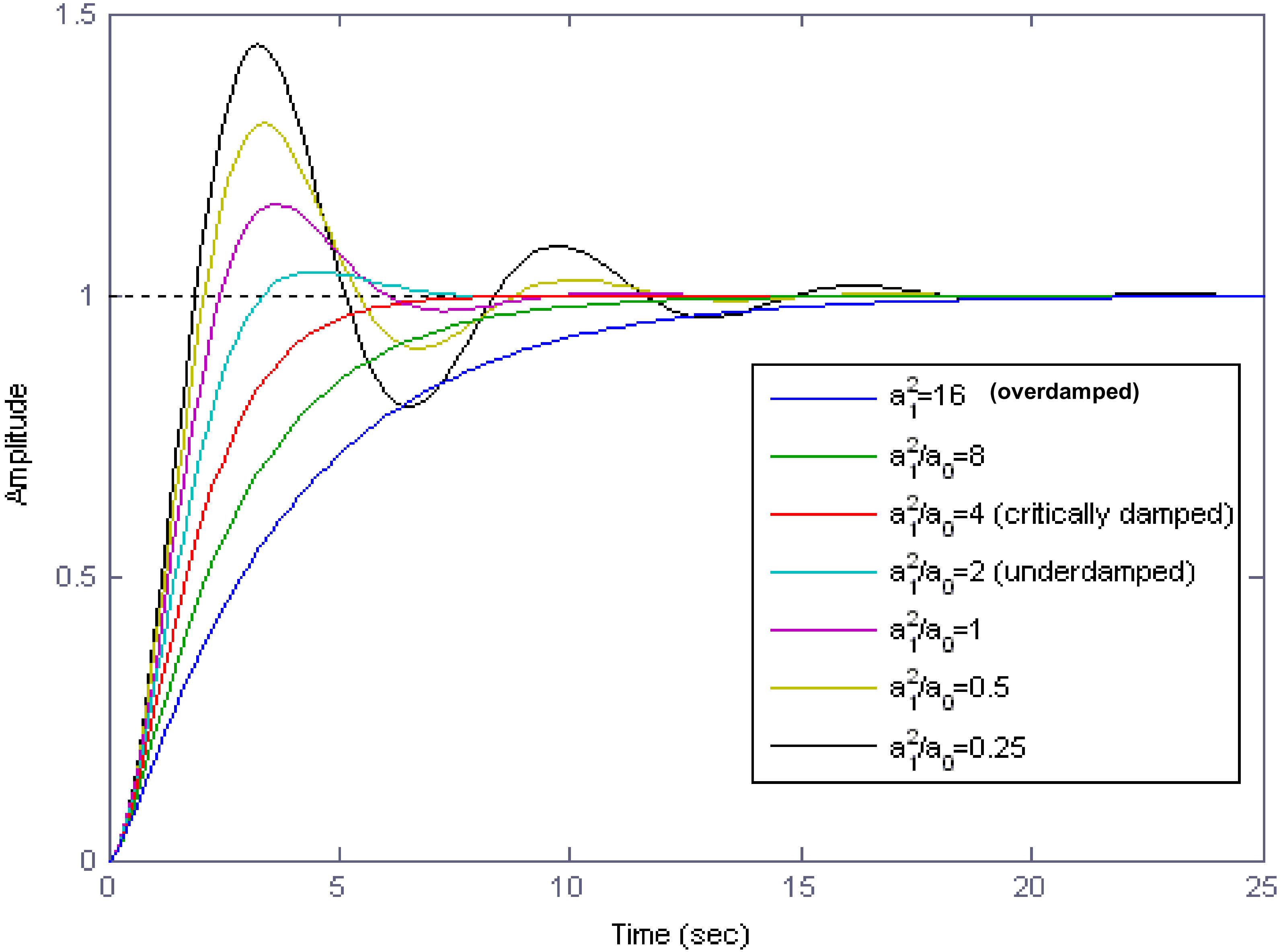
For a given ω_0*, the damping is determined by the value of* ζ

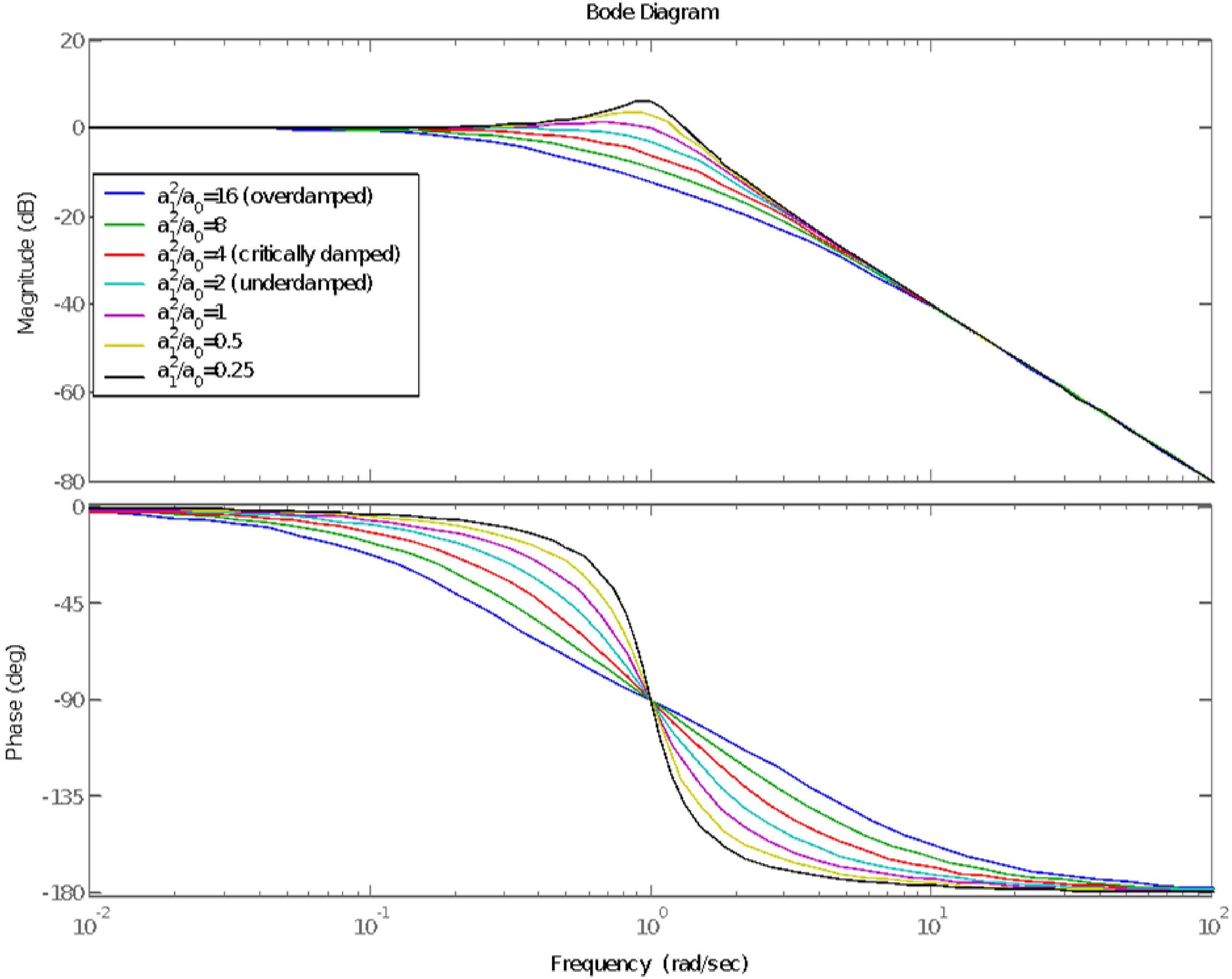
As the damping increases, the peak frequency of the response decreases.

This notation is useful for $\zeta \leq 1$

- *Two pole circuits have two degrees of freedom. One degree sets the system time scale. One degree sets the stability parameter*
- *For a given time scale, the more stable the system, the slower its response. Two pole systems can be separated into three categories*
- *Over-damped system radical is positive, roots are real* $a_1^2/a_0 > 4;\ (\zeta > 1)$
  - *Both poles are real*
  - *No oscillation in step response*
- *Critically damped system radical is zero, roots are real* $a_1^2/a_0 = 4;\ (\zeta = 1)$
  - *Both poles are real and identical*
  - *Fastest step response with no overshoot or oscillation*
- *Under-damped system radical is negative, roots are complex* $a_1^2/a_0 < 4;\ (\zeta < 1)$
  - *Poles are complex conjugates of each other*
  - *Step response is faster than the other two, but has overshoot*

- *For a given ω_0, there is a trade-off between the responses in the time and frequency domain*
    - *A faster response means less damping*
        - *Leads to higher gain near $\omega = \omega_0$*
        - *More overshoot and ringing in step responses*
    - *A slower response means more damping*
        - *Lower gain near $\omega = \omega_0$*
        - *Smoother approach to final values for step responses*
- *At high frequencies, each of the two poles contributes*
    - $20\ \mathrm{dB}$ *per decade attenuation*
    - $90°$ *total phase shift*
- *For a given ω_0, the low and high frequency behavior of all two pole systems are identical*
- *The only difference occurs in the behavior around $\omega \approx \omega_0$*


1.5
1
0.5
0
Amplitude
0
5
10
15
20
25
Time (sec)
$a_1^2=16$ (overdamped)
$a_1^2/a_0=8$
$a_1^2/a_0=4$ (critically damped)
$a_1^2/a_0=2$ (underdamped)
$a_1^2/a_0=1$
$a_1^2/a_0=0.5$
$a_1^2/a_0=0.25$



Bode Diagram
Magnitude (dB)
20
0
-20
-40
-60
-80
a_1^2/a_0=16 (overdamped)
a_1^2/a_0=8
a_1^2/a_0=4 (critically damped)
a_1^2/a_0=2 (underdamped)
a_1^2/a_0=1
a_1^2/a_0=0.5
a_1^2/a_0=0.25
Phase (deg)
0
-45
-90
-135
-180
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)

- *Summarizing*
- *Low and high frequency behavior is almost independent of* a_1
- *At low frequencies the magnitude is constant and the phase approaches* $0°$
- *At high frequencies the magnitude decreases* 40 dB/*decade* (20 dB/*pole) and the Phase approaches* $-180°$ ($-90°$/*pole)*
- *At* ω_0 a_1*determines the attenuation and phase slope*
- *Increased rise time and overshoot are the result of additional response near* ω_0
- *A resonant circuit is a lossless* ($R_S = 0$ *and* $R_L = \infty$ *in diagram) second order circuit often encountered in pulsed-power systems. Real systems have loss (and damping), but can be well approximated by resonant circuits*
- *The resonant frequency is* $f_0 = \frac{1}{2\pi\sqrt{LC}}$

- *Bode plots are a standard way to present properties of feedback systems*
- *Each pole*
  - *Corresponds to a* 6 dB/*octave* (20 dB/*decade*) *roll-off in amplitude above the pole*
  - *Corresponds to a* 90° *phase shift at high frequencies*
- *Represent reasonably accurate magnitude and phase Bode plots with*
  - *Represent magnitude reasonably accurately on a log-log plot with a straight line that has a* 6 dB/*octave kink at the pole*
  - *Represent phase on a log-linear plot with a set of three straight lines*
    - $0°$ *shift for* $f \leq f_C/10$
    - $-45°$ *shift at* $f = f_C$
    - $-90°$ *shift for* $f \geq 10 f_C$

- *Complex conjugate poles are slightly more complex*

  Far from the poles they have the same behavior as two real poles

  Far from the poles they have the same behavior as two real poles

  - $12\ \text{dB}/octave$
  - $180°$ *phase shift*

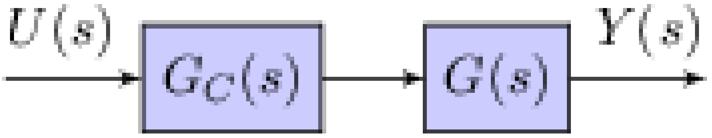
  Near the pole frequency, their behavior depends on the damping factor of the complex pole pair

- *Similar rules exist for transfer function zeros*

  $6\ \text{dB}/octave$ *increase in gain above zero*

  $+45°$ *phase shift at the zero*

- *We are working with power supplies*
  - *Systems that produce an output based on a given input*
- *Use feedback circuits to*
  - *Regulate a system, that is, keep the output fixed at a desired constant value*
  - *Reduce sensitivity to:*
    - *Internal system changes*
    - *External environmental changes*
    - *External disturbance sources*
  - *Control a system, that is, force the output to follow a variable control input*
- *This will only be a brief introduction to control theory (feedback)*
  - *Single input single output (SISO) systems (not MIMO)*
  - *Linear systems (non-linear systems are much more difficult)*
- *Control theory is a very rich subject*
  - *It is a discipline into itself*



- *Our systems have a given input-output relationship (transfer function)* $G(s)$
- *We often put a controller,* $G_C(s)$*, upstream of the system for:*
  - *Amplification*
  - *Filtering*
- *Even with* $G_C(s)$ *both of these systems are "open loop"*
  - *We command how we want the system to behave*
  - *We do not have any mechanism to force its behavior*
- *We want to improve the system behavior by using feedback to "close the loop"*

- *First some terminology.*
  - *Error signal e*
    - *The difference between the input control signal u and the processed output signal fed back into ∑, b*
  - *Forward gain (open loop gain)*
    - *The gain $A(s)$ between the input u and output y with no feedback $(b = 0)$*
  - *Loop gain*
    - *The gain $A(s)B(s)$ around the loop from e to b*
- *Now some algebra (multiplication in the frequency domain instead of convolution in the time domain)*

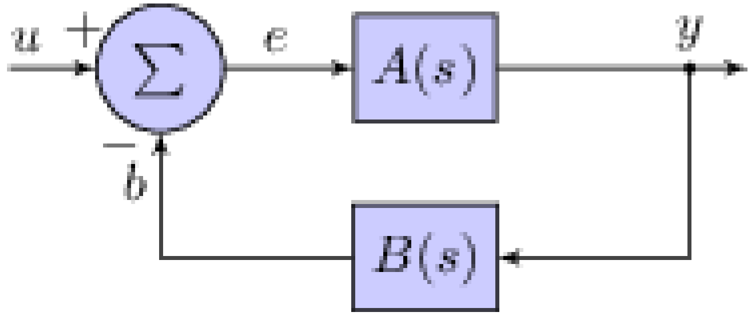
$$y(s) = A(s) \cdot e(s); \quad b(s) = B(s) \cdot y(s) = B(s)A(s) \cdot e(s);$$

$$e(s) = u(s) - b(s) = u(s) - B(s)A(s) \cdot e(s)$$

$$\big(1 + B(s)A(s)\big)e(s) = u(s)$$

$$e(s) = \big(1 + B(s)A(s)\big)^{-1} u(s) = S(s) \cdot u(s)$$

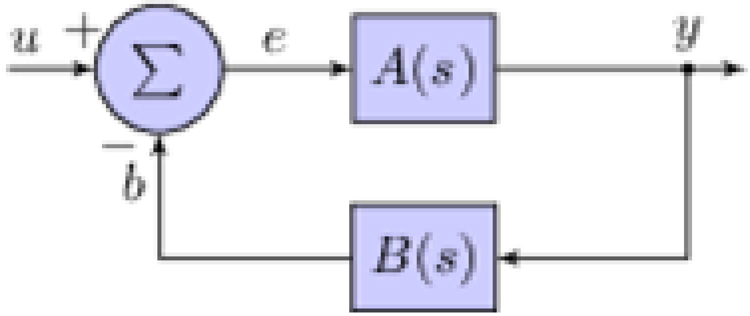
$$\frac{y(s)}{u(s)} = \big(1 + B(s)A(s)\big)^{-1} A(s) = \frac{A(s)}{1 + B(s)A(s)} = \frac{G_{FOR}}{G_{LOOP}}$$



•*The feedback system works to combat output changes by working on the error signal,* $e = u - b$ *to adjust the output*

•$S(s) = \frac{E(s)}{U(s)} = \left(1 + B(s)A(s)\right)^{-1}$ *is the sensitivity*

- •*Want* $S(s)$ *small to minimize the error* $e(s)$



•$TF(s) = \frac{y(s)}{u(s)} = \frac{A(s)}{1+B(s)A(s)}$ *is the transfer function*

- •*Want* $TF(s) = 1$ *in order to track the input*

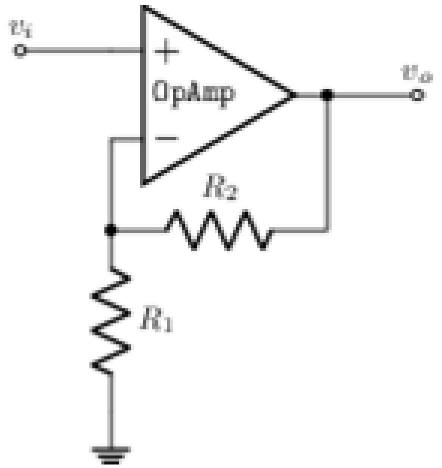
•*If the system is acting as a regulator, the setpoint,* u*, is constant*

- •*A decrease in the output* y*, for any reason, will*
  - •*Decrease* b
  - •*Increase* e
  - •*Increase* y*.*
- •*An increase in* y *will increase* b*, decrease* e*, and also* y

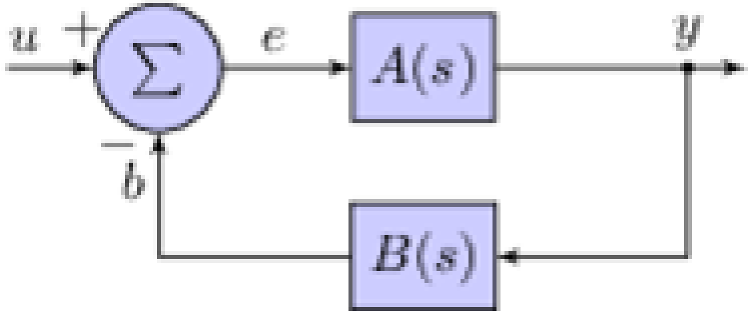
•*If the system is acting as a controller,* u *is adjusted to change the output*

- •*An increase in* u *will increase* e *and therefore increase* y
- •*The system will then regulate the output to keep* y *tracking* u

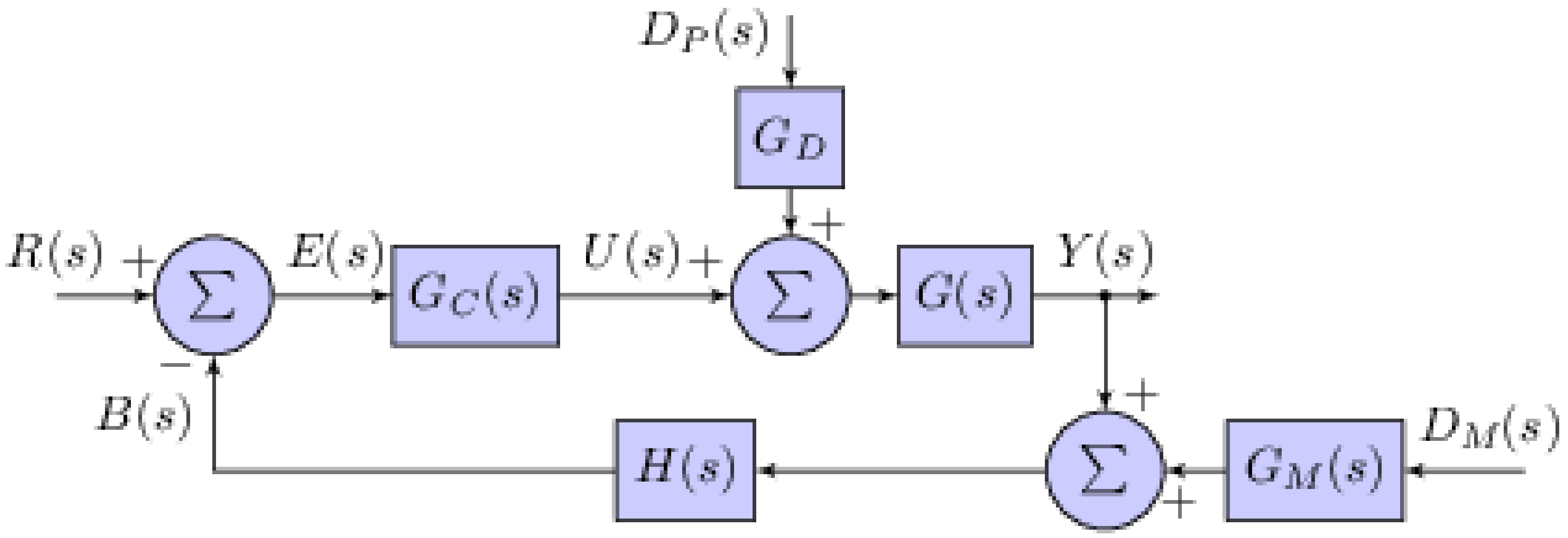
- *Non-inverting DC operational amplifier (op-amp)*
  - $A(s) = A \gg 1; \;\; B(s) = \frac{R_1}{R_1+R_2}$
  - $\frac{v_O}{v_I} = \frac{A}{1+\frac{AR_1}{R_1+R_2}} \approx \frac{A}{\frac{AR_1}{R_1+R_2}} = \frac{R_1+R_2}{R_1} = 1 + \frac{R_2}{R_1}$
  - *As long as* $A \gg 1$*, performance is independent of A*



- *Non-inverting AC operational amplifier (op-amp)*
  - $A(s) = \frac{A_{GBW}}{s+\frac{A_{GBW}}{A_{DC}}}$ *Typical values* $A_{GBW} \approx 2\pi 10^6; \;\; A_{DC} \approx 10^7$
  - $\frac{v_O}{v_I} = \frac{\frac{A_{GBW}}{s+\omega_C}}{1+\left(\frac{A_{GBW}}{s+\omega_C}\right)\left(\frac{R_1}{R_1+R_2}\right)} = \frac{A_{GBW}}{s+\omega_C+\frac{A_{GBW}R_1}{R_1+R_2}}; \quad \omega_C = \frac{A_{GBW}}{A_{DC}}$
  - $\frac{v_O}{v_I} \approx 1 + \frac{R_2}{R_1}, \omega \ll \frac{A_{GBW}R_1}{R_1+R_2}; \;\; \frac{v_O}{v_I} \approx \frac{A_{GBW}}{s}, \;\; \omega \gg \frac{A_{GBW}R_1}{R_1+R_2}$
  - *Ideal performance drops off at high frequency*
    - *Gain decreases as* ω^{-1}
    - $90°$ *phase shift*

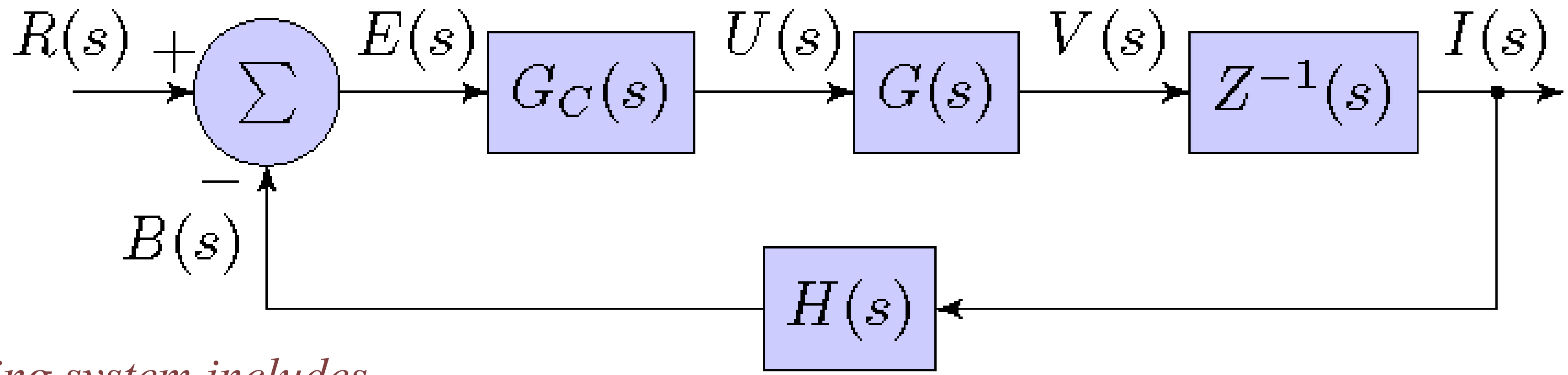


Feedback Example – General Case



- *A general feedback loop has the elements we want*
  - *The system to control $G(s)$ and the measurement system $H(s)$ with* $TF_R = \frac{Y(s)}{R(s)} = \frac{GG_C}{1+HGG_C}$
- *It also has systems that we do not want*
  - *A generalized disturbance $D_P(s)$ with amplification $G_D(s)$ leads to the* $TF_D = \frac{Y(s)}{D_P(s)} = \frac{GG_D}{1+G_CHG}$
  - *A measurement disturbance $D_M(s)$ with amplification $G_M(s)$ leads to the* $TF_M = \frac{Y(s)}{D_M(s)} = \frac{GG_CHG_M}{1+GG_CH}$
- *We want to optimize what we want, TF_R, and minimize what we don't want, TF_D and TF_M*

Feedback – Typical Power Supply Application



Our existing system includes

the power supply $G(s)$

the magnet load and resistance $Z(s) = sL + R$

To this we add

the controller $G_C(s)$

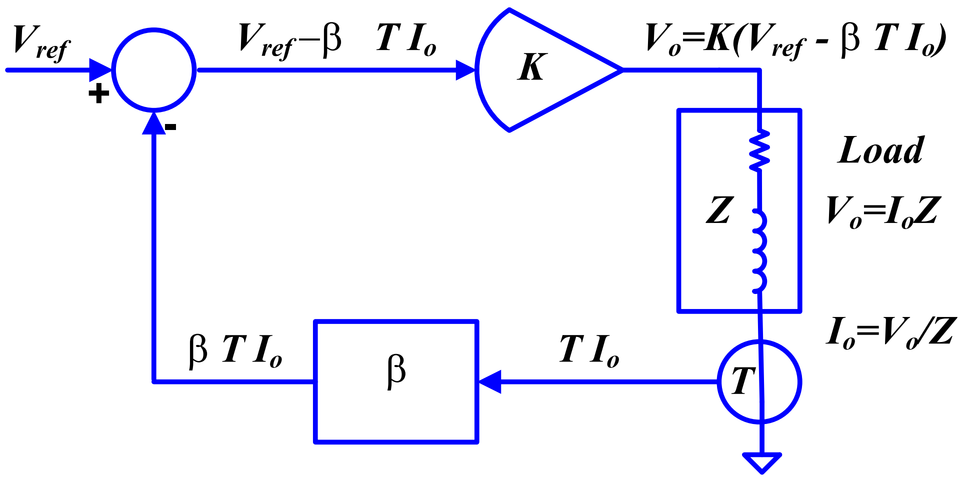
the current transducer and associated electronics $H(s)$

The closed loop transfer function is

$$A_{CL} = \frac{Z^{-1}(s)G(s)G_C(s)}{1 + H(s)Z^{-1}(s)G(s)G_C(s)} \approx \frac{1}{H(s)}$$

when $H(s)Z^{-1}(s)G(s)G_C(s) \gg 1$

(The noise and distortion previously discussed is still there and, in practice, we must address it. But we will largely ignore these terms in our further discussions)



$$V_o = K(V_{ref} - \beta\ T I_o)$$

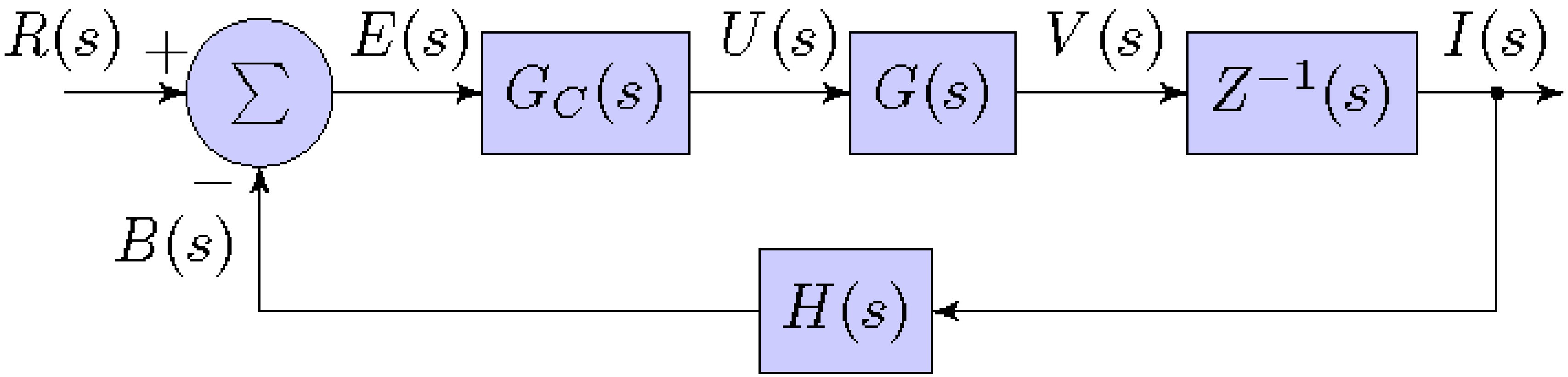
$$I_o Z = K(V_{ref} - \beta\ T I_o)\ \text{rearranging gives}\ \frac{I_o}{V_{ref}} = A_{CL} = \frac{K/Z}{1 + \beta\ T K/Z}$$

- *A_{CL} is called the closed loop gain*
- *For $\beta\ T K/Z >> 1$* $\quad A_{CL} = \dfrac{1}{\beta\ T}$
- *Power amplifier and load characteristics (K, Z) relatively unimportant, gain and stability dependent upon feedback loop $\beta\, T$*

Three Types of Stability

- *Stability against short and long-term output voltage or current drift*
- *Stability (Regulation) against rapid, short changes in line voltage or load characteristics*
- *Stability against oscillations (and large transients)*

Factors Affecting Power Supply Drift Stability



Short-Term (24 hour) Stability - *essentially stability against cyclic or diurnal temperature changes.*

$$A_{CL} = \frac{Z^{-1}(s)G(s)G_C(s)}{1 + H(s)Z^{-1}(s)G(s)G_C(s)} \approx \frac{1}{H(s)}$$

- *Since* $H(s)Z^{-1}(s)G(s)G_C(s) \gg 1$, *the details of the forward gain,* $Z^{-1}(s)G(s)G_C(s)$, *are unimportant*
- *The stability primarily depends on the stability of the*
  - *Voltage reference* $R(s)$
  - *Current transducer and associated electronics* $H(s)$
  - *Error amplifier* $\sum$
- *These are required to be precision components and are often temperature stabilized*

Factors Affecting Short-Term (24 hour) Power Supply Drift Stability

- *The diurnal temperature cycle for the power supply system can be as much as* 40° F (22° C). *(Not all systems can be in temperature controlled environments.)*

- *All parts (resistors, capacitors, semiconductors, op-amps, etc) are temperature dependent. It is important to concentrate on those that make up the sensitive parts of the feedback loop.*

- *The variation of the sensitive components must be characterized over this temperature range.*

- *The load is also temperature dependent and is subject to the same diurnal changes, but this variation is less important if the current measurement is precise*

- *The input line voltage will change during the course of the day as more premises load is consumed or shed. This variation should also be characterized.*

General Techniques

- *Either use low-temperature coefficient (tempco) parts or balance parts with positive tempcos with those that have negative tempcos.*

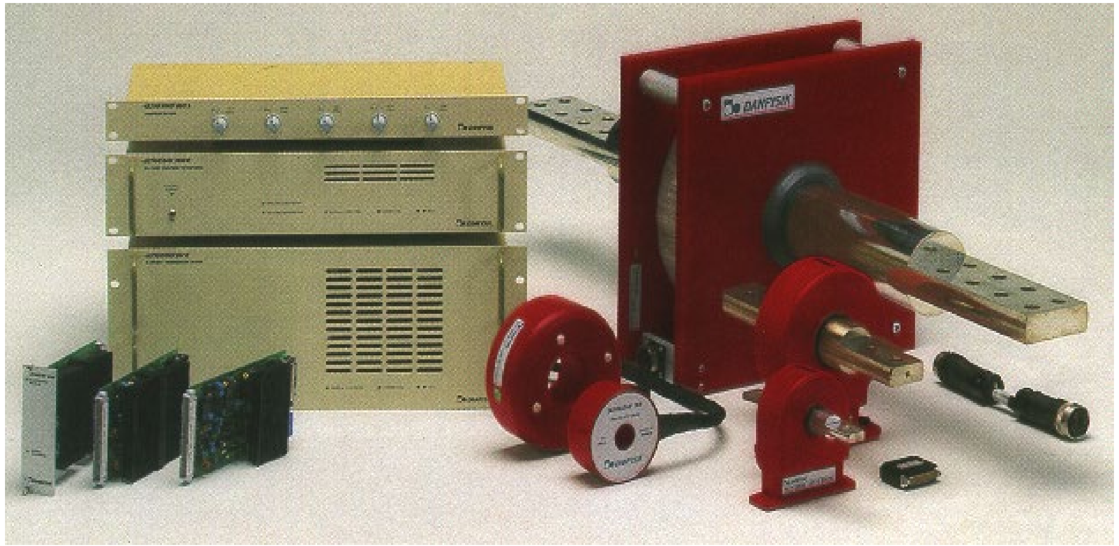
- *Enclose the power supply in a controlled environment where temperature change is held to a minimum*

It is possible to attain 10 to 50 ppm w/o temperature control and 5 to 10 ppm with temperature control

For the read-back monitoring signal used for the feedback loop, use:

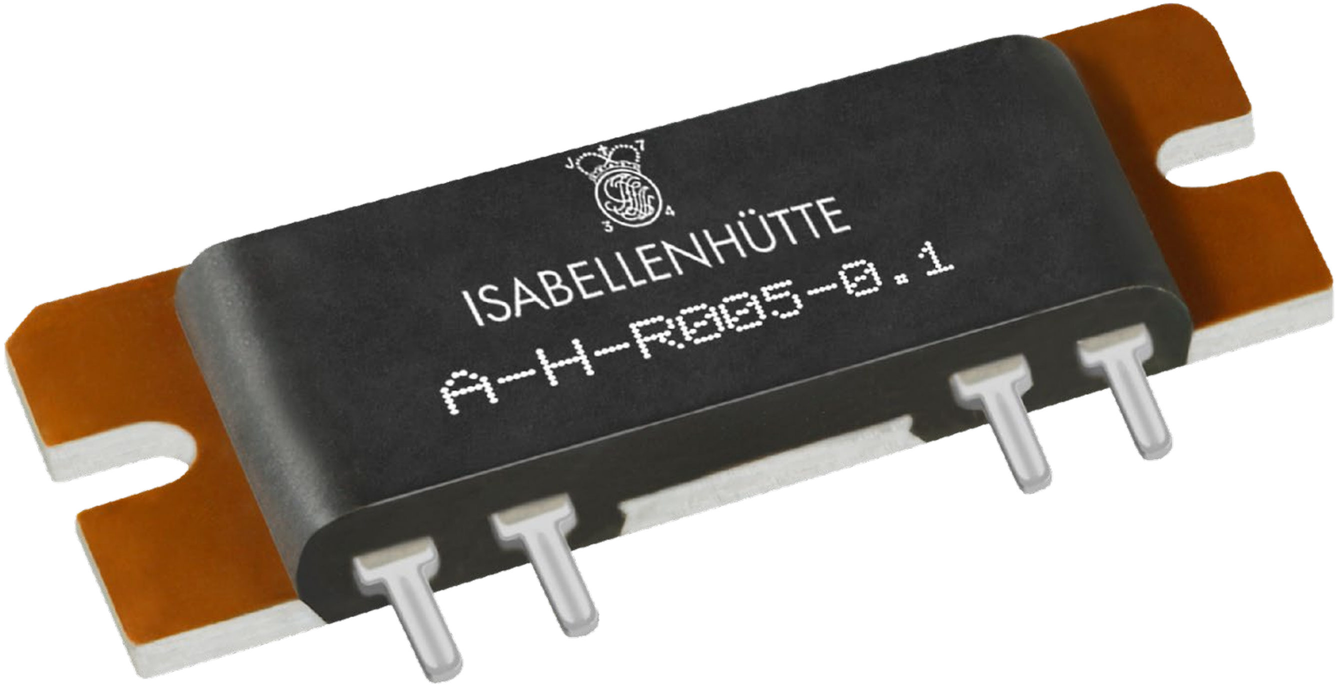
- *Precision, low-temperature coefficient current transductors* $(0.3\ \mathrm{ppm}/°C)$ *with metal film burden resistor* $(0.9\ ppm/°C)$ *to give a total variation of* $(1.2\ ppm/°C)$
- *Precision, low-temperature coefficient resistors for current shunt or voltage read-back give a total variation of about* $10\ \mathrm{ppm}/°C$

LEM (acquired Danfysik)
Model 866
0 - ±600 A
±400 mA out
0.3 ppm / ^{O} C
DC – 100 kHz
10 kA / mS
Separate burden resistor
Danisense also a vendor



LEM (acquired Danfysik) Model 860 Series
0 - ±1000 A, ±2000 A, ±3000 A
±10 V out
0.3 ppm / ^{O} C
DC – 100 kHz
10 kA / mS
Danisense also a vendor

Stability - Isabellenhutte Model A-H, Manganin (Copper-Manesium-Nickel) < 10 ppm/ ^{O}C Shunt Resistor



https://isabellenhuetteusa.com/

Long-Term Stability

- *All parts are subject to aging.*
- *Resistors increase or decrease in value*
- *Capacitor dielectrics breakdown*
- *Capacitor electrolytes dry out or evaporate and leak*
- *Semiconductor bias points change*
- *Op-amp scale, linearity, monotonicity, gain and offsets change with time*

Stability Enhancement

- *Aging follows a "bathtub" curve*
  - *Rapid changes occur at the beginning of use*
  - *Moderate changes for a long time after that*
  - *Rapid changes again before failure*
- *Accelerate initial aging components prior to intended use by baking at elevated temperatures*
- *Accelerate aging by exposure to electron beam*

- *Two types of Regulation – Load and Line*
- *Classic definition of Load Regulation (output changes with the size of the load) (0% is best)*

$$\%V_R = \frac{V_{NL} - V_{FL}}{V_{FL}} 100\% \qquad \%I_R = \frac{I_{NL} - I_{FL}}{I_{FL}} 100\%$$

- *Classic definition employing V_{NL} is usually not applicable. A limited version uses "decreased load or increased load" instead of a no-load condition*

$$\%V_R = \frac{V_{DL} - V_{FL}}{V_{FL}} 100\% \qquad \%I_R = \frac{I_{DL} - I_{FL}}{I_{FL}} 100\%$$

- *In addition, the recovery time for the power supply output voltage or current to return the original condition is also specified*

"The power supply shall have a voltage regulation of 0.5% for load changes of ± 5% from nominal with voltage recovery in ≤ 2 milliseconds"

Line Regulation – Definition (HL= output voltage under high line, NL= output voltage under nominal line, LL = output voltage under low line)

$$\%V_R = \frac{V_{HL} - V_{NL}}{V_{NL}} 100\% \qquad \%I_R = \frac{I_{HL} - I_{NL}}{I_{NL}} 100\%$$

$$\%V_R = \frac{V_{NL} - V_{LL}}{V_{NL}} 100\% \qquad \%I_R = \frac{I_{NL} - I_{LL}}{I_{NL}} 100\%$$

In addition, the recovery time for the power supply output voltage or current to return the original condition is also specified

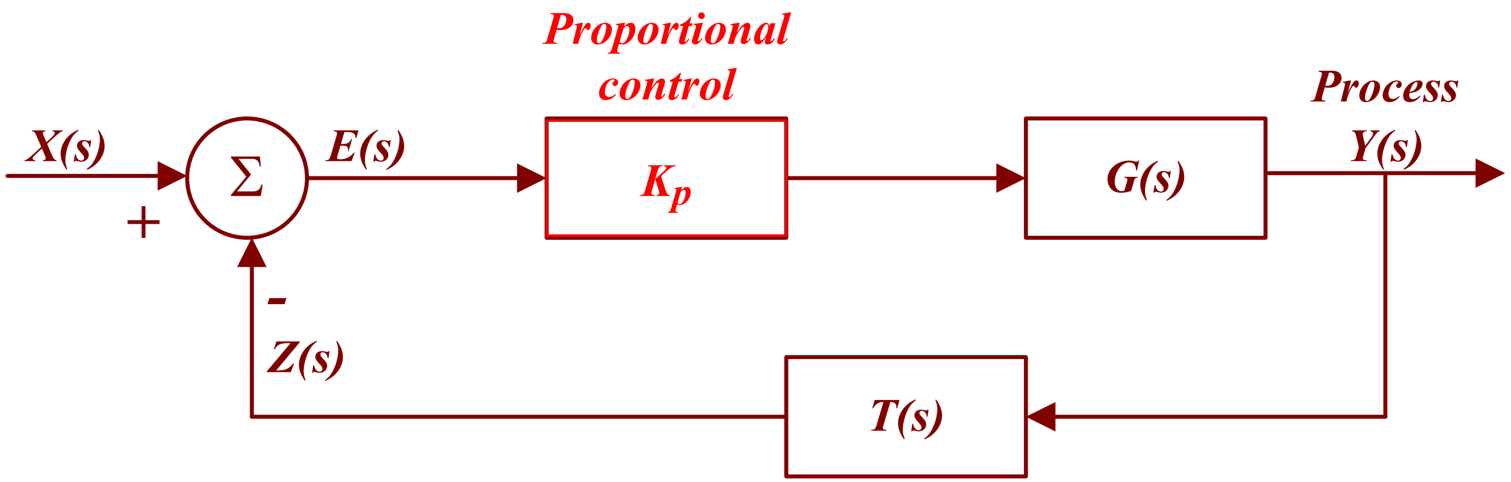
"The power supply shall have a voltage/current regulation of 0.5% for line changes of ± 5% from nominal with voltage/current recovery in ≤ 2 mS"

Factors that Affect Stability (Regulation) Against Transient Effects

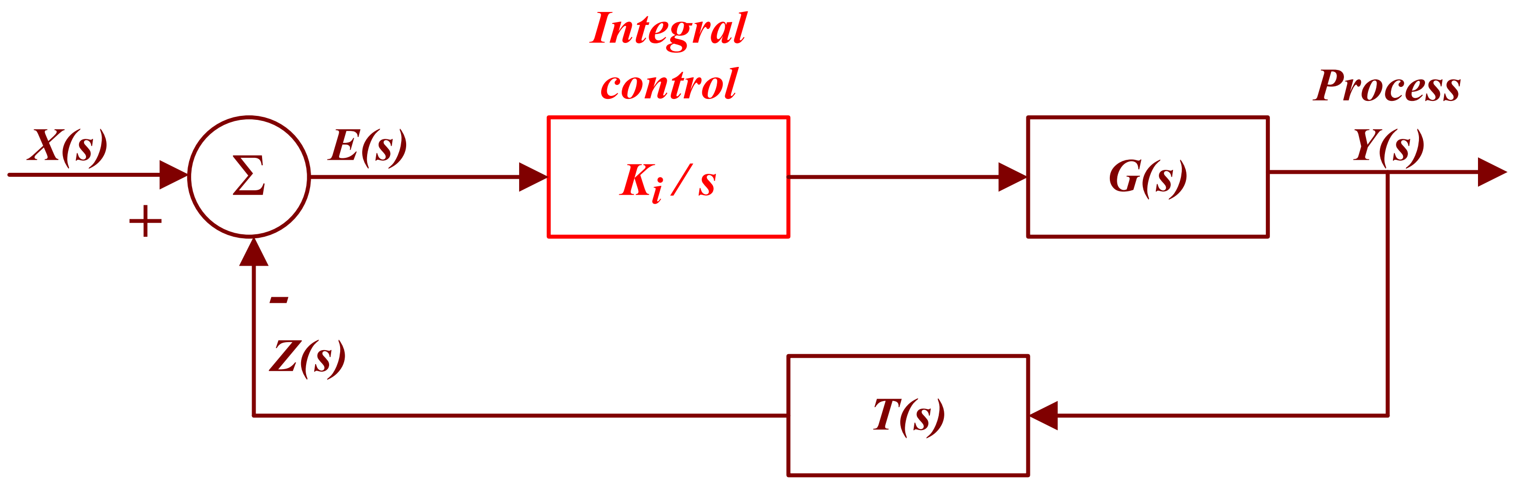
The ability of a power supply to respond to a transient condition depends upon the speed, depth and duration of the transient. The transient can be mitigated by the use of:

- *Large filter capacitors and inductors in the input and output filters to maintain the input and output load voltage and current against line voltage changes and load changes..*
- *Employ fast regulating circuits. Regulating speed should be at least as fast as the fastest expected transient.*

- *Earliest controllers proportional only*
- *Proportional control consists of just a gain*
- *It has good response to instantaneous changes in the process or other cause of error*
- *Control effort is the product of the error and a finite gain Kp*
- *Eventually effort is too small to reduce error to zero*
- *There is always an error - it can never be eliminated*



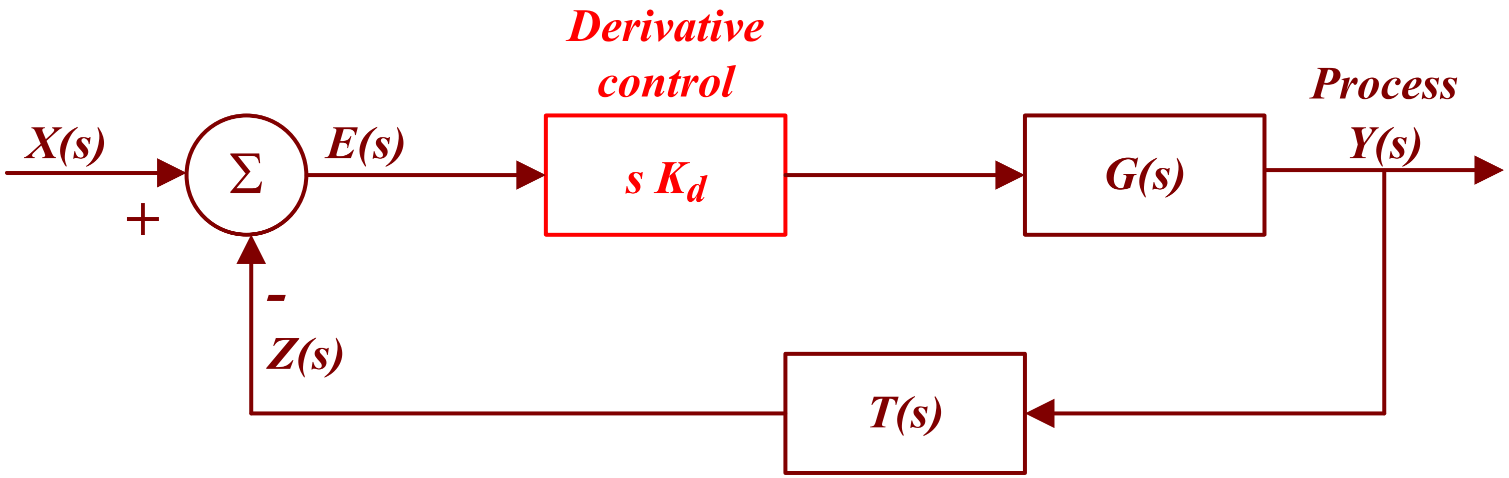
- *Integral control consists of a pure integrator*
- *The control effort is now* $\frac{K_i}{s}E(s) \Rightarrow K_i \int^t e(\tau)d\tau$
- *The main advantage is that it eliminates* DC *errors*
    - *The integrator output can be non-zero when the instantaneous error* $e(t)$ *is zero*
- *There are disadvantages to the use of the integrator*
    - *It limits the high frequency system response*
    - *It introduces a phase delay that slows the response and can lead to an oscillation*



- *Responds to the derivative (change) of the error signal*
- *Control effort increases with frequency of error signal*

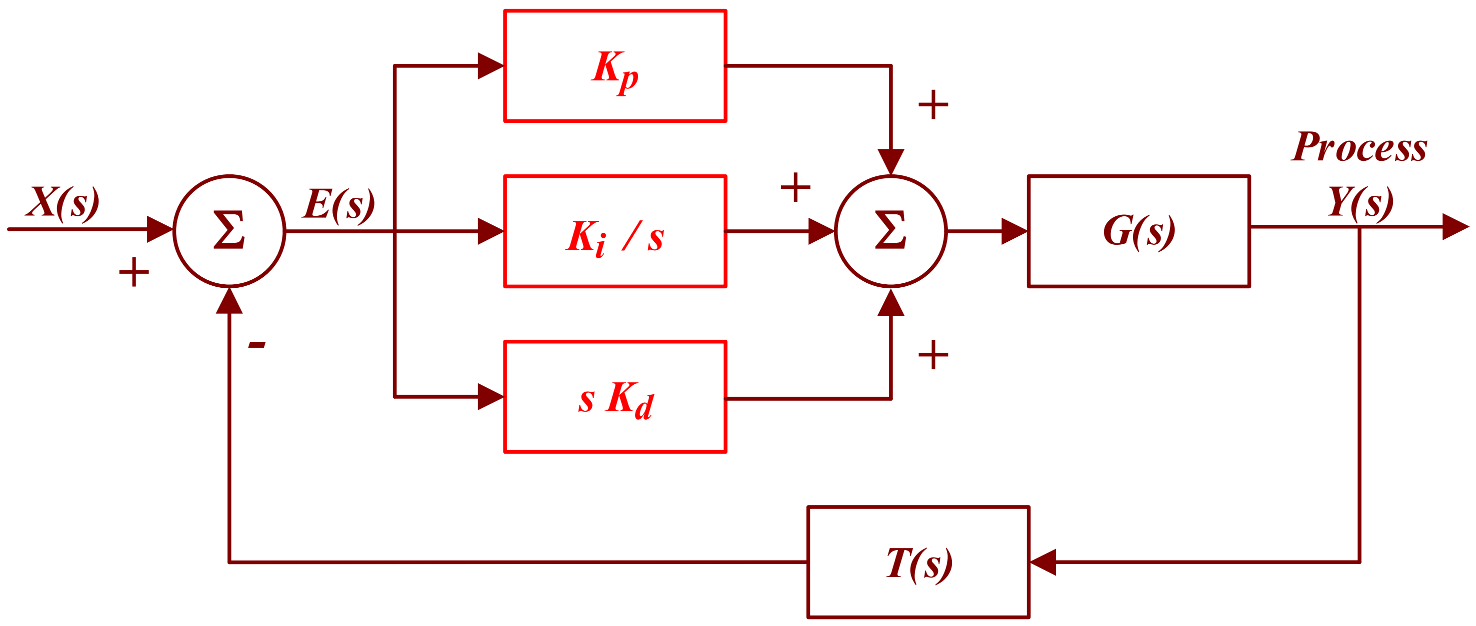
$$sK_d E(s) \Rightarrow K_d \frac{de}{dt}$$

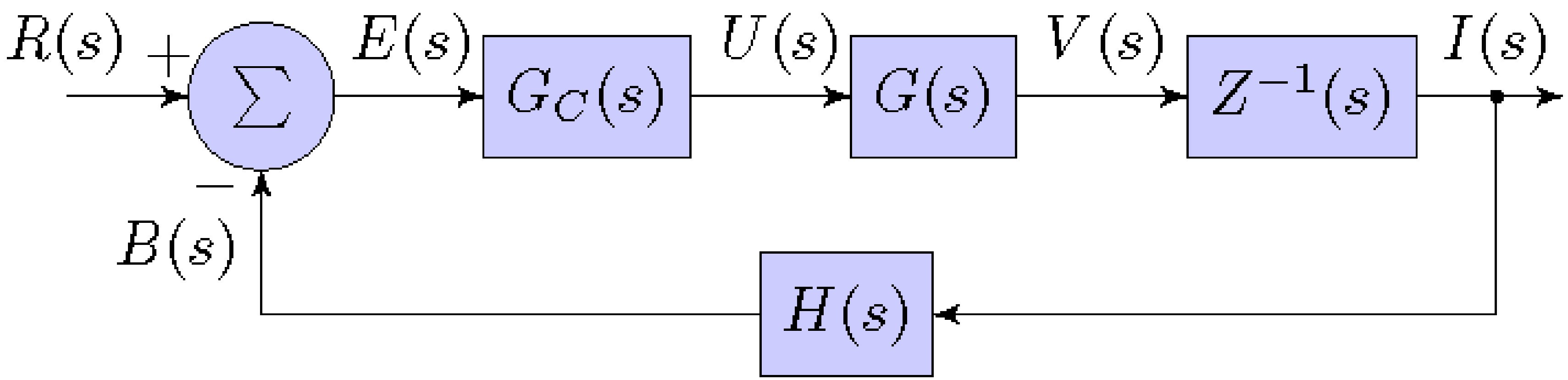
- *Useful either to cancel a pole or to "feedforward" on known future signals*
- *Amplifies high frequency noise and can lead to instability*



PID Loops - Summary

- *PID stands for Proportional, Integral, and Derivative control*
- *Standard, general purpose classical control element*
- *K_p general cancelling of error signals*
- *K_i eliminates DC error*
- *K_d provides nimble circuit for fast changes in the error signal or process*

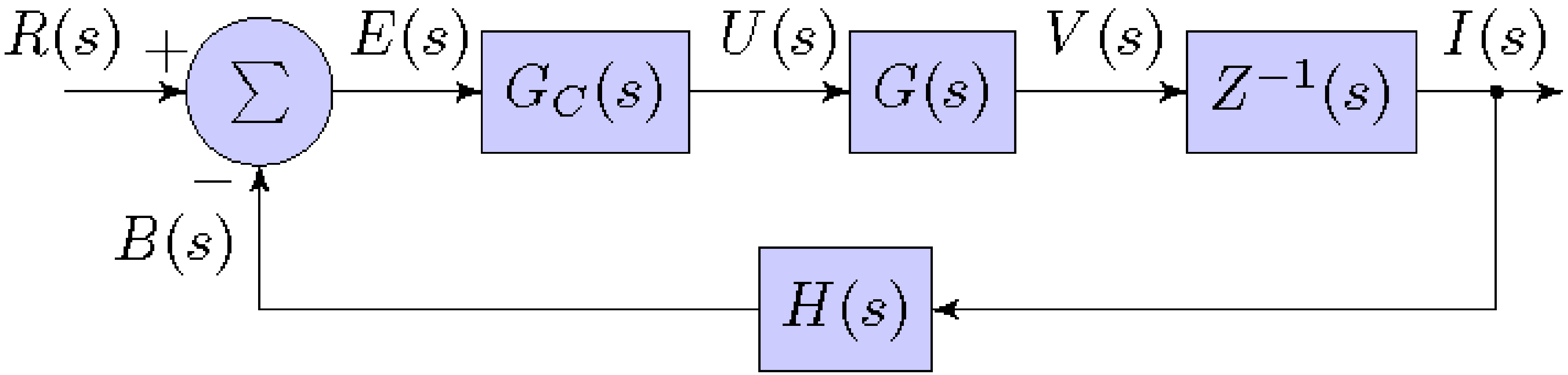




- *We saw that, with just proportional gain, we could decrease the error between set point and readback by further increasing the controller gain.*
- *Why then do we not just increase the gain until the error is at an acceptably small value?*
- *One reason is that increasing the gain would also increase the contributions of the disturbance and measurement error inputs.*
- *However, a more fundamental reason is that increasing the gain, for any but the simplest system, makes the output responses to changes unacceptable, at best, and, more likely, cause oscillations.*
- *The transfer function is*

$$TF = \frac{Z^{-1}(s)G(s)G_C(s)}{1 + H(s)Z^{-1}(s)G(s)G_C(s)}$$

and becomes infinite at all poles of the denominator, that is, when $|H(s)Z^{-1}(s)G(s)G_C(s)| = 1$ *and* $\angle\left(H(s)Z^{-1}(s)G(s)G_C(s)\right) = 180°$



But even if we have a relatively simple two pole system in which

$Z^{-1}(s)G(s)G_C(s) = K\frac{a}{s+a}\frac{b}{s+b}$ *and* $H(s) = 1$ *then*

$$\frac{I(s)}{R(s)} = \frac{Kab}{s^2 + (a+b)s + (K+1)ab}$$

Even though the denominator will never vanish, a large value of K will push its poles so close to the real axis that the system will ring with every change of input or readback, leading to unacceptable performance.

More complicated systems will have poles that cross the imaginary axis as the gain is increased

One way to look at the mechanism of instabilities and oscillations is to consider a simple pendulum.

(This is a non-linear oscillator that can be considered harmonic for small motions.)

An ideal feedback control on the pendulum would sense the motion and command an actuator to apply the correct magnitude force opposite to that motion as the pendulum is passing through its lowest point

However it takes some time to sense the motion, determine the correct magnitude of force to apply, and then have the actuator apply that force. We call that response time τ.

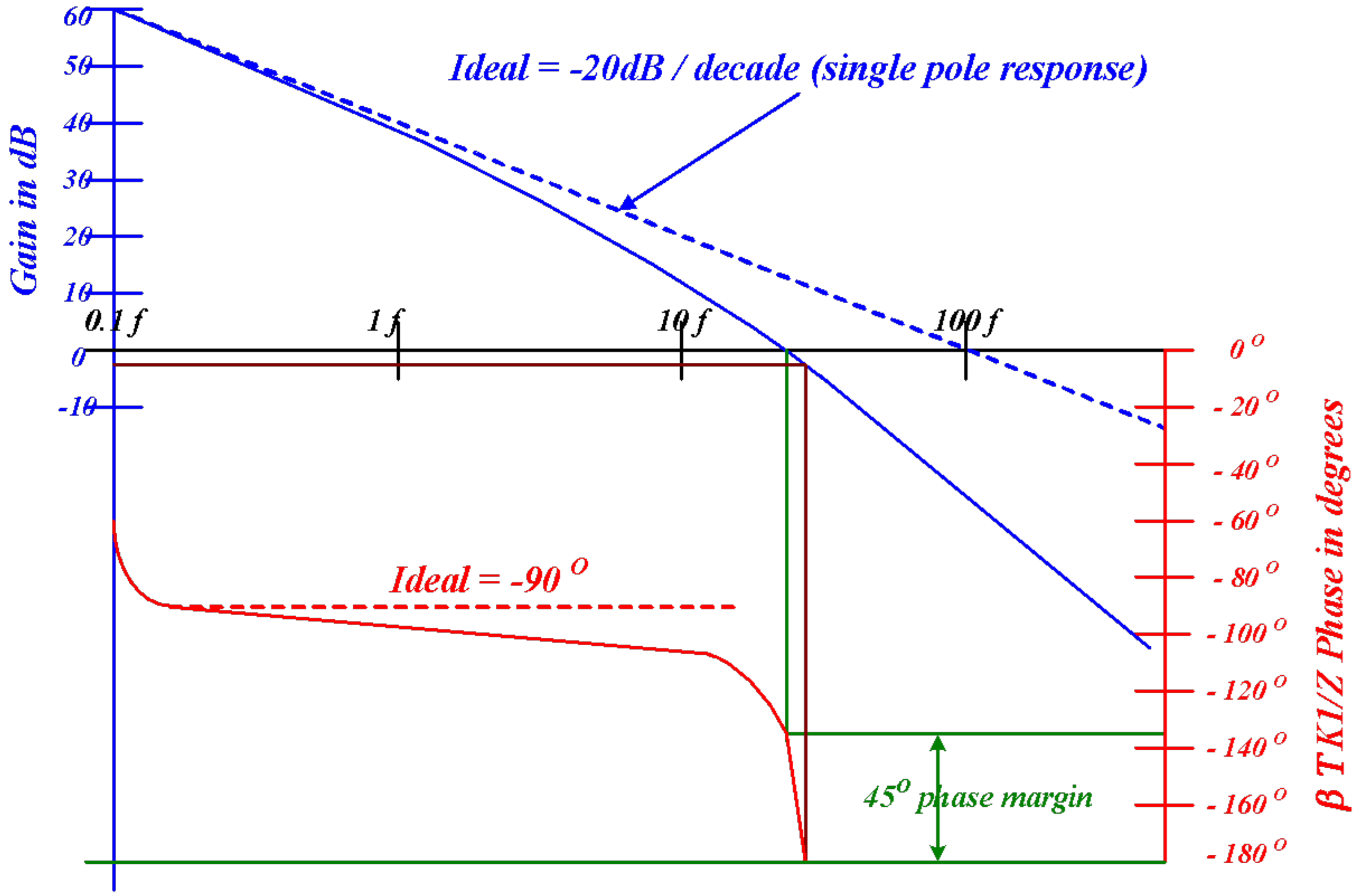
If τ *is short compared to the period of the pendulum,* T, *the feedback works well.*

As the frequency of oscillation increases, T *decreases down to a time when* τ *is comparable to* T.

In fact, when $\tau = T/2$, *instead of the system damping the motion through negative feedback, it amplifies the oscillation through positive feedback.*

Each pole in the system adds delay so we must ensure that the system gain is sufficiently weak at the frequency at which the feedback control would provide positive feedback.

One can determine the stability of a system by defining gain margins and phase margins and using one of several visual graphical methods, such as a Bode plot, root-locus plot, etc., to determine these margins.



- *For stability, the phase shift must be < 180 ^{O} when the |gain| =1*
- *For stability, the |gain| must be < 1 when the phase shift is 180 ^{O}*

- *The study of feedback systems is known as control theory.*
- *"Classical" control theory started in the 1920s and continued through the early 1950s*
  - *Its motivation was for the amplification of communication signals (telephone and radio signals)*
  - *No computers existed so linear systems theory, Laplace transforms, complex analysis, etc., were studied to investigate stability and determine circuit designs that achieved the desired control*
  - *Famous names were Bode, Nyquist, Routh, Hurwitz, Nichols, Ziegler, …*
  - *Work accelerated in WWII to achieve control of aircraft and simple rockets.*
  - *Techniques were developed to design compensators, e.g. PID loops, to obtain the desired control*
- *"Modern" control theory began in the 1950s*
  - *The "space race" required much more sophisticated control system for modern aircraft and rockets.*
  - *Computers were becoming available for numerical calculations*
  - *Analysis using matrix linear algebra extended the Laplace techniques to higher dimensional systems*
  - *Using these techniques, modern control theory can design feedback controllers in a more straight-forward matter.*

Feedback Design Procedure

- *Characterize the system*
  - *Model the system elements*
  - *Write the system equations that define the dynamics (KVL and KCL)*
- *Define the desired performance*
  - *DC stability*
  - *AC response time and acceptable overshoot*
- *Determine the available controls*
  - *Available voltage and current capabilities and limits*
- *Determine the available monitors*
  - *Monitor as many of the important (state) variables as possible*
- *Include other constraints on the system*
  - *Optimize performance with technical limits on voltage, current, power, etc.*
- *Model the performance of the system with controller*
- *Prototype, test, and iterate*

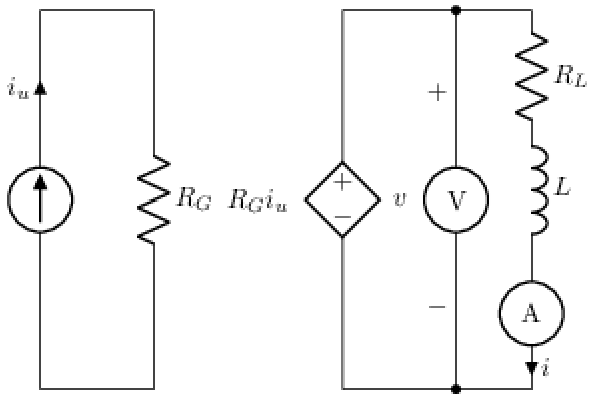
State Feedback - Introduction

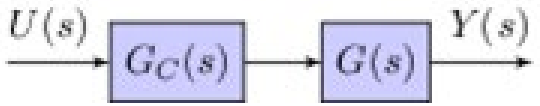
- *We have been discussing "classical" control theory*
  - *Insert control elements in the forward path to compensate for the plant dynamics*
- *Now we will have a brief introduction to "modern" control theory*
  - *Identify the states of the system*
    - *One state exists for each order of differential equation that describes the plant*
    - *Typically one state for each reactive component:* $di_L/dt = v_L/L$; $dv_C/dt = i_C/C$
    - *States are not unique; only the number of states are*
      - *Sometimes it is convenient to use the "natural" states of voltages and currents*
      - *Other times, for large circuits, it is more convenient to transform to an equivalent set*
  - *Feedback on each state, with appropriate weighting, in order to compensate for the plant dynamics*
  - *System states need to be "observable" and "controllable" (Ours usually are.)*
- *We will work through some simple examples*
  - *Simple power supply*
  - *Damping of a resonant circuit*
    - *Compare with Praeg filter damping*

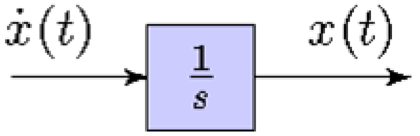
- *We want to control a power supply that drives a magnet*
  - *The power supply is a controlled linear amplifier*
    - *We assume its switching frequency is much greater than any other frequency of interest*
  - *The load is a series magnet and its associated resistance*
    - *The load has a natural frequency* $\omega_L = R/L \quad (\tau_L = L/R)$
- *Our goal is to design a stable system that has a response time* $\tau \ll \tau_L$
- *We will add a controller,* $G_C(s)$ *that is just an integrator, to give us a controller with no DC error*
  - *An inverting integrator is easily synthesized with an opamp*

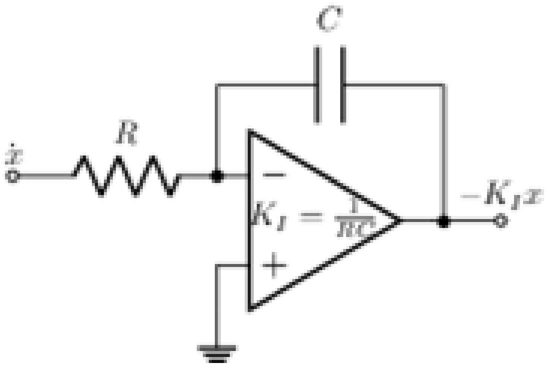
$$x_{out} = -\frac{1}{sCR}x_{in} = -\frac{1}{RC}\int^{t} x_{in}(t')\,dt'$$

- *Our goal is to construct the desired system using just linear gain stages and summing junctions acting on easily accessible measurements*
  - *The output voltage of the supply*
  - *The output current of the supply (the magnet current)*









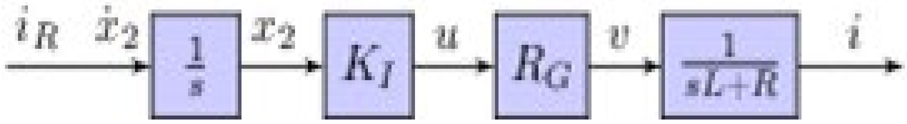
State Feedback – Overview

- *We start with the open loop transfer function of our system*

$$\frac{I(s)}{I_R(s)} = \frac{K_I R_G}{s(sL+R)} = \frac{K_I R_G/L}{s(s+R/L)} = \frac{K_I R_G/L}{s^2+(R/L)s}$$

- *This corresponds to a second order differential equation*

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} = \frac{K_I R_G}{L} i_R$$



- *We know how to use Laplace transforms to solve first order equations*

$$\frac{di}{dt} = -\frac{R}{L} i + \frac{1}{L} i_R \Rightarrow (s+R/L)I(s) = i(0) + (1/L)I_R(s)$$

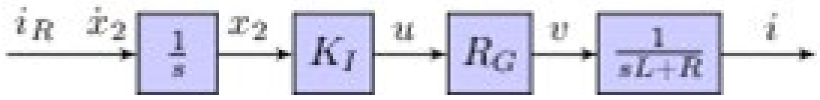
$$I(s) = (s+R/L)^{-1} i(0) + (s+R/L)^{-1}(1/L)I_R(s)$$

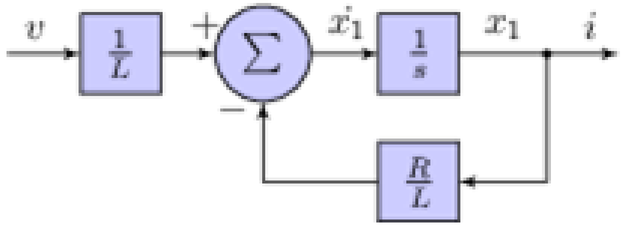
- *Then we invert the transform to get, for example, for a step function* $I_0 \frac{1}{s}$

$$i(t) = e^{-\frac{R}{L}t} i(0) + \left(1 - e^{-\frac{R}{L}t}\right) I_0$$

- *It turns out that this same mathematics works for linear systems if , instead of working with scalars, we work with vectors and matrices*
- *It is beyond the scope of the course to prove or derive this, but, having motivated our intention, we will use these mathematics.*

- *Our state space system is a second order differential equation (diffeq)*
  - *Our system has two states*
- *State space feedback works only on first order systems*
  - *We rewrite our single second order diffeq as two coupled first order diffeqs*
    - *We preserve the number of states*
    - *Since we operate on first order diffeqs*
      - *We can apply matrix algebra to our known techniques*





- *Define two state variables, x_1 and x_2 and translate the second order diffeq*

$$x_1 = i; \ \ \dot{x}_1 = -(R/L)x_1 + (1/L)v = -(R/L)x_1 + (K_I R_G/L)x_2$$
$$\dot{x}_2 = i_R$$

- *Create a column vector $\boldsymbol{x} = (x_1 \ \ x_2)^T$, where the superscript T denotes a transpose and write as a matrix*

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -R/L & K_I R_G/L \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} i_R \Rightarrow \dot{\boldsymbol{x}} = \boldsymbol{A}_{\boldsymbol{x}} \boldsymbol{x} + \boldsymbol{B}_{\boldsymbol{x}} i_R$$

- *It is easier and better to use i and v as state variables rather than x_1 and x_2; same system, same dynamics*

$$i = x_1; \quad v = K_I R_G x_2$$
$$\begin{pmatrix} \dot{i} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} -R/L & 1/L \\ 0 & 0 \end{pmatrix} \begin{pmatrix} i \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ K_I R_G \end{pmatrix} i_R \Rightarrow \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B} i_R$$

- *The solution we found for a scalar first order equation*

$$I(s) = (s + R/L)^{-1} i(0) + (s + R/L)^{-1}(1/L) I_R(s)$$

now becomes, in matrix notation

$$\boldsymbol{X}(s) = (s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{x}(0) + (s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} I_R(s)$$

where $(s\boldsymbol{I} - \boldsymbol{A})^{-1}$ *is the matrix inverse of* $(s\boldsymbol{I} - \boldsymbol{A})$ *such that* $(s\boldsymbol{I} - \boldsymbol{A})^{-1}(s\boldsymbol{I} - \boldsymbol{A}) = \boldsymbol{I}$

- *The output* $Y(s)$ *(when* $x(0) = 0$*) is a linear combination of the vector* $\boldsymbol{X}(s)$ *and the input* $I_R(s)$

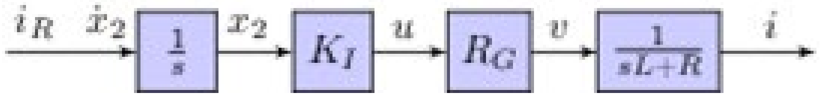
$$Y(s) = \boldsymbol{C}\boldsymbol{X}(s) + \boldsymbol{D} I_R(s) = (\boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}) I_R(s)$$

- *In order for* $\boldsymbol{A}^{-1}$ *to exist,* $\boldsymbol{A}$ *must be a square matrix and have a non-vanishing determinant*
  - *The general definition is* $\boldsymbol{A}^{-1} = \text{Adjoint}(\boldsymbol{A})/\det(\boldsymbol{A})$
  - *For a two-dimensional array* $\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ *(all we will need),* $\boldsymbol{A}^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$
- *Our open loop transfer function, with supply, load, and controller, is, for*

$$s\boldsymbol{I} - \boldsymbol{A} = \begin{pmatrix} s + R/L & -1/L \\ 0 & s \end{pmatrix}; \quad \boldsymbol{B} = \begin{pmatrix} 0 \\ K_I R_G \end{pmatrix}; \quad \boldsymbol{C} = (1 \quad 0); \quad \boldsymbol{D} = 0$$

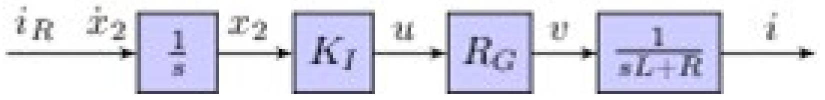
$$G(s) = \frac{1}{s(s + R/L)} (1 \quad 0) \begin{pmatrix} s & -1/L \\ 0 & s + R/L \end{pmatrix} \begin{pmatrix} 0 \\ K_I R_G \end{pmatrix} = \frac{K_I R_G / L}{s(s + R/L)}$$

which is what we get by inspection.

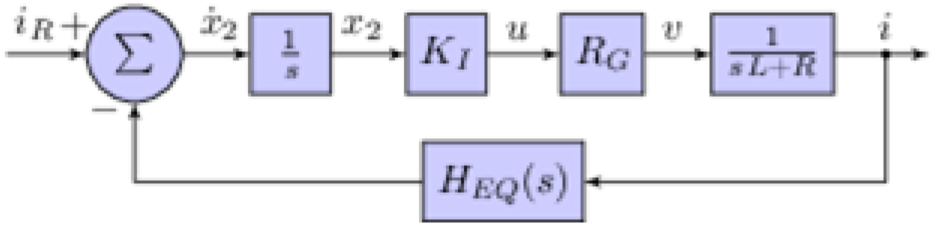


State Feedback – Advantages

- *We have just spent almost two slides deriving what we could read off a diagram*
- *The advantages of state space are much clearer for more complicated systems*
  - *Construction of the system equations builds quickly from equations for smaller subsets*
  - *Well known matrix techniques established to calculate system responses*
  - *Many software tools available, both commercial (MATLAB) and free (LAPACK, Scilab, Octave)*
  - *More straightforward and algorithmic to design controllers*
- *Our controller design using state space techniques is*
  - *Describe existing system. Find matrices* $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$
  - *Characterize the dynamics of the system*
    - *Find system poles from solving* $\det(s\boldsymbol{I} - \boldsymbol{A}) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = 0$
  - *Choose the poles that give the desired system performance* $s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0 = 0$
  - *Use algorithmic techniques to determine the required gains from each state to achieve the desired* α_k
  - *Amplify and add the outputs of the states to implement the controller*
- *Our example is a second order system, completely characterized by* ω_0 *and* ζ
  - *We know everything there is to know about the time and frequency responses of this linear system*
  - *We just have to choose the values we want for* ω_0 *and* ζ



- *We start with our open loop system*
- *We want to create a closed loop system with the desired performance*
- *Assume some reasonable values*
  - $L = 10$ mH; $R = 160$ m$\Omega \Rightarrow \tau_n = 0.0625$; $\omega_n = 16$
  - *Our goal is to extend the response to* $\omega_0 = 2\pi 10^3$



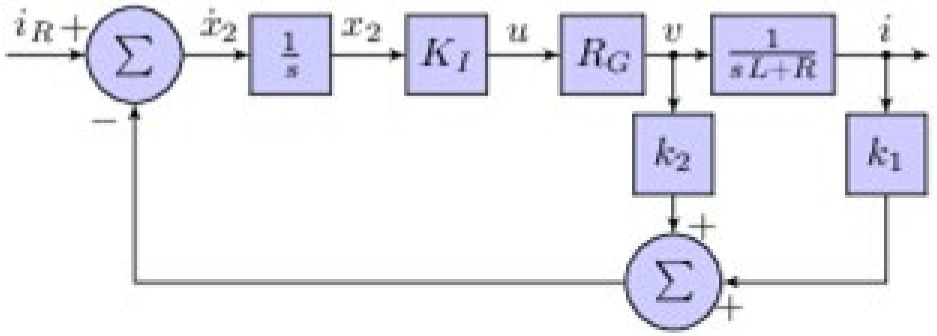
- *The transfer function is*

$$\frac{I(s)}{I_R(s)} = \frac{K_I R_G/L}{s^2 + (R/L)s + (K_I R_G/L)H_{EQ}(s)}$$

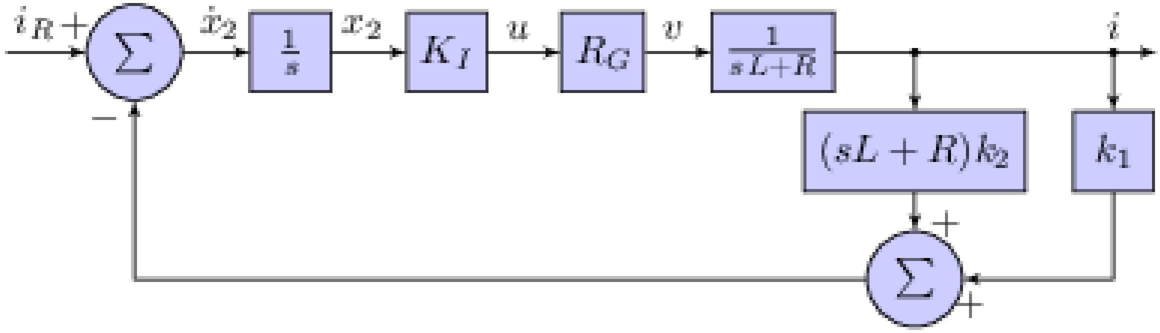
- *We want the transfer function to be, choosing* $\zeta = \sqrt{2}/2$, *the standard Butterworth response,*

$$\frac{I(s)}{I_R(s)} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

- $\omega_0 = 2\pi 10^3 \Rightarrow K_I R_G \approx 4 \times 10^5$ *We need the very large system gain to increase the frequency response*
- *Since our desired* $2\zeta\omega_0 \approx 4443 \gg R/L = 16$, *if we feed back only on the current*
  - *We need to add a differentiator to* $H_{EQ}(s)$
  - *But differentiators amplify high frequency noise, an undesirable feature*



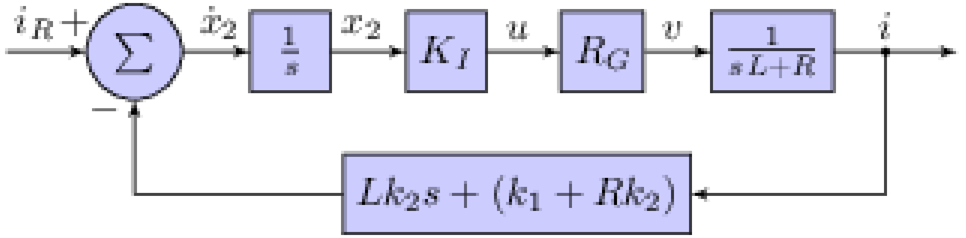
- *But the voltage is related to the derivative of the current*
  - *By appropriately feeding back on the voltage*
    - *We can feed back on the derivative of the current without amplifying noise*
- *There are well-defined matrix techniques to find k_1 and k_2*
  - *Select k_1 and k_2 to place the roots of $\det(s\boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{BK})) = 0$ where desired, where $\boldsymbol{K} = (k_1 \quad k_2)$*



- *We can solve this with simpler algebra*
- *If, to calculate, we push the k_2 gain to the output*
  - *It gets multiplied by $1/(sL+R)$*
  - *To compensate, we need to multiply by $sL+R$*
- *Now $H_{EQ}(s) = Lk_2 s + (k_1 + Rk_2)$ and*

$$\frac{I(s)}{I_R(s)} = \frac{G(s)}{1+G(s)H_{EQ}(s)} = \frac{\frac{K_I R_G/L}{s(s+R/L)}}{1+\frac{K_I R_G/L}{s(s+R/L)}(Lk_2 s+(k_1+Rk_2))}$$

$$\frac{I(s)}{I_R(s)} = \frac{K_I R_G/L}{s^2+\left(\frac{R}{L}+K_I R_G k_2\right)s+\frac{K_I R_G}{L}(k_1+Rk_2)}$$

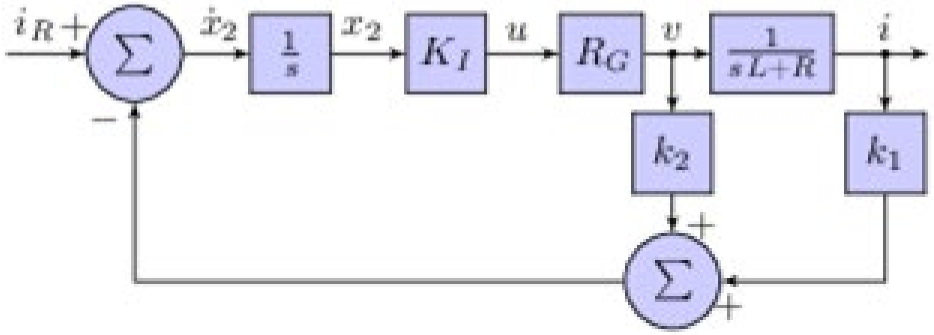


$$\frac{I(s)}{I_R(s)} = \frac{K_I R_G/L}{s^2 + \left(\frac{R}{L} + K_I R_G k_2\right)s + \frac{K_I R_G}{L}(k_1 + R k_2)} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

• *We already know that we select* $K_I R_G$ *such that* $K_I R_G = \omega_0^2 L$

• *In order to get our desired damping* $\frac{R}{L} + K_I R_G k_2 = \frac{R}{L} + \omega_0^2 L k_2 = 2\zeta\omega_0$

$$k_2 = \frac{2\zeta\omega_0 - \frac{R}{L}}{\omega_0^2 L}$$



• *In order to get no DC error,* $k_1 + R k_2 = 1$

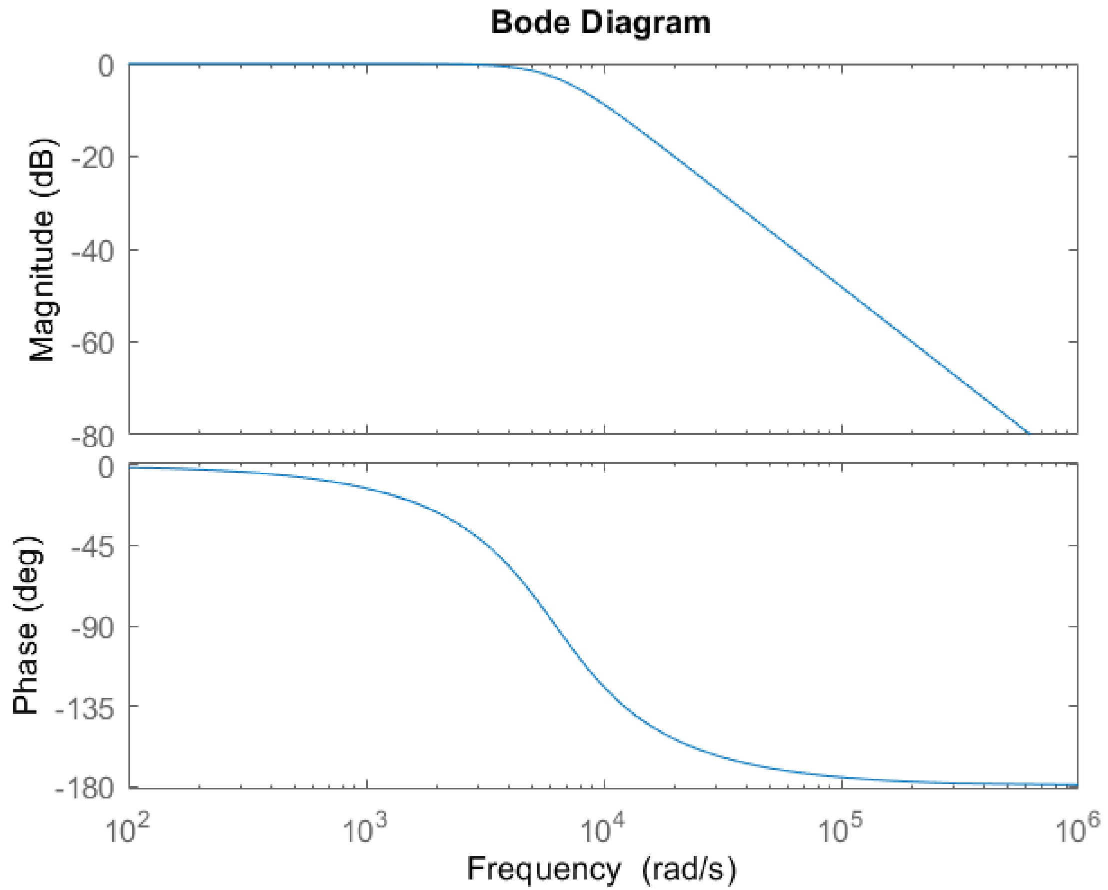
$$k_1 = 1 - R\,\frac{2\zeta\omega_0 - \frac{R}{L}}{\omega_0^2 L}$$

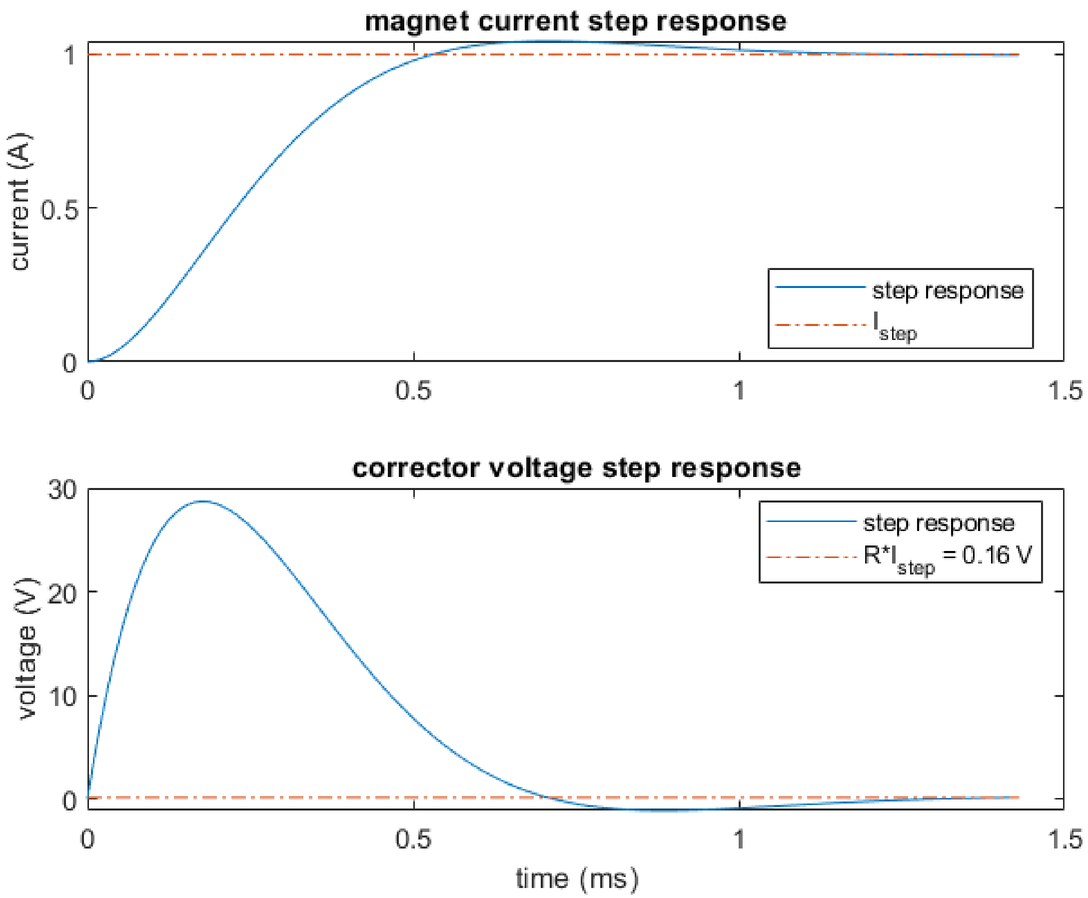
• *For our system we select three gains*

$$K_I R_G \approx 394800$$
$$k_1 \approx 0.9964$$
$$k_2 \approx 0.02247$$

and create our controller with amplifiers and summing junctions.

State Feedback – Corrector Power Supply



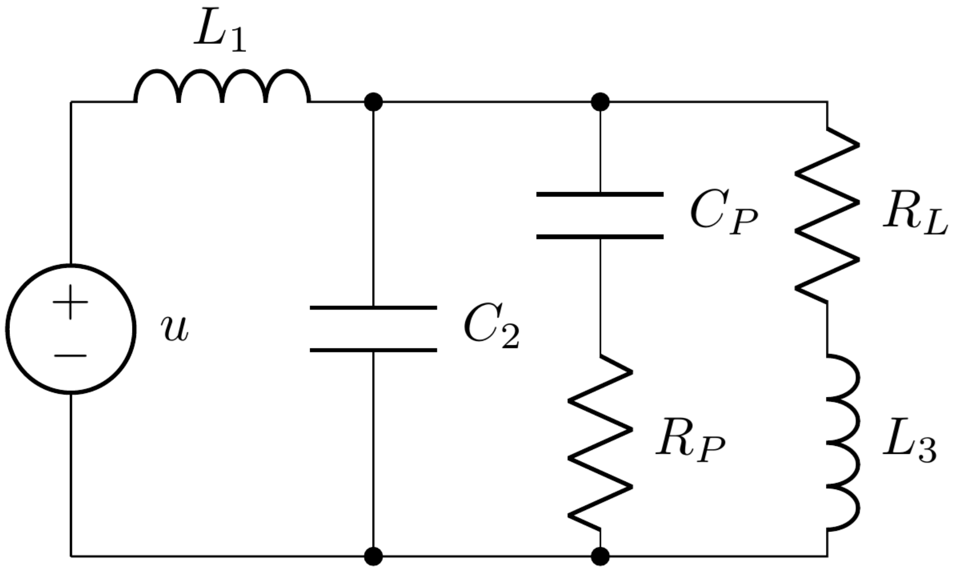


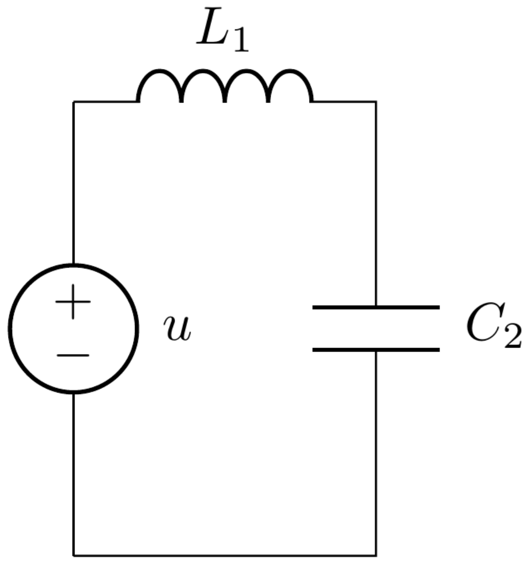
- *We have built our desired feedback control so that it has maximally flat (Butterworth) response to* 1 *kHz*
- *The step response has the desired fast rise time with the acceptably small overshoot*
- *In order to achieve this high bandwidth we needed very high gain and an overhead factor of* 180 *on the corrector voltage. (High bandwidth is often specified for small signals and not from* i_{MIN} *to* i_{MAX}*)*

State Feedback – Corrector Power Supply

- *We have met our specifications, but to meet them, the design is very aggressive*
  - *Our integrator gain* $K_I \approx 2.5 \times 10^6$
  - *We need a large voltage reserve in the power supply to supply the voltage for* $L \cdot di/dt$
- *The reason for this is the mismatch between the natural* R/L *frequency and the desired* ω_0 *bandwidth*
- *Things that can be negotiated with other groups in the accelerator design to relax the requirements*
  - *Decrease the corrector inductance*
    - *Can a lower inductance magnet provide the required field strength? (Accelerator Physics)*
    - *Can the magnet be made physically smaller? (Magnet Design)*
    - *Can the vacuum chamber inside the corrector be made smaller? (Vacuum Design, Acc Phys)*
  - *Decrease the required frequency response*
    - *What are the expected disturbance frequencies? (Mechanical Design, Acc Phys)*
    - *What definition of bandwidth was used for the specification, single pole or double pole? (AP)*
    - *What is the frequency cutoff of the magnetic fields in the vacuum chamber? (Vacuum Design)*
- *The optimization should be system wide; the power supply is only one element of the system.*

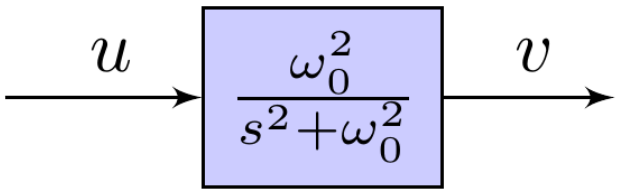
- Our supply has a high fundamental frequency, ω_S
- Our load is a magnet of inductance L_3 and resistance R_L
    - The L_3/R_L time constant is very long compared to $2\pi/\omega_S$
    - But not long enough to reject enough of the ripple from u
- We introduce a two pole $L - C$ low pass filter to provide additional rejection of ω_S
    - We set the frequency of this filter, $\omega_0 = 1/\sqrt{L_1 C_2}$, such that
$$\omega_S \gg \omega_0 \gg R_L/L_3$$
- R_L only lightly damps the $L - C$ filter
    - L_1, C_2, R_L, L_3 circuit exhibits much ringing at ω_0
- A standard solution is to introduce a Praeg filter, a series $C_P - R_P$, to damp the ω_0 oscillations
- We will design an alternate damping scheme using feedback
- As with the Praeg, because of the separation in the frequency domain, we only need to analyze the $L_1 - C_2$ resonator.





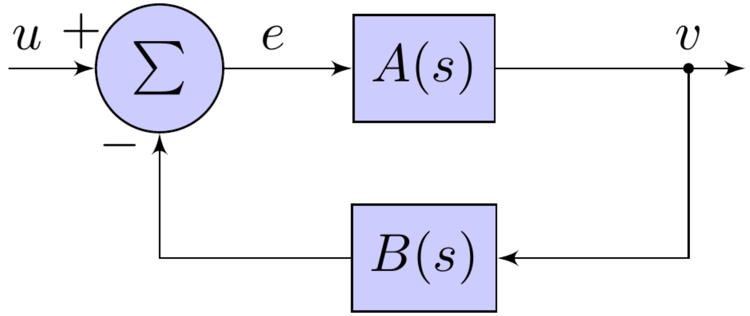
•*This problem is very similar to the corrector problem we just solved.*

  •*It is a second order system for which we want to add damping*



•*Recall that the transfer function of our $L - C$ low-pass filter is*

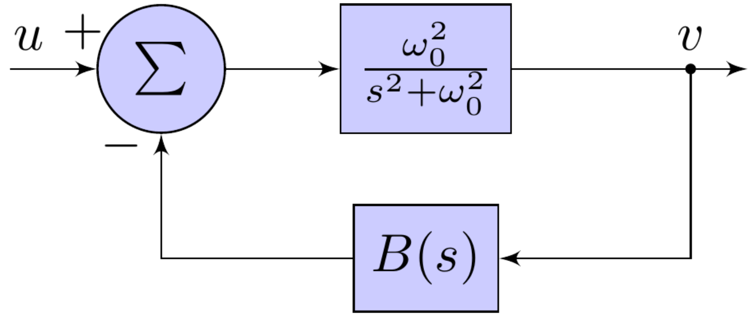
$$\frac{V(s)}{U(s)} = A(s) = \frac{\omega_0^2}{s^2 + \omega_0^2}$$



•*Also recall that the general equation for the transfer function of a closed loop negative feedback system is*

$$H(s) = \frac{G_{FORWARD}}{1 + G_{LOOP}} = \frac{A(s)}{1 + B(s)A(s)}$$

•*If we just place a loop around the oscillator (no controller in the forward loop)*



$$H(s) = \frac{\frac{\omega_0^2}{s^2 + \omega_0^2}}{1 + \frac{B(s)\omega_0^2}{s^2 + \omega_0^2}} = \frac{\omega_0^2}{s^2 + \left(1 + B(s)\right)\omega_0^2}$$

•*If $B(s)$ is just a constant, we can increase the frequency, but not provide any damping.*

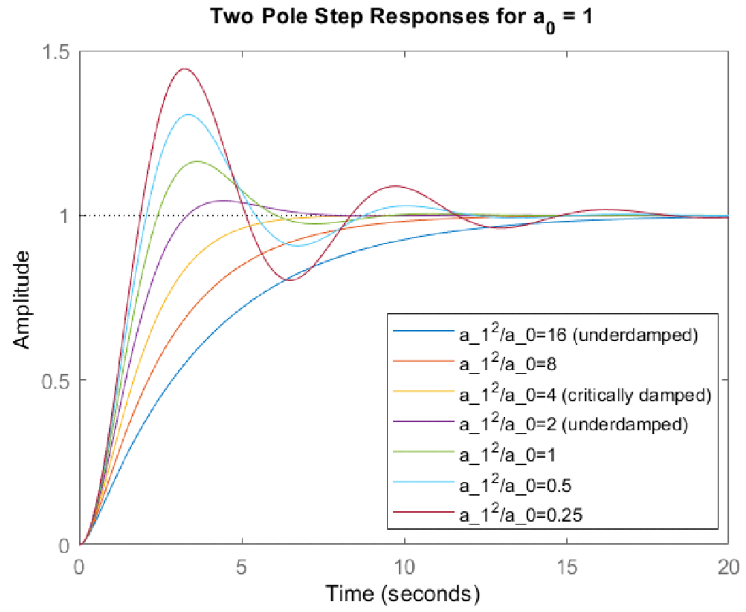
- *We want to change the transfer function to give us a damped response*

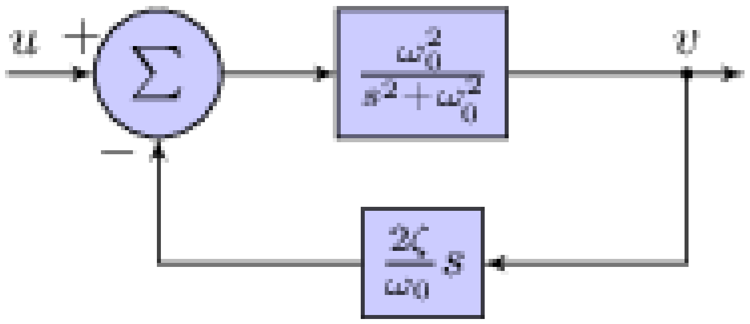
$$H(s) = \frac{a_0}{s^2 + a_1 s + a_0}$$

- *To do this, we choose $B(s) = 2\zeta s/\omega_0$, with, for example, $\zeta = \sqrt{2}/2$. Then*

$$H(s) = \frac{\omega_0^2}{s^2 + (1 + B(s))\omega_0^2} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{\omega_0^2}{s^2 + \sqrt{2}\omega_0 s + \omega_0^2}$$

- *But this choice of $B(s)$ is a differentiator*
- *Inserting a differentiator in the feedback loop is problematic*
  - *A differentiator amplifies noise in the system*
  - *(An integrator reduces noise)*
- *Can we instead extract $\dot{v}$ from the plant?*
  - *No. We do not have access to $\dot{v}$*
  - *But we do have access to $i = C_2\dot{v}$*



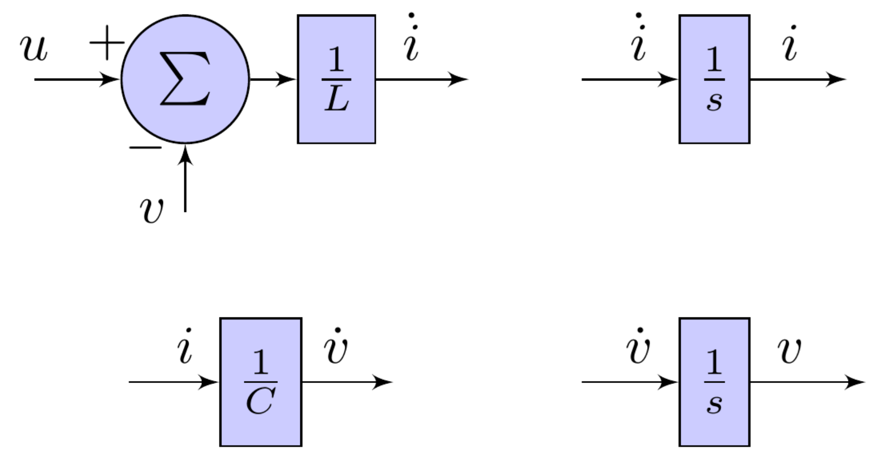


State Feedback – State Equations/State Diagram for an Oscillator

- *Starting with our standard state equations*

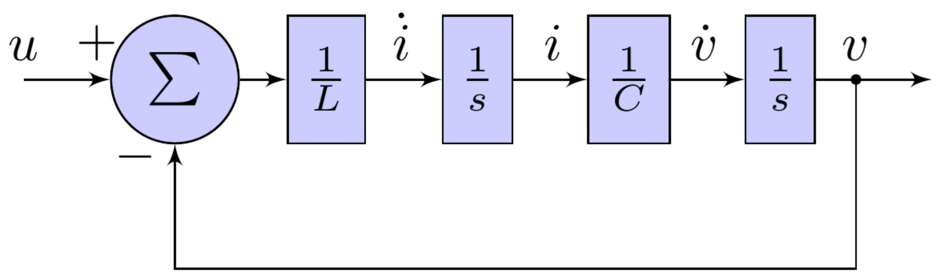
$$L\frac{di}{dt} = u - v;\quad C\frac{dv}{dt} = i$$

- *We draw state diagrams for the derivatives of the state variables*
- *We draw additional diagrams that integrate the derivatives to obtain the state variables*
- *Finally we connect the blocks to form the oscillator feedback loop*



- *We verify the transfer function* ($\omega_0 = 1/\sqrt{LC}$)

$$H(s) = \frac{\frac{1}{L}\frac{1}{s}\frac{1}{C}\frac{1}{s}}{1+\frac{1}{L}\frac{1}{s}\frac{1}{C}\frac{1}{s}} = \frac{\frac{\omega_0^2}{s^2}}{1+\frac{\omega_0^2}{s^2}} = \frac{\omega_0^2}{s^2+\omega_0^2}$$

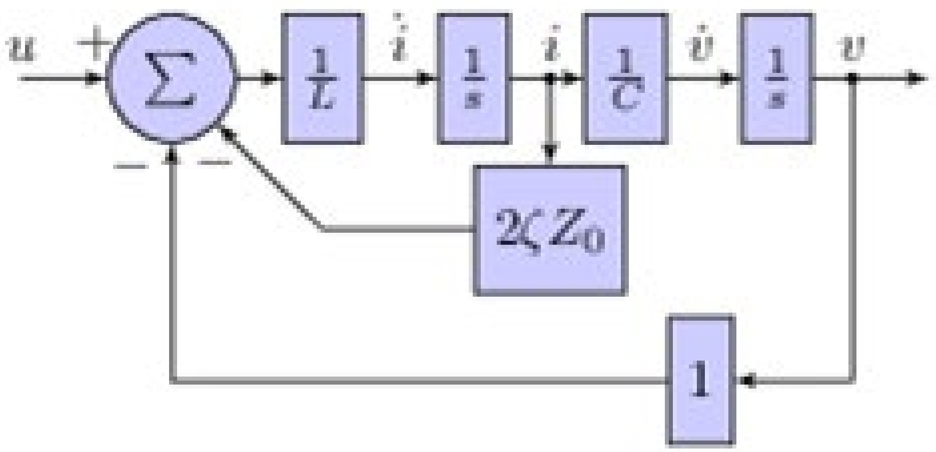


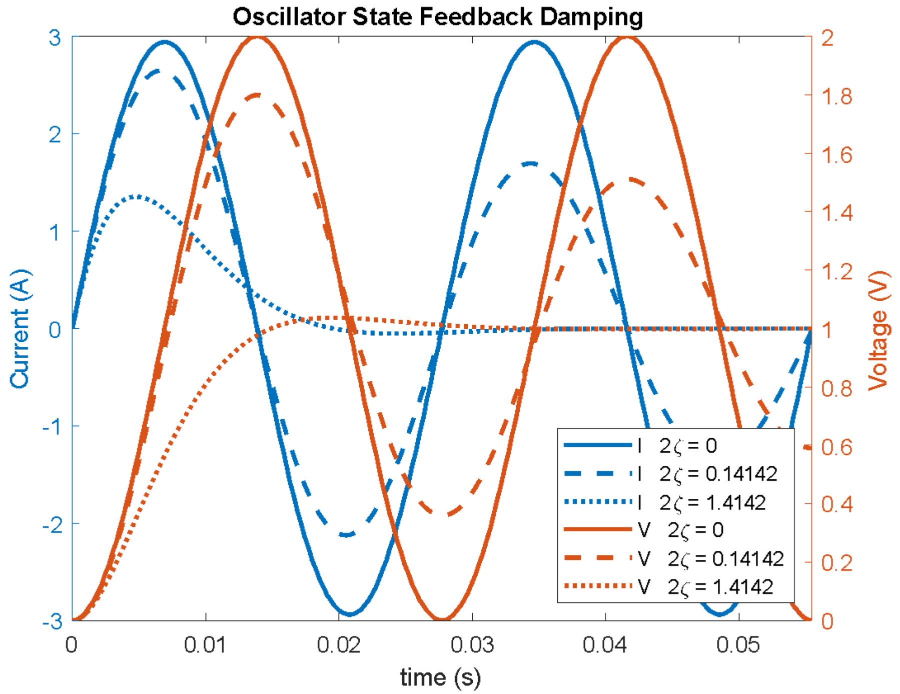
- *We cannot access* $\dot{v}$, *but multiplying* i *by* $1/C$ *gives us* $\dot{v}$

$$\frac{2\zeta}{\omega_0}\dot{v} = \frac{2\zeta}{\omega_0}\frac{1}{C}\,i = 2\zeta Z_0 i$$

where $Z_0 = \sqrt{L/C}$

- *Feeding back on the intermediate variable (i) enables us to damp the oscillation of i.*
  - *i and v are in quadrature ⇒ they oscillate together*
    - *Damping i oscillation also damps v oscillation*
  - *i grows faster* ($\sim \sin\omega_0 t$) *than v* ($\sim (1 - \cos\omega_0 t)$)
    - *Faster feedback response*
- *Proper choice of 2ζ gives desired response*
- *State feedback loop is now complete*
  - *Determined desired system response (system poles)*
  - *Determined coefficients of denominator polynomial*
  - *Related states to system derivatives*
  - *Implemented required gains on each state variable*





State Feedback vs Praeg Filter

- *Praeg filter*
  - *Advantages*
    - *Passive solution*
    - *No requirements on control circuitry*
  - *Disadvantages*
    - *Large capacitance required in series with damping resistor*
    - *Power dissipation/heat required to damp system*
- *State filter*
  - *Advantages*
    - *Minimal extra hardware (current sensor)*
    - *All control at low power*
  - *Disadvantages*
    - *Need sufficient bandwidth of power supply controller*
      - *Bandwidth much greater than* ω_0

Feedback and Stability Summary

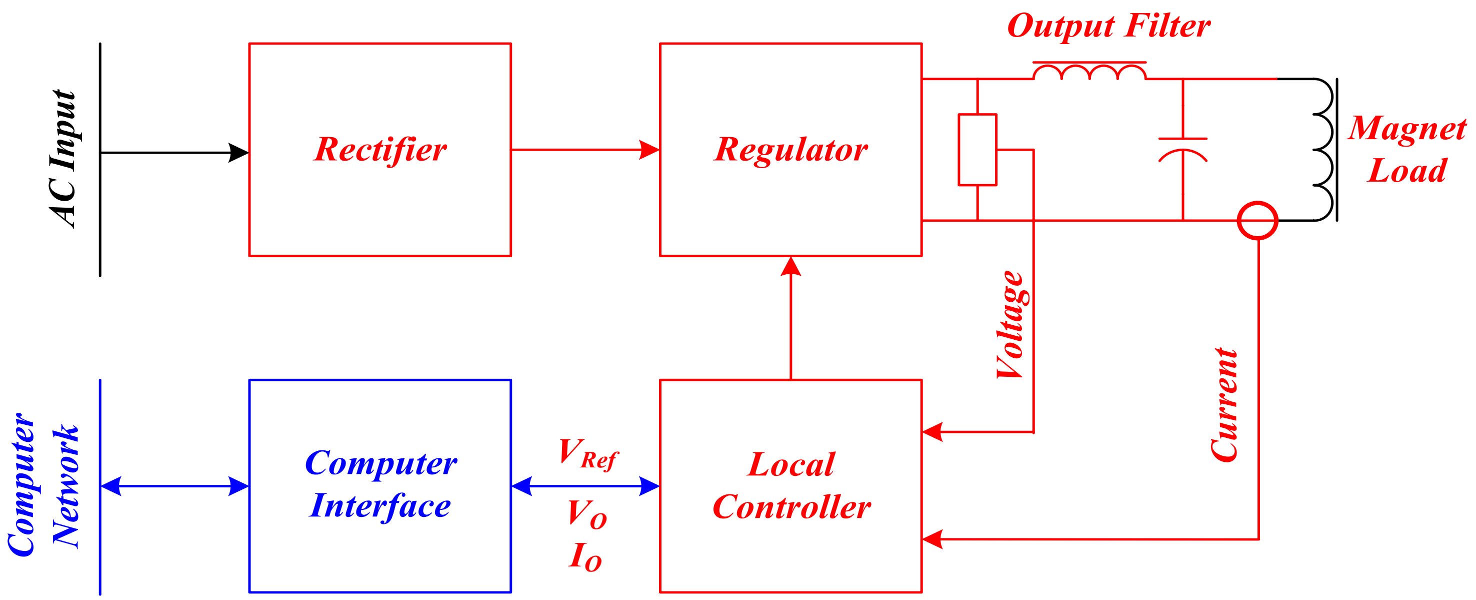
- *The transfer function is the relation between the input, x, and the output, y*
- *By increasing feedback gain, y more closely approaches the desired output*
- *The efficiency of feedback for a dynamic (time-varying) system involves not only the gains, but also the speed of the system response. Some common terms that characterize the dynamics are*
  - *Bandwidth is the frequency range over which the feedback achieves (close) to its nominal gain (3 dB point)*
  - *DC Response is a measure of how closely the system tracks a constant input. Improve the DC Response by increasing the loop gain*
  - *Step Response is the action of the system in response to an input step*
  - *Settling Time is how long it takes to settle to within a certain fraction of its final value*
  - *Overshoot is any ringing occurs as the system achieves its final value*
  - *Ramp response is a measure of how well the system follows an input ramp command*

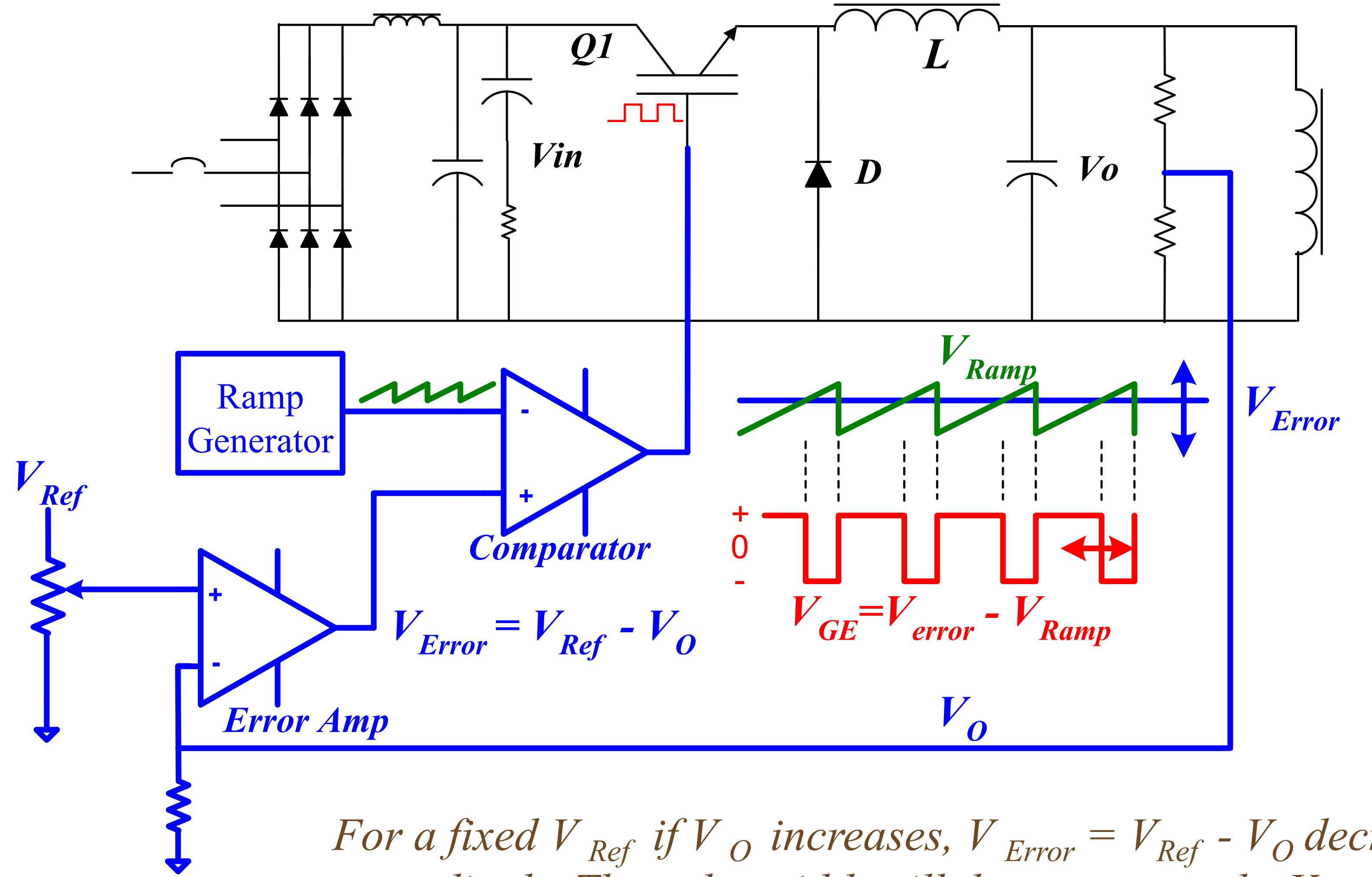
Feedback References

- *Feedback Control of Dynamic Systems*, Gene F. Franklin, J. David Powell, and Abbas Emami-Naeini
- *Modern Control Engineering*, Katsuhiko Ogata
- *Modern Control Systems*, Richard C. Dorf and Robert H. Bishop
- *Automatic Control Systems*, Benjamin Kuo and Farid Golnaraghi
- *Control System Design*, Bernard Friedland
- *Feedback Control Theory*, John Doyle, Bruce Francis, and Allen Tannenbaum
- *Feedback Systems*, Karl Johan Astrom, Richard M. Murray
- *Linear Control System Analysis & Design*, John J. D'Azzo and Constantine H. Houpis
- *Multivariable Feedback Control*, Sigurd Skogestad and Ian Postlethwaite
- *Digital Control of Dynamic Systems*, Gene F. Franklin, J. David Powell, and Michael Workman
- *Digital Control System Analysis and Design*, Charles L. Phillips and H. Troy Nagle
- *Introduction to Linear Algebra*, Gilbert Strang

Purposes

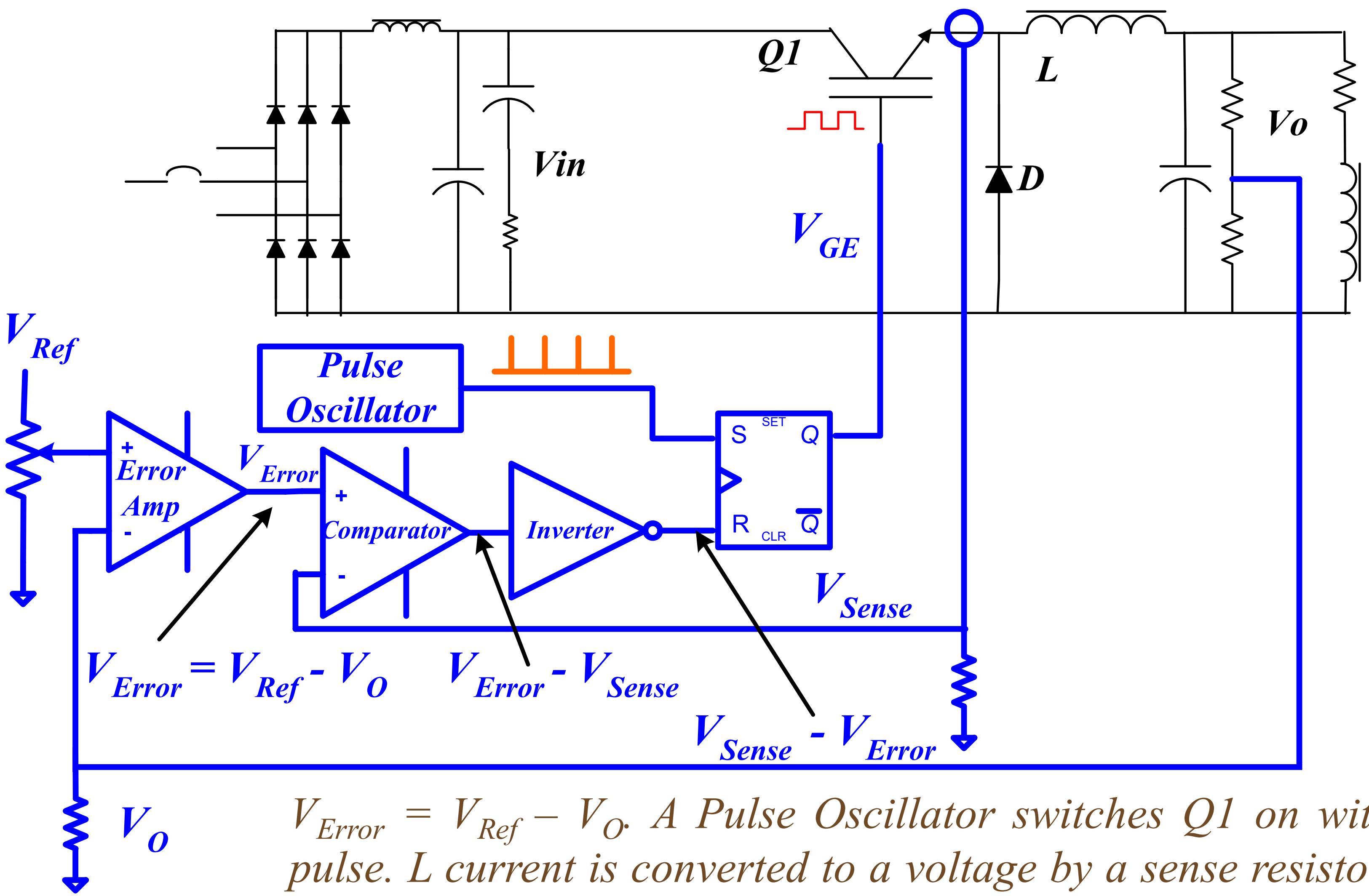
- *Sets the output voltage or current to a desired value*
- *Regulates the output voltage or current to the desired value in the presence of line, load and temperature changes*
- *Monitors load and power supply actual versus desired performance*



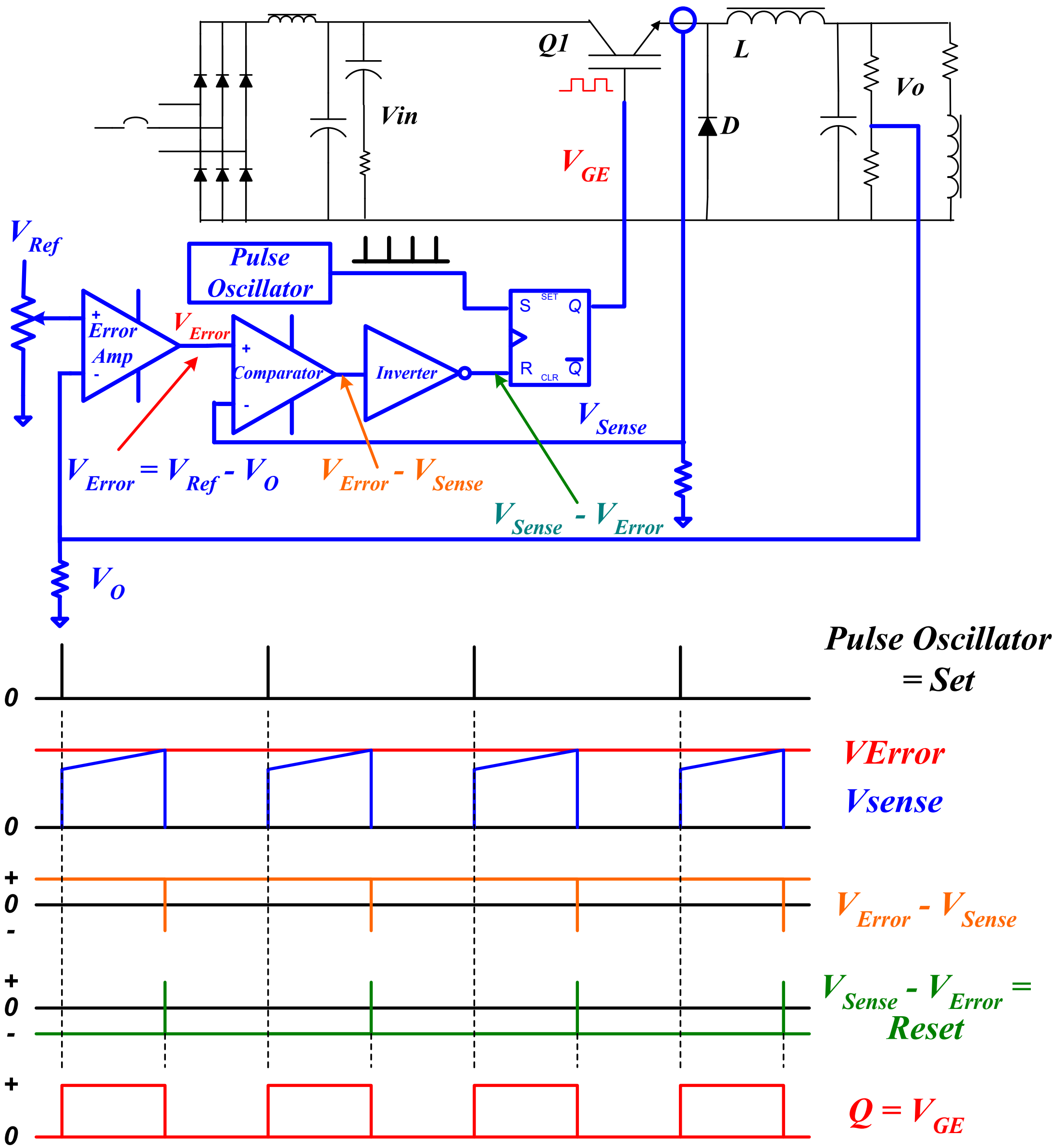


For a fixed V_{Ref} if V_O increases, $V_{Error} = V_{Ref} - V_O$ decreases accordingly. The pulse width will decrease to make $V_O = V_{Ref}$.

If V_O decreases, V_{Error} increases accordingly. The pulse width will increase to keep $V_O = V_{Ref}$.



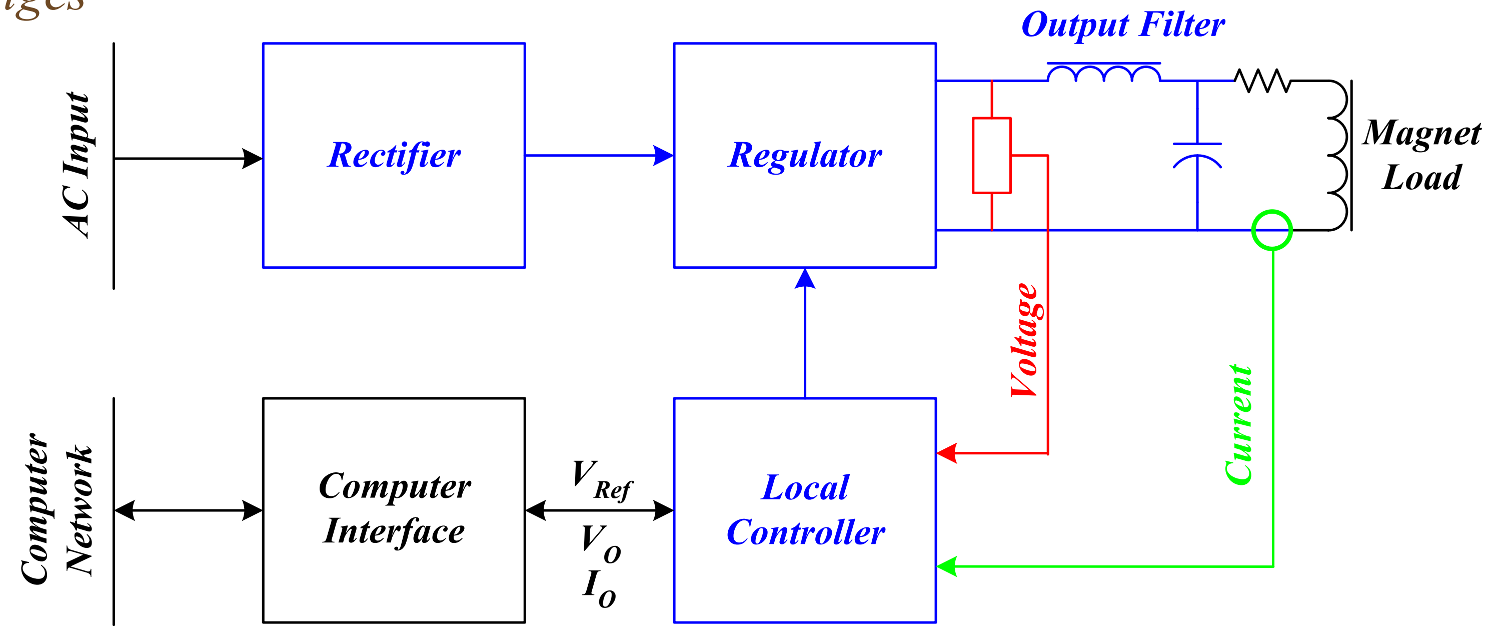
$V_{Error} = V_{Ref} - V_O$. A Pulse Oscillator switches Q1 on with every pulse. L current is converted to a voltage by a sense resistor. The L current builds up to the threshold set by the error voltage which then turns off Q1 in order to keep the output voltage or current constant.

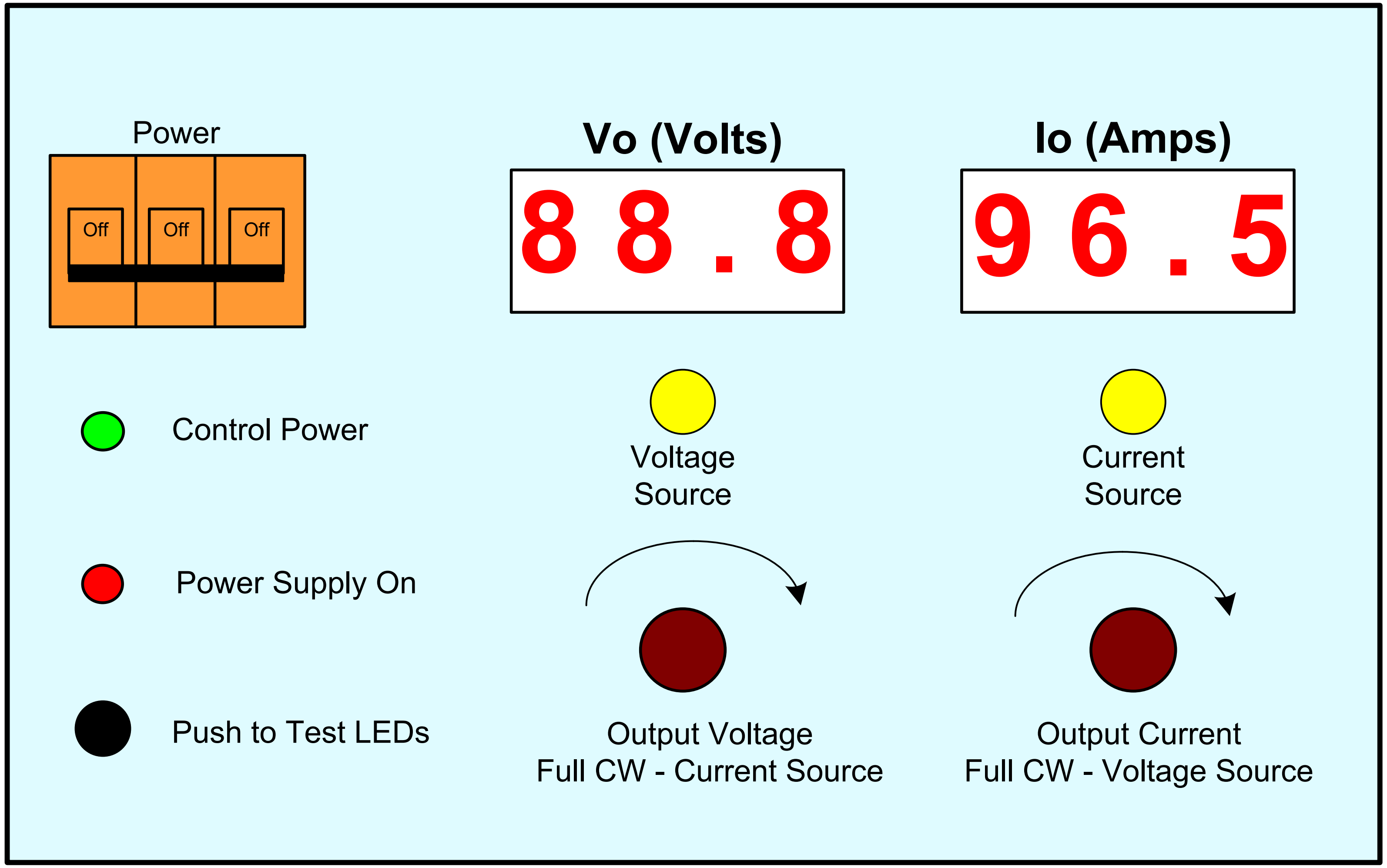


Q1
L
Vo
Vin
D
V GE
V Ref
Pulse Oscillator
Error Amp
V Error
Comparator
Inverter
S SET Q
R CLR Q
V Sense
V Error = V Ref - V O
V Error - V Sense
V Sense - V Error
V O
Pulse Oscillator = Set
VError
Vsense
V Error - V Sense
V Sense - V Error = Reset
Q = V GE

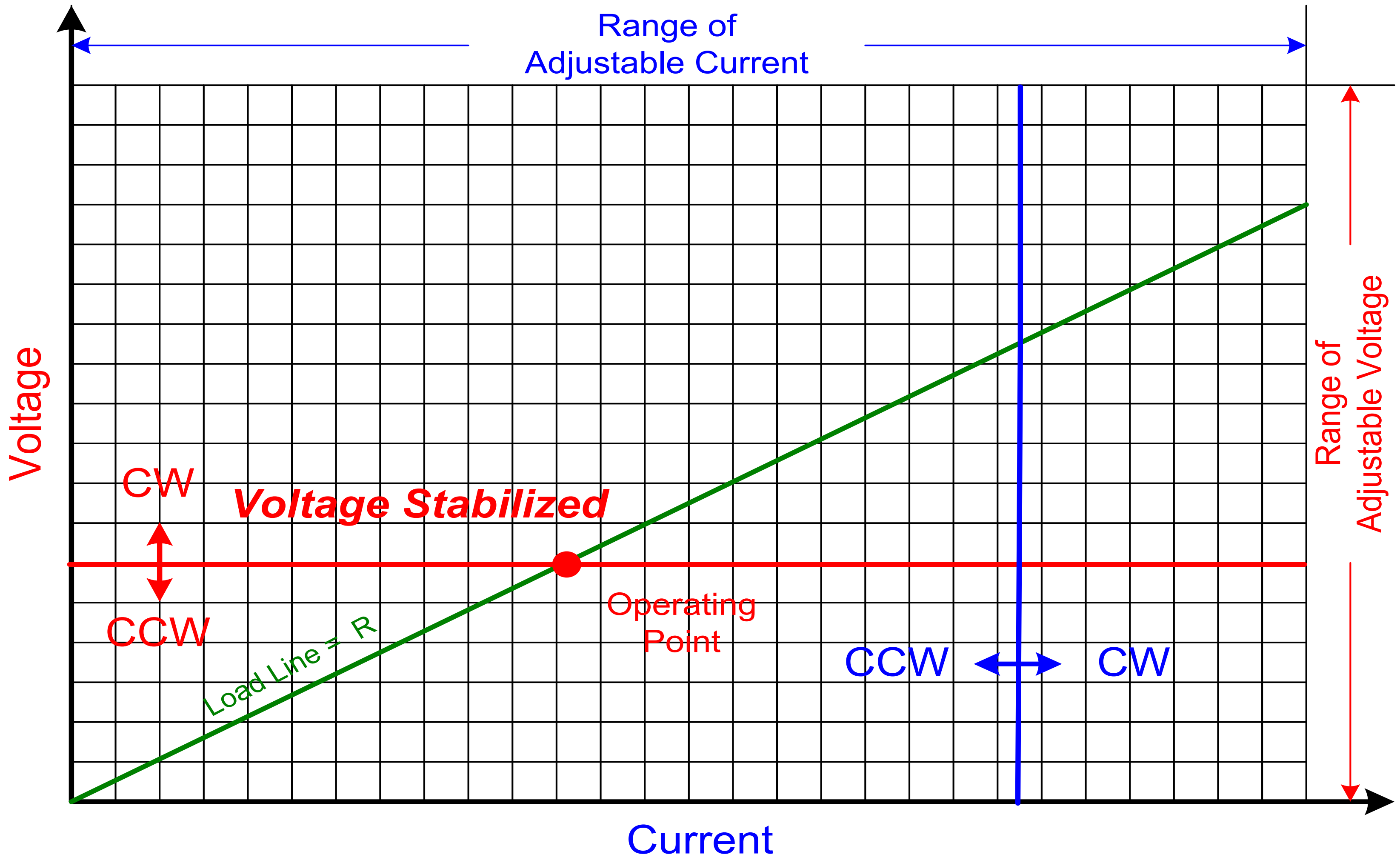
Summary

- Typically 2 control loops – voltage and current
- The outer loop defines the source type – voltage or current stabilized
- The outer loop has lower BW and corrects for drift due to slow temperature changes and aging effects
- The inner loop has higher BW and compensates for fast transients, AC line changes

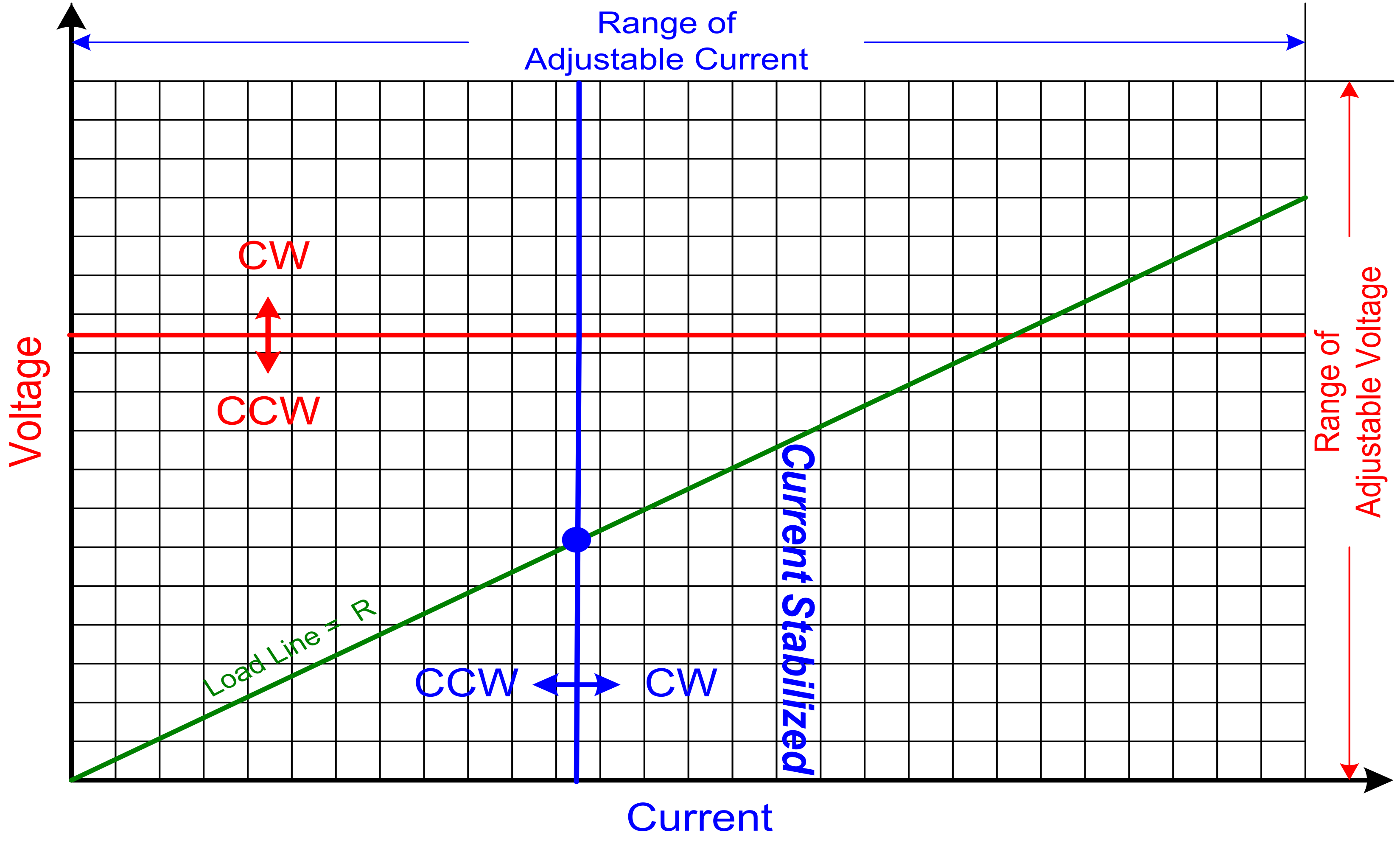




Power Supply Front Panel

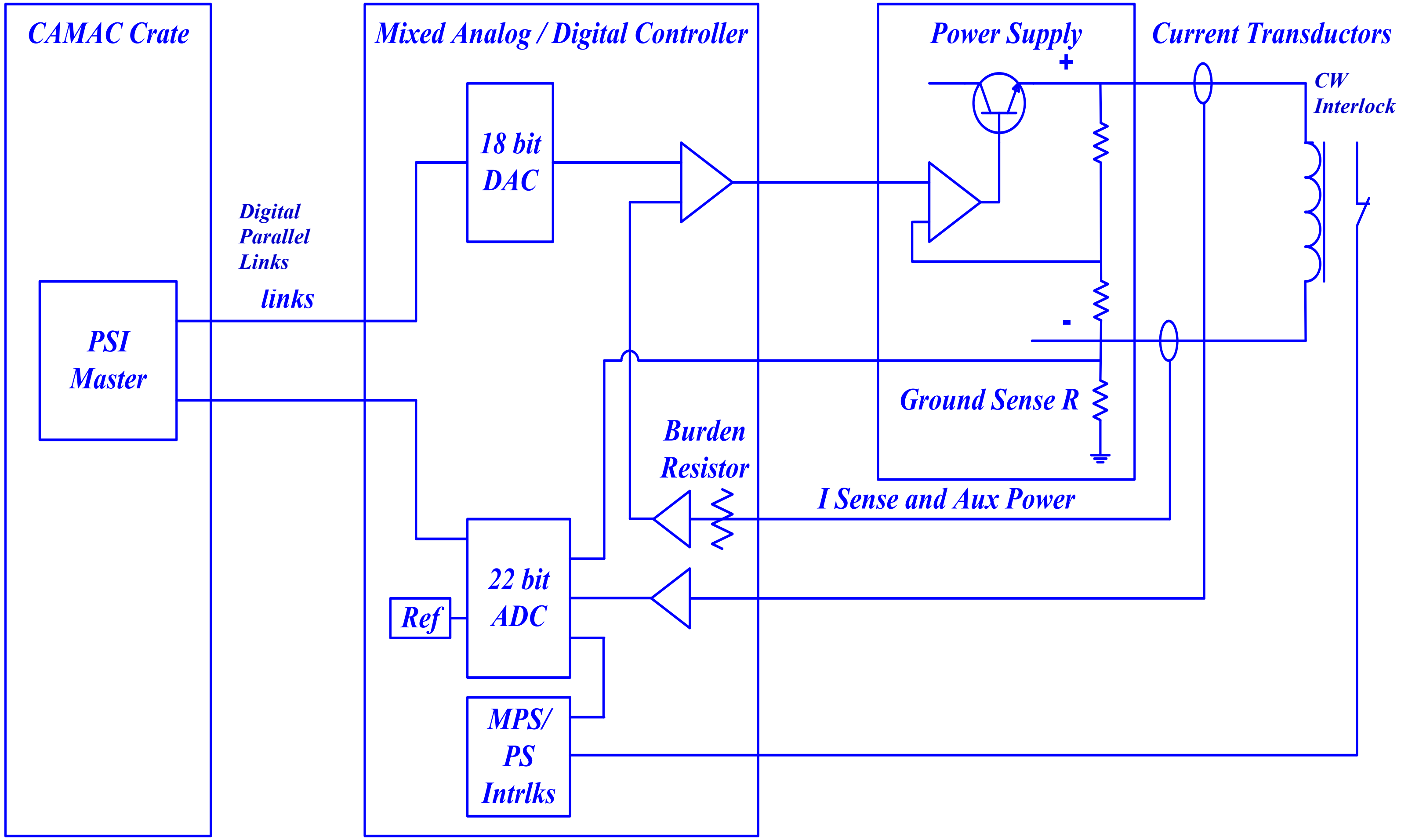


Constant Voltage Mode. *The power supply will operate in this mode whenever the current demanded by the load is less than that defined by the front panel current control. The output voltage is set by the front panel voltage control. The output current is set by the load resistance and the Vset.*

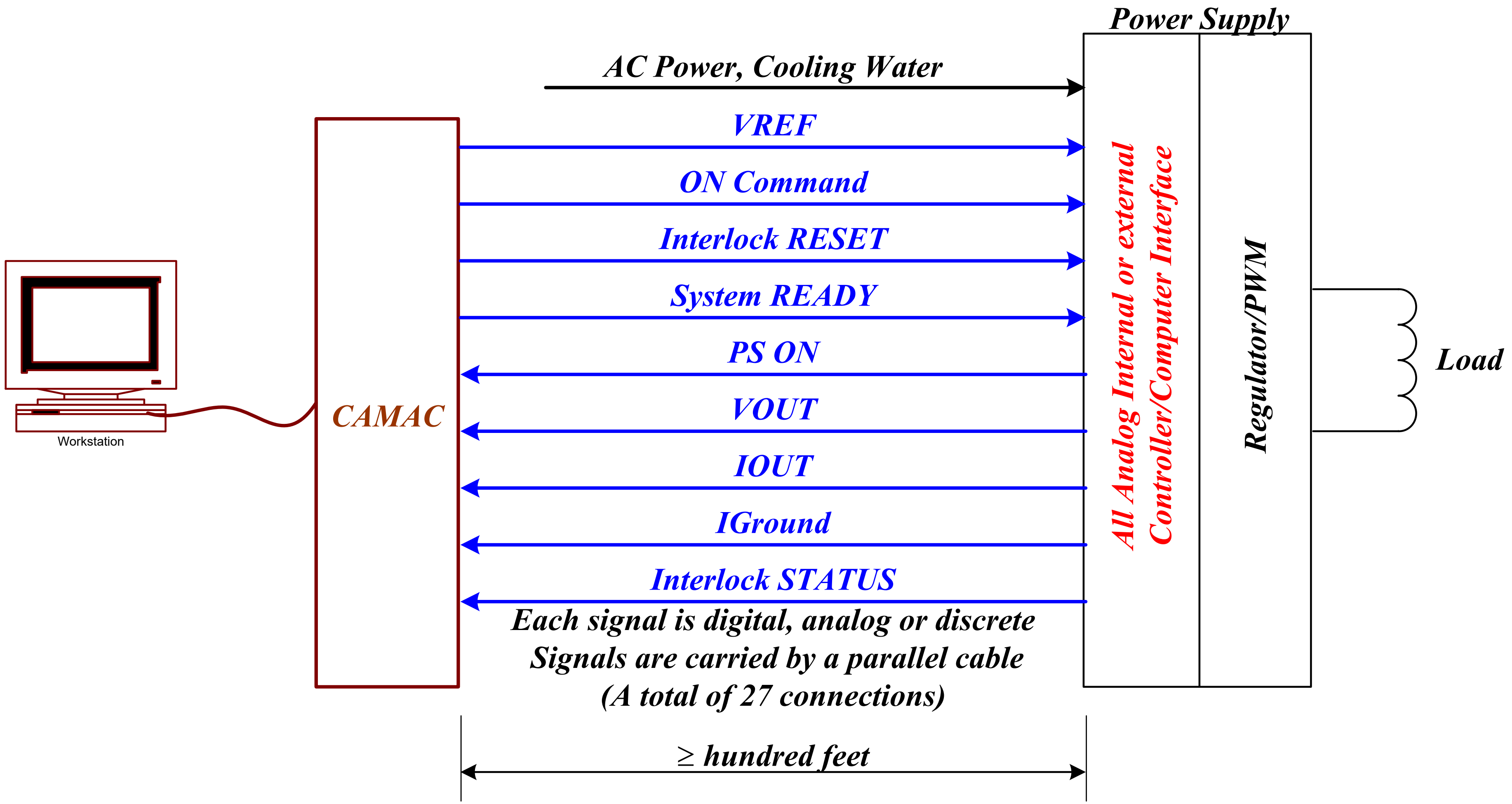


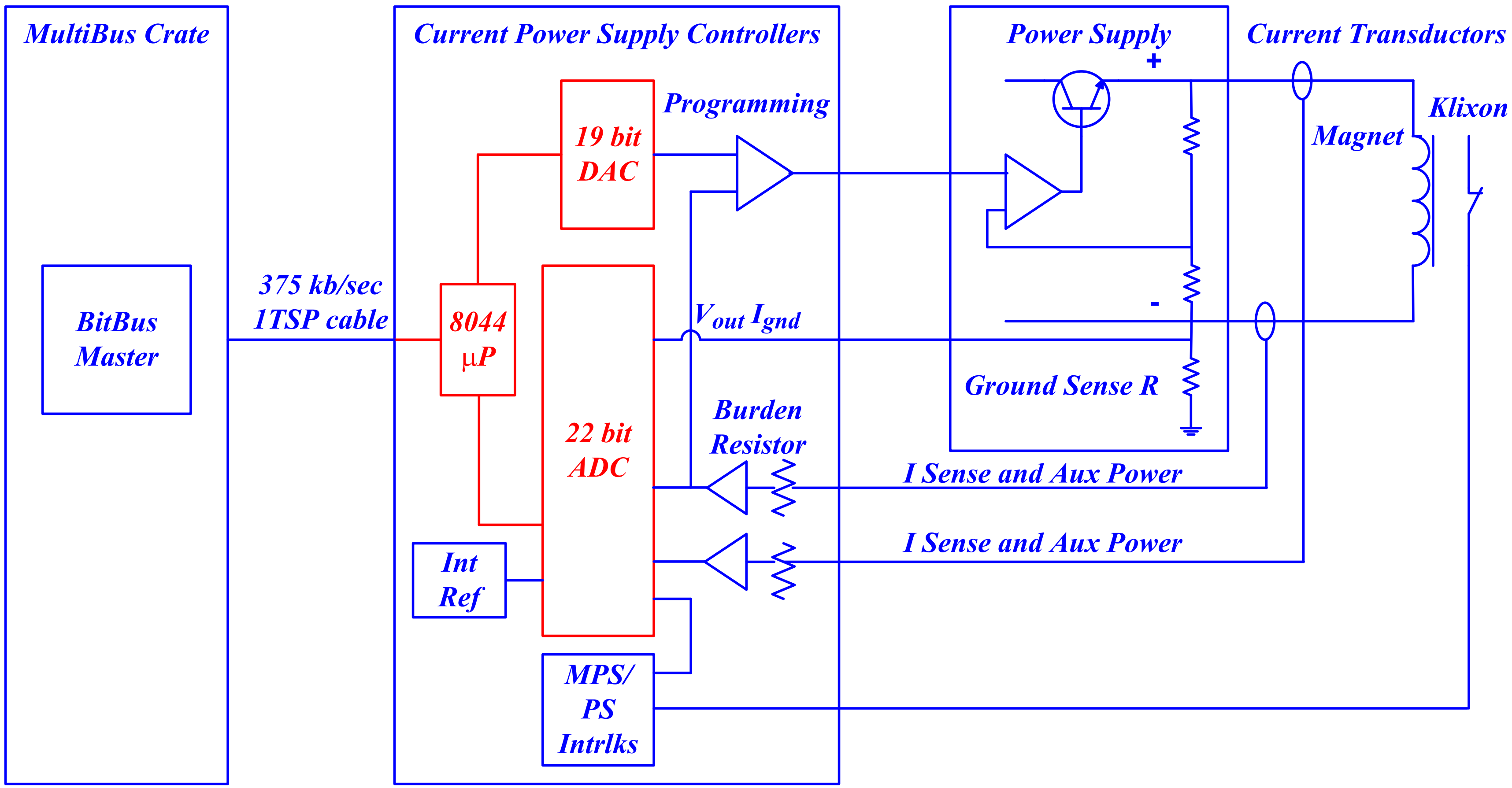
Constant Current Mode. *The power supply will operate in this mode whenever the voltage demanded by the load is less than that defined by the front panel voltage control. The output current is set by the front panel current control. The output voltage is set by the load resistance and the I set.*

All-Analog Power Supply Controllers – Circa 1970s to 1980s

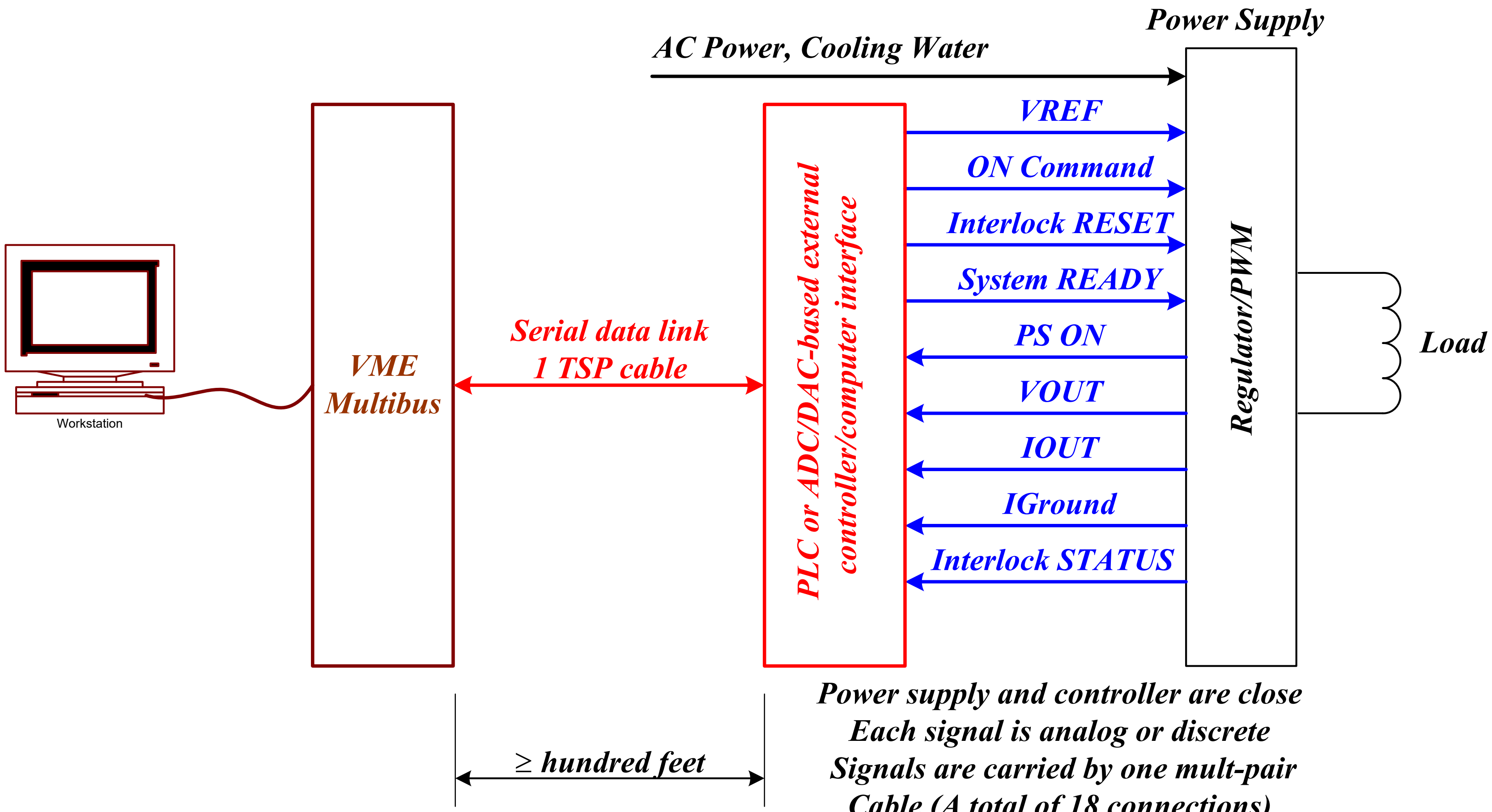


All-Analog Power Supply Controllers – Circa 1970s to 1980s



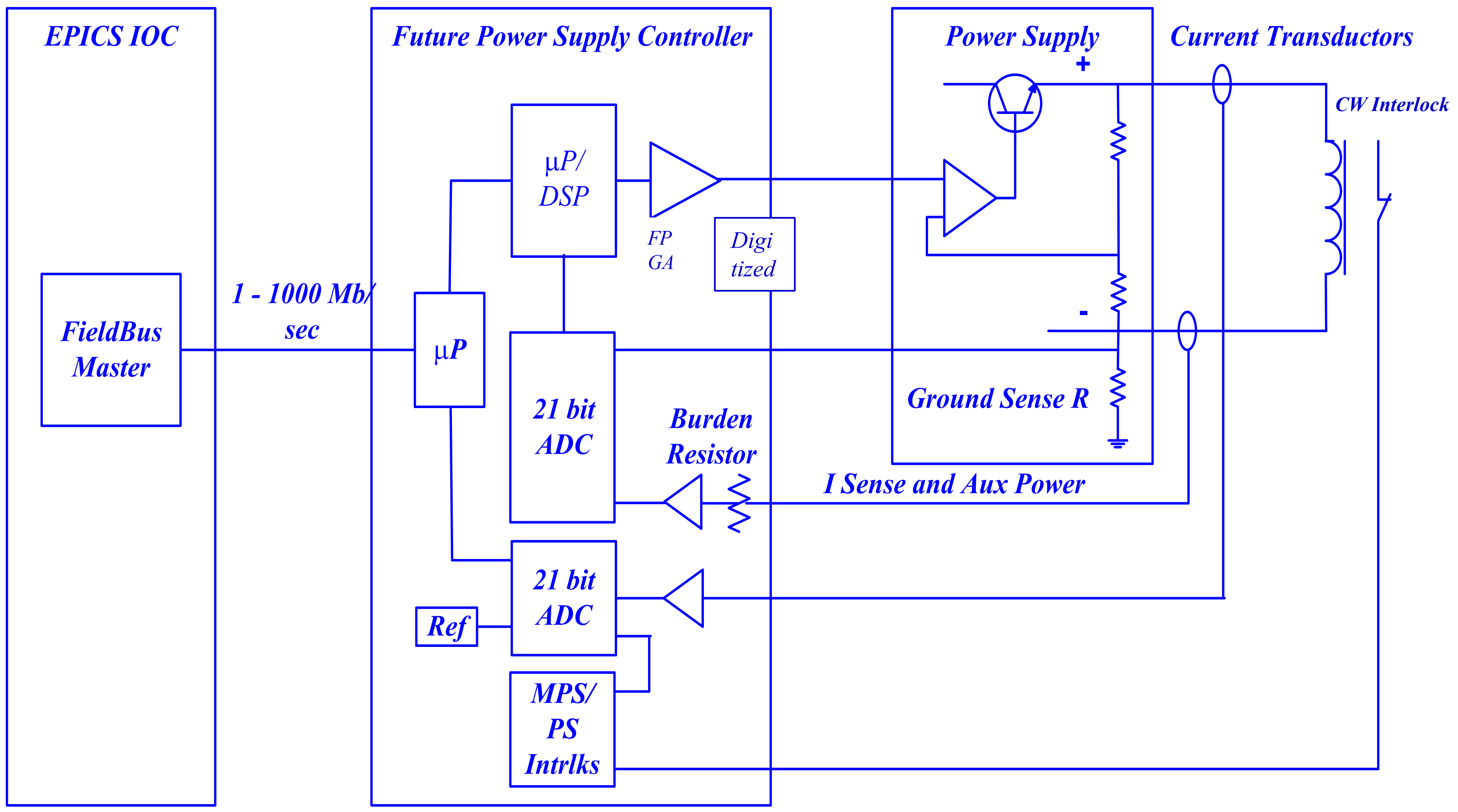

MultiBus Crate
BitBus
Master
375 kb/sec
1TSP cable
Current Power Supply Controllers
19 bit
DAC
Programming
8044
μP
22 bit
ADC
Vout Ignd
Burden
Resistor
Int
Ref
MPS/
PS
Intrlks
Power Supply
+
-
Ground Sense R
Current Transductors
Magnet
Klixon
I Sense and Aux Power
I Sense and Aux Power

Hybrid Analog/Digital Power Supply Controllers – Circa 1980s to Present

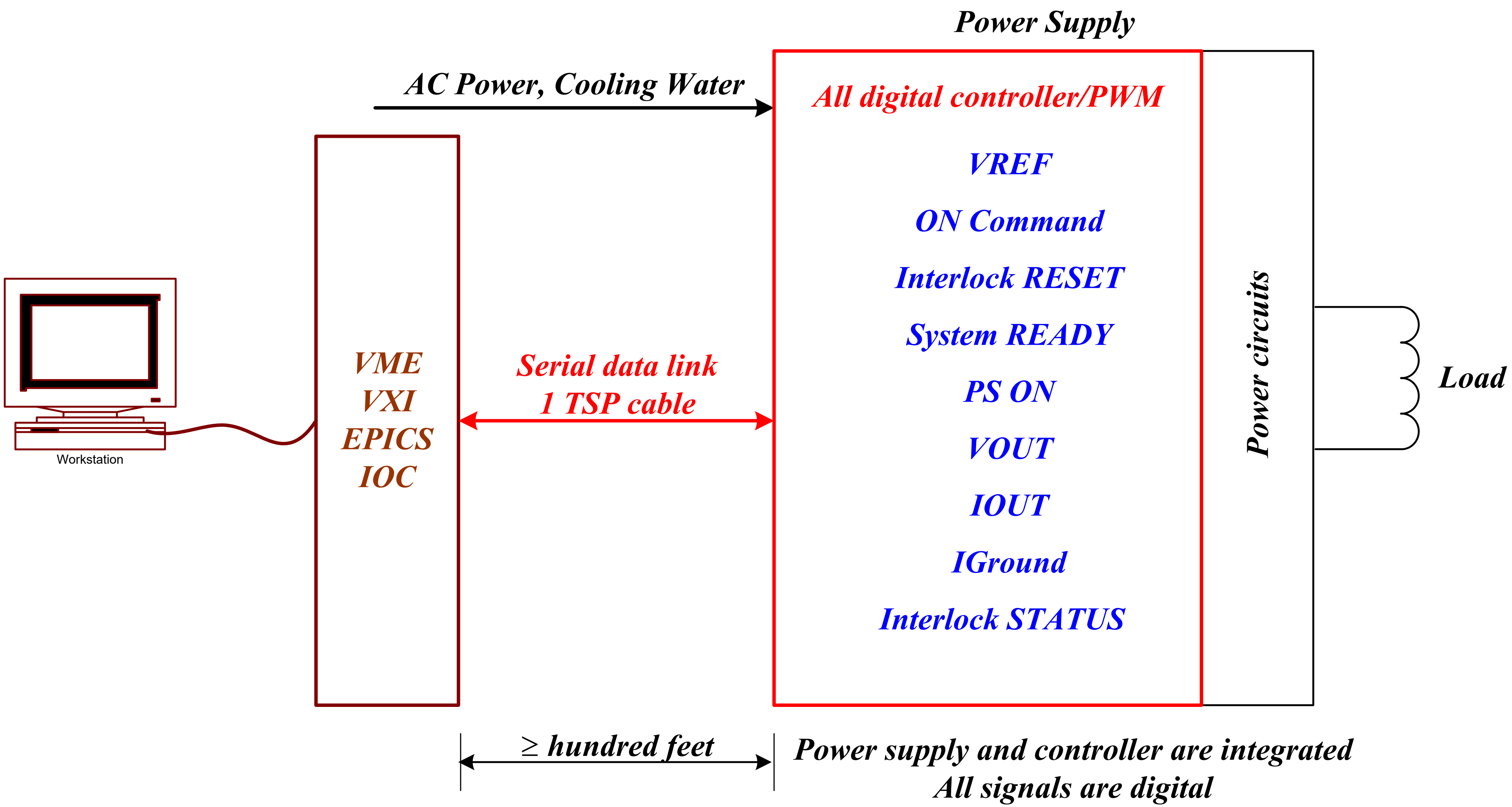


Power supply and controller are close
Each signal is analog or discrete
Signals are carried by one mult-pair
Cable (A total of 18 connections)

All Digital Power Supply Controllers – Circa the Future

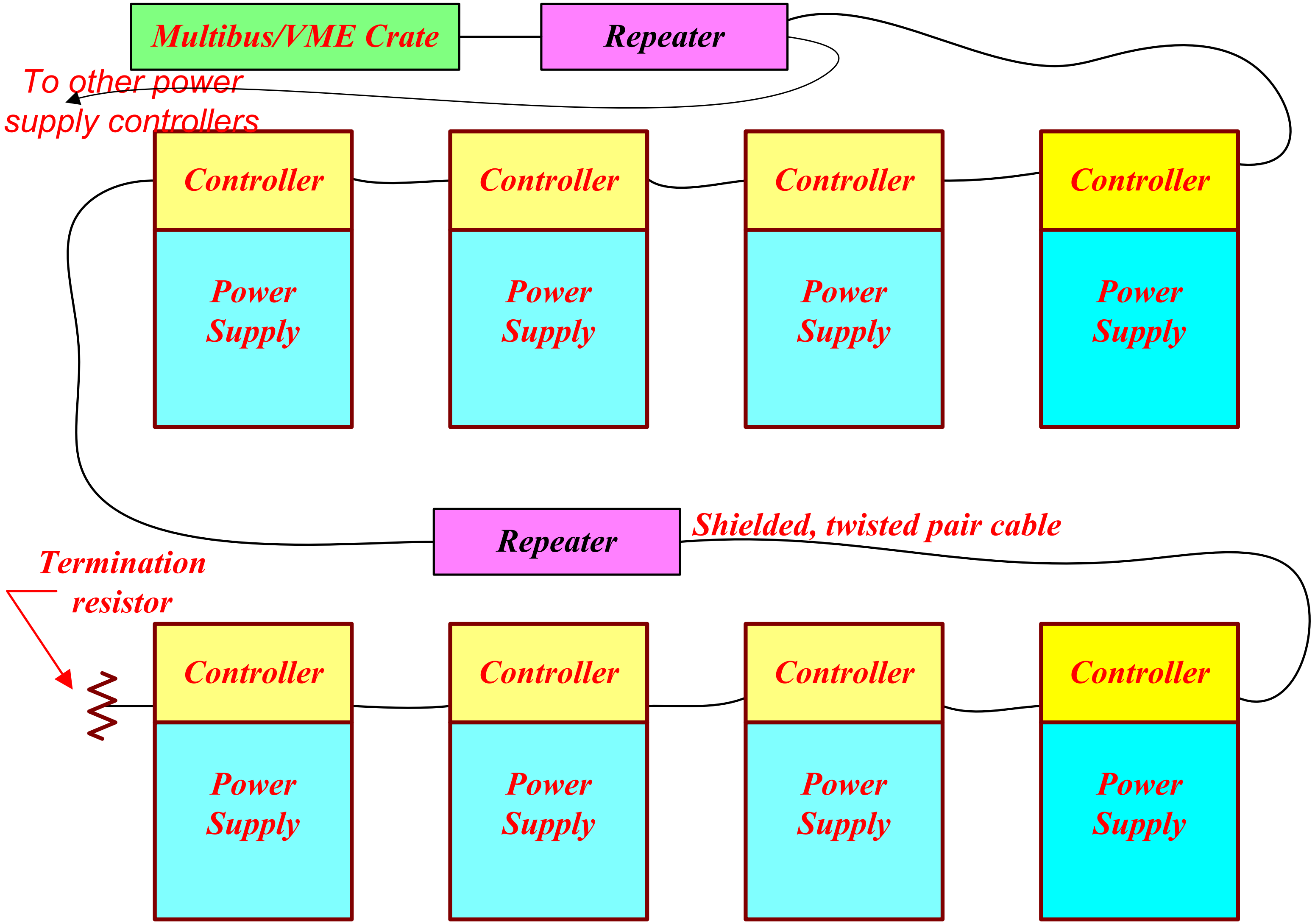


All Digital Power Supply Controllers – Circa the Future



Power supply and controller are integrated
All signals are digital

Daisy - Chaining of Power Supply Controllers



Controls Type	*Characteristics*
All analog controls	• *Long, expensive multi-conductor cable* • *Cables subject to noise pickup, ground loops, losses in signal strength* • *Installation rigid, difficult to modify*
Hybrid analog/digital controls	• *PLCs, ADCs / DACs subject to noise pickup, ground loops, must keep out of power supply* • *Serial data cable can be daisy-chained* • *Installation rigid, difficult to modify*
All digital controls	• *Integrated high level digital signals exhibit greater immunity to noise pickup, ground loops* • *Serial data cable can be daisy-chained* • *Installation flexible, control system can be modified in software or firmware* • *Will require novel implementation of interlocks, voltage and current transductors*

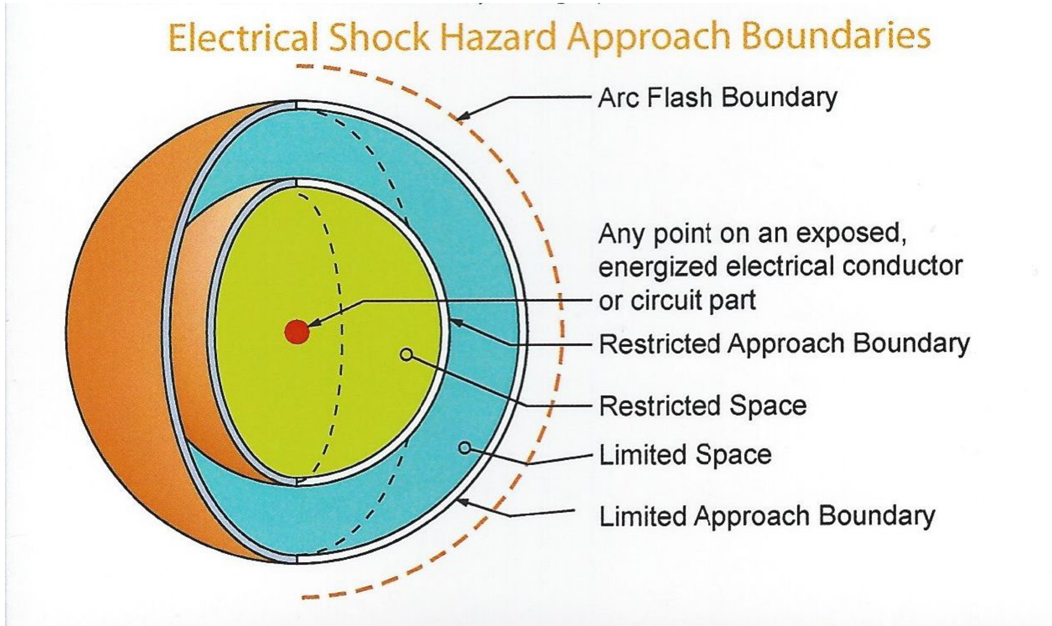
Some Communication Busses

Bus Type	Single / Differential	Data Transmittal	Data Rate	Length	Connector	Comments
RS232	-12 →+12V SE	Serial	115kb/s	5m	25 /15/9pin sub D	Inexpensive wiring
BitBus IEEE 1118	0-5V Differential	Serial	375kb/s	300m	9 pin sub D	Inexpensive wiring
IEEE488 GPIB		Parallel	8Mb/s	20m	24 pin	Measurement Equipment
Ethernet	Optical/SE Differential	Serial	1Gb/s		RJ8, RJ45 Optical	Move lots of data packets
USB 2.0		Serial	12Mb/s	5m	4 pin USB	Hot-swappable
Firewire IEEE1394	3.3V Differential	Serial	800Mb/s	46m	4 pin / 6 pin Optical	Hot-swappable
SCSI	3.3V Diff/ Optical	Parallel	1.28Gb/s	12m	68 pin 80 pin	
eSATA		Serial	3Gb/s			Hot-swappable

Section 11 – Personnel and Equipment Safety

NFPA 70E - 2018 - Standard for Electrical Safety in the Workplace

- *Addresses employer and employee safety in the workplace*
- *Focus is on procedures, personnel protective equipment*
- *Attempts to mitigate effects of three major electrical hazard types – shock, arc flash and arc blast*



- Limited approach boundary is the distance from an exposed live part within which a shock hazard exists

- Restricted approach boundary is the distance from an exposed live part within which there is an increased risk of shock, due to electrical arcover for personnel working in proximity to the live part

NFPA 70E - Approach boundaries – AC, Table 130.4(D)(a)

Nominal Voltage, Phase to Phase	Limited Approach Boundary		Restricted Approach Boundary
	Exposed Moveable Conductor	Exposed Fixed Circuit Part	
Less than 50	Not Specified	Not Specified	Not Specified
50 to 150	10 ft 0 in.	3 ft 6 in.	Avoid Contact
151 to 750	10 ft 0 in.	3 ft 6 in.	1 ft 0 in.
751 to 15 kV	10 ft 0 in.	5 ft 0 in.	2 ft 2 in.
15.1 kV to 36 kV	10 ft 0 in.	6 ft 0 in.	2 ft 7 in.
36.1 kV to 46 kV	10 ft 0 in.	8 ft 0 in.	2 ft 9 in.
46.1 kV to 72.5 kV	10 ft 0 in.	8 ft 0 in.	3 ft 3 in.
72.6 kV to 121 kV	10 ft 8 in.	8 ft 0 in.	3 ft 4 in.
138 kV to 145 kV	11 ft 0 in.	10 ft 0 in.	3 ft 10 in.
161 kV to 169 kV	11 ft 8 in.	11 ft 8 in.	4 ft 3 in.
230 kV to 242 kV	13 ft 0 in.	13 ft 0 in.	5 ft 8 in.
345 kV to 362 kV	15 ft 4 in.	15 ft 4 in.	9 ft 2 in.
500 kV to 550 kV	19 ft 0 in.	19 ft 0 in.	11 ft 10 in.
765 kV to 800 kV	23 ft 9 in.	23 ft 9 in.	15 ft 11 in.

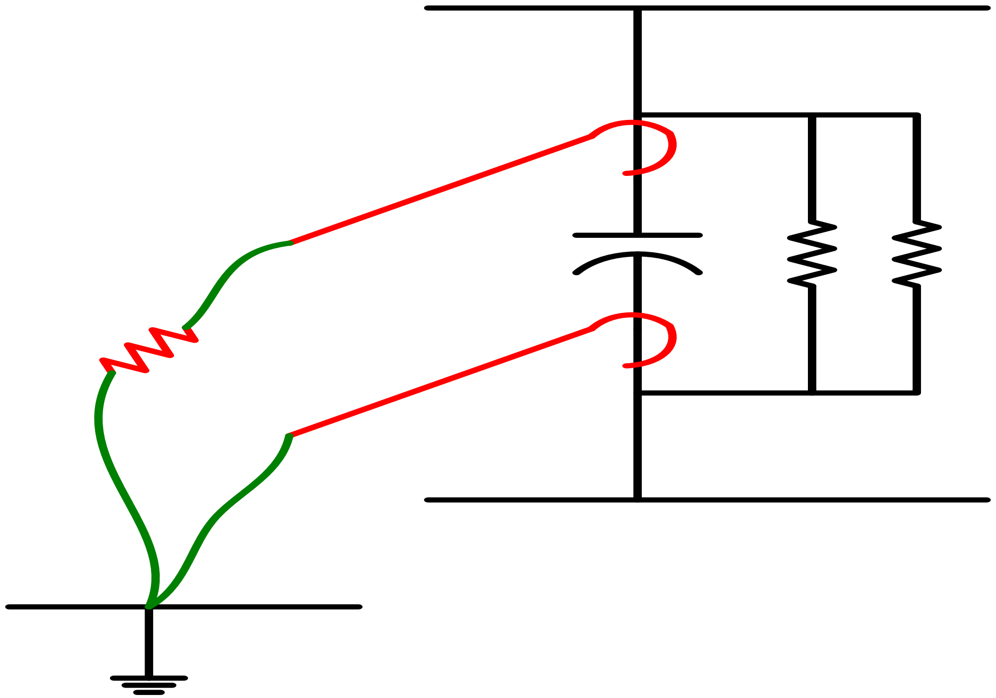
NFPA 70E - Approach boundaries – DC, Table 130.4(D)(b)

(1)	(2)	(3)	(4)
	Limited Approach Boundary		Restricted Approach Boundary; Includes Inadvertent Movement Adder
Nominal Potential Difference	Exposed Movable Conductor*	Exposed Fixed Circuit Part	
Less than 50 V	Not specified	Not specified	Not specified
50 V–300 V	3.0 m (10 ft 0 in.)	1.0 m (3 ft 6 in.)	Avoid contact
301 V–1 kV	3.0 m (10 ft 0 in.)	1.0 m (3 ft 6 in.)	0.3 m (1 ft 0 in.)
1.1 kV–5 kV	3.0 m (10 ft 0 in.)	1.5 m (5 ft 0 in.)	0.5 m (1 ft 5 in.)
5 kV–15 kV	3.0 m (10 ft 0 in.)	1.5 m (5 ft 0 in.)	0.7 m (2 ft 2 in.)
15.1 kV–45 kV	3.0 m (10 ft 0 in.)	2.5 m (8 ft 0 in.)	0.8 m (2 ft 9 in.)
45.1 kV– 75 kV	3.0 m (10 ft 0 in.)	2.5 m (8 ft 0 in.)	1.0 m (3 ft 6 in.)
75.1 kV–150 kV	3.3 m (10 ft 8 in.)	3.0 m (10 ft 0 in.)	1.2 m (3 ft 10 in.)
150.1 kV–250 kV	3.6 m (11 ft 8 in.)	3.6 m (11 ft 8 in.)	1.6 m (5 ft 3 in.)
250.1 kV–500 kV	6.0 m (20 ft 0 in.)	6.0 m (20 ft 0 in.)	3.5 m (11 ft 6 in.)
500.1 kV–800 kV	8.0 m (26 ft 0 in.)	8.0 m (26 ft 0 in.)	5.0 m (16 ft 5 in.)

Mitigating Voltage Hazard - Rubber Electrical Insulating Gloves

- They are marked with the class appropriate for the voltage, and should be subject to periodic electrical tests
- Leather protective gloves should be worn outside the rubber gloves to provide protection from cuts, abrasions, or punctures
- Before each use, check for signs of damage or color change. Replace if contamination or any physical damage is evident
- Gloves should be stored in a closed, dry container

The possibility of residual voltage on capacitors is high. Use one or more ground stick to remove the voltage (stored energy)



- *Short circuit through air*
- *Caused when circuit insulation or isolation is compromised*
- *A burn and explosion hazard, not an electrocution hazard*
- *Temperature can greatly exceed 5000 F*
- *Instantaneous, almost too fast for the eye to comprehend*
- *Arc flashes occur 5 – 10 times a day in electric equipment in US alone.*

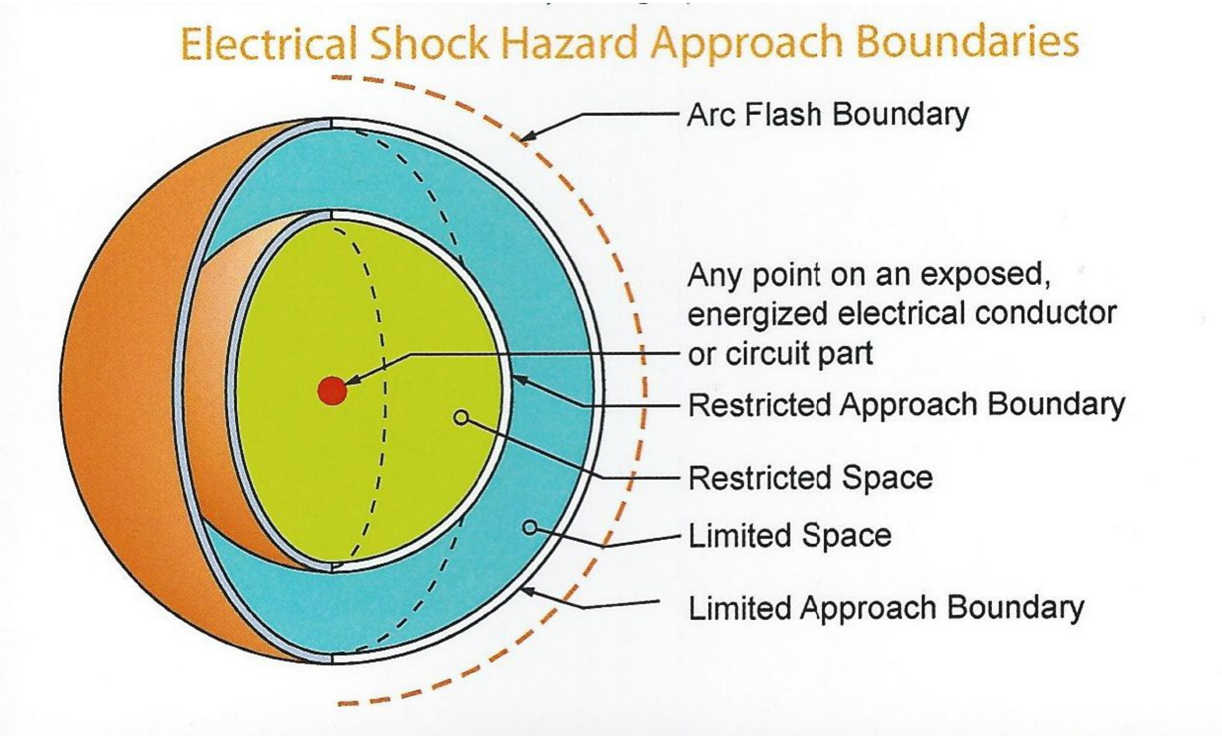
NFPA 70E - Possible Causes of Arc Flash

- *Tool inserted or dropped into a breaker or service area*
- *Equipment cover removal causes a short*
- *Loose connections on bus work*
- *Improper bus work fabrication*
- *Insulation breakdown due to environmental factors or equipment aging*
- *Failure to ensure equipment is de-energized before work*
- *Primarily applications above 208 VAC*

Injuries Associated with Arc Flash

- *Third Degree Burns, Blindness, Hearing Loss, Nerve Damage, Cardiac Arrest, Concussion, Death*

NFPA 70E - The Arc Flash Hazard



- *Arc flash hazard - a dangerous condition associated with the release of electrical energy caused by an electrical arc. Typically due to the molten plasma formed by the melting of conductors during an electrical short circuit*

- *Arc flash protection boundary - The distance from exposed live parts within which a person could receive a second degree (curable) burn (1.2 cal/cm ^{2} = 5 J/cm ^{2})*

- *An arc generates power that radiates out from a fault*

$$P_{arc} = V_{arc} * I_{arc}$$

- *The total energy is the product of the arc power and duration of the arc*

$$E_{arc} = P_{arc} * t$$

- *The energy density decreases with distance from the arc*
- *An arc-flash hazard occurs when the energy density on the torso or face exceeds 1.2 cal/cm^2, the energy density at which a second degree burn occurs. Note: This is comparable to holding the flame from a cigarette lighter on your skin for 1 second*
- *Flash protection boundaries and energies are calculated using NFPA 70E [example Table 130.7(C)(9)(a)] and IEEE1584*
- *The calculations entail knowing the voltage class of the equipment, some details about its manufacture, the available short circuit and the opening times of the protective circuit breaker(s)*

- *The hazard/risk category is determined by selecting the row for which $E_{min} \leq E < E_{max}$ at the working distance.*

E_{min} (cal/cm²)	**E_{max} (cal/cm²)**	**Hazard/Risk Category**
1.2	4	1
4	8	2
8	25	3
25	40	4

- *The allowable working distances are determined from:*
  - *Table 130.7(C)(15)(a) for AC systems*
  - *Table 130.7(C)(15)(b) for DC systems*

- *The appropriate Personal Protective Equipment (PPE) is determined from*
  - *Table 130.7(C)(15)(c)*

- *Decrease available energy by using smaller upstream transformer (lower short circuit current)*
- *Decrease clearing time*
  - *Size breaker trip units more aggressively*
  - *Choose breakers for instantaneous trip times (smaller frame sizes generally trip faster than larger frame sizes)*
  - *Choose breakers with adjustable trip units including adjustments for instantaneous trips*
- *Protective devices upstream of transformers need to allow "inrush" current when transformer is energized. Using only upstream sensors, it is difficult to be as aggressive as desirable for arc-flash protection downstream of transformer. Add overcurrent devices on transformer secondary*

- *Insert fast acting breakers or fuses in separate enclosures between the transformer and the equipment that needs to be operated. In general, separate the enclosures contain arc-flash generated in that enclosure*
- *Increase distance between worker and source of arc-flash*
  - *Use remote controls to operate high arc-flash hazard devices*
  - *Use extension handles on breakers to increase working distance of operation*
  - *Install meters to use for verification that system is de-energized if work is required on system*
  - *Install IR view-ports on panels that need to be monitored for over-temperature*
- *Install protective devices that sense arcs and not just overcurrent*

More information

- http://ieeexplore.ieee.org/servlet/opac?punumber=8088
- NFPA 70E 2021 Edition
- http://www.eaton.com/ecm/idcplg?IdcService=GET_FILE&dID=12075
- http://www.eaton.com/ecm/idcplg?IdcService=GET_FILE&dID=118182

National Electrical Code NFPA 70

- *Deals with hardware design, inspection and installation*
- *Most Articles do not pertain directly to power systems, but some examples that do are:*

1. *Sizing of raceways and conduits to carry power and control cables.*
2. *Sizing of power cables for ampacity.*
3. *Discharge of stored energy in capacitors*

Example of cable ampacity sizing

A power supply provides 375A to a magnet via cables. The ambient temperature is 45C (104F), maximum and the cables are installed in cable tray. The cable tray fill conforms to the requirements of NEC Article 392.

Use NEC Table 310-15(B)(17) for single conductor cables in free air at 30C. The derating for the 45C ambient is 0.87. The derating for the single copper conductor with 90C insulation and 600V rating in a cable tray is 0.65 if placed touching other cables in the cable tray. The required amapcity is

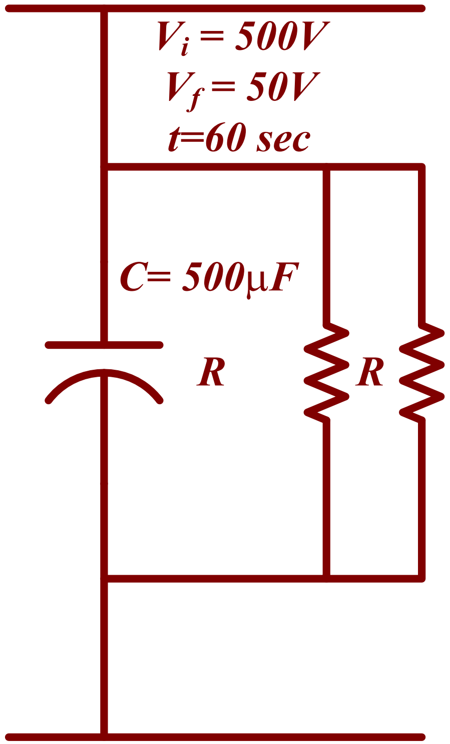
$$Ampacity = \frac{I_{PS}}{deratings} = \frac{375\,A}{0.87 * 0.65} = 663\,A$$

From Table 310-15(B)(17) the basic amapcity of 500kcmil cable is 700A > 663A.

Use two 1/C500kcmil cables to connect the PS to the magnet

Example of capacitor bleeder resistor sizing per NEC Article 460. Code requires permanent fixed energy discharge devices on capacitors operating at > 50V working voltage

- $\leq$ 1,000 V , discharge to 50 V or less in 1 minute
- > 1,000 V , discharge to 50 V or less in 5 minutes
- Redundant bleeder resistors recommended



$$V_f = V_i e^{\frac{-t}{RC}}$$

$$R = \frac{-t}{C \ln(V_f / V_i)} = \frac{-60 * sec}{500\,\mu F \ln(50V / 500V)}$$

$$R = 50\,kohm$$

$$P_R = \frac{V_i^2}{R} = \frac{(500V)^2}{50k\Omega} = 5W$$

Use two 5W, 100kΩ resistors in parallel

3 Types

- *Personnel Protection System (PPS)*
- *Load Protection - Machine or Magnet Protection System (MPS)*
- *Power Supply Protection – Power Supply Internal Interlocks*

Personnel Protection System (PPS) at SLAC

- *Protection from hazards external to power supply (example accelerator housing door opened)*

- *Hazards are defined as AC voltages > 50 V, and currents > 5m A, DC voltages > 100V, and currents > 40mA.*

- *Capacitor energy storage 100V and 100 J, or 400V and 1J, or 0.25J*

- *Must be hardwired (recently SLAC introduced PLC-based PPS)*

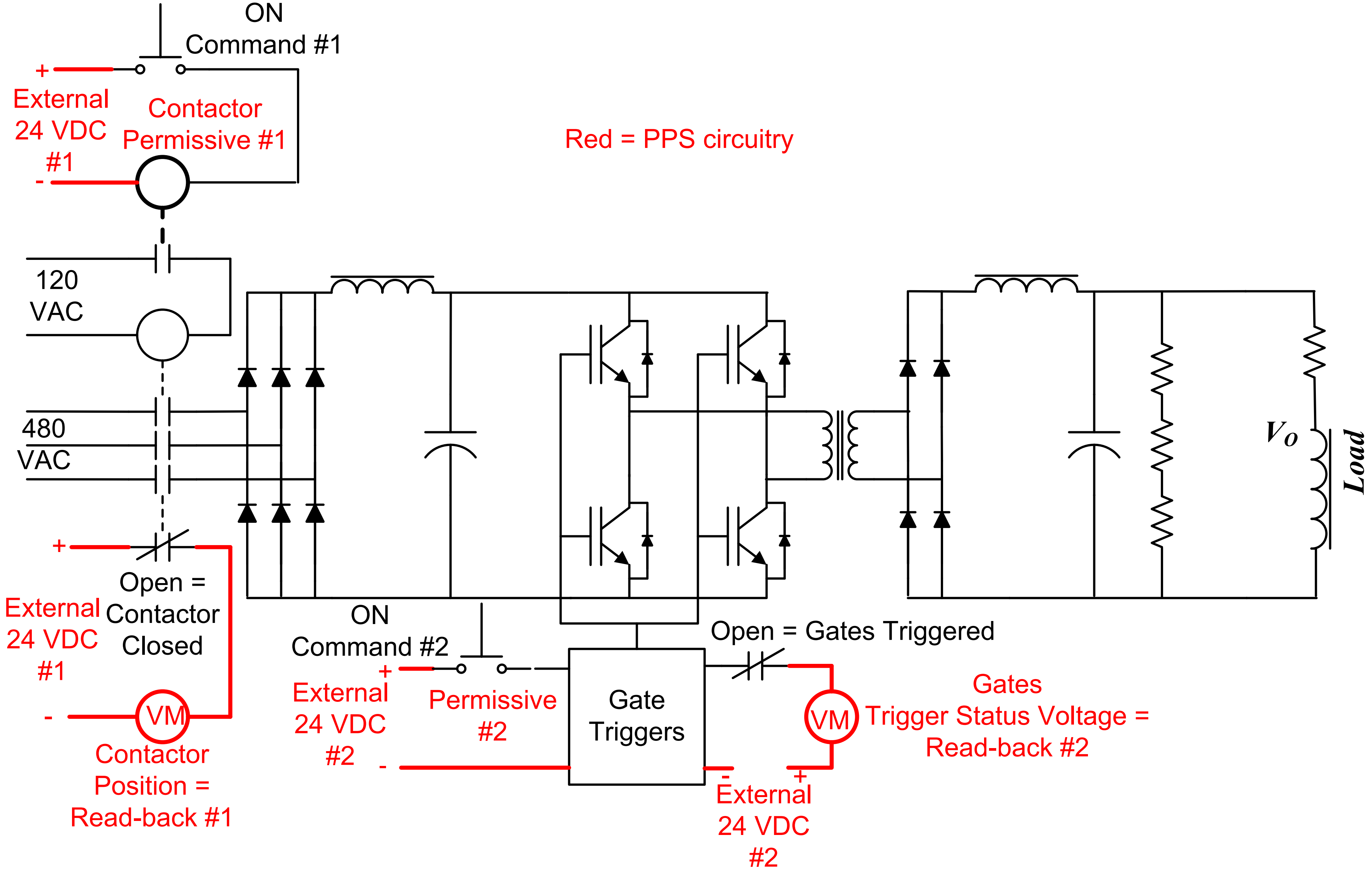
- *Two (2) PPS permissives are needed for power supply turn-on*

- *Two (2) separate and different read-backs are required*

- *Permissives and read-backs are usually 24 VDC systems*

- *Permissives and read-backs must be fail-safe*

- *If PPS is not practical, then energized equipment must be enclosed or live terminals covered*



Many variations of this example

Machine (or magnet) protection systems protect loads from damage.

Magnet Cooling Water Temperature / Flow Sensors

- *Usually employ a simple normally closed (NC) contact that opens when a pre- determined temperature has been reached.*

- *Water flow monitoring switches open when flow drops below a pre-established safe value*

- *Temperature / Flow switches are wired to the source power supply. If the water temperature is too high or if the flow drops the contacts open and turn the power supply off*

Vacuum Interlock System

- *Sensors are similar to that described in the magnet cooling water system*

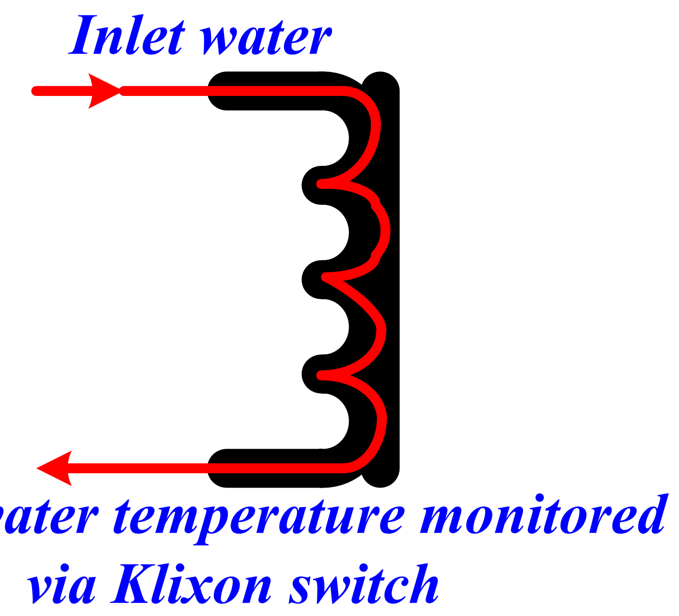
Orbit Interlock System

- *Sensors consist of Beam Position Monitors and switches. Function is essentially the same in the magnet cooling water system*

• *Thermal switches - Klixons (a trade name) are NC contact bimetal switches mounted on the load cooling water outlet line. Their contacts open when temperature exceeds a pre-established safe value*

• *Multiple-winding, multiple water path magnets employ simple series connected Klixons.*

• *Klixons are wired to the source power supply. If the load overheats, the contacts open and turn off the power supply*

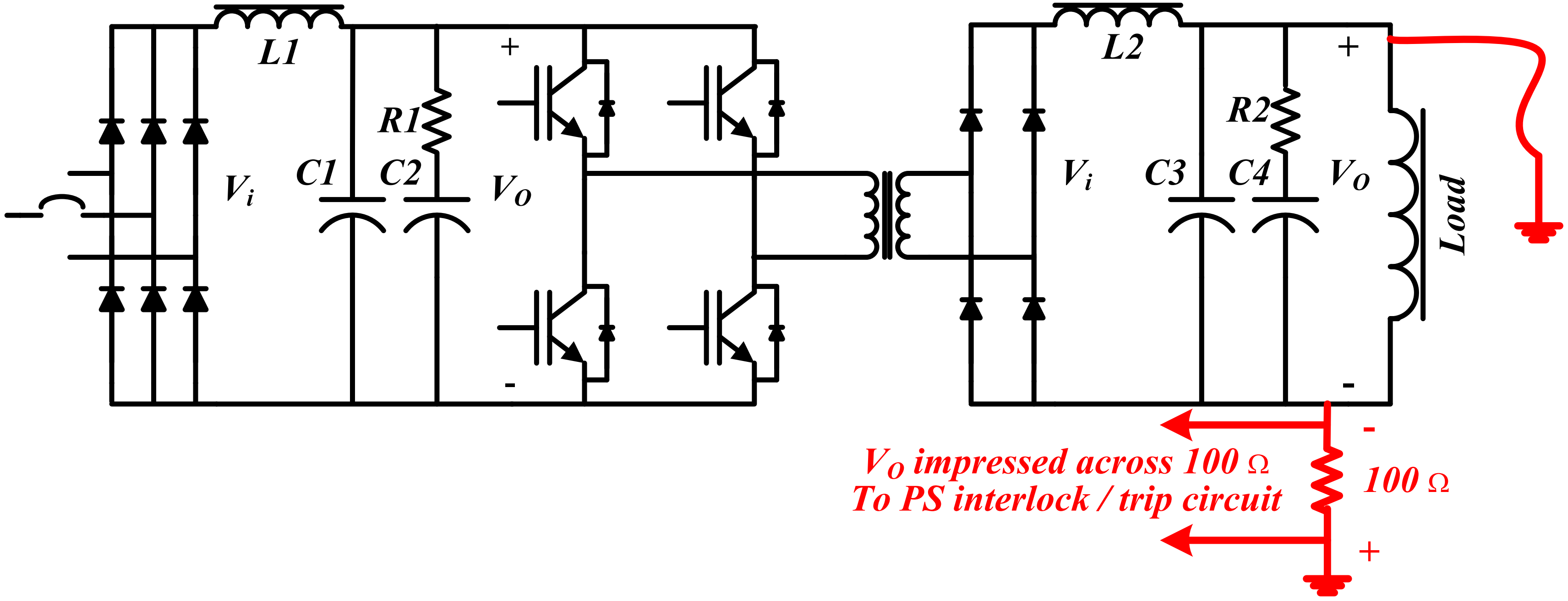


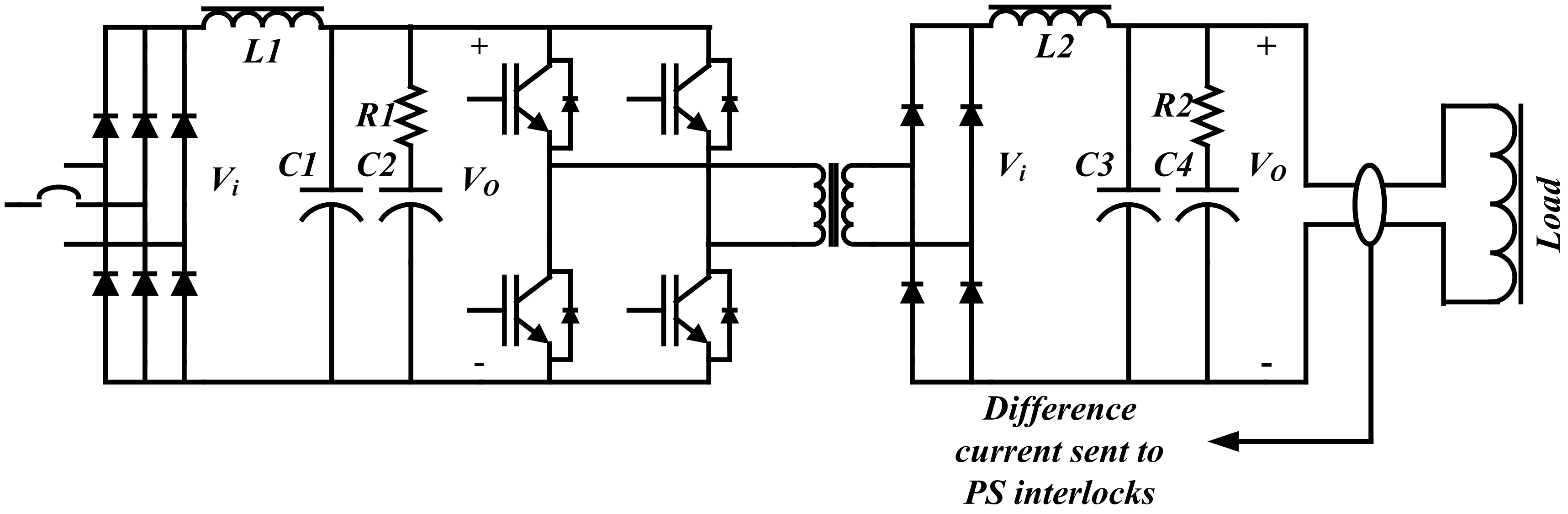
Klixon switches

Ground Fault Detection / Protection Systems

• Loads are usually located in crowded, dense areas with a multitude of other equipment. This makes them vulnerable to ground faults

• Power supplies are usually isolated from ground so that a single ground fault does not cause load-catastrophic ground fault current. Fix first fault before the second fault occurs




L1
R1
C1
C2
V_i
V_O
+
-
L2
R2
C3
C4
V_i
V_O
+
-
Load
Difference current sent to PS interlocks

Some Internal Interlocks

Internal interlocks protect the power supply itself

- *Low input supply voltage*
- *Phase loss detection*
- *Output DC over-current*
- *Low frequency filter inductor temperature*
- *Heat-sink temperature or heat-sink cooling water flow*
- *IGBT temperature*
- *IGBT over-current*
- *Ground Fault current*
- *Output over-voltage*
- *Cabinet or chassis over-temperature*

Example of a PLC and its Use

Manufacturers are many

•Allen-Bradley

•Rockwell International (AB)

• Siemens

• General Electric

• IDEC

Programming logic

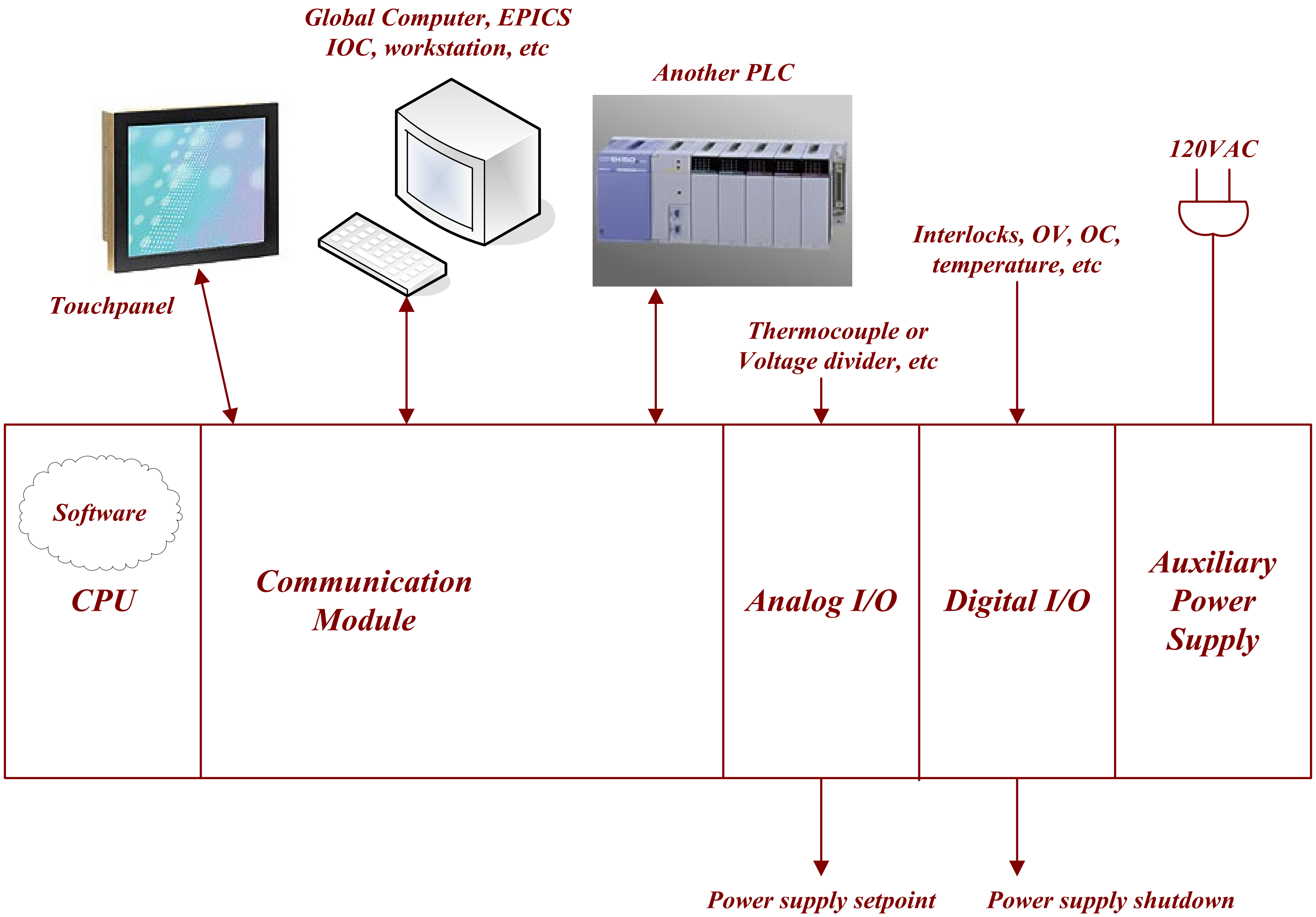
• Ladder logic

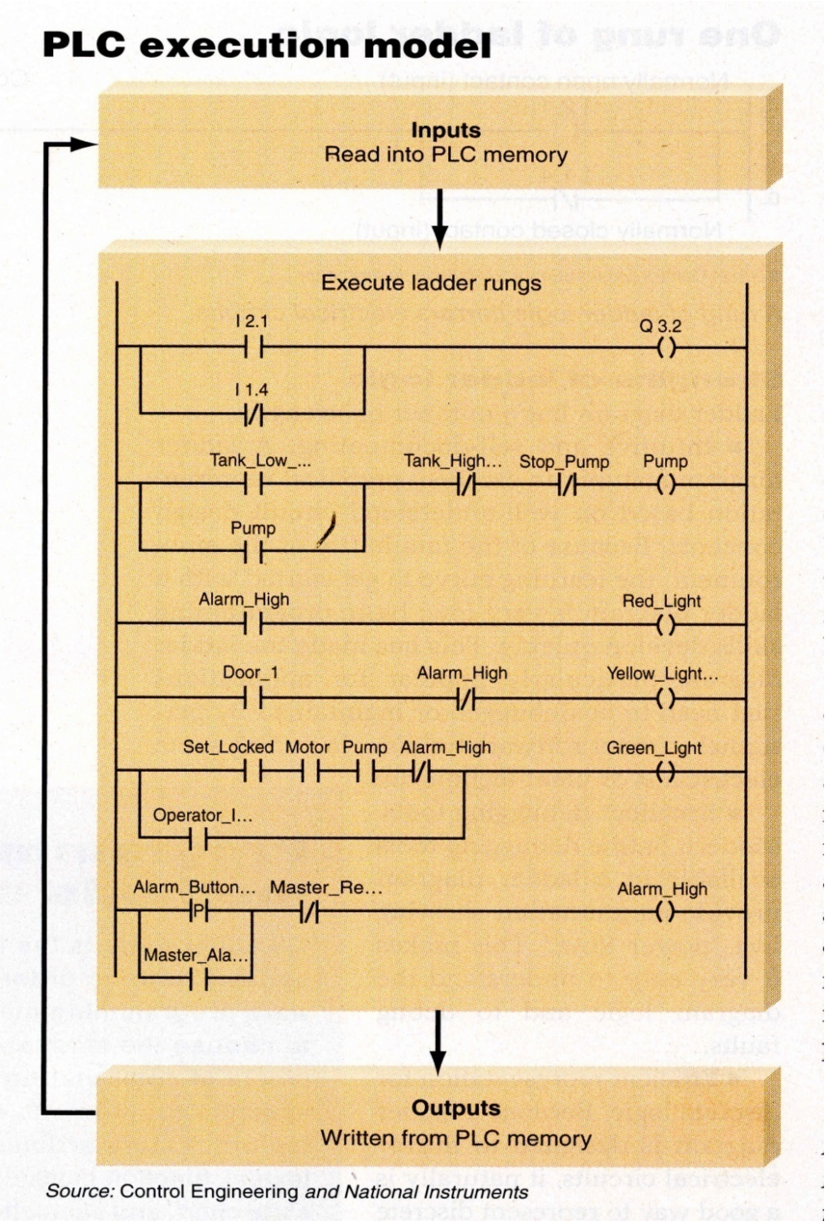
• C language

• LabView

• Functional block diagrams

• Structured text


Global Computer, EPICS IOC, workstation, etc
Another PLC
120VAC
Interlocks, OV, OC, temperature, etc
Touchpanel
Thermocouple or Voltage divider, etc
Software
CPU
Communication Module
Analog I/O
Digital I/O
Auxiliary Power Supply
Power supply setpoint
Power supply shutdown



Ladder diagrams evolved in the 1960s when the automobile industry needed a more flexible and self-documenting alternative to relay and timing cabinets. A microprocessor was added and software designed to mimic the relay panels.

Left rail is the "power bus". The right rail is the "ground bus". Power flows through NO or NC contacts to power coils.

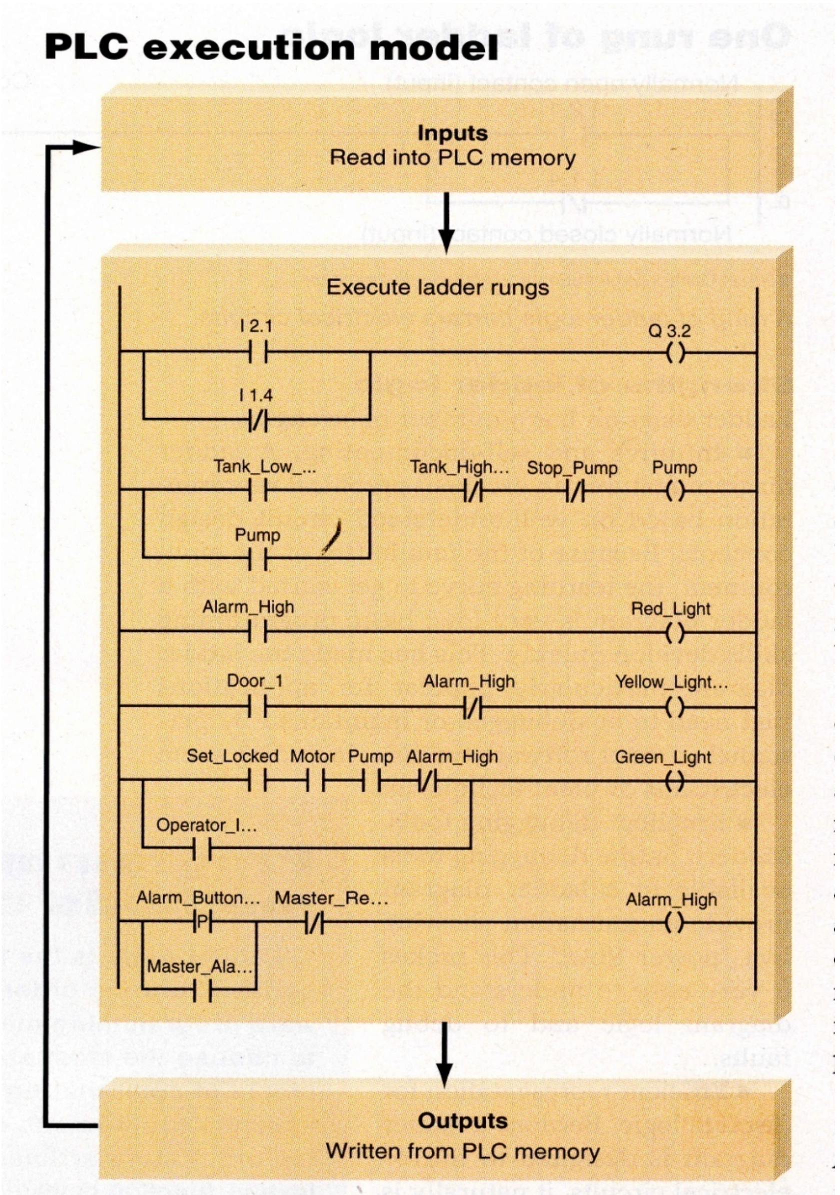
Each contact and coil is linked to a Boolean memory location.

Series contacts look like "AND" and parallel contacts look like "OR"

Execution is left to right and top to bottom

Ladder Logic

Most widely used to program PLCs



Source: Control Engineering and National Instruments

Strengths

- *Intuitive – can be learned very quickly by with little or no software training*
- *Excellent debugging tools, include animation showing live "power flow". This makes the logic easy to understand and debug*
- *Efficient representation for discrete logic*

Weaknesses

- *Hierarchical data and logic flow.*
- *Poor data structure. Rungs are executed in a left-to-right, top-to-bottom order. Timing is limited by the PLC processor speed*
- *Limited execution control*
- *Arithmetic operations are limited*

PLCs implement specific functions such as:

I/O control	*Timing*	*Report generation*	*Arithmetic*
Logic	*Communication*	*Data file manipulation*	*Counting*

PLC Versus Programmable Automation Controllers (PAC)

Consider a PAC upgrade if your application requires:

- *advanced control algorithms*
- *extensive database manipulation*
- *HMI functionality in one platform*
- *Integrated custom control routines*
- *complex process simulation*
- *very fast CPU processing*
- *memory requirements that exceed PLC specifications*

Lock & Tag for Personnel Safety During Maintenance

- *Procedures and requirements for servicing and maintaining machines and equipment*
- *Provision for locking off source power, the discharge of stored energy prior and the total de-energization of equipment before working on exposed electrical circuits or other hazardous equipment in which unexpected energization, startup or release of energy could cause injury to personnel*

Required by

- *Occupational Safety and Health Administration (OSHA) under 29CFR1910.147*

Applicability

- *For working on exposed electrical circuits that would expose personnel to any electrical hazard as defined by the Codes. All types of equipment containing electrical, mechanical, hydraulic, pneumatic, chemical and/or thermal active or stored energy*

Items Locked Out (Off) – Tagged Out (Off)

The power source or power device

Application by

Authorized employee trained in LOTO and qualified to lock-off the equipment

Interlocks As LOTO

Interlocks are not used as a substitute for lock and tag

For Locking and Tagging

- *Padlocks, usually red-colored for personal use. Yellow-colored for administrative lock-out*
- *Tags*
- *Specialty locks (Kirk-Key Locks) for complex systems*
- *Master lock boxes*

Section 12 - Reliability, Availability and Maintainability

Reliability

According to IEEE Standard 90, reliability is the ability of a system or component to perform its required functions under stated conditions for a specified period of time

Availability

The degree to which a system, subsystem, or equipment is operable and in a committable state during a mission (accelerator operation).

The ratio of the time a unit is functional during a given interval to the length of the interval. $Availability = MTBF/(MTBF + MTTR)$

Importance

Accelerators are expensive. They are expected to perform to justify their cost. Reliability is important because accelerators are expected to perform like industrial factories; i.e., to be on-line at all times. In particular, accelerator power supplies are expected to be available when needed, day after day, year after year. Reliability must be considered when subsystems are complex or when they contain a large part count. An accelerator composed of a large number of systems or parts simply will not function without considering reliability.

Consequences

Failures lead to annoyance, inconvenience and a lasting user dissatisfaction that can play havoc with the accelerator's reputation. Frequent failure occurrences can have a devastating effect on project performance and funding.

PDF, CDF and MTBF

In this section we attempt to estimate the lifetime of complex systems. Each component of these systems will fail at a random time . Knowing the failure rates of the components, we use probability theory to estimate the system reliability (probability of success) and lifetime

We begin by introducing the non-negative probability density funtion (PDF), f(t). We then define a cumulative distribution function (CDF), F(t)which has specific properties

- *There is no probability that the component has failed before being built, so* $F(-\infty)=0$
- *It is certain that at some point in time the component will fail, so with F(t) normalized,* $F(\infty)=1$
- *F(t) is an increasing function of t.*
- *Lastly* $0 \le F(t) \le 1$

The CDFcan be expressed in terms of the PDF, $F(t)=\int_{-\infty}^{t} f(t)\,dt$ *or more typically* $F(t)=\int_{0}^{t} f(t)\,dt$

f(t) is normalized such that $F(t)=\int_{-\infty}^{\infty} f(t)\,dt=1$

The probability that the component (hence system) has failed between t_1 *and* t_2 *is* $\int_{t_1}^{t_2} f(t)\,dt = F(t_2)-F(t_1)$

The average value of time that components of this type will fail is given by $\langle t\rangle = t\int_{0}^{t} f(t)\,dt = MTBF = MTTF$

where MTBF and MTTF are the mean time between failure or mean time to fail, respectively

One probability density (distribution) function is the exponential distribution. It accurately predicts the lifetime of a component with an exponential decay, e.g., the lifetime of radioactive particles. Although there are other distributions that might be more appropriate, the exponential works reasonably well for a large class of components and is easy to use.

$f(t) = \lambda e^{-\lambda t}$ *where* λ = *failure rate of the component (number of failures / time)*

$$\int_0^{\infty} \lambda e^{-\lambda t}\, dt = 1$$

$$F(t) = \int_0^{t} \lambda e^{-\lambda t} dt = 1 - e^{-\lambda t}$$

where $1 - e^{-\lambda t}$ = *probability of failure*

lastly $\langle t \rangle = 1/\lambda$ = *MTBF = time (usually hours)*

We now define the reliability $R_i(t)$ of the i^{th} component as the probability that the component is still functioning after a time t. We also define a complementary function $Q_i(t)$ that gives the probility that the component has failed

$Q_i(t) = 1 - e^{-\lambda t}$ *and since probability of failure=1 - reliability we see that*

$R_i(t) = e^{-\lambda t}$ = *reliability (probability of success)*

A series system is such that all subsystems or elements must work in order for the entire system to work. For such a system the total system reliability is the product of the individual component reliabilities

$R_T = R_1 * R_2 * .... * R_n = \prod_1^n R_i =$ *probability of system success*

The probability of system failure is

$Q_T = 1 - R_T = 1 - \prod_1^n R_i = 1 - \prod_1^n (1 - Q_i)$

For a two component system $R_T = R_1 * R_2$

and $Q_T = 1 - (1 - Q_1)(1 - Q_2) = Q_1 + Q_2 - Q_1 * Q_2$

The probability of system failure is less than the sums of the probabilities for each component because of the subtraction of the failure probability products

A parallel system is such that only one subsystem or element must work in order for the entire system to work. For such a system it is easier to calculate the total system reliability by first calculating the probability of the total system failure, since all elements must fail in order for the entire system to fail. Therefore

$$R_T = 1 - Q_T = 1 - \prod_1^n Q_i = 1 - \prod_1^n (1 - R_i)$$

General Systems

A general system will not be simply series or parallel. It might have some redundancy, meaning that some, but not all, of the subsystems need to work for the entire system to be functional. We break the system into individual components and examine every possible combination of the states, working or failed. These combinations are all mutually exclusive, so we just sum the probabilies of each functioning combination to get the probability of system success.

Consider a parallel system of 3 identical units requiring 2 to work for a functioning system

There are $2^n = 8$ *mutually exclusive states to examine*

$Q_1 * Q_2 * Q_3, \;\; Q_1 * Q_2 * R_3, \; Q_1 * R_2 * Q_3, \;\; Q_1 * R_2 * R_3,$

$R_1 * Q_2 * Q_3, \;\; R_1 * Q_2 * R_3, \; R_1 * R_2 * Q_3, \;\; R_1 * R_2 * R_3$

Of these states the fourth, sixth, seventh and eighth describe a functing system. Therefore the total system reliability is

$R_T = Q_1 * R_2 * R_3 + R_1 * Q_2 * R_3 + R_1 * R_2 * Q_3 + R_1 * R_2 * R_3$

Recognizing that $Q_i + R_i = 1$

$R_T = Q_1 * R_2 * R_3 + R_1 * Q_2 * R_3 + R_1 * R_2 * Q_3 + R_1 * R_2 * (1 - Q_3)$

$R_T = Q_1 * R_2 * R_3 + R_1 * Q_2 * R_3 + R_1 * R_2$

The counting on the previous page gets complicated very quickly. Fortunately the calculations can be expressed in a combinational formula which gives the system reliability for m of n components connected in parallel

$$R_T = \sum_{k=m}^{n} \frac{n!}{(n-k)!k!}(R_k)^k (Q_k)^{n-k}$$

For a system described by an exponential distribution

$$R_T = \sum_{k=m}^{n} \frac{n!}{(n-k)!k!}(e^{-\lambda_k t})^k (1-e^{-\lambda_k t})^{n-k}$$

Failure rate is constant	λ	(hr^{-1})
Mission time	t	(hr)
Probability Density Function (PDF)	$f(t) = \lambda e^{-\lambda t}$	$(dimensionless)$
Cumulative Density Function (CDF)	$F(t) = 1 - e^{-\lambda t}$	$(dimensionless)$
Reliability (Success probability)	$R(t) = e^{-\lambda t}$	$(dimensionless)$
Expected time to failure (MTBF)	$E(T) = \int_{-\infty}^{\infty} t\, f(t)\, dt = \frac{1}{\lambda}$	(hr)

Glossary - Math Expressions

Failure rate of N series critical components $\lambda_{composite} = \sum_{i=1}^{N} \lambda_i$ (hr^{-1})

Reliability of N series components $R_T(t) = \prod_{i=1}^{N} e^{-\lambda_i t} = \prod_{i=1}^{N} R_i(t)$ (dimensionless)

Failure N series components $Q_T(t) = 1 - R_T(t) = 1 - \prod_{i=1}^{N}(1 - Q_i(t))$ (dimensionless)

Reliability of N parallel components $R_T(t) = 1 - \prod_{i=1}^{N}(1 - R_i(t))$ (dimensionless)

The reliability of parallel connected m out of n components

$$R_{system}(t) = \sum_{k=m}^{n} \left(\frac{n!}{(n-k)!\,k!}\right)\left(e^{-\lambda_k t}\right)^{k}\left(1 - e^{-\lambda_k t}\right)^{n-k} \quad \text{(dimensionless)}$$

λ_k = constant = failure rate of individual component

k= index counter, m= minimum number of components needed for operation

n = total number of components in the system

Special cases occurs when $m = n$ or when $m=n=1$

$R(t) = e^{-n\lambda t}$ $R(t) = e^{-\lambda t}$

MTBF of series critical components	$MTBF = 1/\lambda_{composite}$	(hr)
MTBF of N series identical components	$MTBF_{composite} = MTBF_i / N$	(hr)
Mean time to repair or recover is	$MTTR$	(hr)
Availability is	$A = \dfrac{MTBF}{MTBF + MTTR}$	(dimensionless)
Availabilty of series components	$A_{composite} = \prod_{i=1}^{N} A_i$	(dimensionless)
Availbilty of identical components	$A_{composite} = A^N$	(dimensionless)

Glossary of Terms and Definitions

Availability	*Ratio of operating time to operating + downtime A=MTBF/(MTBF+MTTR). This is a dimensionless number*
MTBF	*Mean time between failures in hours*
$MTBF_O$	*The increased MTBF in hours that considers equipment operation at lower than rated power levels*
$MTBF_R$	*MTBF with operation at ratings - in hours*
MTTR	*The mean time to repair and recover beam in hours*
R(t)	*Reliability or probability of success over the mission time* *(Typically 9 months = 6600hours)*
λ, λ_O, λ_R	*Failure rates in* hr^{-1}*. These are the reciprocals of the MTBFs*
1/1	*One full rated power supply. Rated power = delivered power*
1/2	*One out of two redundant power module configuration*
2/3	*Two out of three redundant power module configuration*
3/4	*Three out of four redundant power module configuration*
4/5	*Four out of five redundant power module configuration*

Homework Problem # 17

A. At least 1 of 4 parallel identical power supplies in an accelerator must continue to operate for the system to be successful. Let R_i =0.9. Find the probability of success.

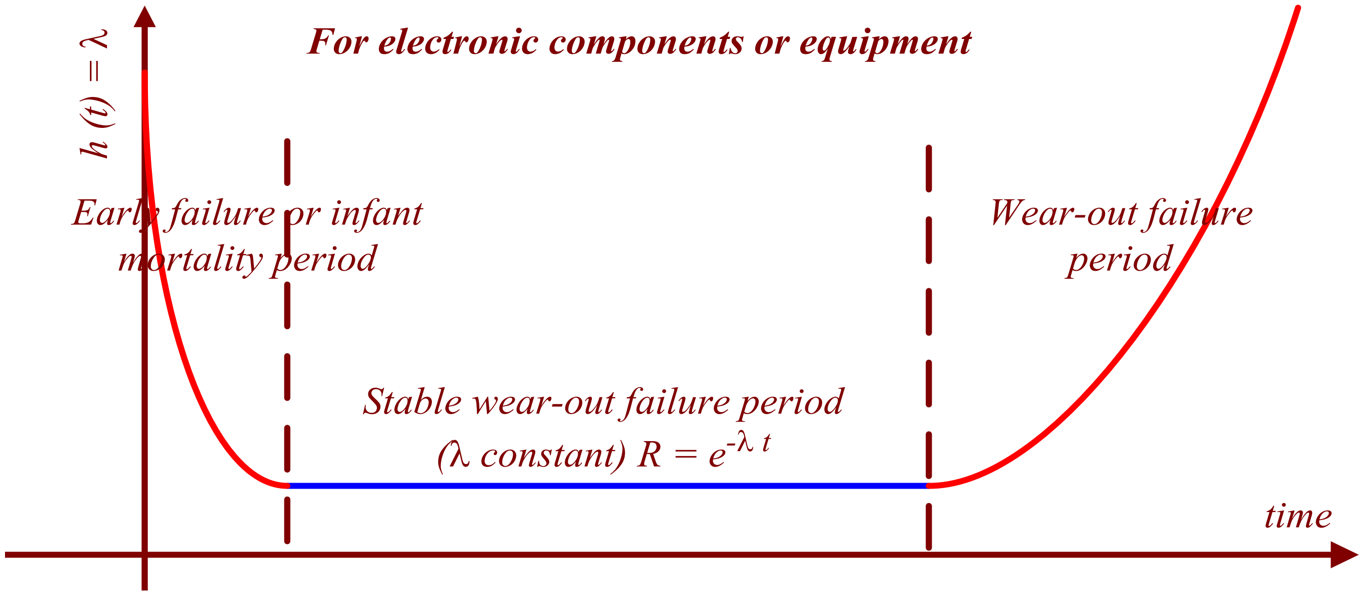
B. Repeat for at least 2 out of 4 success

C. Repeat for at least 3 out of 4 success

D. Repeat for 4 out of 4 success

Solution:

Glossary - Failure Rate Curve



Weibull Distribution
Bathtub Curve

- ***Infant mortality*** *manufacturing defects, dirt, impurities. Infant mortality reduced for customer by burn-in and stress-screening*
- ***Stable wear-out*** *statistics, manufacturing anomalies, out-of tolerance conditions*
- ***Wear-out failure*** *dry electrolytic capacitors, aged and cracked cable insulation*

Reliability Calculation Standards

MIL-HDBK-217F (USA)	• *Internationally used* • *Parts count* • *Parts stress* • *Broad in scope* • *Pessimistic*
Telcordia (Bellcore) (USA)	• *National use* • *Parts count* • *Parts stress* • *Narrow scope (telecommunications)* • *Optimistic*
CNET 93 (France)	• *Limited to France* • *Parts count* • *Parts stress* • *Broad in scope*
HRD5 (UK)	• *Limited to UK* • *Parts count* • *Parts stress* • *Broad in scope*

Parts Count

- Appropriate failure rate is assigned to each part in the subsystem (power supply) that is mission critical
- Failure rates are functions of environment (Ground fixed Π_{GF} /Ground benign Π_{GB} /Ground mobile, Π_{GM}) and ambient temperature (Π_T)
- The parts count method is simple and used early in system design when detailed information is unknown
- Failure rates are summed and the following information is obtained

$$MTBF = \frac{1}{\sum \lambda} \qquad R(t) = e^{-\sum \lambda t}$$

Parts Stress *– Same as the Parts Count method, except it takes into account more detailed information about the components and their operating stresses. The detailed information is implemented via additional Π reliability factors, such as:*

Π_{GB} = ground benign $0 < \Pi_{GB} < \infty$

Π_{T} = ambient temperature $0 < \Pi_{T} < \infty$

Π_{MQ} = manufacturing quality $0 < \Pi_{MQ} < \infty$

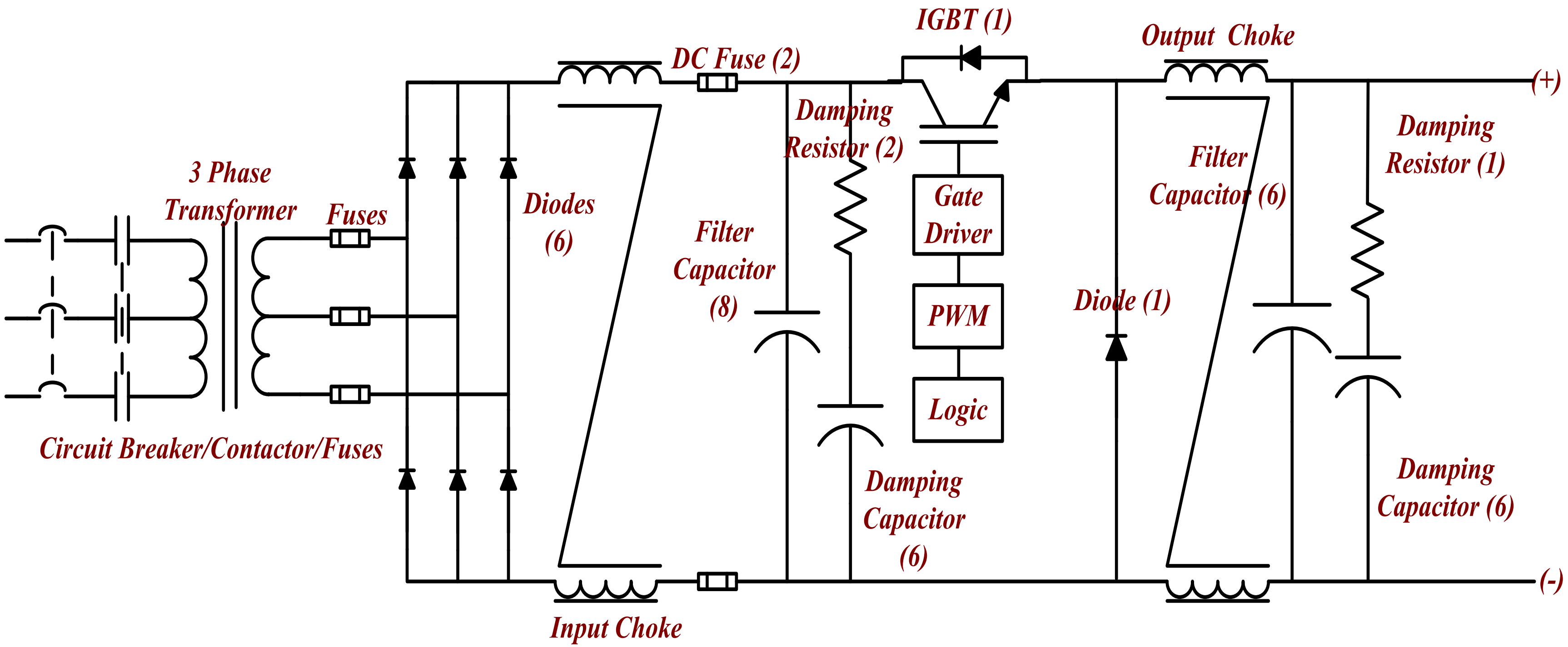
Π_{VS} = voltage stress factor $0 < \Pi_{VS} < \infty$

Π_{IS} = current stress factor $0 < \Pi_{IS} < \infty$

Π_{PS} = power stress factor $0 < \Pi_{PS} < \infty$

$$\lambda_{resultant} = \lambda_{initial} * \Pi_{GB} * \Pi_{T} * \Pi_{MQ} * \Pi_{VS} * \Pi_{IS} * \Pi_{PS}$$

Example of Reliability Calculation – Power Supply



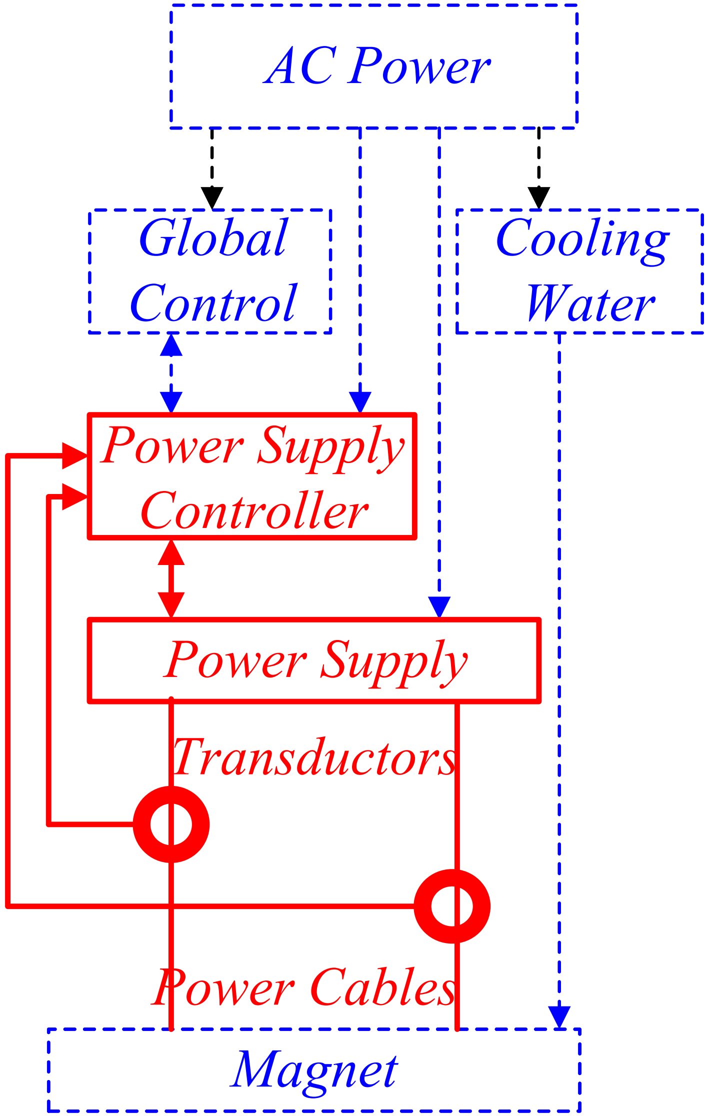
Example of Reliability Calculation – Power Supply

Component Description	Qty	λ	π_{GB}	π_T	π_{MQ}	π_{VS}	π_{IS}	π_{PS}	Mission Loss	Total Rate $\lambda_T\ 10^{-6}$
Circuit Breaker/Contactor/Fuse	5	0.42	1.00	1.10	1.00	1.01	1.05	1.10	Yes	2.695
3 Phase Transformer	1	0.05	1.00	1.10	1.00	1.50	1.50	1.50	Yes	0.186
Input/Output Filter Choke	2	0.02	1.00	1.10	1.10	1.42	1.60	1.75	Yes	0.144
Secondary/DC Link Fuse	2	0.08	1.00	1.10	1.89	1.02	0.95	0.90	Yes	0.291
Main Filter Capacitor	8	0.23	1.00	1.12	1.50	1.25	1.25	1.05	Yes	5.057
Damping Capacitors/Resistor	15	0.02	1.00	1.10	1.00	1.00	1.00	1.00	No	0.000
IGBT/Diode	8	0.03	1.00	1.10	1.50	1.00	1.00	1.00	Yes	0.330
Heatsink Assembly	1	0.01	1.00	1.10	1.00	1.00	1.00	1.00	Yes	0.011
Gate Driver/PWM	2	0.50	1.00	1.10	1.00	1.10	1.10	1.15	Yes	1.524
Logic Board	1	3.50	1.00	1.10	1.00	1.00	1.00	1.00	Yes	3.850
Output Filter Capacitor	6	0.25	1.00	1.10	1.00	1.25	1.25	1.00	Yes	2.578
MTBF and Total Failure Rate								60,000		16.667

Homework Problem # 18

A "typical commercial" 5 kW, switch-mode power supply consists of the components below with the listed failure rates. It also has critical electromechanical safety features amounting to 10% of the total number of components. The power supply operates at 50C ambient temperature. Assuming no derating for the elevated ambient temperature or other stress factors, calculate the power supply MTBF.

- *2 each ICs, plastic linear, λ = 3.64 failures per million hours each*
- *1 each opto-isolator, λ = 1.32 failures per million hours each*
- *2 each hermetic sealed power switch transistors, λ = 0.033 failures per million hours each*
- *2 each plastic power transistors, λ = 0.026 failures per million hours each*
- *4 each plastic signal transistors, λ = 0.0052 failures per million hours each*
- *2 each hermetic sealed power diodes, λ = 0.064 failures per million hours each*
- *8 each plastic power diodes, λ = 0.019 failures per million hours each*
- *6 each hermetic sealed switch diodes, λ = 0.0024 failures per million hours each*
- *32 each composition resistors, λ = 0.0032 failures per million hours each*
- *3 each potentiometers, commercial, λ = 0.3 failures per million hours each*
- *8 each pulse type magnets, 130C rated, λ = 0.044 failures per million hours each*
- *12 each ceramic capacitors, commercial, λ = 0.042 failures per million hours each*
- *3 each film capacitors, commercial, λ = 0.2 failures per million hours each*
- *9 each Al electrolytics, commercial, λ = 0.48 failures per million hours each*


AC Power
Global Control
Cooling Water
Power Supply Controller
Power Supply
Transductors
Power Cables
Magnet

Example of Reliability Calculation – Power System

Single System Availabilty

Component	MTBF	Availability
PS Controller	110,000	0.9999818
Power Supply	60,000	0.9999667
Transductor 1	381,500	0.9999948
Transductor 2	381,500	0.9999948
Cables	14,000,000	0.9999999
System	32,184	0.9999379

t=6574 hrs/year MTTR=2 hrs components/system

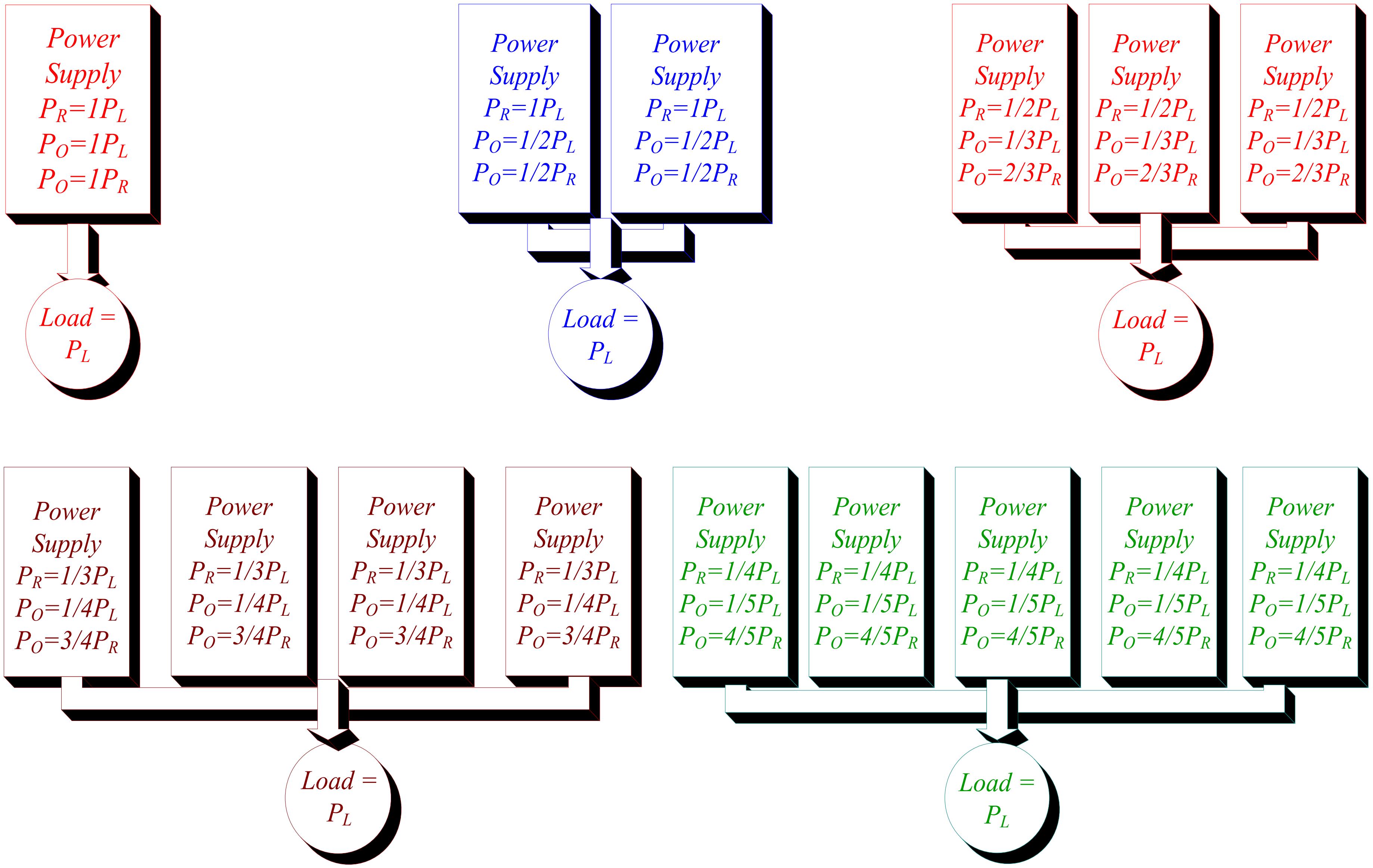
Relex by Relex Software

See Reference Appendix for web link to this manufacturer's products

RelCalc by T-Cubed

See Reference Appendix for web link to this manufacturer's products

Reliability/Availability Improvement By Redundancy



Two types - Standby and Active

1. *Standby - the redundant parts are off and only operate when the first part fails. This requires more vigilance on the part of the control system and is not covered here.*

2. *Active - the redundant part(s) are on, albeit operating at a reduced power level until asked to assume increased or full load. This is easier to implement than Standby redundancy and is the more common method. We will examine this further*

The general, exponential form of the Binomial Distribution for m out of n parts is

$$R(t) = \sum_{k=m}^{n} \left(\frac{n!}{(n-k)!k!} \right) \left(e^{-\lambda t} \right)^{k} \left(1 - e^{-\lambda t} \right)^{n-k}$$

λ = constant = failure rate

k = index counter

m = minimum number of power modules needed for operation

n = total number of power modules in the system

Special cases occurs when m = n or when m=n=1

$R(t) = e^{-n\lambda t}$ $\qquad$ $R(t) = e^{-\lambda t}$

Availability Improvement By Oversizing and Redundancy

Binomial Expansion 2 out of 3 example

$$R_{2/3}(t) = \sum_{k=m=2}^{n=3} \left(\frac{n!}{(n-k)!k!}\right)(e^{-\lambda t})^{k}(1-e^{-\lambda t})^{n-k}$$

$k = 2$

$$\frac{3!}{1!2!} e^{-2\lambda t}(1-e^{-\lambda t}) = 3 \; e^{-2\lambda t} \; (1-e^{-\lambda t})$$

3 cases, probability of success, probability of failure

S	S	F
S	F	S
F	S	S

3 Cases

$k=3$

$$\frac{3!}{0!3!} e^{-3\lambda t}(1-e^{-\lambda t})^{0} = 1 \; e^{-3\lambda t}$$

1 case, probability of success, no failure

S	S	S

1 Case

$$R_{2/3}(t) = 3e^{-2\lambda t} - 2e^{-3\lambda t}$$

Derivation

When λ(t) is a function of time

General form $R(t) = e^{-\lambda(t)t}$

$$\frac{dR(t)}{dt} = -\frac{d\lambda(t)}{dt} e^{-\lambda(t)t} - \lambda(t) e^{-\lambda(t)t}$$

$$\frac{d\lambda(t)}{dt} \text{ is} << \lambda(t)$$

$$\frac{dR(t)}{dt} = -\lambda(t) e^{-\lambda(t)t} \quad \text{but } e^{-\lambda(t)t} = R(t)$$

$$\lambda(t) = \frac{-\frac{dR(t)}{dt}}{R(t)}$$ *If λ is a constant then the above reduces to λ(t)=λ*

$$MTBF(t) = \frac{R(t)}{-\frac{dR(t)}{dt}}$$

For the m out of n case, where $m \neq n$

n quantity of $\frac{m}{n}$ *rated power supplies. Each power supply operates at* $\frac{m}{n}$ *rated* P_R

$$P_O = \frac{m}{n} P_R$$

$$MTBF_O = \frac{P_R}{P_O} MTBF_R = \frac{n}{m} MTBF_R \qquad \lambda_O = \frac{m}{n} \lambda_R$$ *linear relationship is conservative*

$$R_{Om/n}(t) = \sum_{k=m}^{n} \left(\frac{n!}{(n-k)!k!} \right) \left(e^{-\lambda_O t} \right)^k \left(1 - e^{-\lambda_O t} \right)^{n-k} = n e^{-m\lambda_O t} - m e^{-n\lambda_O t}$$

$$MTBF_{Om/n}(t) = \frac{n e^{-m\lambda_O t} - m e^{-n\lambda_O t}}{mn\lambda_O e^{-m\lambda_O t} - mn\lambda_O e^{-n\lambda_O t}}$$ (Using $MTBF = \frac{R(t)}{-\frac{dR(t)}{dt}}$)

$$A_{Om/n}(t) = \frac{MTBF_{Om/n}(t)}{MTBF_{Om/n}(t) + MTTR}$$

For the case of 1 power supply with a power rating equal to the required operational power

$$P_R = P_O$$

$$MTBF_R = MTBF_O$$

$$\lambda_R = \lambda_O$$

$$R_O = e^{-\lambda_O t} = e^{-\lambda_R t}$$

$$A_O = \frac{MTBF_O}{MTBF_O + MTTR} = \frac{MTBF_R}{MTBF_R + MTTR}$$

For the m=1 out of n=2 case

2- full rated rated power supplies. Each power supply operates at $\frac{1}{2}$ rated P_R

$$MTBF_O = \frac{P_R}{P_O} MTBF_R = 2\, MTBF_R \qquad \lambda_O = \frac{1}{2}\lambda_R$$

$$R_{O1/2}(t) = 2e^{-\lambda_O t} - e^{-2\lambda_O t}$$

$$MTBF_{O1/2}(t) = \frac{2e^{-\lambda_O t} - e^{-2\lambda_O t}}{2\lambda_O e^{-\lambda_O t} - 2\lambda_O e^{-2\lambda_O t}}$$

$$A_{O1/2}(t) = \frac{MTBF_{O1/2}(t)}{MTBF_{O1/2}(t) + MTTR}$$

For the m=2 out of n=3 case

3-1/2 rated power supplies. Each power supply operates at 2/3 rated P_R

$$MTBF_O = \frac{P_R}{P_O} MTBF_R = \frac{3}{2} MTBF_R \qquad \lambda_O = \frac{2}{3}\lambda_R$$

$$R_{O2/3}(t) = 3e^{-2\lambda_O t} - 2e^{-3\lambda_O t}$$

$$MTBF_{O2/3}(t) = \frac{3e^{-2\lambda_O t} - 2e^{-3\lambda_O t}}{6\lambda_O e^{-2\lambda_O t} - 6\lambda_O e^{-3\lambda_O t}}$$

$$A_{O2/3}(t) = \frac{MTBF_{O2/3}(t)}{MTBF_{O2/3}(t) + MTTR}$$

For the m=3 out of n=4 case

4-3/4 rated power supplies. Each power supply operates at 3/4 rated P_R

$$MTBF_O = \frac{P_R}{P_O} MTBF_R = \frac{4}{3} MTBF_R \qquad \lambda_O = \frac{3}{4}\lambda_R$$

$$R_{O3/4}(t) = 4e^{-3\lambda_O t} - 3e^{-4\lambda_O t}$$

$$MTBF_{O3/4}(t) = \frac{4e^{-3\lambda_O t} - 3e^{-4\lambda_O t}}{12\lambda_O e^{-3\lambda_O t} - 12\lambda_O e^{-4\lambda_O t}}$$

$$A_{O3/4}(t) = \frac{MTBF_{O3/4}(t)}{MTBF_{O3/4}(t) + MTTR}$$

For the m=4 out of n=5 case

5-4/5 rated power supplies. Each power supply operates at 4/5 rated P_R

$$MTBF_O = \frac{P_R}{P_O} MTBF_R = \frac{5}{4} MTBF_R \qquad \lambda_O = \frac{4}{5}\lambda_R$$

$$R_{O4/5}(t) = 5e^{-4\lambda_O t} - 4e^{-5\lambda_O t}$$

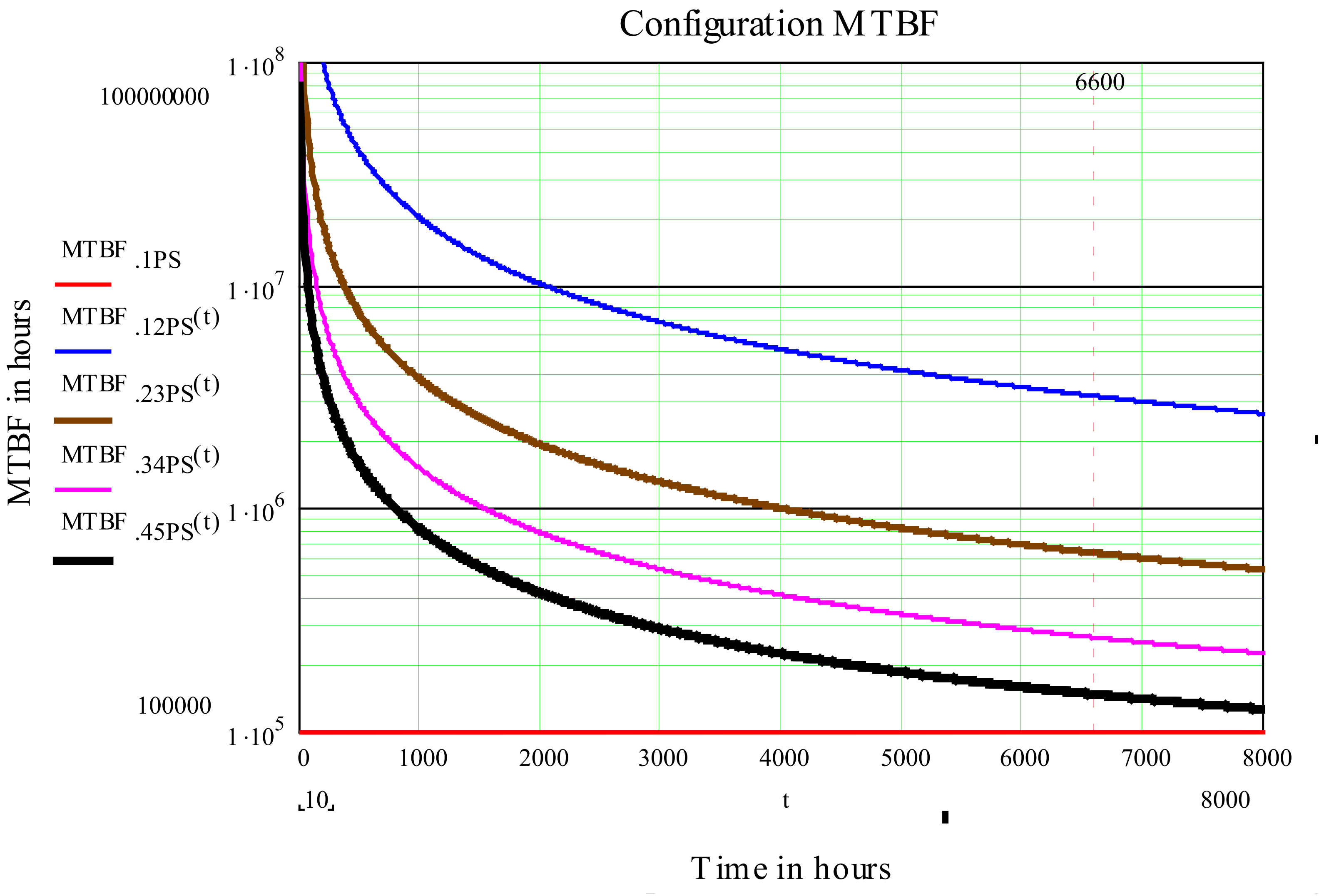
$$MTBF_{O4/5}(t) = \frac{5e^{-4\lambda_O t} - 4e^{-5\lambda_O t}}{20\lambda_O e^{-4\lambda_O t} - 20\lambda_O e^{-5\lambda_O t}}$$

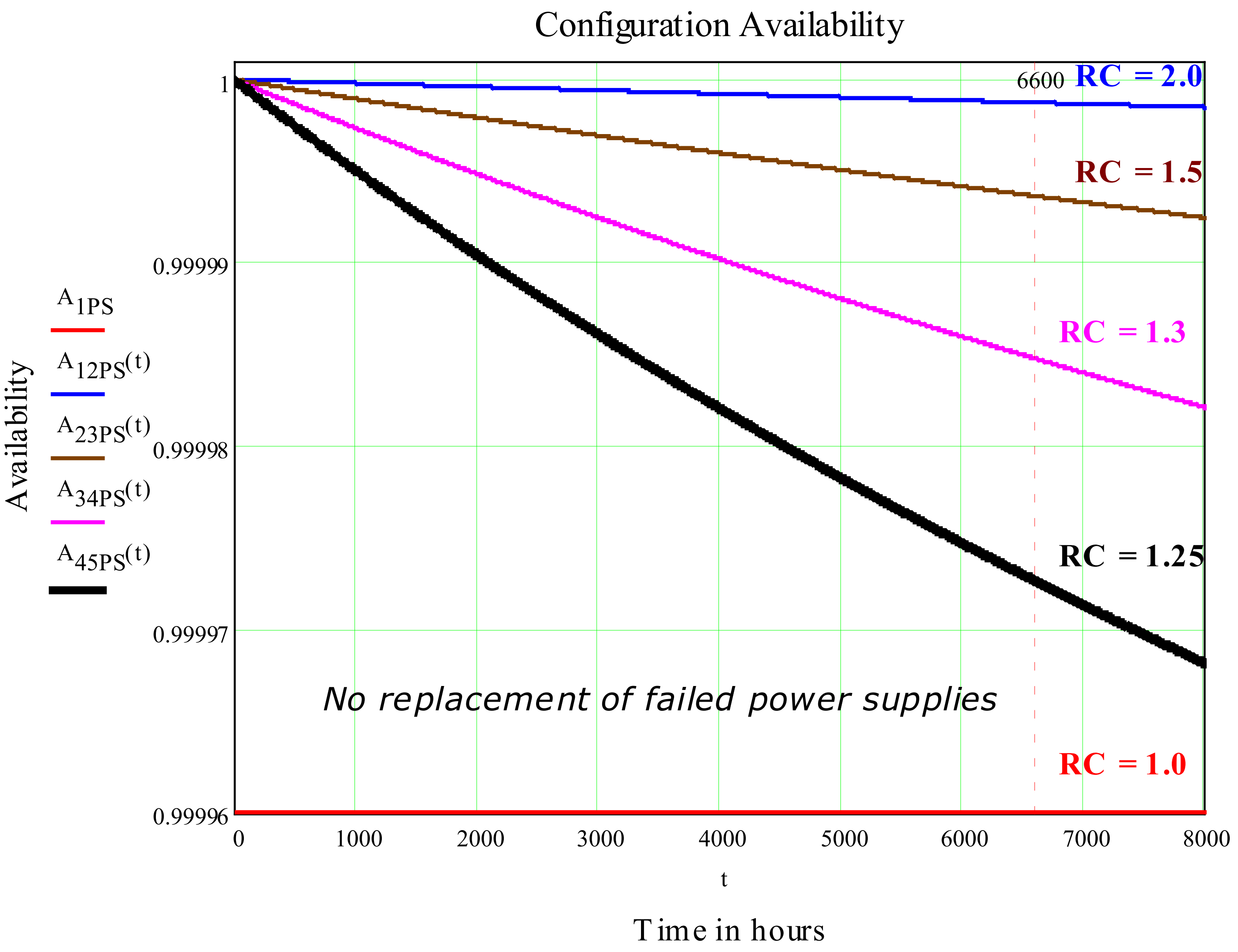
$$A_{O4/5}(t) = \frac{MTBF_{O4/5}(t)}{MTBF_{O4/5}(t) + MTTR}$$

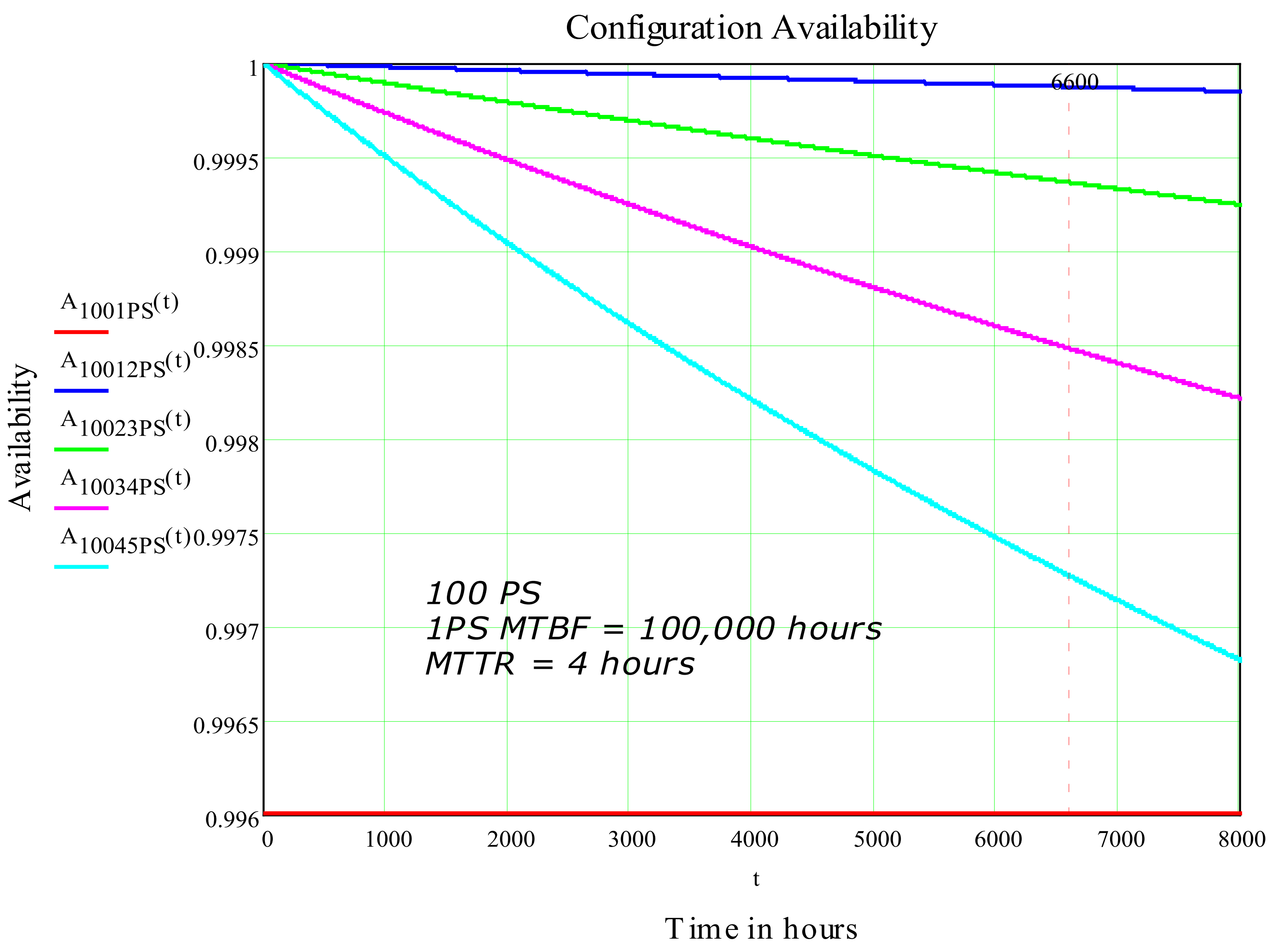
Active Redundancy Power Supply Reliability Summary

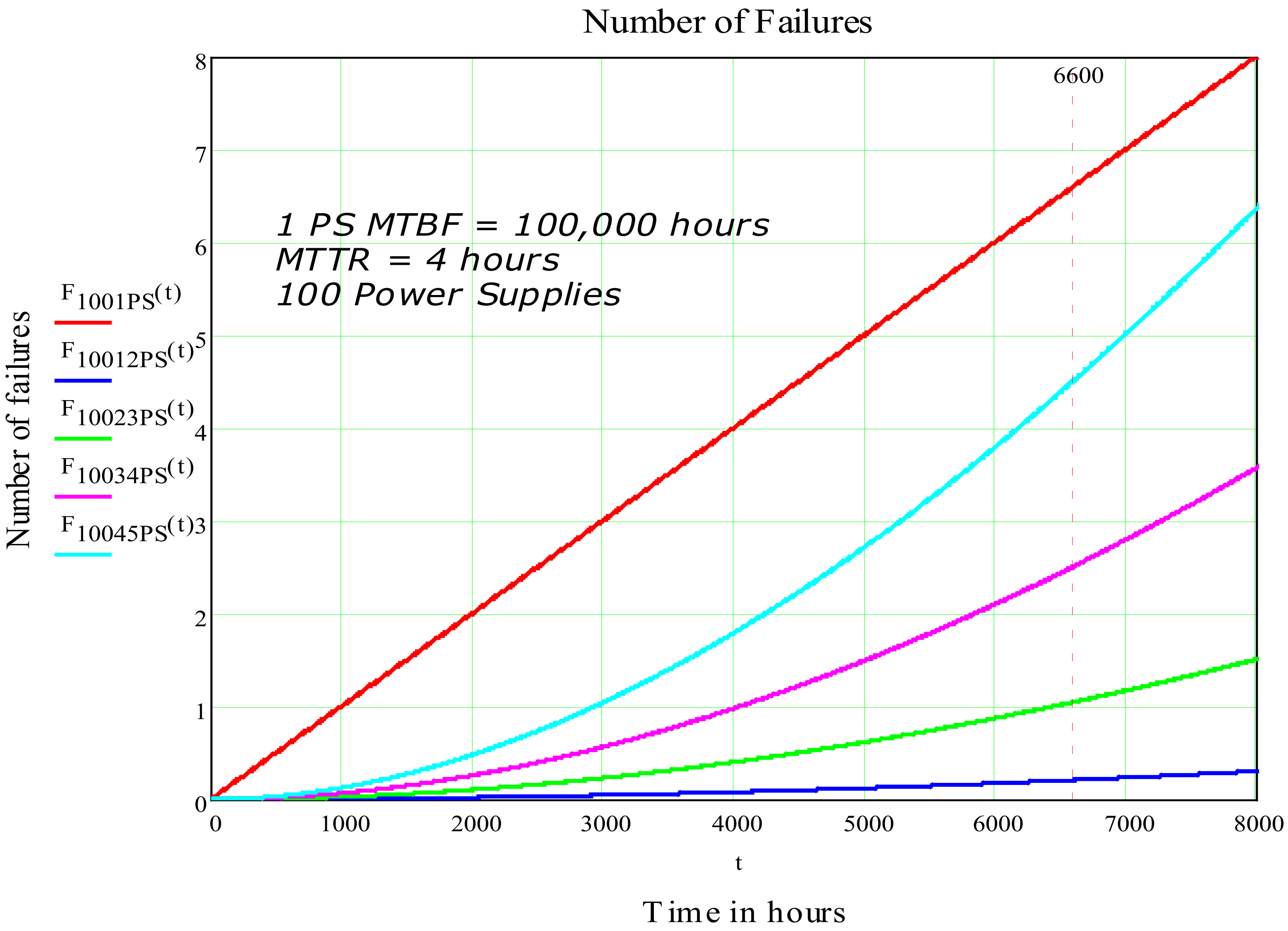
PS		Redundant Power Supplies		
1FR	$\lambda_O = \lambda_R$	$R_O = e^{-\lambda_O t}$	$MTBF_O = MTBF_R$	$A_O = \frac{MTBF_O}{MTBF_O + MTTR}$
1/2	$\lambda_O = \frac{1}{2}\lambda_R$	$R_{O1/2} = 2e^{-\lambda_O t} - e^{-2\lambda_O t}$	$MTBF_{O1/2}(t) = \frac{2e^{-\lambda_O t} - e^{-2\lambda_O t}}{2\lambda_O e^{-\lambda_O t} - 2\lambda_O e^{-2\lambda_O t}}$	$A_{O1/2}(t) = \frac{MTBF_{O1/2}(t)}{MTBF_{O1/2}(t) + MTTR}$
2/3	$\lambda_O = \frac{2}{3}\lambda_R$	$R_{O2/3} = 3e^{-2\lambda_O t} - 2e^{-3\lambda_O t}$	$MTBF_{O2/3}(t) = \frac{3e^{-2\lambda_O t} - 2e^{-3\lambda_O t}}{6\lambda_O e^{-2\lambda_O t} - 6\lambda_O e^{-3\lambda_O t}}$	$A_{O2/3}(t) = \frac{MTBF_{O2/3}(t)}{MTBF_{O2/3}(t) + MTTR}$
3/4	$\lambda_O = \frac{3}{4}\lambda_R$	$R_{O3/4} = 4e^{-3\lambda_O t} - 3e^{-4\lambda_O t}$	$MTBF_{O3/4}(t) = \frac{4e^{-3\lambda_O t} - 3e^{-4\lambda_O t}}{12\lambda_O e^{-3\lambda_O t} - 12\lambda_O e^{-4\lambda_O t}}$	$A_{O3/4}(t) = \frac{MTBF_{O3/4}(t)}{MTBF_{O3/4}(t) + MTTR}$
4/5	$\lambda_O = \frac{4}{5}\lambda_R$	$R_{O4/5} = 5e^{-4\lambda_O t} - 4e^{-5\lambda_O t}$	$MTBF_{O4/5}(t) = \frac{5e^{-4\lambda_O t} - 4e^{-5\lambda_O t}}{20\lambda_O e^{-4\lambda_O t} - 20\lambda_O e^{-5\lambda_O t}}$	$A_{O4/5}(t) = \frac{MTBF_{O4/5}(t)}{MTBF_{O4/5}(t) + MTTR}$

Active Redundancy MTBF Plot




Configuration Availability
1
0.99999
0.99998
0.99997
0.99996
0
1000
2000
3000
4000
5000
6000
7000
8000
t
Time in hours
Availability
A1PS
A12PS(t)
A23PS(t)
A34PS(t)
A45PS(t)
6600
RC = 2.0
RC = 1.5
RC = 1.3
RC = 1.25
RC = 1.0
No replacement of failed power supplies


Configuration Availability
1
0.9995
0.999
0.9985
0.998
0.9975
0.997
0.9965
0.996
Availability
A1001PS(t)
A10012PS(t)
A10023PS(t)
A10034PS(t)
A10045PS(t)
6600
100 PS
1PS MTBF = 100,000 hours
MTTR = 4 hours
0
1000
2000
3000
4000
5000
6000
7000
8000
t
Time in hours


Number of Failures
6600
1 PS MTBF = 100,000 hours
MTTR = 4 hours
100 Power Supplies
F1001PS(t)
F10012PS(t)
F10023PS(t)
F10034PS(t)
F10045PS(t)
Number of failures
0
1
2
3
4
5
6
7
8
0
1000
2000
3000
4000
5000
6000
7000
8000
t
Time in hours

Two inverter stages in an uninterruptible power supply are to be connected in parallel. each is capable of full-load capability. The calculated failure rate of each stage is l = 200 failures per million hours.

A. What is the probability that each inverter will remain failure free for a mission time of 1000 hours and

B. What is the probability that the system will operate failure free for 1000 hours?

Solution:

Homework Problem # 20

For a critical mission, 3 power supplies, each capable of supplying the total required output, are to be paralleled. The power supplies are also decoupled such that a failure of any power supply will not affect the output. The calculated failure rate of each power supply is 4 per million hours.

A. What is the probability that each power supply will operate failure free for 5 years?

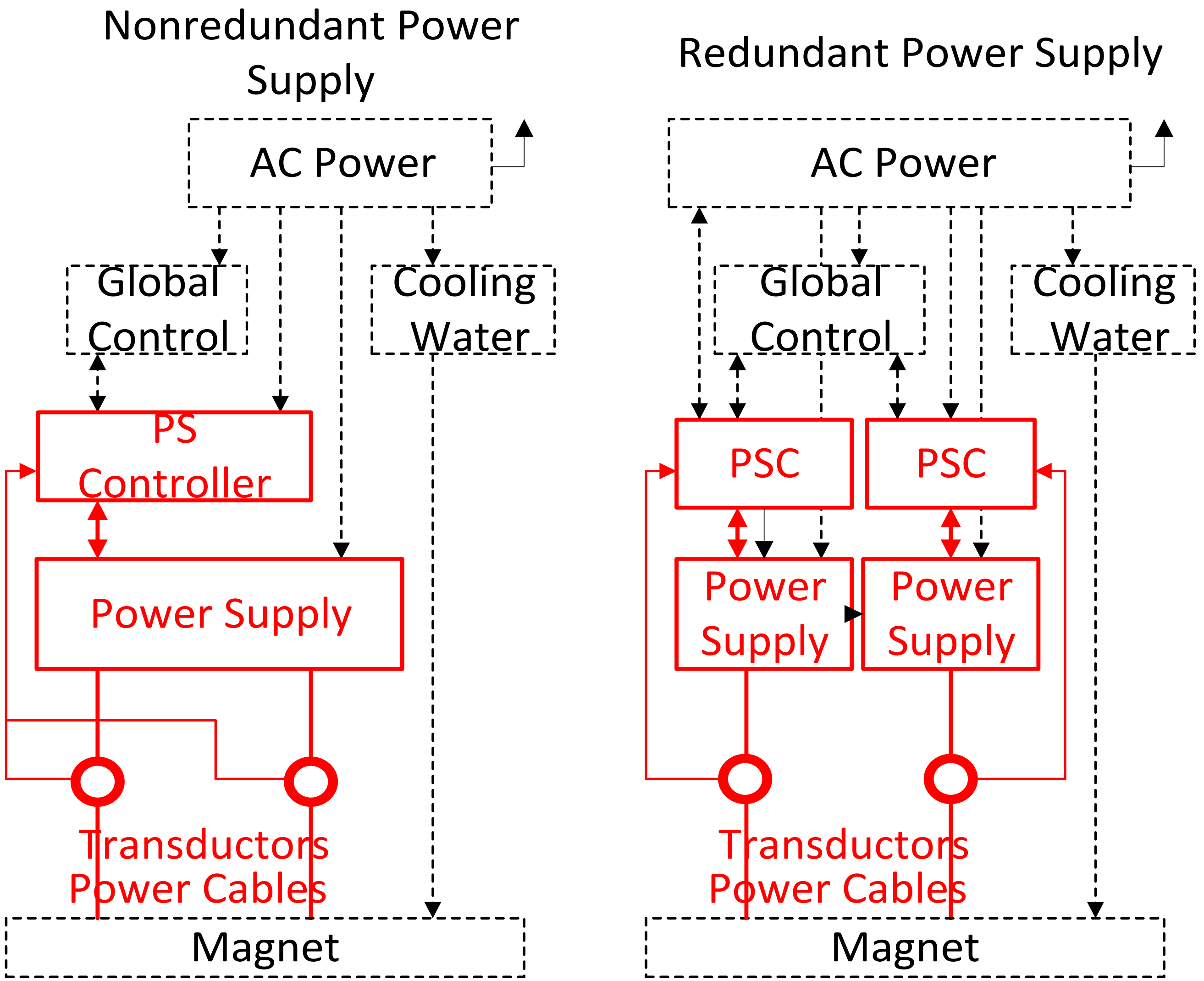
B. What is the probability that the system will operate failure free for 5 years? That is, only 1 out of the 3 power supplies is needed in order for the system to operate.

SLAC Next-Generation High Availability Power Supply

Dave MacNair

SLAC National Accelerator Laboratory

Power Conversion Department (PCD)



Item	***MTBF kHrs***
PSC	*110*
PS	*60*
Transductor	*381.5*
Cables/ Connectors	*14000*

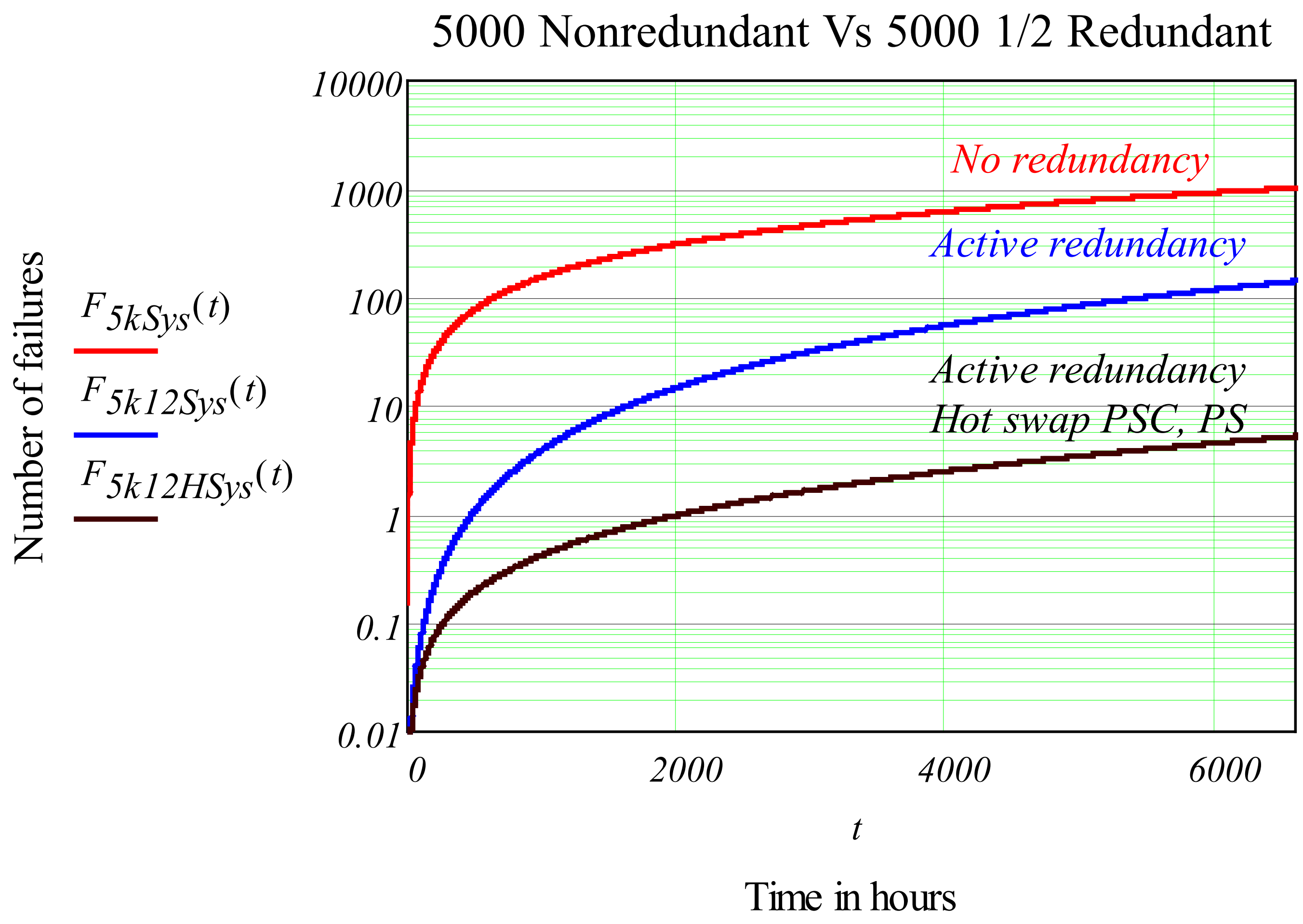
MTTR = 2 Hours

PAC 2001 Chicago, Illinois

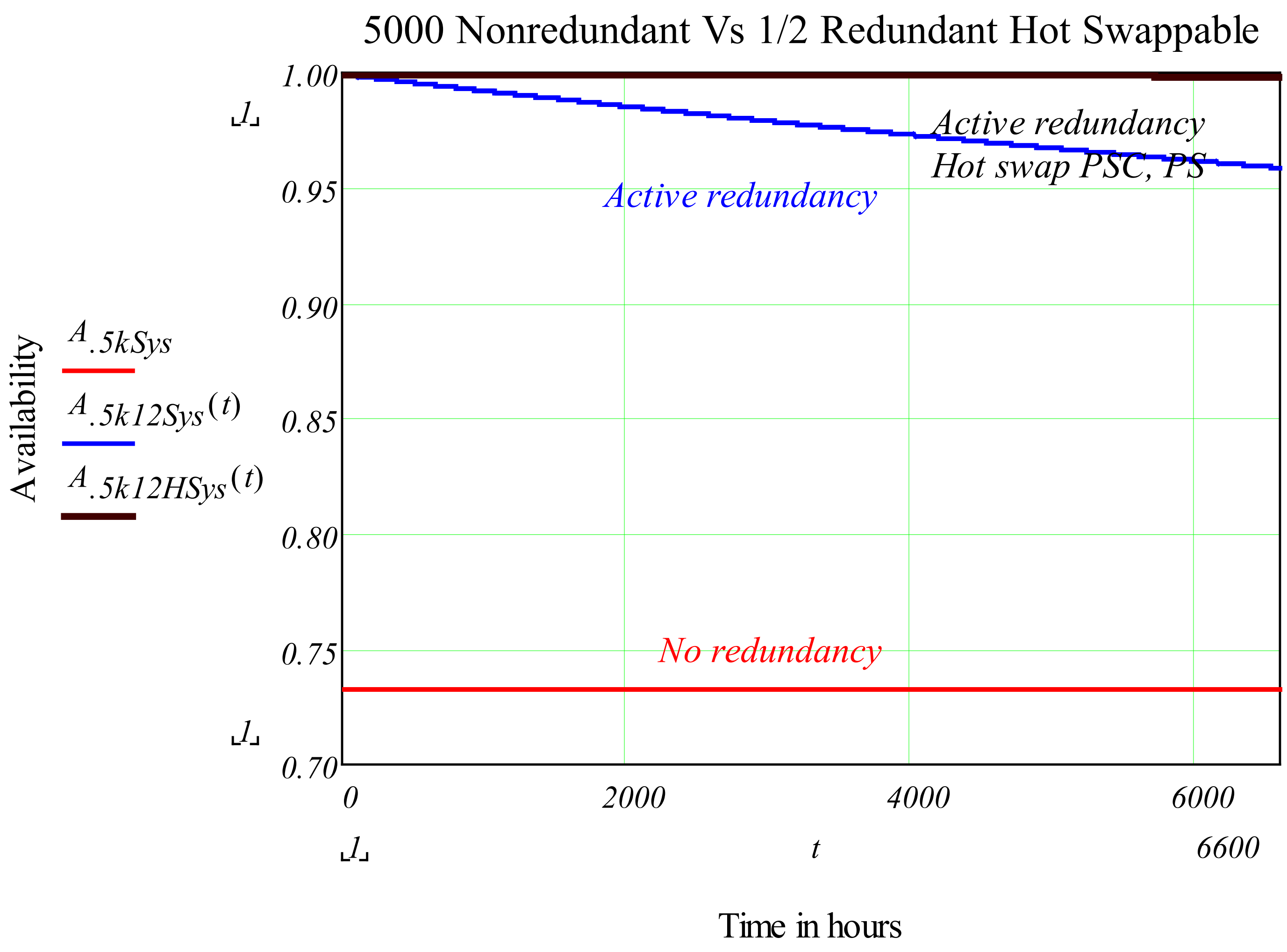
Bellomo, Donaldson, MacNair

Assumptions

- *MTTR and MTBFs of components on previous slide*
- *Only power supply is redundant*
- *For one case the power supply and PSC are hot swappable*



Hot Swap is Essential



Assumptions

•MTTR and MTBFs of components on two slides back

•Only power supply is redundant

It is clear that redundancy and hot swap are needed

SLAC Projects with Non-redundant or Redundant Power Supplies

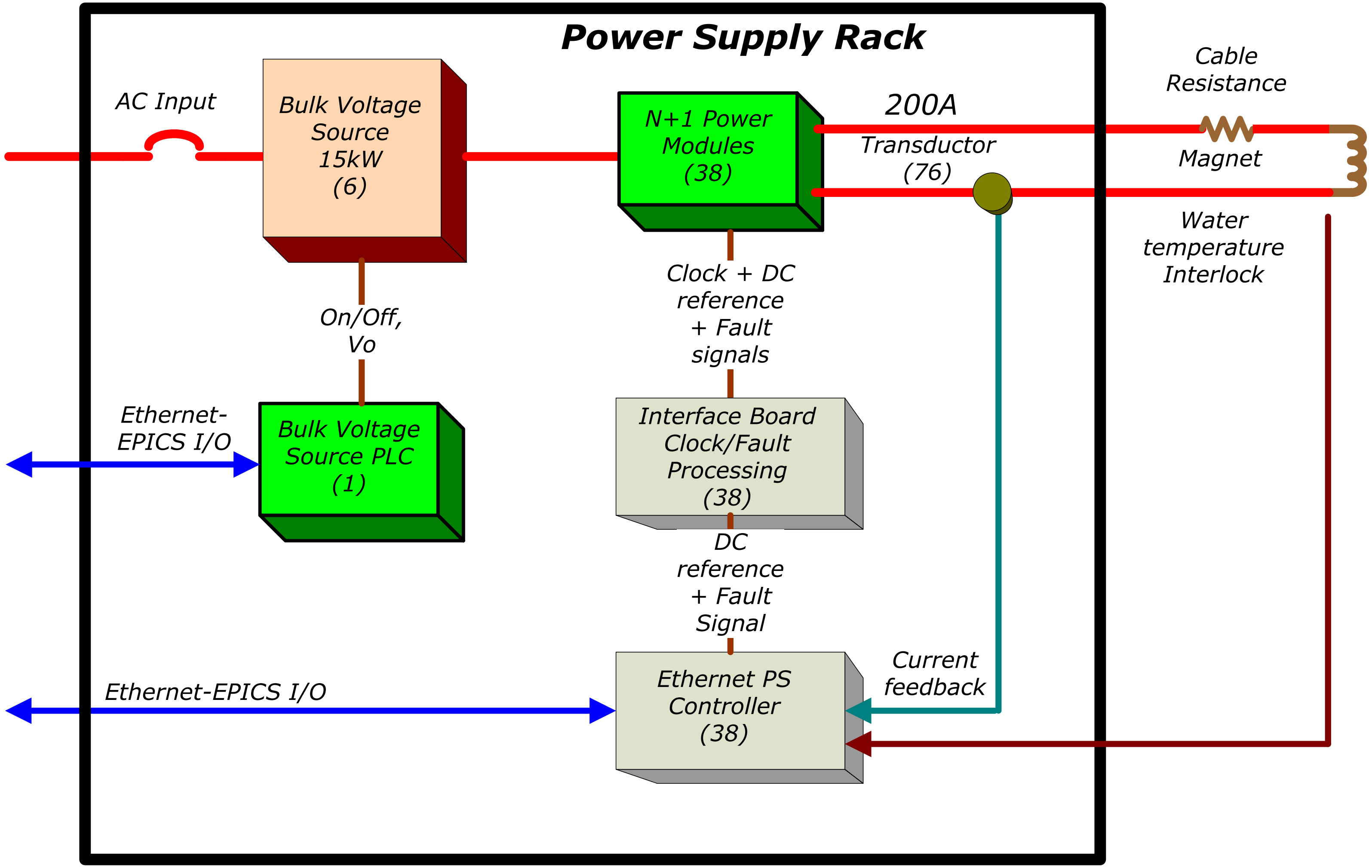
Non-redundant - PEP II, SPEAR 3, LCLS (1994 – 2006)

- *Power supply quantity is hundreds, not thousands*
- *Power supply availability budget is modest 98%*
- *Non-redundant supplies satisfied availability budget*
- *Redundant power systems not readily available from industry*
- *Redundant systems would not fit within cost and schedule constraints*

Redundant - KEK ATF 2 (2006 – 2008)

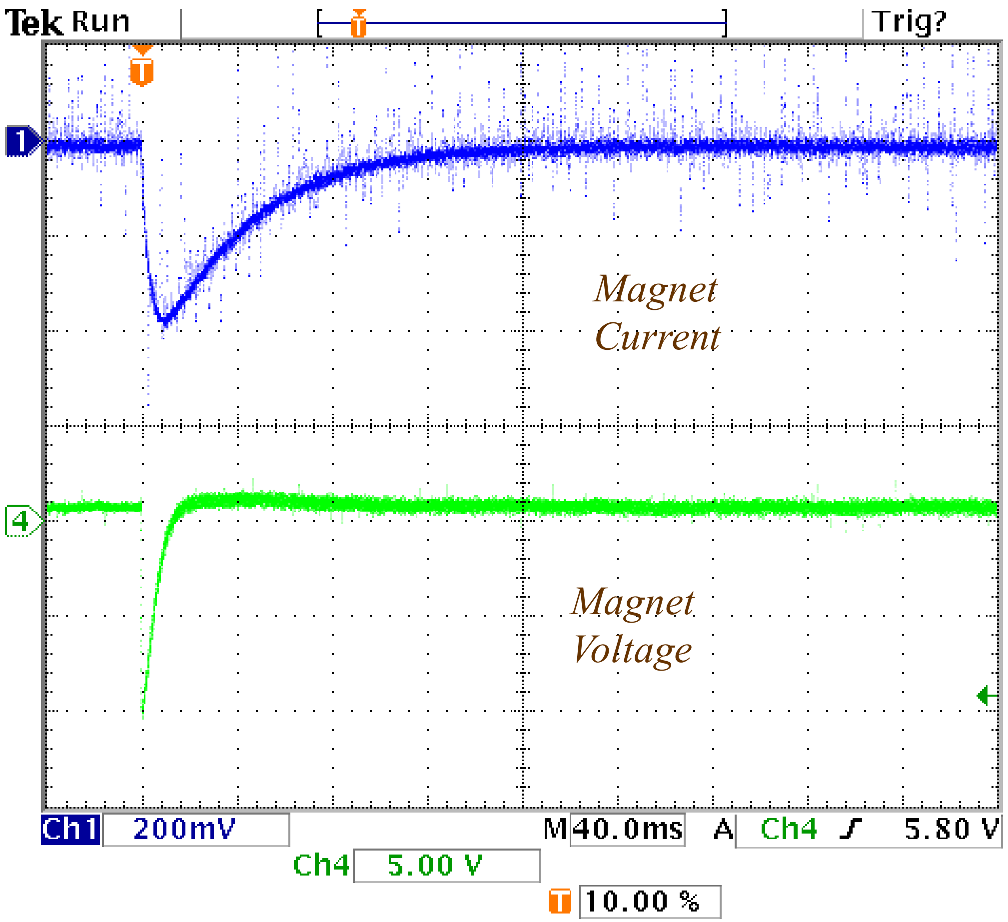
- *Mock-up of ILC Final Focus accelerator*
- *Magnet power supplies ILC-like*

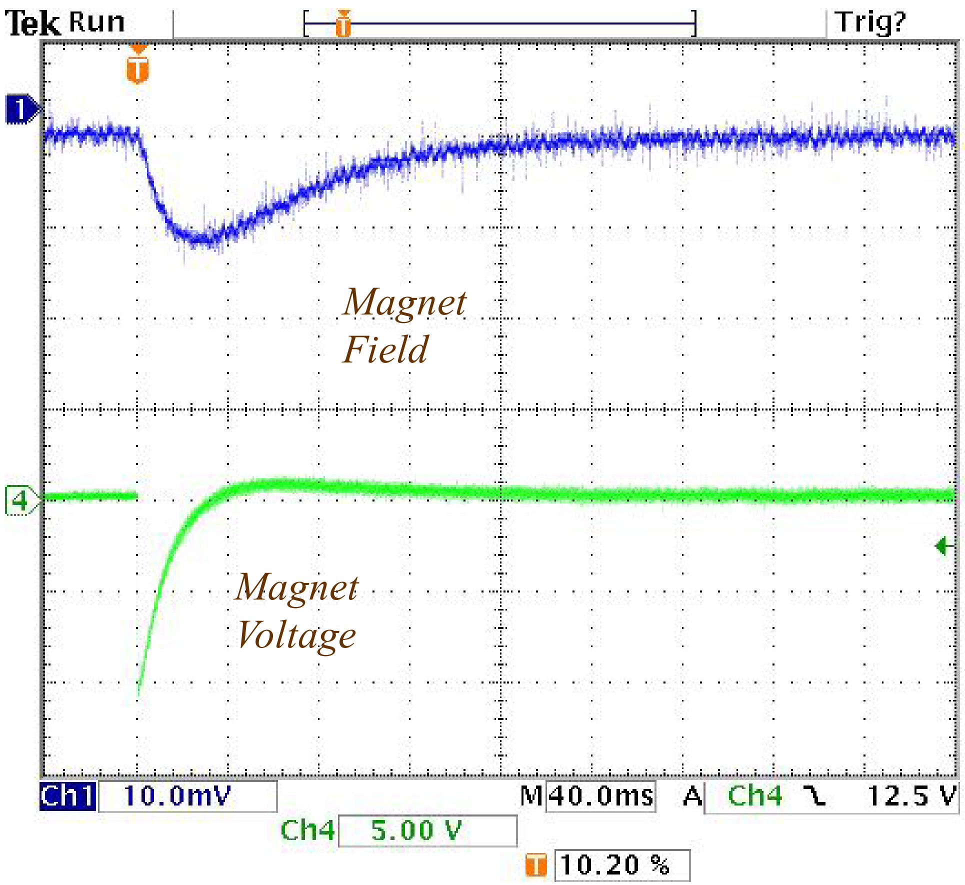
ATF2 Block Diagram



ATF2 – at KEK

ATF2 Current and Field Recovery Plots





- *During power module loss measured 6A magnet current drop at 150A*
- *100 Gauss drop at 3.1 kilogauss. 200mS recovery with no overshoot, no re-standardize needed*

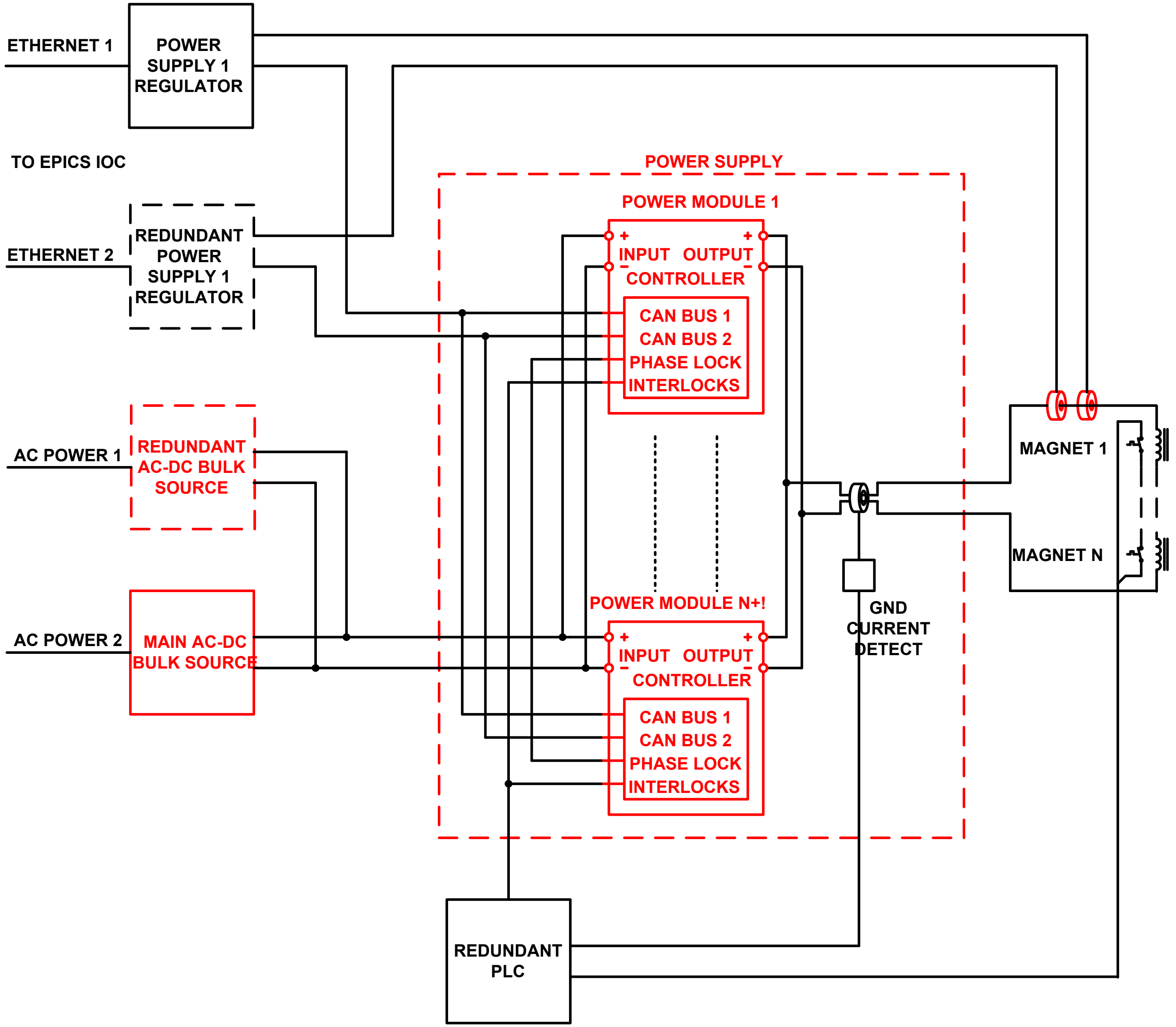
Goals

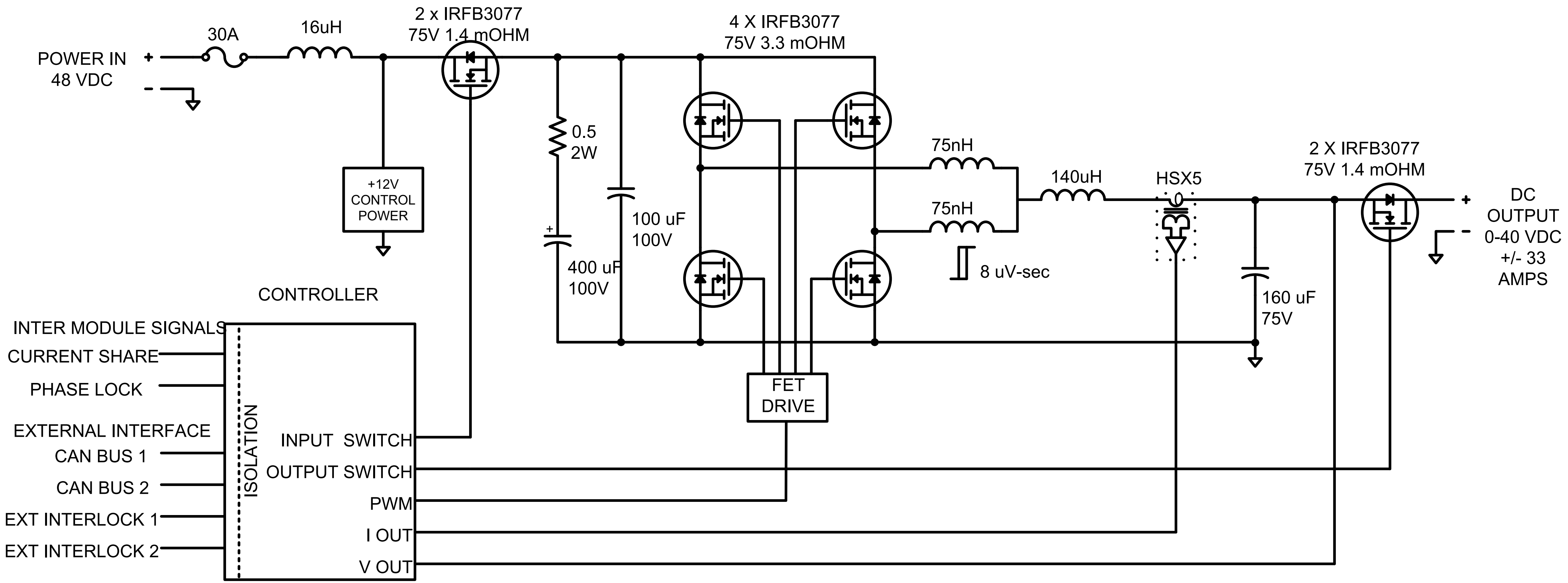
- All components N+1 modular and redundant
- Power module hot-swappable
- Unipolar or bipolar output from a single unipolar bulk voltage source
- Imbedded controller with digital current regulation
- Capable of driving superconducting magnets
- High bandwidth for use in BBA or closed orbit correction systems
- High stability and precision output current
- High accuracy read-backs
- Scalable to higher output levels

Applications

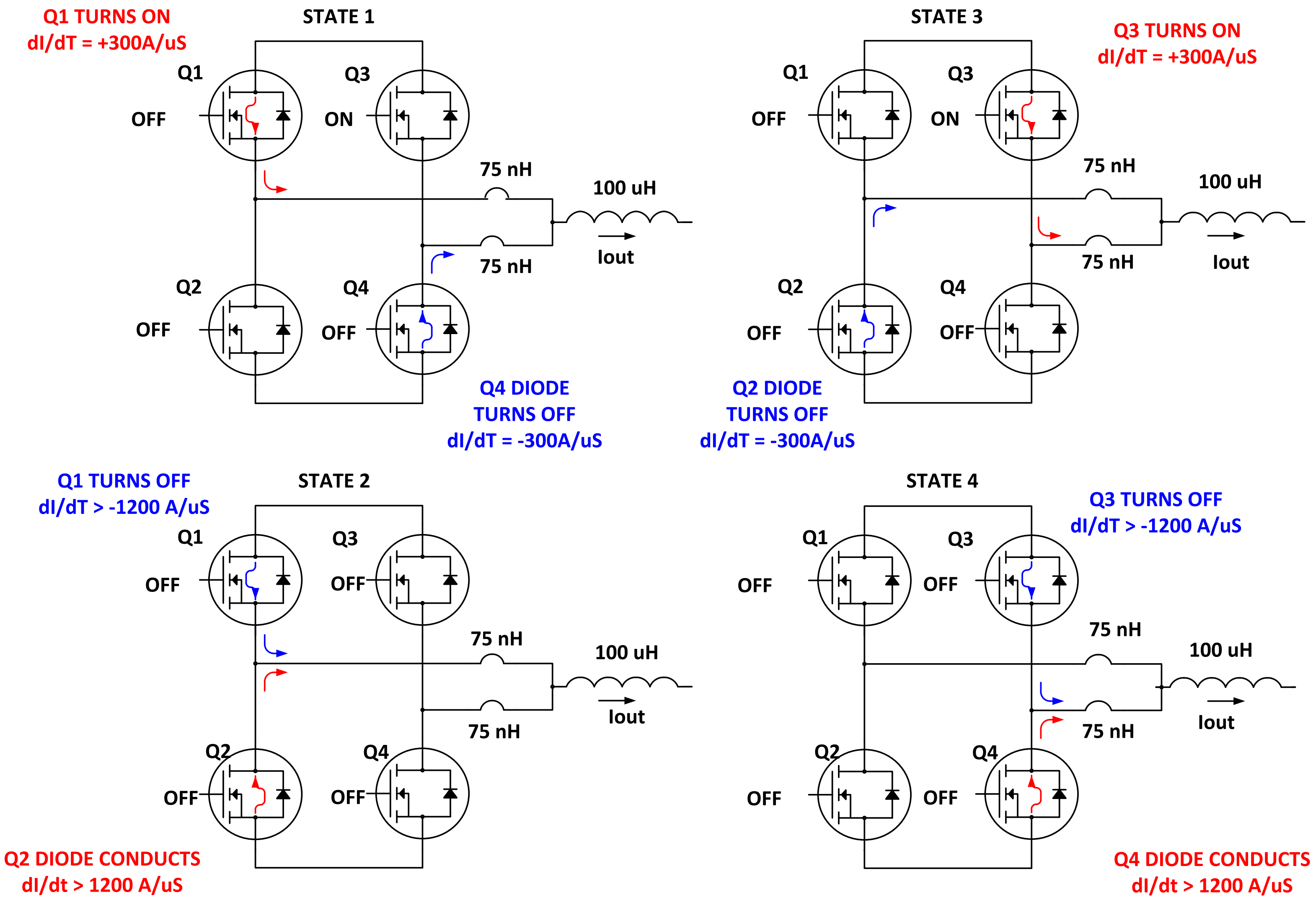
- ILC and other future accelerators

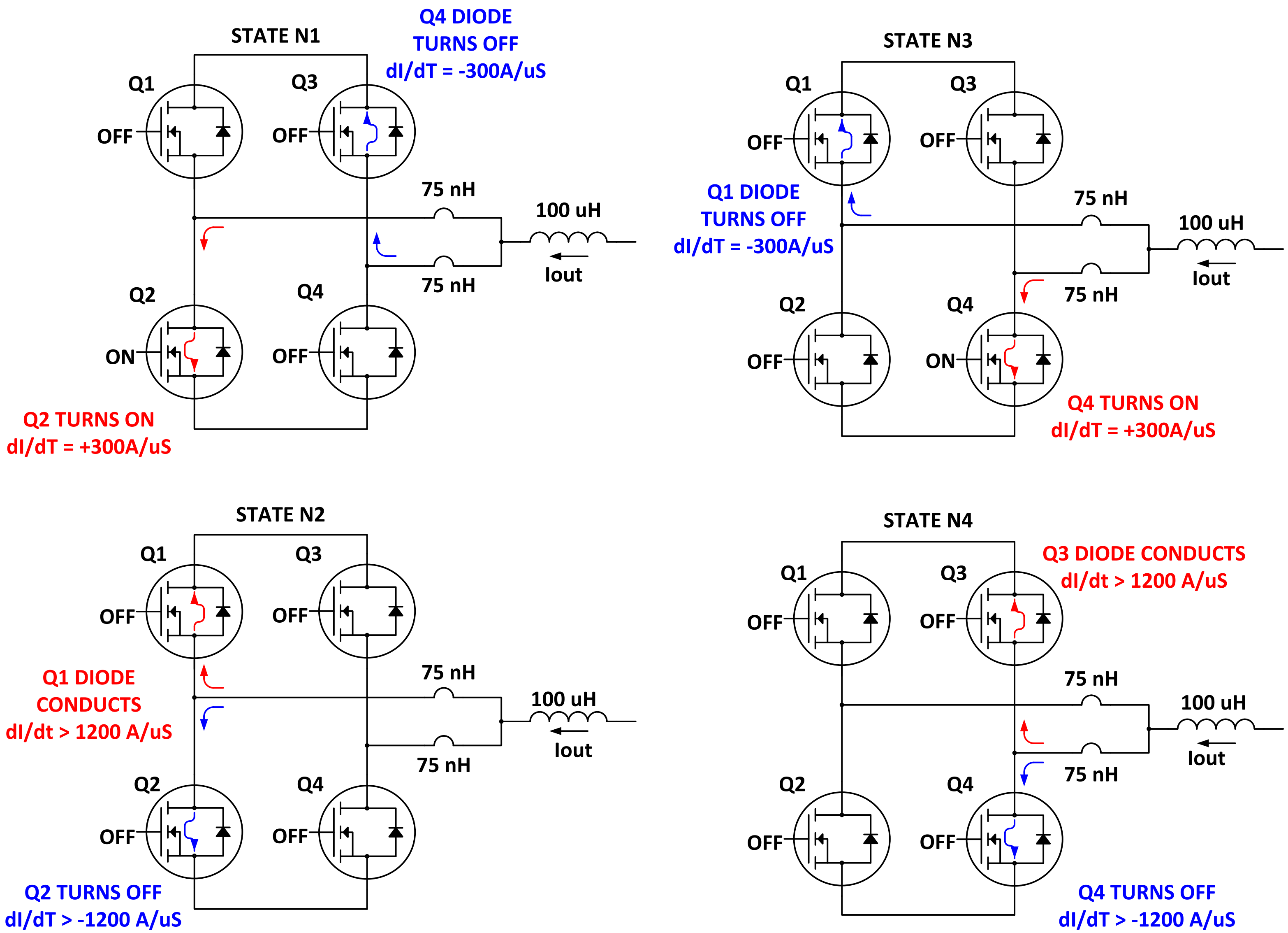
Next Generation Power Supply Block Diagram

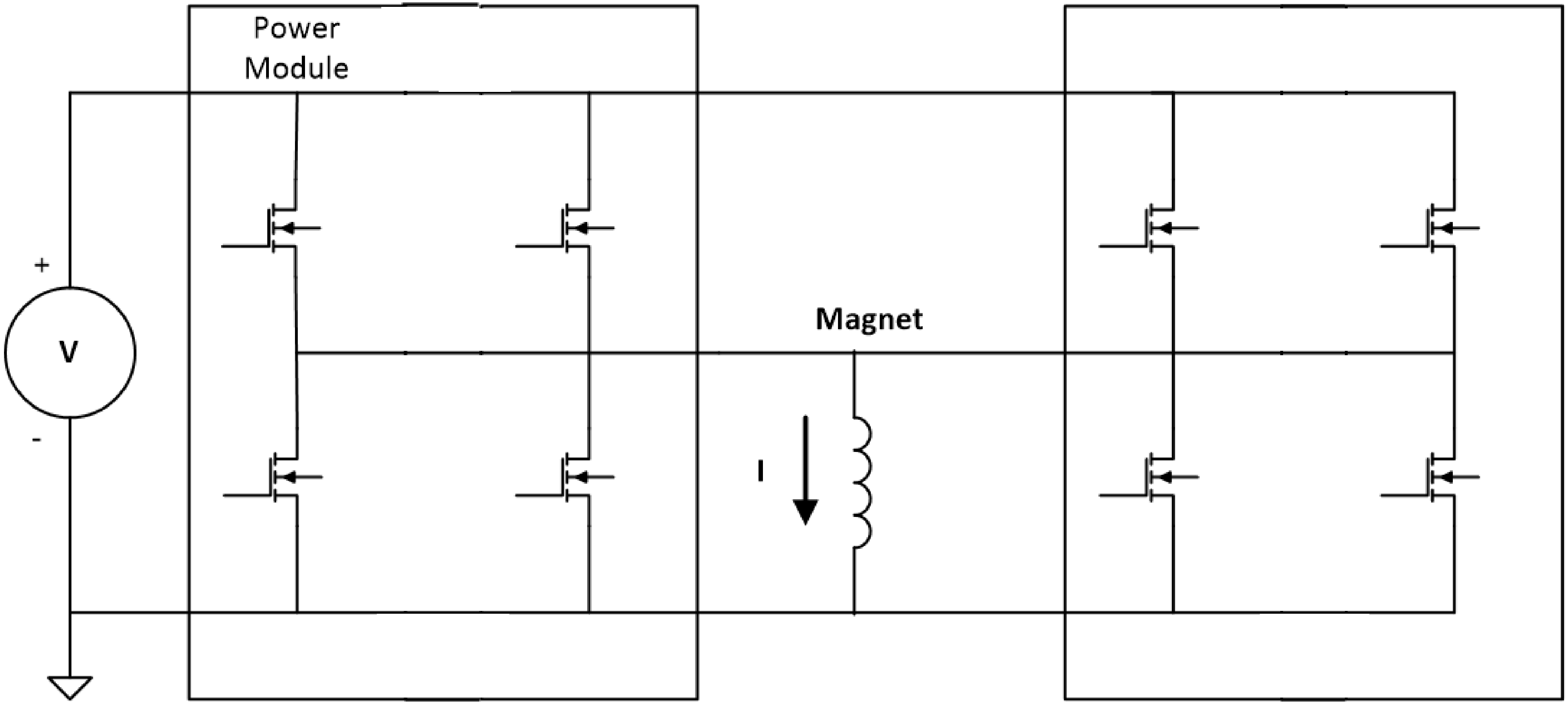


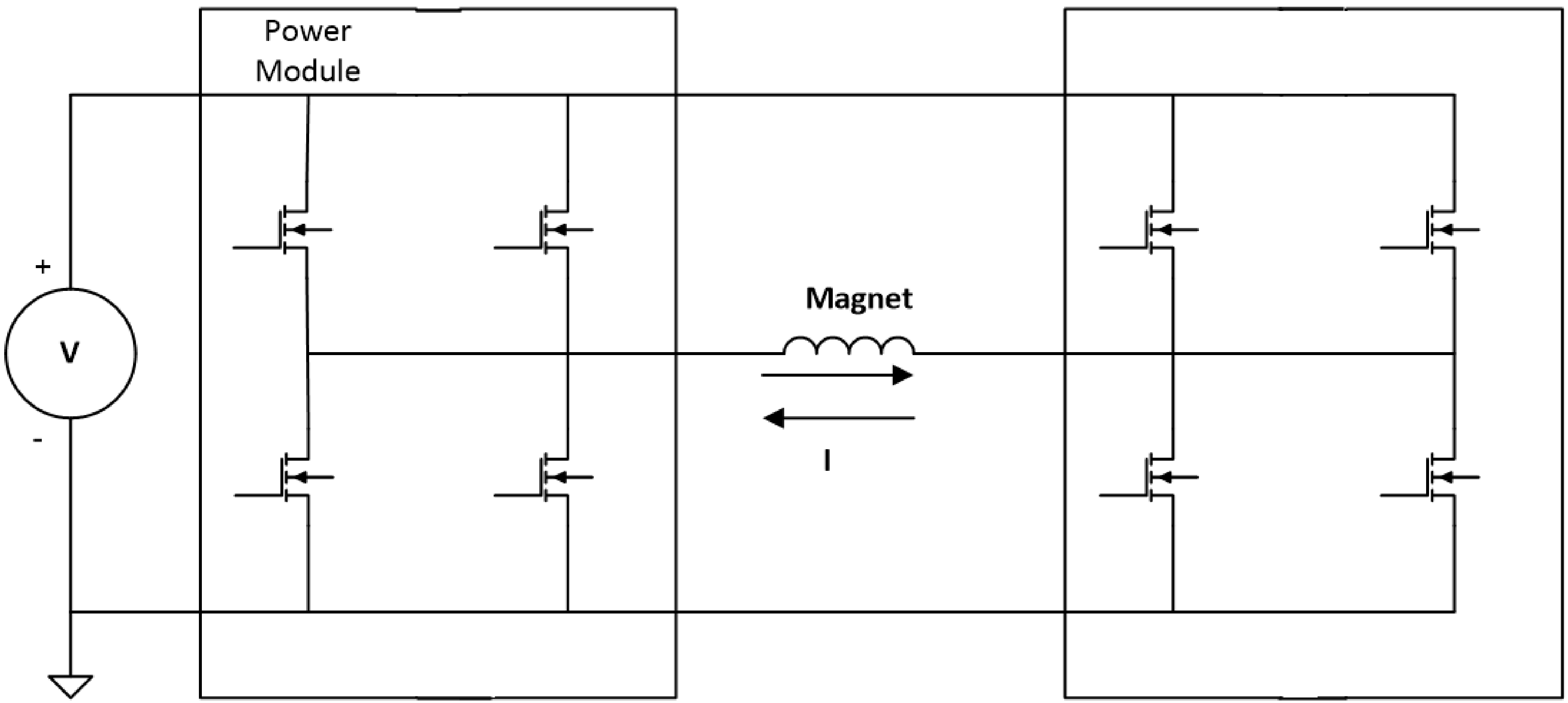

POWER IN
48 VDC
+
-
30A
16uH
2 x IRFB3077
75V 1.4 mOHM
4 X IRFB3077
75V 3.3 mOHM
0.5
2W
+12V
CONTROL
POWER
+
400 uF
100V
100 uF
100V
75nH
75nH
8 uV-sec
140uH
HSX5
2 X IRFB3077
75V 1.4 mOHM
160 uF
75V
DC
OUTPUT
0-40 VDC
+/- 33
AMPS
CONTROLLER
INTER MODULE SIGNALS
CURRENT SHARE
PHASE LOCK
EXTERNAL INTERFACE
CAN BUS 1
CAN BUS 2
EXT INTERLOCK 1
EXT INTERLOCK 2
ISOLATION
INPUT SWITCH
OUTPUT SWITCH
PWM
I OUT
V OUT
FET
DRIVE

Next Generation Positive Output Current (Q1 – Q2 – Q3 – Q4)




STATE N1
Q4 DIODE
TURNS OFF
dI/dT = -300A/uS
Q1
Q3
OFF
OFF
75 nH
100 uH
75 nH
Iout
Q2
Q4
ON
OFF
Q2 TURNS ON
dI/dT = +300A/uS
STATE N3
Q1
Q3
OFF
OFF
Q1 DIODE
TURNS OFF
dI/dT = -300A/uS
75 nH
100 uH
75 nH
Iout
Q2
Q4
OFF
ON
Q4 TURNS ON
dI/dT = +300A/uS
STATE N2
Q1
Q3
OFF
OFF
Q1 DIODE
CONDUCTS
dI/dt > 1200 A/uS
75 nH
100 uH
75 nH
Iout
Q2
Q4
OFF
OFF
Q2 TURNS OFF
dI/dT > -1200 A/uS
STATE N4
Q3 DIODE CONDUCTS
dI/dt > 1200 A/uS
Q1
Q3
OFF
OFF
75 nH
100 uH
75 nH
Iout
Q2
Q4
OFF
OFF
Q4 TURNS OFF
dI/dT > -1200 A/uS

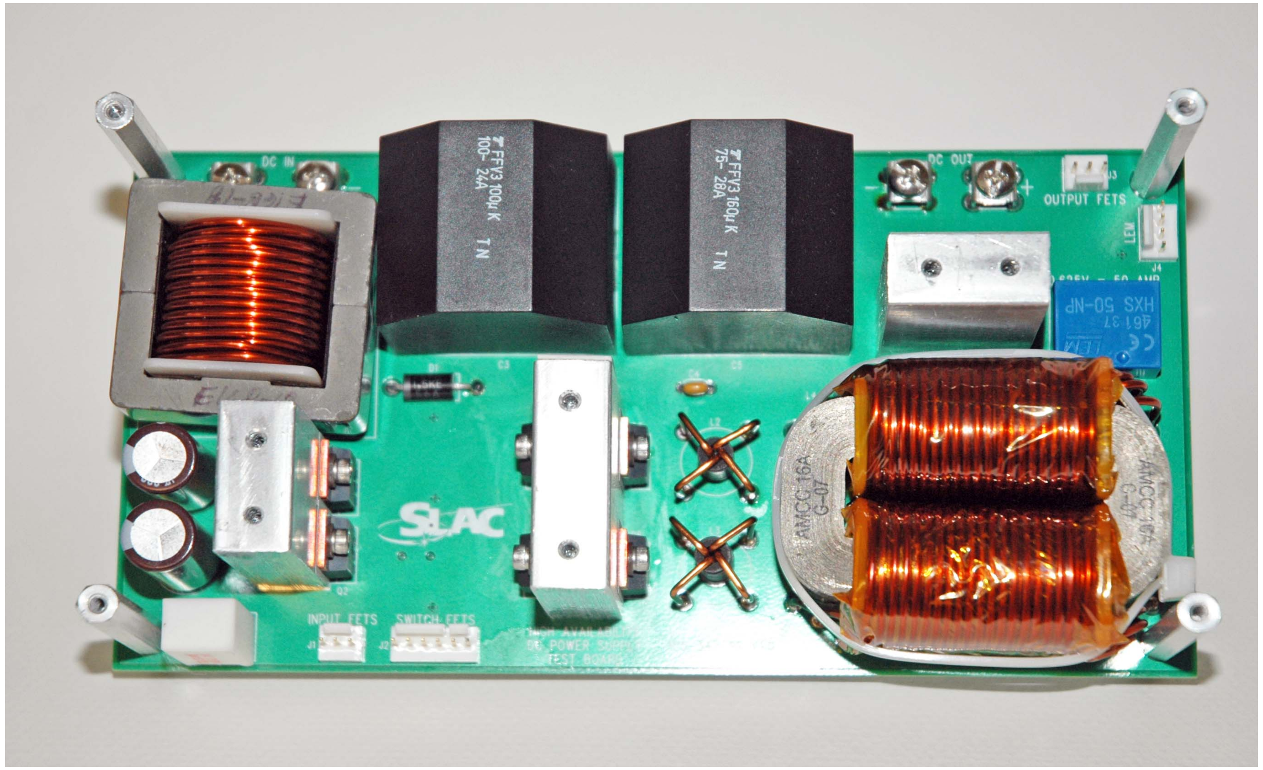

Power
Module
+
V
-
Magnet
I


Power
Module
+
V
-
Magnet
I

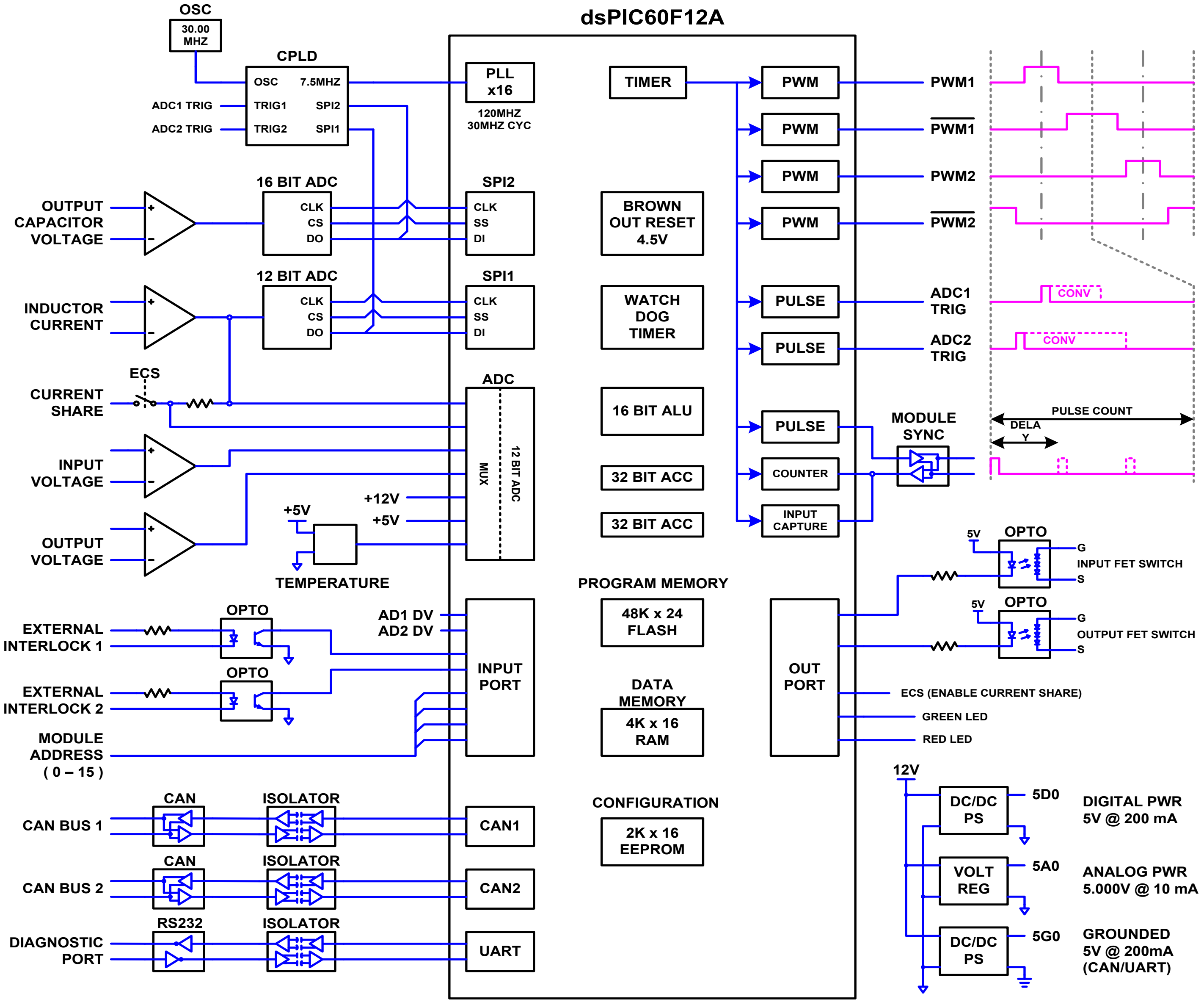
Next Generation Power Modules are "Bricks"

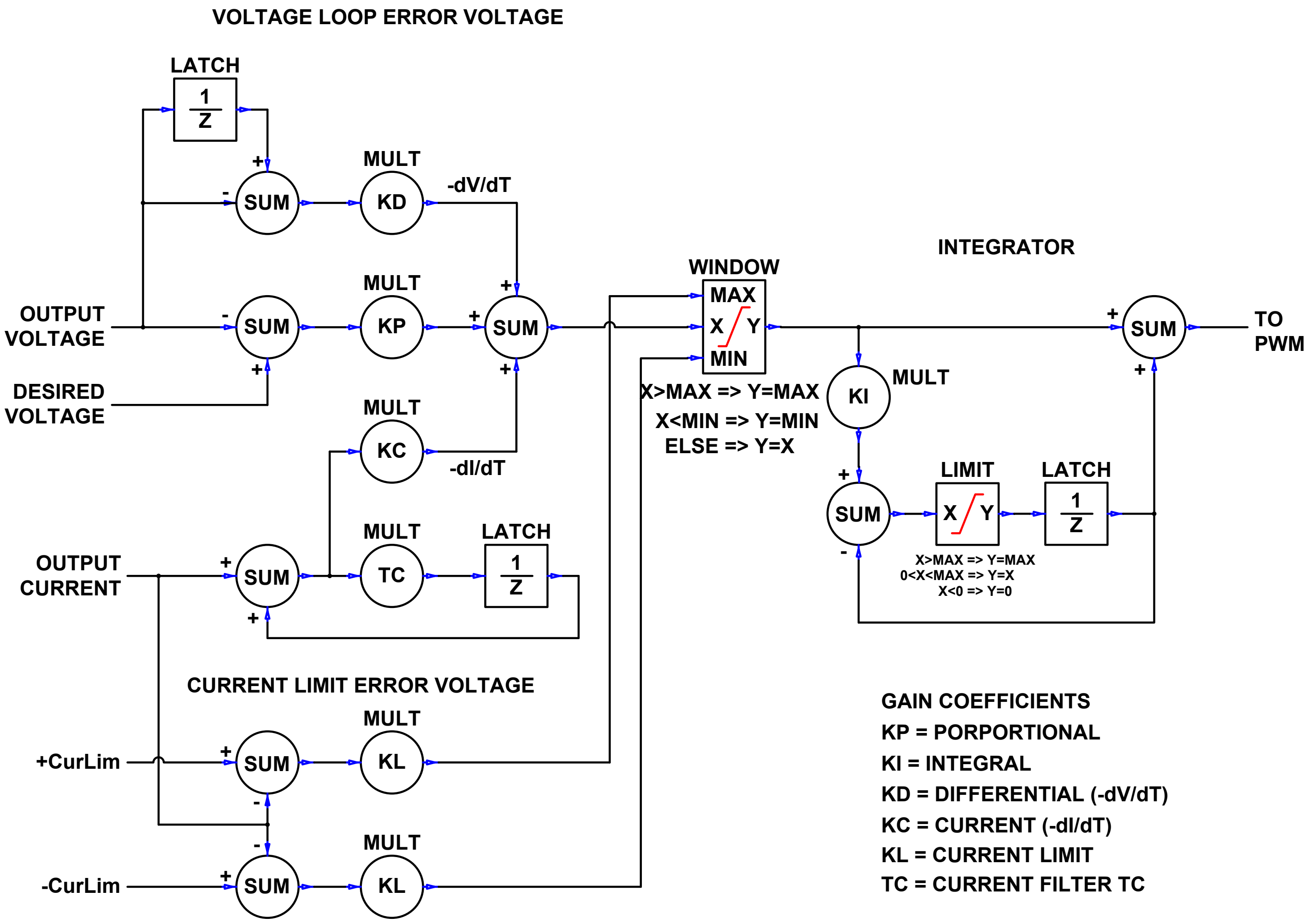
- *Input: 48V*
- *Output V: 0 to 40V*
- *Output I: 0 to 33A*
- *Output P: 0 to 1,320W*
- *2" X 4" X 8"*

Next Generation Power Module Layout



Next Generation Controller





GAIN COEFFICIENTS

KP = PORPORTIONAL

KI = INTEGRAL

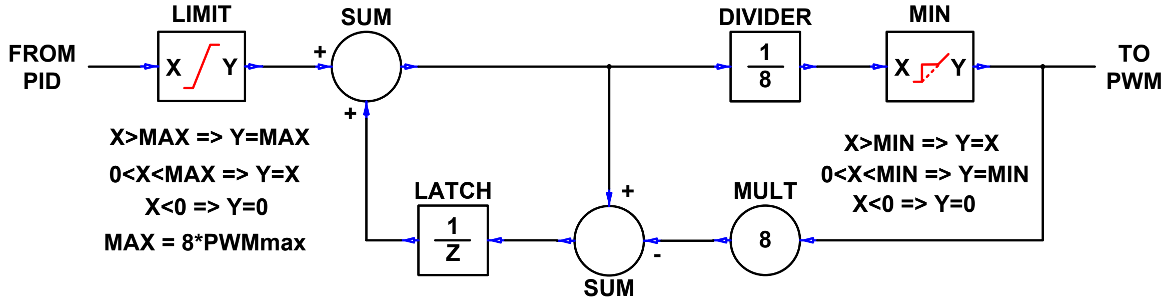
KD = DIFFERENTIAL (-dV/dT)

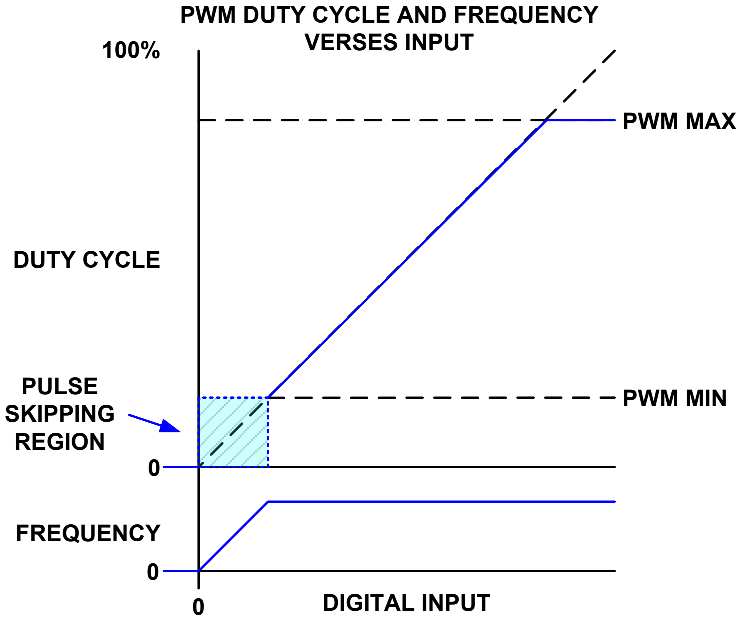
KC = CURRENT (-dI/dT)

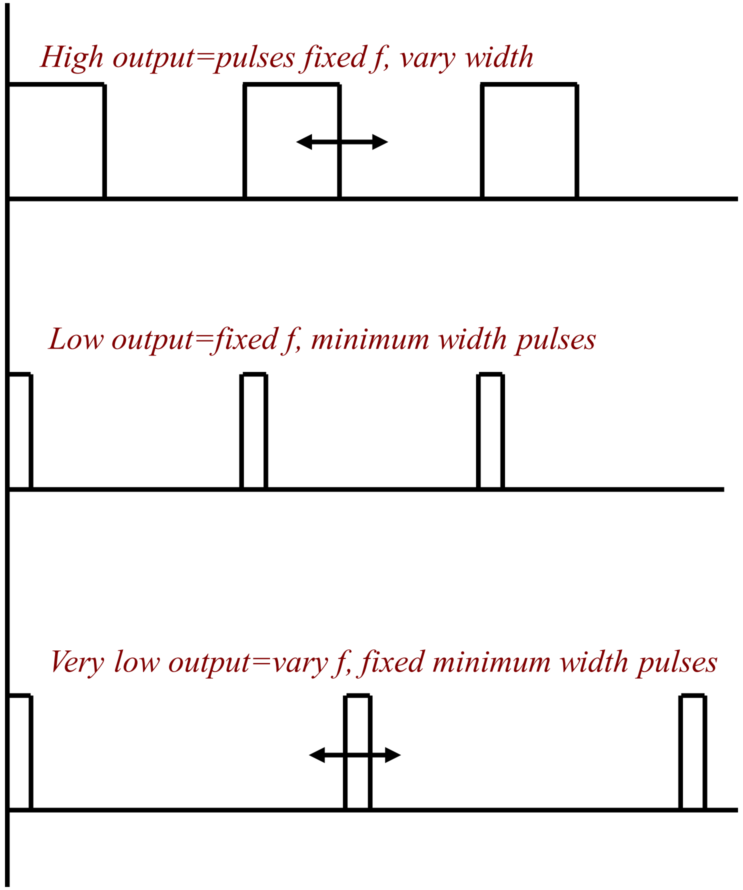
KL = CURRENT LIMIT

TC = CURRENT FILTER TC

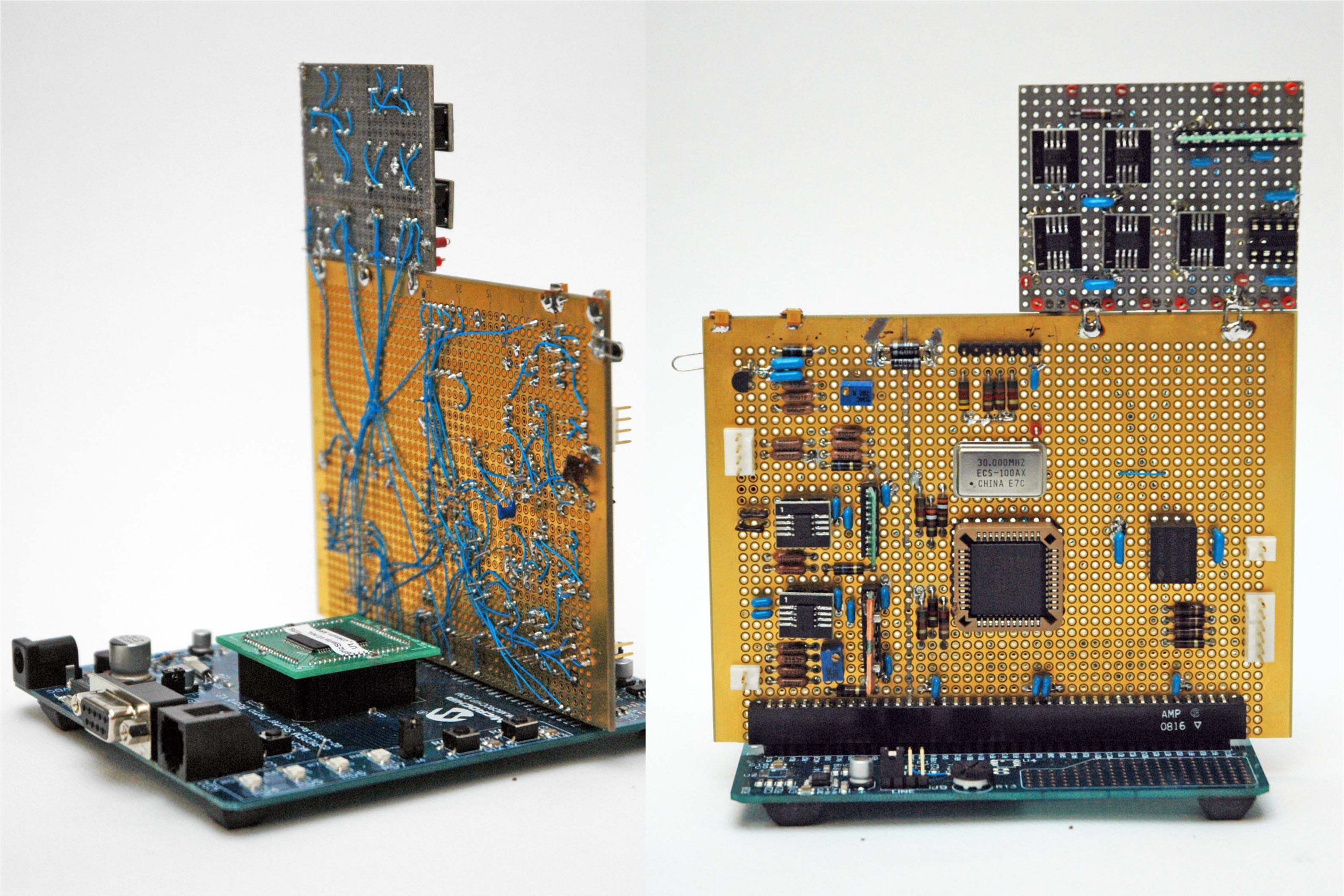
Next Generation PWM Control



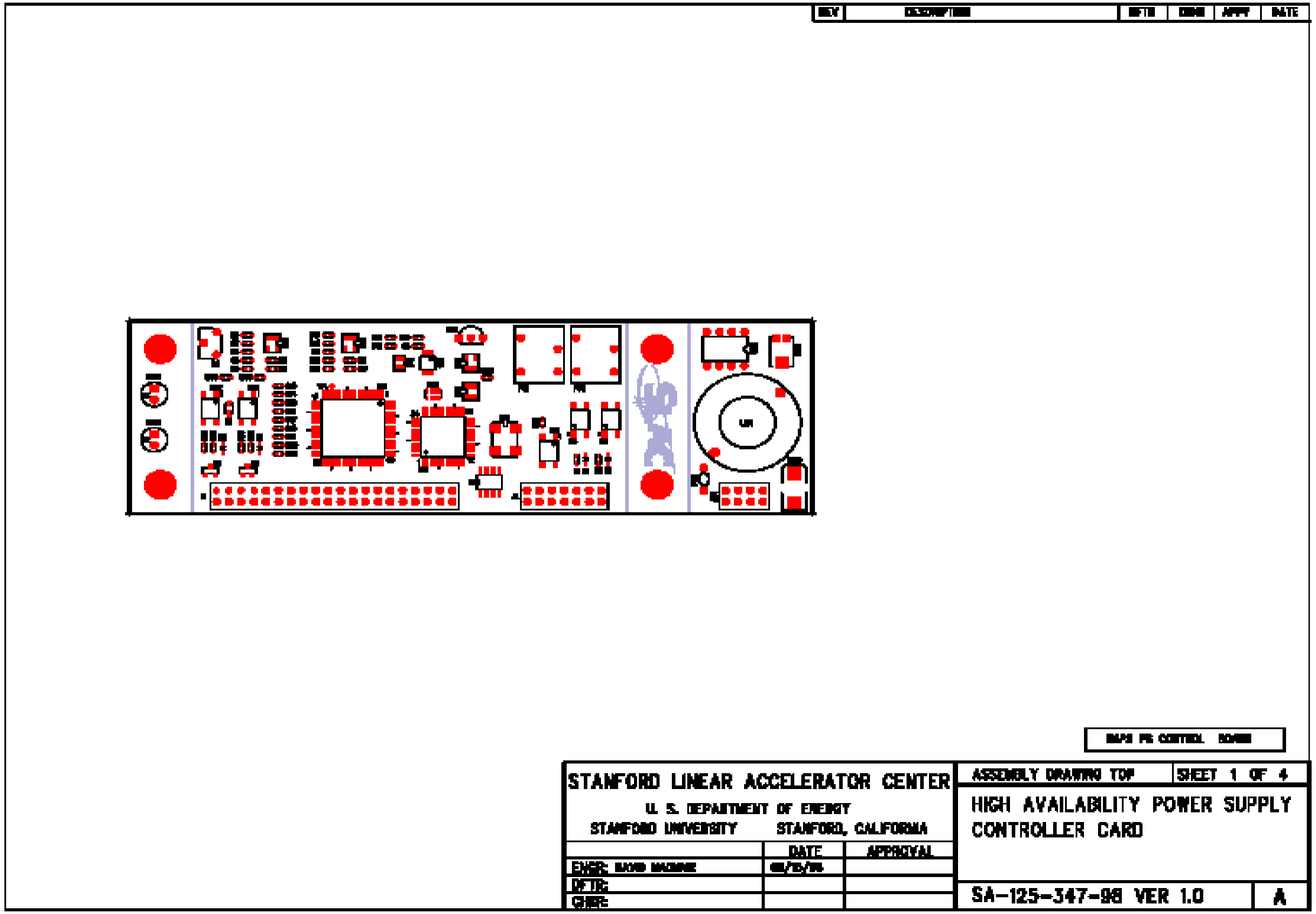


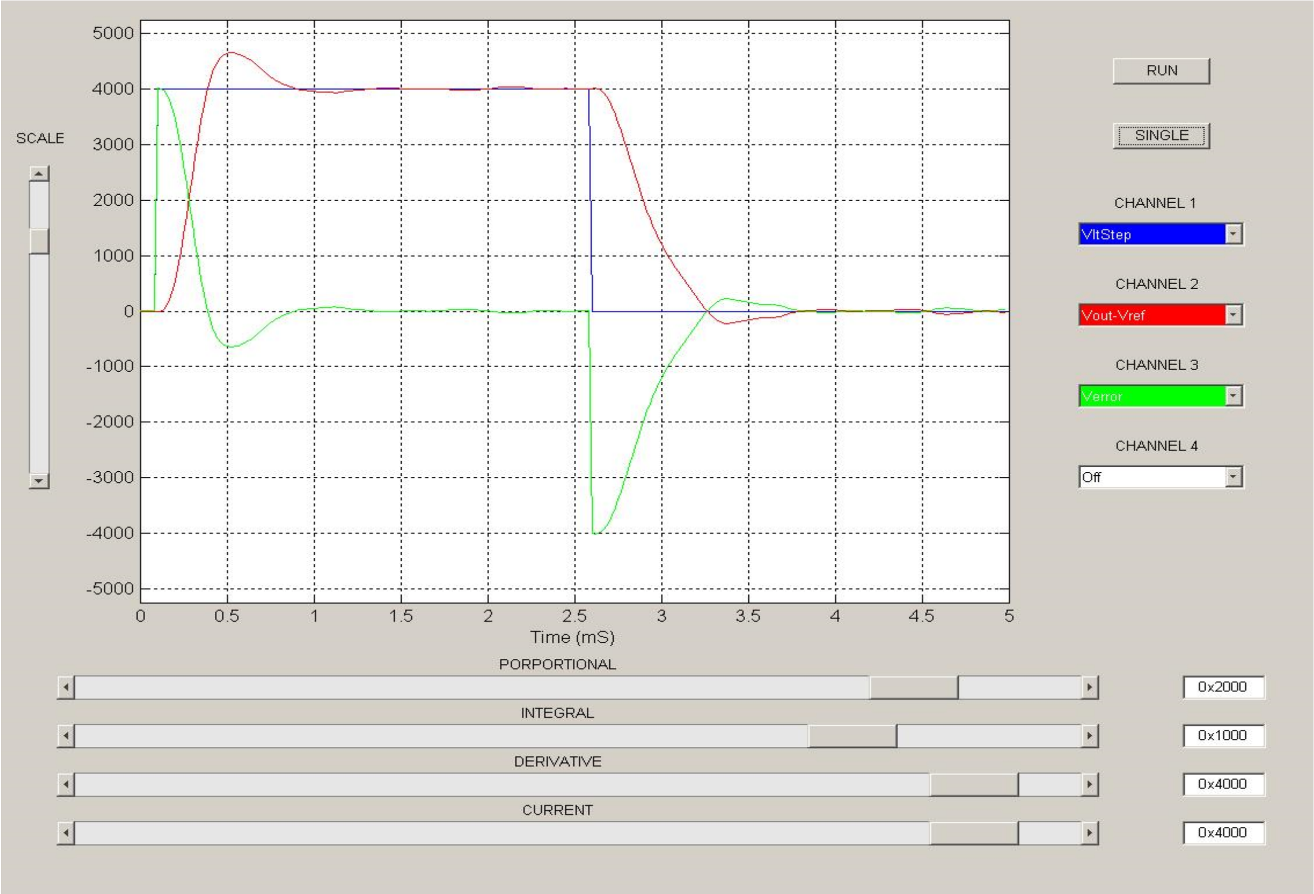


Next Generation Prototype Controller with Development Board



Next Generation – Controller Card




SCALE
5000
4000
3000
2000
1000
0
-1000
-2000
-3000
-4000
-5000
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time (mS)
RUN
SINGLE
CHANNEL 1
VltStep
CHANNEL 2
Vout-Vref
CHANNEL 3
Verror
CHANNEL 4
Off
PORPORTIONAL
0x2000
INTEGRAL
0x1000
DERIVATIVE
0x4000
CURRENT
0x4000

Program Status – As of January 2010

To date

- *Five power modules with embedded controllers have been built*
- *The modules have been tested individually and run as pairs*
- *Demonstrated*
    - *4 modules, 40V, 100A, 4,000W unipolar output then reconfigure*
    - *4 modules, 40V, 33A, 1,320W bipolar output*

Future

- *Design the outer current control loop components*
- *Demonstrate operation of a completely redundant power supply*

Confidence Levels

- *MTBF previously discussed relates to the laws of large quantities and 50% confidence limits*
- *Confidence intervals are bounded with upper and lower limits. The broader the limits, the higher the confidence*
- *Electronic equipment, a one-sided, lower limit is appropriate*

t= *time in hours*

f=*number of failures*

$$MTBF_{Predicted} = t / f$$

K_L *from chi-square distribution*

$$MTBF_{LL} = MTBF_{Predicted} * K_L$$

KL Multipliers For MTBF Confidence Levels

Failures	Lower Limit K_L				
f	60%	70%	80%	90%	95%
1	0.620	0.530	0.434	0.333	0.270
2	0.667	0.600	0.515	0.422	0.360
3	0.698	0.630	0.565	0.476	0.420
4	0.724	0.662	0.598	0.515	0.455
5	0.746	0.680	0.625	0.546	0.480
500	0.965	0.954	0.942	0.930	0.915

Excerpted and abridged from W. Grant Ireson, Reliability Handbook, McGraw-Hill, NY 1966

Confidence Limit Example

If a power supply is to operate for 3 years before the first failure, what is the MTBF prediction for an 80% confidence level? Repeat for a 90% confidence level.

Solution:

$3 \ \ years = 26280\, hours = MTBF$

From the confidence limit table $K_L = 0.434$ for 80% and $f = 1$

Therefore, $MTBF_{80\%} = MTBF \ * 0.434 \geq 11{,}406\ hours$

For $MTBF_{90\%} = MTBF \ * 0.333 \geq 8{,}751\ hours$

Homework Problem # 21

It is desired to claim with 90% confidence that the actual MTBF of a power supply is 2500 hours. What must be the predicted MTBF?

FMECA is

- *A systematic way to prioritize the addressing of system "weak links".*
- *An inductive, bottoms-up method of analyzing a system design or manufacturing process in order to properly evaluate the potential for failures*

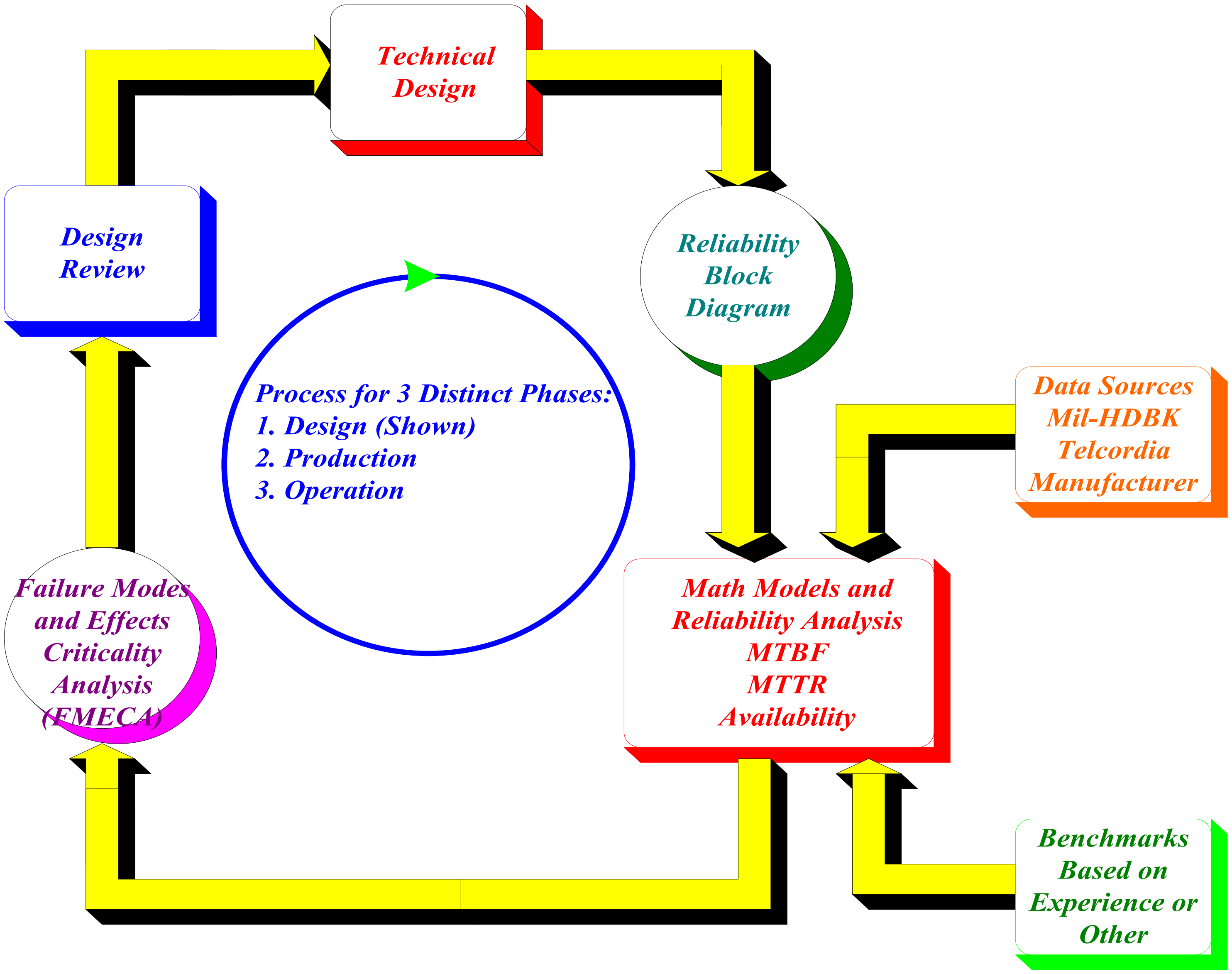
It Involves

- *Identifying all potential failure modes, determining the end effect of each potential failure mode, and determining the criticality of that failure effect.*

3 Major Iterations

- *Used in the Design, Fabrication and Operation Stages*

Reliability Process Diagram



SEV=*Severity*

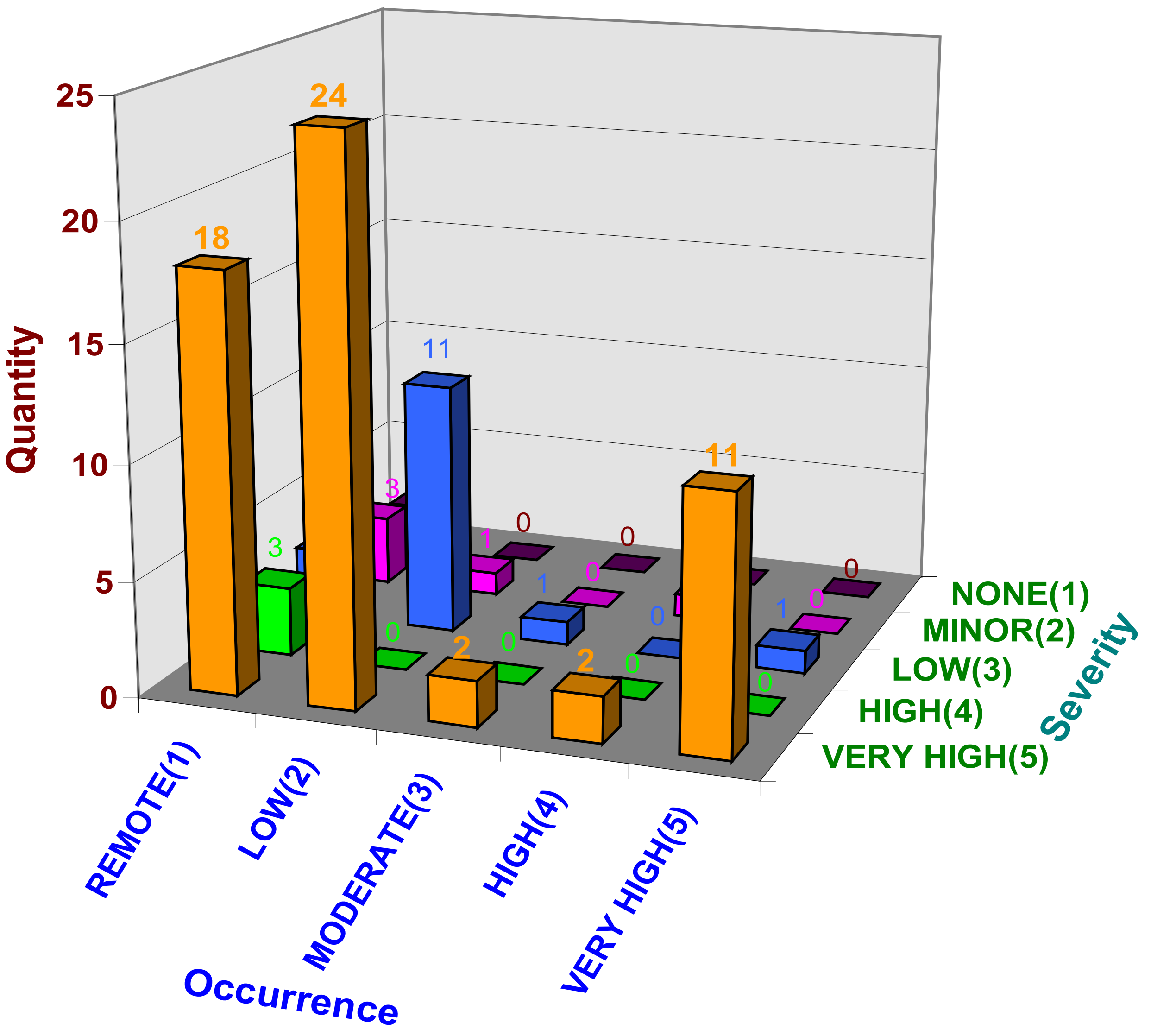
OCC=*Occurrence*

DET=*Detection (A numerical subjective estimate of the effectiveness of the controls to detect the cause or failure mode 10=uncertain, 1 absolute certainty)*

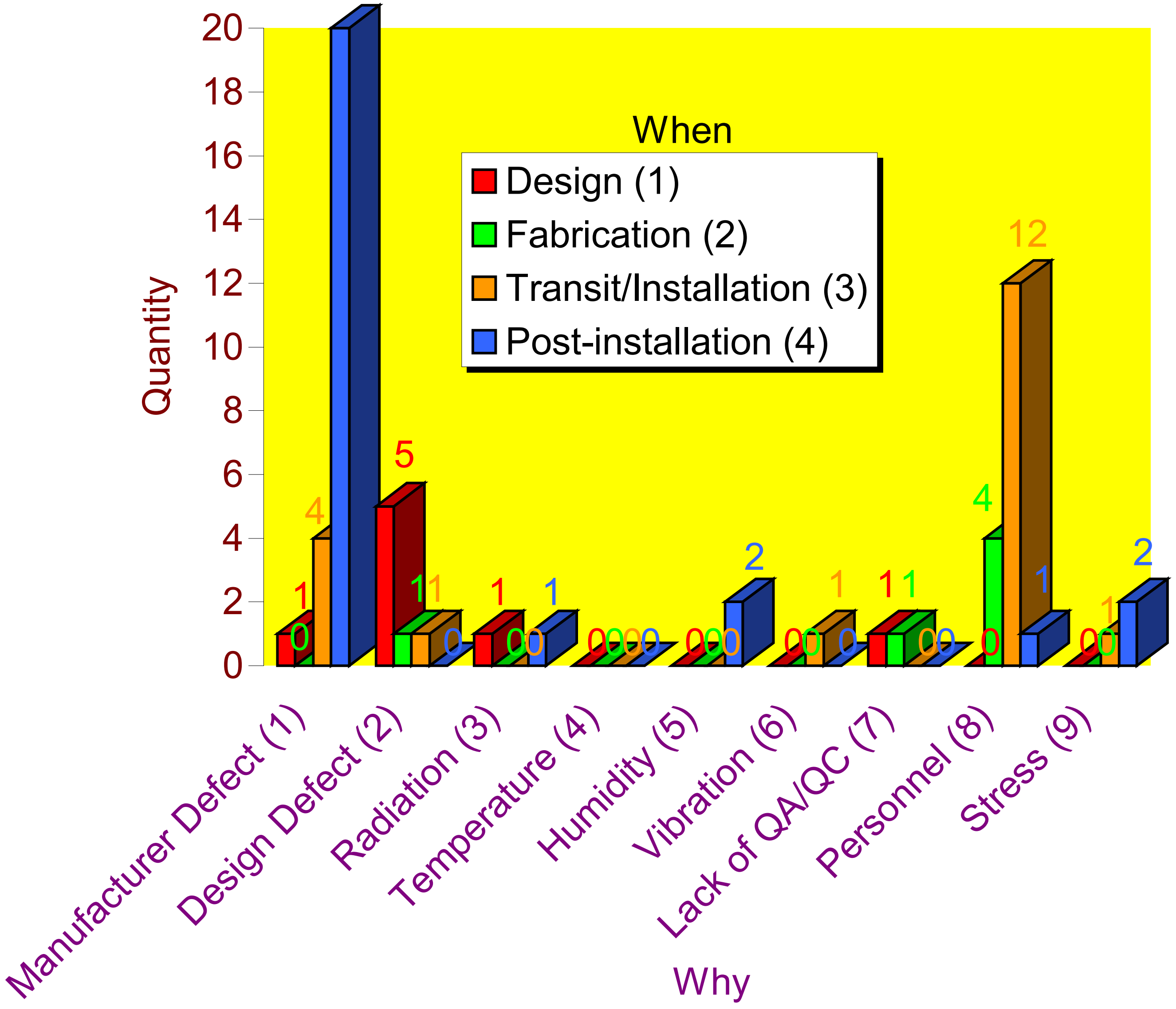
RPN=*Risk Priority Number=SEV*OCC*DET*

Part Name/ #	Part Function	Potential Failure Mode	Potential Effects of Failure	SEV	Potential Causes of Failure	OCC	Design Evaluation Technique	DET	RPN	WHEN	WHY
Coils	Provide magnetic field	coil to coil or coil to magnet steel short	magnet goes off line	5	coils moved during installation of magnet or adjacent beamline component, or alignment of magnet	5	protype test	1	25	3	2
Coils	Provide magnetic field	klixon trip due to overheating	magnet goes off line	5	inadequate water pressure differential across magnet	5	prototype test, calculation	1	25	1	2
Coils	Provide magnetic field	klixon trip due to overheating	magnet goes off line	5	too many loads on water circuit	5	prototype test, calculation	1	25	1	2
Coils	Provide magnetic field	klixon trip due to overheating	magnet goes off line	5	conducter sclerosis	3	n/a	1	15	4	9
Coils	Provide magnetic field	klixon trip due to overheating	magnet goes off line	5	foreign object in water line or coil which blocks water flow	2	n/a	1	10	4	8
Coils	Provide magnetic field	klixon trip due to overheating	magnet goes off line	5	damaged (crimped) coil which restricts water flow	2	n/a	1	10	3	8
Coils	Provide magnetic field	water leak	magnet goes off line due to ground fault	5	water hose brakes because of radiation damage	5	n/a	1	25	4	3
Coils	Provide magnetic field	water leak	magnet goes off line due to ground fault	5	corrosion in aluminum/copper conductor	2	n/a	1	10	4	9
Coils	Provide magnetic field	water leak	magnet goes off line due to ground fault	5	erosion of coil from excess water velocity	4	n/a	1	20	4	2
Coils	Provide magnetic field	water leak	magnet goes off line due to ground fault	5	break in braze joint between copper block and coil	3	prototype test	1	15	3	8
Fittings	Make water connection	water leak	magnet goes off line due to ground fault	5	cracked fittings from incorrect installation procedure	4	n/a	1	20	3	8
Jumpers	Connection between coils	short at jumper	magnet goes off line due to ground fault	5	sloppy installation	5	n/a	1	25	3	8
Jumpers	Connection between coils	short at jumper	magnet goes off line due to ground fault	5	poor design	5	design review, prototype	1	25	1	2
Jumpers	Connection between coils	loose jumpers	excessively high temperatures leading to melting of materials	5	poor design or incorrect procedures used at installation	5	n/a	1	25	3	8

Fault Modes And Effects Criticality Analysis (FMECA)



FMECA When and Why Plot

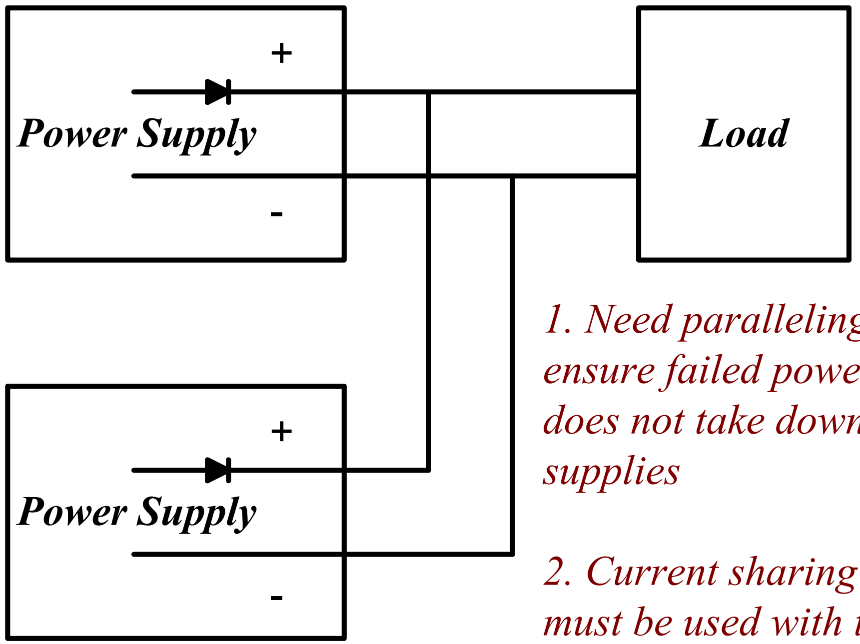


Maintainability

Cold swap *– input bus and power supply must be off when it is exchanged*

Warm swap *– input bus is on but power supply is off when exchanged*

Hot swap *– input bus is on and power supply is on when exchanged. Typically used with redundant, full rated power supplies*



1. Need paralleling diodes to ensure failed power supply does not take down both power supplies

2. Current sharing circuitry must be used with the power supplies

Section 13 - Power Supply Specifications

List of Specifications to be given to the Power Supply Designer	
Requirement	**Example**
1. Site conditions	Elevation, ambient temperature range, humidity, seismic requirements
2. Intended use and system	Storage ring accelerator dipole magnet power supply
3. Function	DC or pulsed, voltage or current source
4. Load parameters and description	Inductance, capacitance and resistance
5. Output ratings	Maximum voltage, current, operating or pulse time, pulse width and repetition rate
6. Input voltage and phases	208V, 1 ϕ 208V, 3 ϕ 480V, 3 ϕ
7. Efficiency	Up to 94% achievable at full load output

List of Specifications to be given to the Power Supply Designer	
Requirement	**Example**
8. Input power factor	Up to 0.99 achievable for 1 phase PS with active PF correction Up to 0.95 achievable for 6 pulse Up to 0.97 achievable for 12 pulse
9. Input line THD	< 5% voltage < 24% current
10. Conducted EMI 10kHz to 30MHz	MIL-STD-461E FCC Class A Industrial FCC Class B Residential
11. Line regulation	0.05 % of rated output voltage change for a 5% line voltage change. Recovery in 500μ S
12. Short-term (1 to 24 hour) stability	Allowable voltage or current deviation - 10s of ppm achievable
13. Output voltage ripple (PARD)	DC to 1 MHz, peak-to-peak, 0.05 % of rated voltage output

List of Specifications to be given to the Power Supply Designer	
Requirement	**Example**
14. Output pulse amplitude stability	
15. Output pulse – to pulse deviation in time (jitter)	1 nanosecond for solid-state converters. 10s of nanoseconds for thyratron triggers
16. Load regulation	0.05 % of rated output voltage change for 10 % line change. Recovery in 500μ S
17. Type of control system	Analog, mixed analog-digital, all digital Communication bus
18. Interlocks	Turn off power supply if: •Low input voltage - loss of input phase •Output over voltage – over current •Excessive ground current •Insufficient cooling air flow – cabinet over temperature

List of Specifications to be given to the Power Supply Designer	
Requirement	***Example***
18. Interlocks (continued)	*•Insufficient cooling water flow – cooling water over temperature* *•MPS fault* *•PPS violated* *•Cabinet doors open*
19. Cooling methods	*Water cooling for biggest power dissipating devices (IGBTs, rectifiers, chokes)* *< 50 kW – all air cooled* *> 50kW – some measure of water cooling*
20. Front panel controls	*•Local / remote operation* *•Output voltage or current* *•Ground current limit* *•Output current limit*

List of Specifications to be given to the Power Supply Designer	
Requirement	**Example**
21. Front panel displays	•Output voltage •Output current •Ground current •Voltage or current mode • Current limited operation
22. Component deratings	Voltage, current and power
23. Mean time between failure (MTBF)	MTBF = 1/ (sum of all parts failure rates)
24. Mean time to repair or beam recovery (MTTR)	Establish from MTBF and operational Availability requirement
25. Availability	Establish from MTBF MTTR
26. Maintainability	Replace or repair in the field or repair in the shop

List of Specifications to be given to the Power Supply Designer	
Requirement	**Example**
27. Physical size	Based on output power – typically 1 to 4 W / cu in
28. Rack or free-standing	< 17kW rack-mounted > 17kW free-standing
29. Compliance with UL or other nationally-recognized inspection/test laboratories	Underwriters Laboratories - UL National Recognized Test Laboratory - NRTL
30. Seismic	Must satisfy site earthquake design criteria Damage criteria and response spectra curves - separate or combined accelerations
31. Quality Assurance	Must satisfy project quality assurance/quality control criteria

Section 14 - References

References And Useful Textbooks

References	*Used in*
Fundamentals of Power Electronics by Robert W. Erickson and Dragan Maksimovic, Springer, Third Edition, 2020	*Textbook*
Elements of Power Electronics; Philip T. Krein, Oxford University Press, 2015	*Textbook*
Power Electronics: Converters, Applications, and Design; Mohan, Undeland, Robbins, Wiley, 2002	*Textbook*
Principles of Power Electronics; Kassakian, Schlecht, Verghese, Addison-Wesley, 1991	*Textbook*
"Power Electronic Converter Harmonics", Derek Paice, Wiley-IEEE Press, 1999 ISBN: 978-0-7803-5394-7	*Textbook*
Rectifier Circuits Theory And Design, Johannes Schaefer, John Wiley 1965	*Textbook*
Switchmode Power Supply Handbook, Keith Billings, McGraw-Hill, February 1999, ISBN 0070067198	*Textbook*

References And Useful Textbooks

References	**Used in**
Elements Of Power System Analysis, Stevenson, McGraw-Hill	*Textbook*
Power System Analysis and Design, Glover, Overbye, and Sarma, Cengage Learning, 2017	*Textbook*
Power Systems Analysis, Bergen and Vittal, Prentice-Hall, 2000	*Textbook*
Electric Machinery, Fitzgerald, Kingsley, Jr., and Umans, McGraw-Hill, 1990	*Textbook*
IEEE 610 - IEEE Standard Computer Dictionary: A Compilation of IEEE Standard Computer Glossaries. Institute of Electrical and Electronics Engineers. New York, NY: 1990	*Textbook*

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References	Used in
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Advanced Engineering Mathematics; Edwin Kreyszig, John Wiley, 2011	*Textbook*
Linear Control System Analysis and Design; D'Azzo and Houpis, McGraw-Hill, 1988	*Textbook*
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References	Used in
Introduction to Linear Algebra; Gilbert Strang, SIAM, 2016	Textbook

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References	Used in
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EMI Control Methodology and Procedures, Michelle Mardiguian, Interference Control Technologies, 1989	Section 3
IEEE 519 – 2014 "Standard Practices and Requirements for Harmonic Control in Electrical Power Systems"	Section 3
Circuit Techniques for Improving the Switching Loci of Transistor Switches in Switching Regulators, E.T. Calkin and B.H. Hamilton, IEEE Transactions On Industry Applications, July 1976	Section 4
How to Select a Heatsink https://www.electronics-cooling.com/1995/06/how-to-select-a-heat-sink/	Section 4
IGBT Theory https://en.wikipedia.org/wiki/Insulated-gate_bipolar_transistor	Section 4
Magnetics Designer for Transformers, chokes and inductors, Intusoft Corporation http://www.i-t.com/engsw/intusoft/magdesgn.htm	Section 4

References And Useful Textbooks

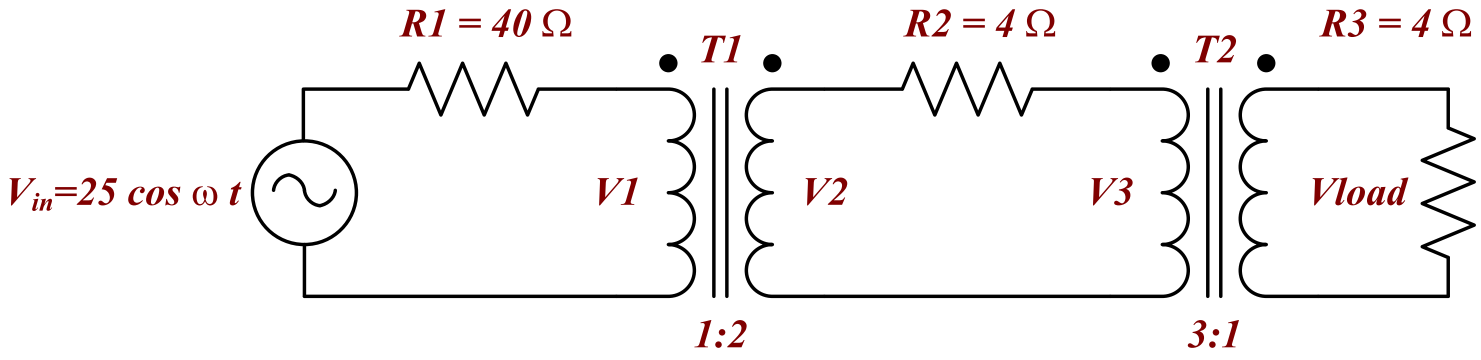
References	*Used in*
Power Electronics Modeling Software, Integrated Engineering Software, CASPOC *http://www. integratedsoft.com*	*Section 4*
LTspice simulator for switching regulators, Linear Technologies, https://www.analog.com/en/design-center/design-tools-and-calculators/ltspice-simulator.html	*Section 4*
PSPICE circuit simulator, Micro-Cap, Spectrum Software, *http://www.spectrum-soft.com*	*Section 4*
Zero Voltage Switching Resonant Power Conversion *http://www.ti.com/lit/an/slua159/slua159.pdf*	*Section 4*
SCSI Technology *https://allpinouts.org/pinouts/cables/parallel/scsi-technology/*	*Section 5*
Table of Laplace Transforms *http://www.vibrationdata.com/Laplace.htm*	*Section 6*
Table of Fourier Transforms *http://mathworld.wolfram.com/FourierTransform.html*	*Section 6*

References And Useful Textbooks

References	Used in
MIL-STD-1629A "Procedures for Performing a Failure Mode, Effects, and Criticality Analysis" 1980	Section 7
RelCalc by T-Cubed	Section 7
Relex by Relex Software	Section 7

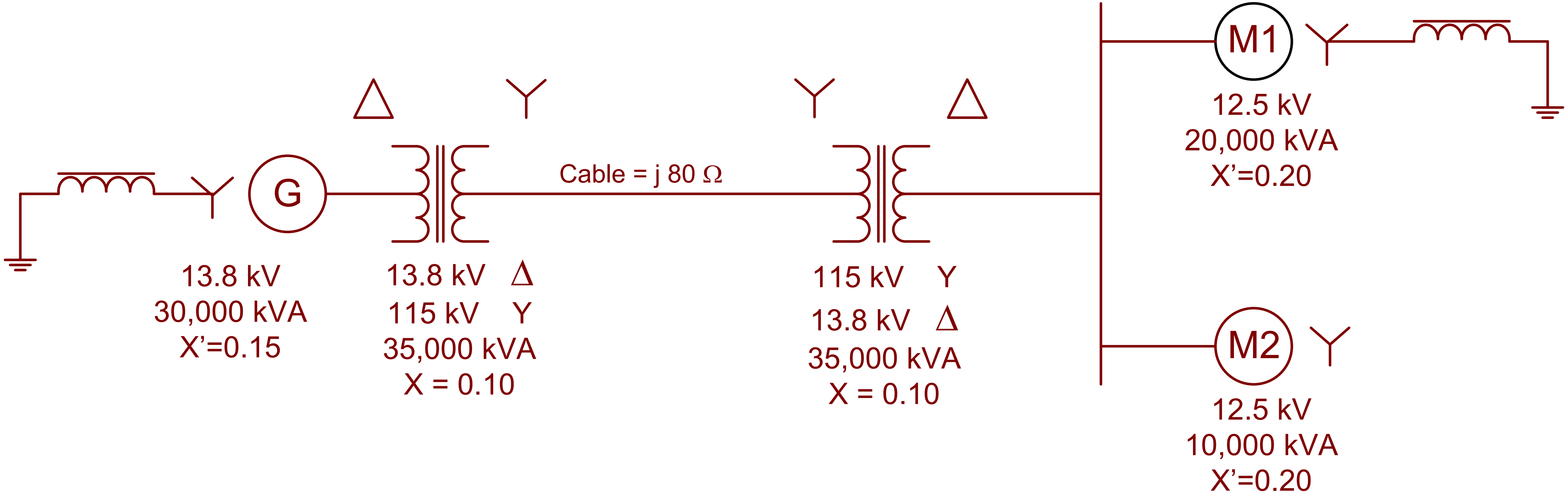
Section 15 - Homework Problems

Calculate the output voltage in the circuit shown below.



Referring to the one-line diagram below, determine the line currents in the:

A. Generator *B. Transmission Line* *C. M1* *D. M2*



Homework Problem # 3

A 1000kVA, 12.47kV to 480V, 60Hz three phase transformer has an impedance of 5%. Calculate:

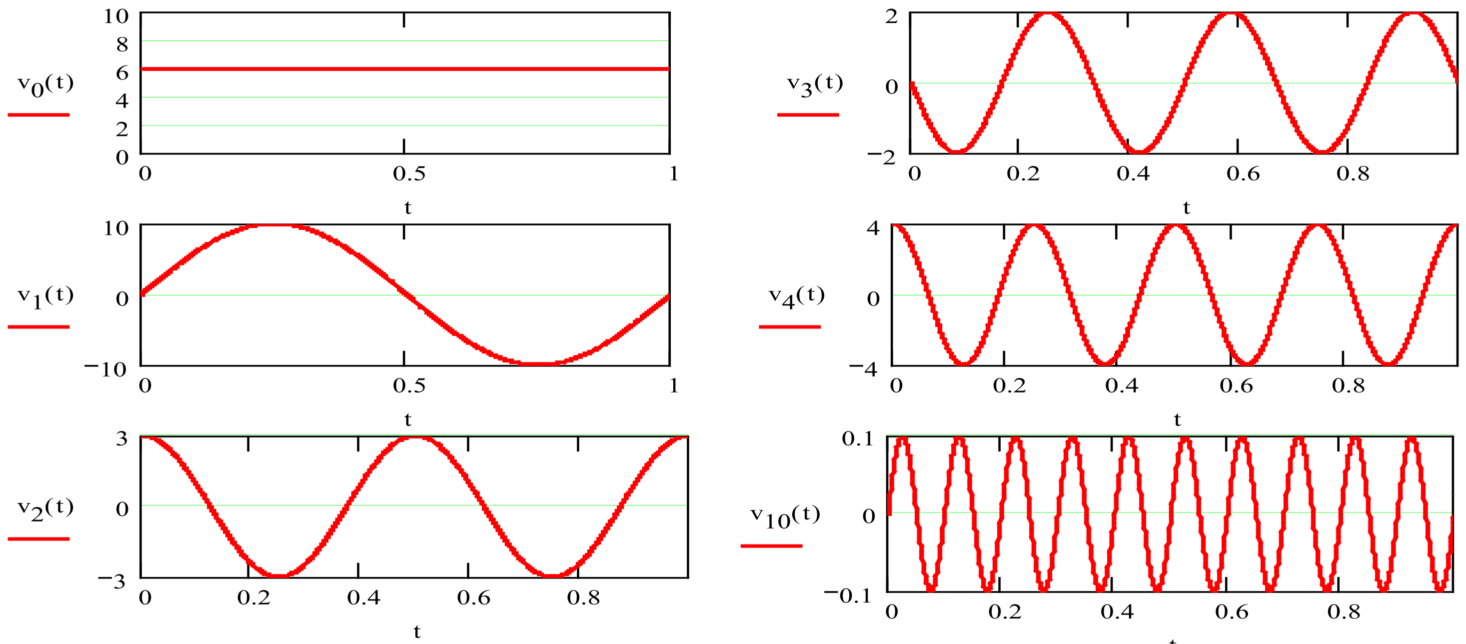
a. The actual impedance and leakage inductance referred to the primary winding

b. The actual impedance and leakage inductance referred to the secondary winding

c. The magnetizing inductance referred to the primary winding

Homework Problem # 4

A waveform v(t) was analyzed and found to consist of 6 components as shown here.



*a. Write the mathematical expression for each component in terms of ω=(2*π)/T*

b. Show the harmonic content graphically by plotting the frequency spectrum

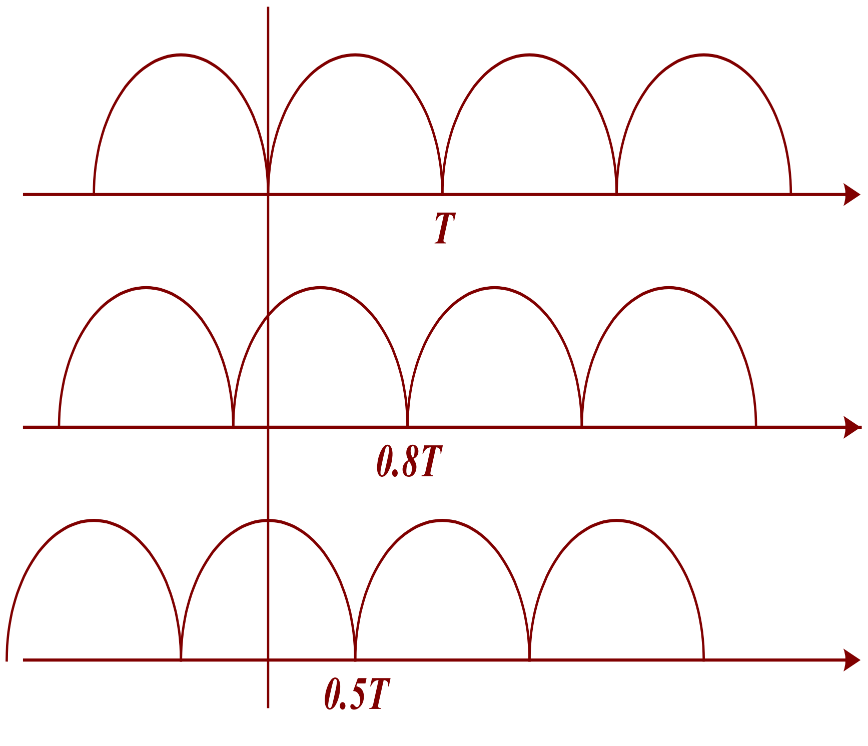
c. Give the numerical result of

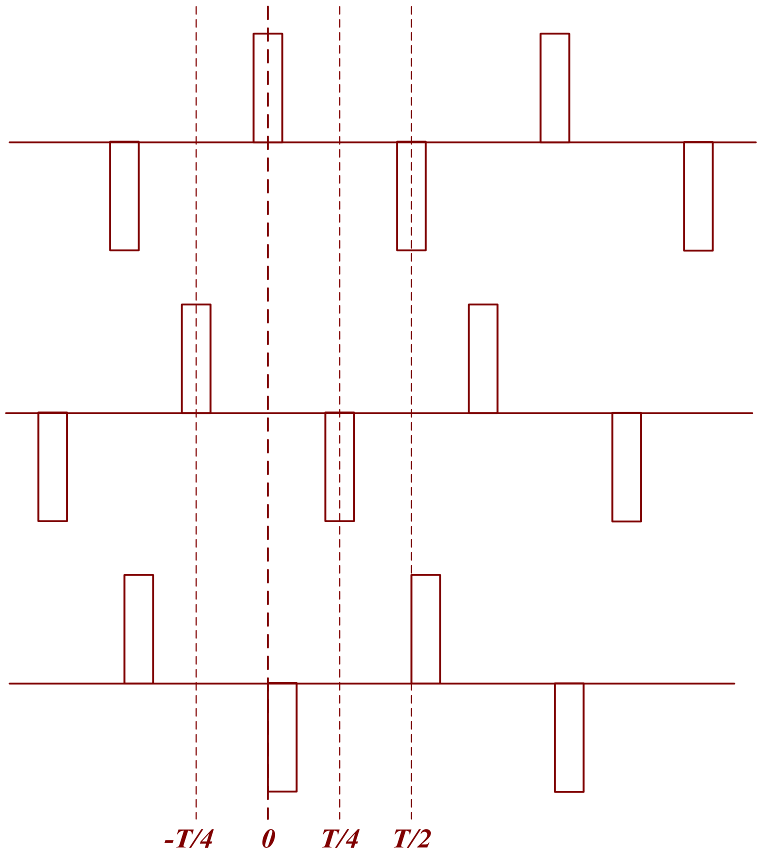
$$b_3 = \frac{2}{T}\int_0^T v(t)\sin 3\omega t\,dt \qquad \textbf{Help :} \int \sin^2(3\omega t)\,dt = \frac{t}{2} - \frac{\sin 6\omega t}{12\omega}$$

$$b_4 = \frac{2}{T}\int_0^T v(t)\sin 4\omega t\,dt \qquad \textbf{Help :} \int \cos(4\omega t)\sin(4\omega t)\,dt = \frac{\sin(4\omega t)^2}{8\omega}$$

Homework Problem # 5

Each waveform below can be written as a Fourier series. The result depends upon the choice of origin. For each of the 6 cases, state the type of symmetry present, non-zero coefficients and the expected harmonics.



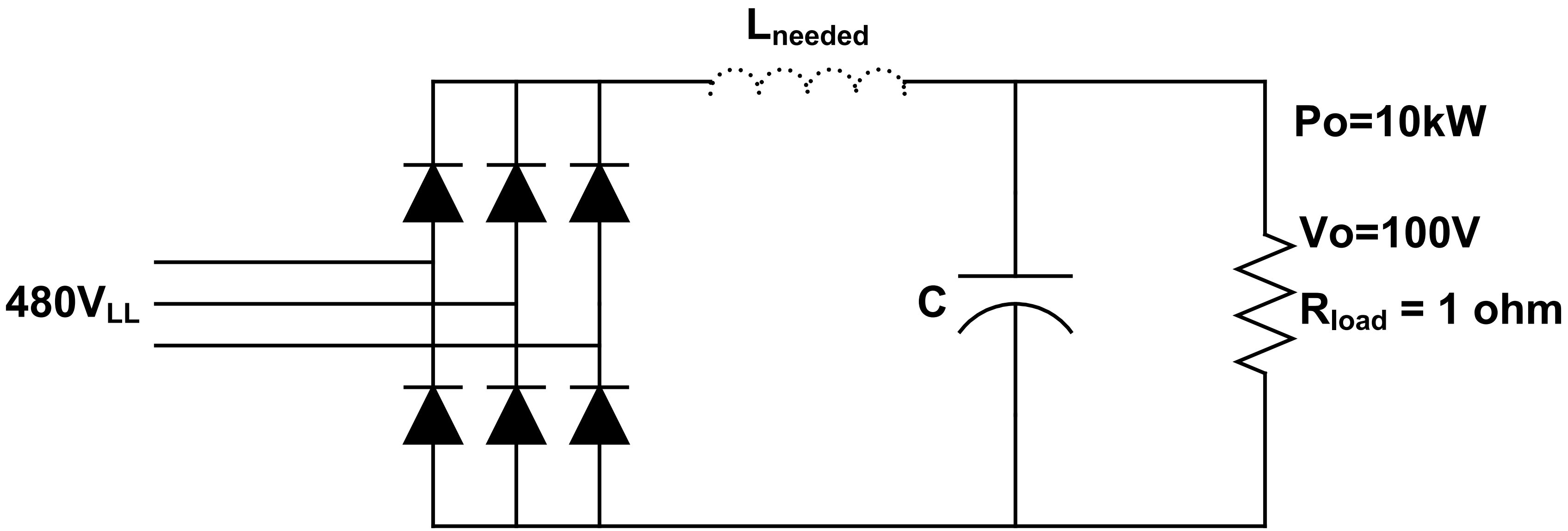


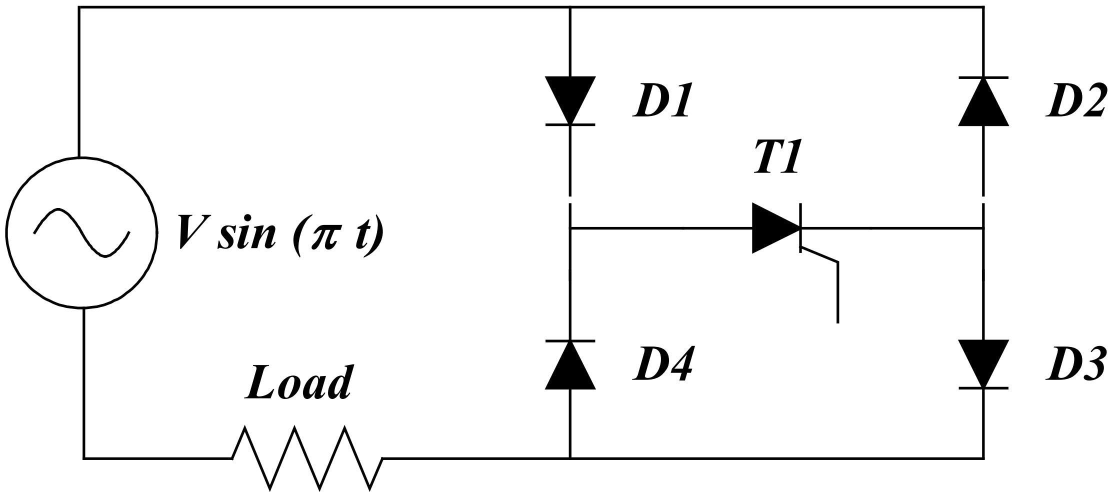
Homework Problem # 6

A uniform magnetic field B is normal to the plane of a circular ring 10 cm in diameter made of #10 AWG copper wire having a diameter of 0.10 inches. At what rate must B change with time if an induced current of 10 A is to appear in the ring? The resistivity of copper is about 1.67 $\mu\,\Omega - cm$*.*

Note: Use the 10cm dimension as the ring diameter.

A 10kW power supply with 3-phase 480V input has an efficiency of 90% and operates with a leading power factor of 0.8. The power supply output is 100V. Determine the size of an added inductor to improve the power factor to 1.00. Below is the circuit diagram.





Assume ideal components in the phase-controlled circuit above. For a purely resistive load:

A. Explain how the circuit operates

B. Draw the load voltage waveform and determine the boundary conditions of the delay angle α

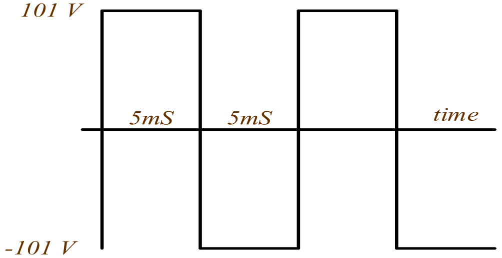
C. Calculate the average load voltage and average load current as a function of α

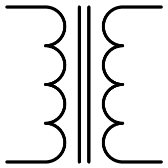
D. Find the RMS value of the load current.

Rectifiers - Homework Problem # 9

Given the following:

- Input voltage waveform

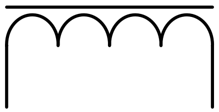




- Losses transformer

- Two SCRs, two diodes, each with conducting voltage drop of 1V.

- Inductor, lossless, with very large inductance



- Resistor, 10 ohms, capable of very large power dissipation

- Circuit operating under steady-state conditions (i.e. all transients have subsided)

Problem

A. With the SCRs triggering retard angle at zero degrees, arrange the circuit to provide a full-wave, rectified, and properly low-pass filtered DC output of 200V into the 10ohm load resistor.

B. Calculate the load current and power

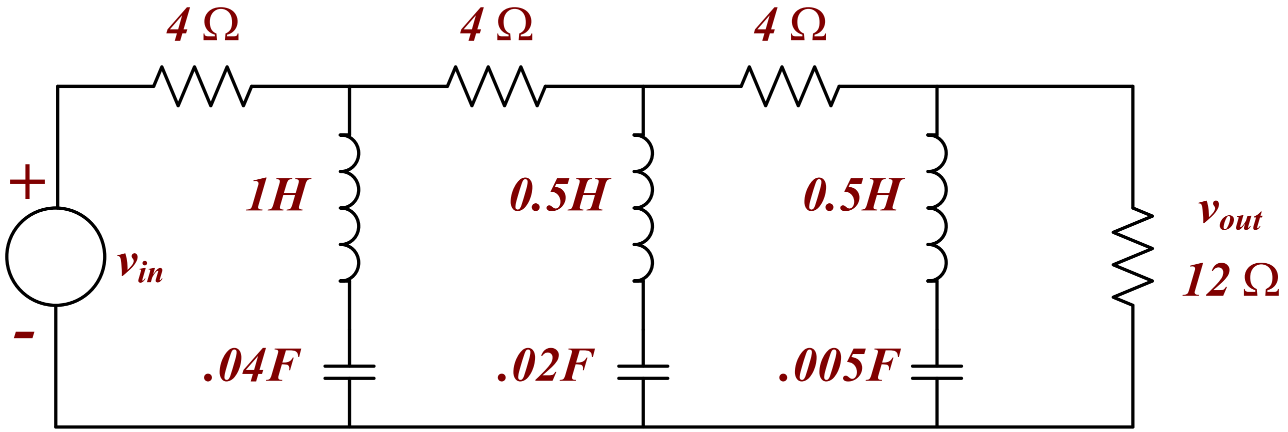
C. Determine the needed transformer turns ratio.

D. Calculate the circuit efficiency

Increase the SCRs trigger retard angle to 90 degrees and
F. Calculate the new output voltage, current, and power

G. Determine the new circuit efficiency

Given the circuit below:



$$h(t) = \frac{v_{out}(t)}{v_{in}(t)} \qquad H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$$

Sketch $|H(j\omega)|$ *versus* ω

Homework Problem # 11

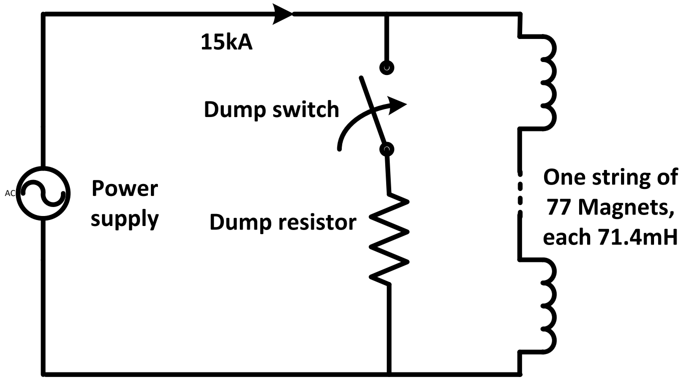
A 100kW power supply is 80% efficient. Approximately 50% of the power supply heat loss is removed by cooling water.

- *How much heat is dissipated to building air and how much heat is removed by the water system.*
- *Calculate the water flow rate needed to limit the water temperature rise to $8^{O}C$ maximum.*

Homework Problem # 12

A collider has several equal strings of 77 superconducting magnets, each with 71.4mH inductance, carrying 15kA of current. If one, or more quenches, all the energy from the other magnets will dissipate their energies into the quenched magnet, thus destroying it. Design a switched dump resistor to discharge the current at a maximum rate, dI/dt, of 300A/s to prevent damage to the superconducting magnet in the event of a quench. Refer to the circuit diagram below.

1. *What is the energy stored in each magnet and in the string when running at its design value?*
2. *What is the total inductance of the string?*
3. *Write the equation that describes the resistor current after closing the switch.*
4. *Find the resistor value to limit the maximum rate of decrease of current in the magnets to 150A/s*
5. *What is the maximum voltage generated across the resistor?*
6. *What is the time constant of this circuit?*
7. *Design a steel dump resistor that has little thermal conductance to the outside world (adiabatic system). Calculate how much steel mass (weight) will limit the temperature increase of the resistor to 500°K.*



Help

$Q = M\ C_p\ \Delta T$

$Q =$ *heat (energy) into the system expressed in joules*

M= mass or weight of the resistor

$C_p = \text{specific heat of material} = 0.466 \dfrac{J}{gm * {}^oK}$ *for steel*

$\Delta T = \text{Temperature rise of the resistor}$

Based on "LHC Magnet Quench Protection System, L.Coull, et.al, 13th International Conference on Magnet Technology, Victoria, Canada, 1993

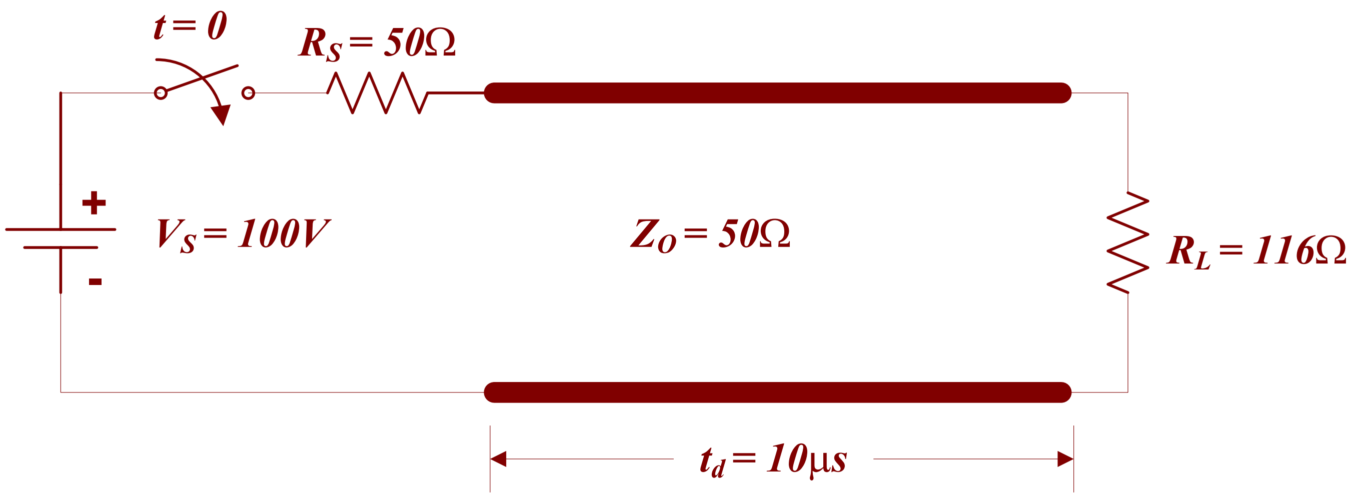
Homework Problem #13

A. *A transmission line can be formed using lumped Ls and Cs. Calculate the delay of a line composed of 8 sections of inductances L=4mH per section and capacitance C=40pF per section.*

B. *The frequency of a signal applied to a two-wire transmission cable is 3GHz. What is the signal wavelength if the cable dielectric is air? Hint – relative permittivity of air is 1*

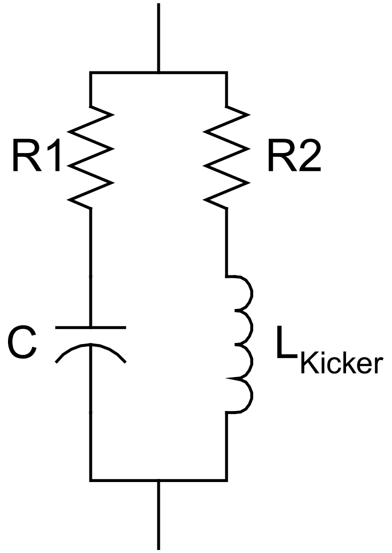
C. *What is the signal wavelength if the cable dielectric has a relative permittivity of 3.6?*

For the transmission line shown below, calculate the Reflection Coefficients Γ, the reflected voltages and the voltage and current along the line versus time.



A controlled impedance transmission line often drives a kicker. The kicker is usually well modeled as an inductor. A matching circuit can be built around the kicker and its inductance so that this circuit, including the kicker magnet, has constant, frequency independent, impedance which is matched to the transmission line.

Assuming that the transmission line impedance is Z_0 and the kicker inductance is L_{Kicker} derive the values of R1, R2, and C necessary to make a frequency independent (constant) impedance Z_0.



A. What is the significance of the value $\sqrt{\dfrac{\mu_O}{\varepsilon_O}}$?

B. What is the significance of the values $\dfrac{1}{\sqrt{\mu_O \varepsilon_O}}$ and $\sqrt{L * C}$?

C. Calculate the speed of light in mediums with dielectric constants of:
$\varepsilon_r = 1$ $\varepsilon_r = 2$ $\varepsilon_r = 4$ $\varepsilon_r = 8$ $\varepsilon_r = 16$

A. At least 1 of 4 parallel identical power supplies in an accelerator must continue to operate for the system to be successful. Let $R_i = 0.9$. Find the probability of success.

B. Repeat for at least 2 out of 4 succes

C. Repeat for at least 3 out of 4 success

D. Repeat for 4 out of 4 success

Solution:

Homework Problem # 18

A "typical commercial" 5 kW, switch-mode power supply consists of the components below with the listed failure rates. It also has critical electromechanical safety features amounting to 10% of the total number of components. The power supply operates at 50C ambient temperature. Assuming no derating for the elevated ambient temperature or other stress factors, calculate the power supply MTBF.

- *2 each ICs, plastic linear, l = 3.64*
- *1 each opto-isolator, l = 1.32*
- *2 each hermetic sealed power switch transistors, l = 0.033*
- *2 each plastic power transistors, l = 0.026*
- *4 each plastic signal transistors, l = 0.0052*
- *2 each hermetic sealed power diodes, l = 0.064*
- *8 each plastic power diodes, l = 0.019*
- *6 each hermetic sealed switch diodes, l = 0.0024*
- *32 each composition resistors, l = 0.0032*
- *3 each potentiometers, commercial, l = 0.3*
- *8 each pulse type magnets, 130C rated, l = 0.044*
- *12 each ceramic capacitors, commercial, l = 0.042*
- *3 each film capacitors, commercial, l = 0.2*
- *9 each Al electrolytics, commercial, l = 0.48*

Homework Problem # 19

Two inverter stages in an uninterruptible power supply are to be connected in parallel. each is capable of full-load capability. The calculated failure rate of each stage is l = 200 failures per million hours.

A. What is the probability that each inverter will remain failure free for a mission time of 1000 hours and

B. What is the probability that the system will operate failure free for 1000 hours?

Homework Problem # 20

For a critical mission, 3 power supplies, each capable of supplying the total required output, are to be paralleled. The power supplies are also decoupled such that a failure of any power supply will not affect the output. The calculated failure rate of each power supply is 4 per million hours.

A. What is the probability that each power supply will operate failure free for 5 years?

B. What is the probability that the system will operate failure free for 5 years?

Homework Problem # 21

It is desired to claim with 90% confidence that the actual MTBF of a power supply is 2500 hours. What must be the predicted MTBF?